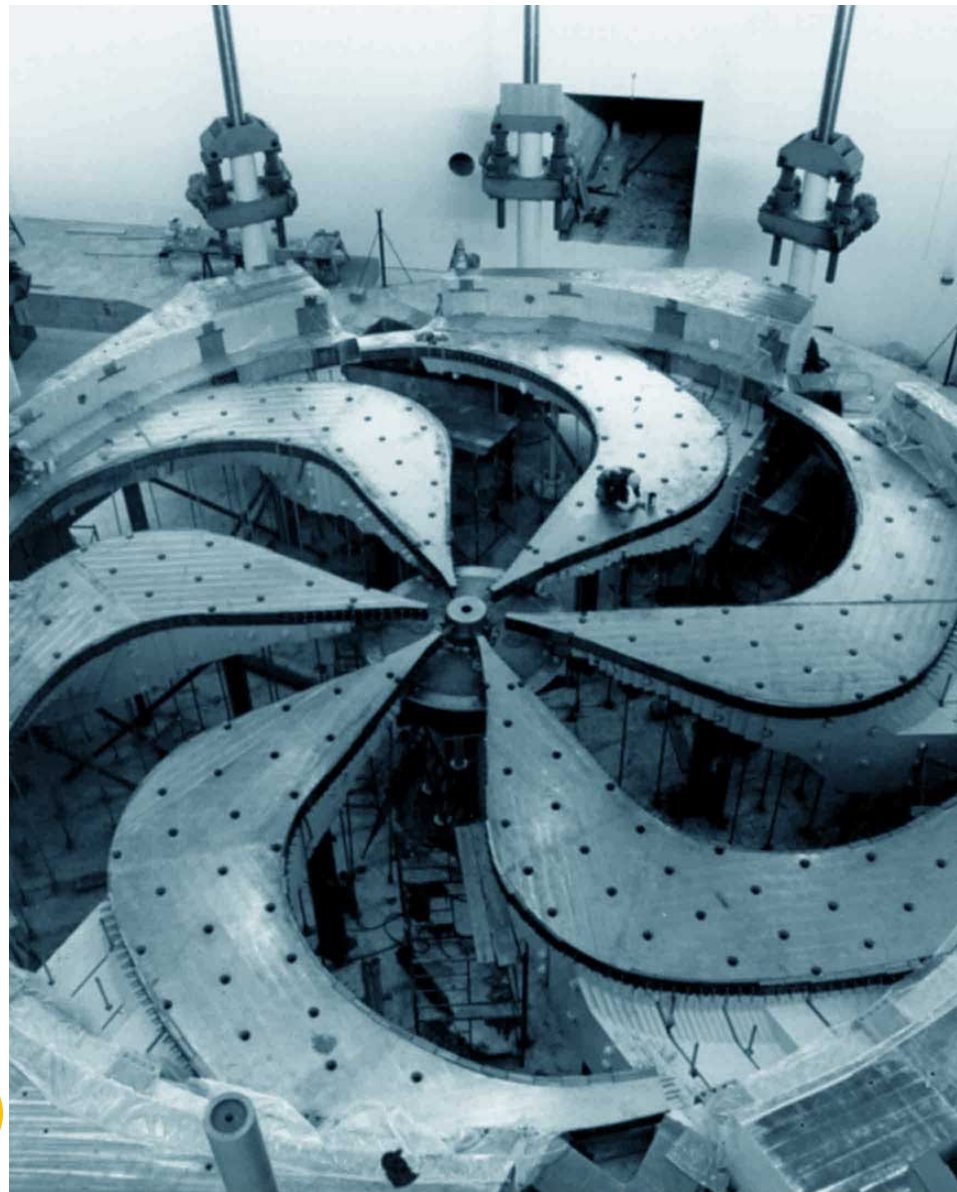


# Global ab initio calculations for exotic and heavy nuclei

Jason D. Holt  
TRIUMF, Theory Department  
Theory Alliance Meeting, FRIB  
May 18, 2023





# Major RIB Facilities Worldwide

Next-generation RIB facilities: unprecedented era of nuclear science

Thousands of new isotopes to be produced: **How does our field maximize this opportunity?**





# Major RIB Facilities Worldwide

Next-generation RIB facilities: unprecedented era of nuclear science

Thousands of new isotopes to be produced: **Meaningful interaction with theory!**



\$4-5B worldwide investment

**What is the fundamental, exciting physics?**



# Major RIB Facilities Worldwide

Next-generation RIB facilities: unprecedented era of nuclear science

Thousands of new isotopes to be produced: **Meaningful interaction with theory!**



## Role of theory

**Motivation:** robust predictions (**with uncertainties!**) where no data exists

**Interpretation:** model independent, connect to underlying forces of nature

Spon. fission

ISOL



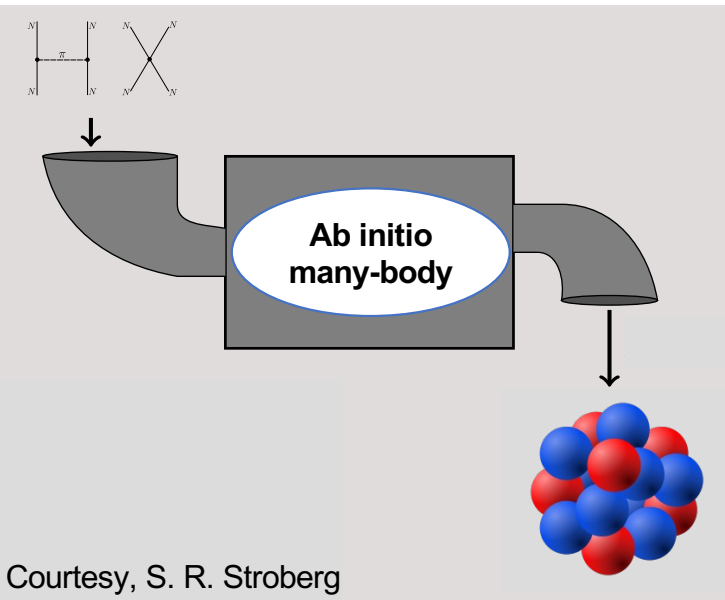
\$4-5B worldwide investment

**What is the fundamental, exciting physics?**

Aim of modern nuclear theory: develop unified *first-principles* picture of structure and reactions

(Approximately) solve nonrelativistic Schrödinger equation

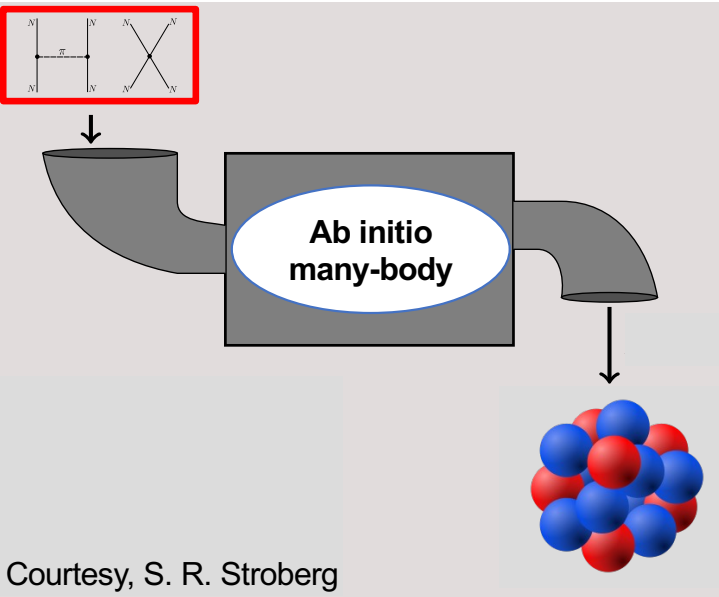
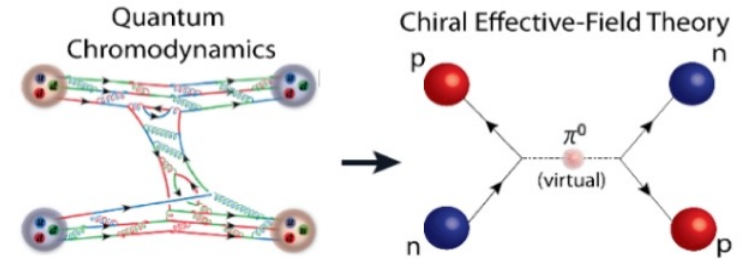
$$H\psi_n = E_n\psi_n$$



Aim of modern nuclear theory: develop unified *first-principles* picture of structure and reactions

(Approximately) solve nonrelativistic Schrödinger equation

$$\boxed{H}\psi_n = E_n\psi_n$$



Courtesy, S. R. Stroberg

## Chiral Effective Field Theory

- Consistent treatment of
- 2N, 3N, 4N, ... forces
- Electroweak physics

Quantifiable uncertainties

### Interactions

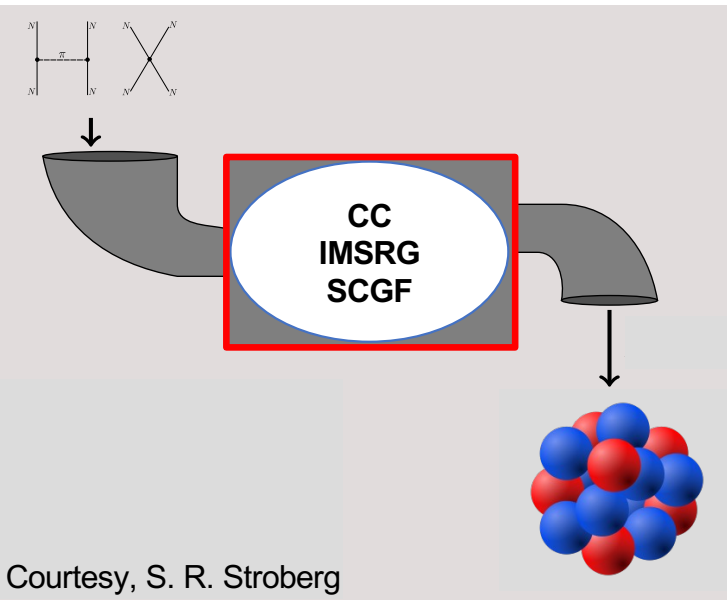
1.8/2.0, N2LO<sub>GO</sub>, N3LO<sub>LNL</sub>  
 (2.0/2.0, N4LO<sub>LNL</sub>)  
**34 non-implausible**

	NN force	NNN force	NNNN force
Q <sup>0</sup> LO			
Q <sup>2</sup> NLO			
Q <sup>3</sup> N <sup>2</sup> LO			
Q <sup>4</sup> N <sup>3</sup> LO			

Aim of modern nuclear theory: develop unified *first-principles* picture of structure and reactions

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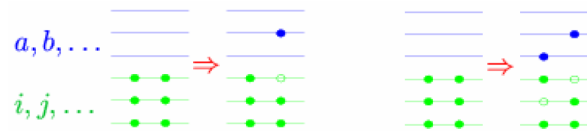
$$H\psi_n = E_n\psi_n$$



Courtesy, S. R. Stroberg

## Ab Initio Cheat Sheet (polynomial scaling methods)

**CC:** Coupled cluster theory



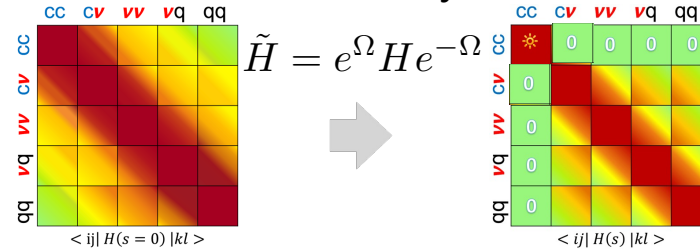
$$E = \langle \Phi | \bar{H} | \Phi \rangle$$

$$0 = \langle \Phi_i^a | \bar{H} | \Phi \rangle$$

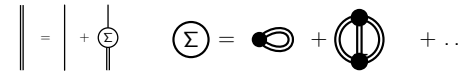
$$0 = \langle \Phi_{ij}^{ab} | \bar{H} | \Phi \rangle$$

$$\bar{H} \equiv e^{-T} H e^T = (H e^T)_c$$

**IMSRG:** In-medium similarity renormalization group



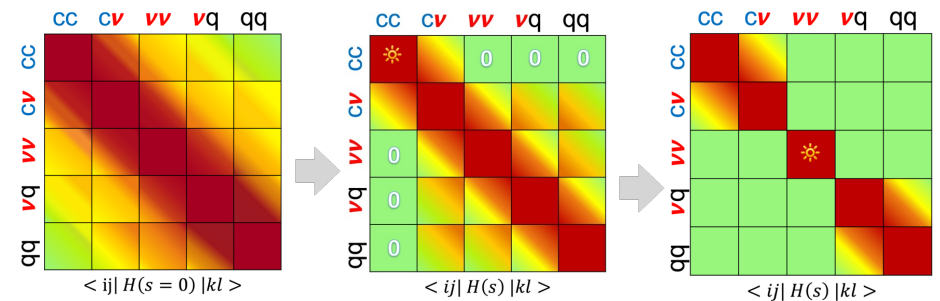
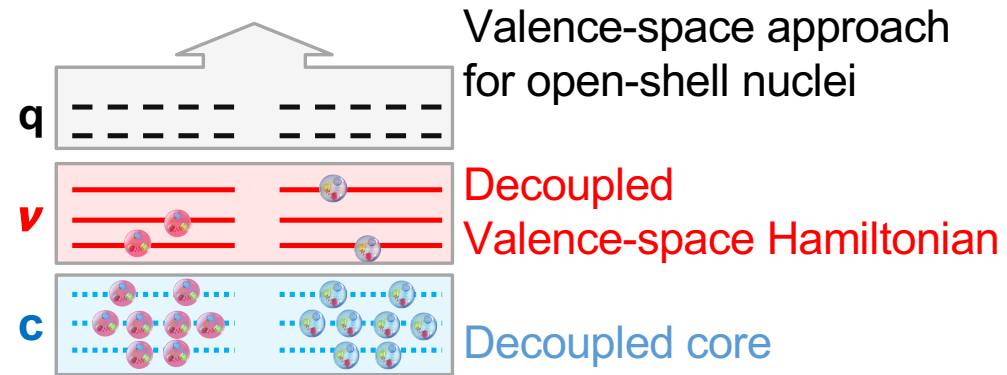
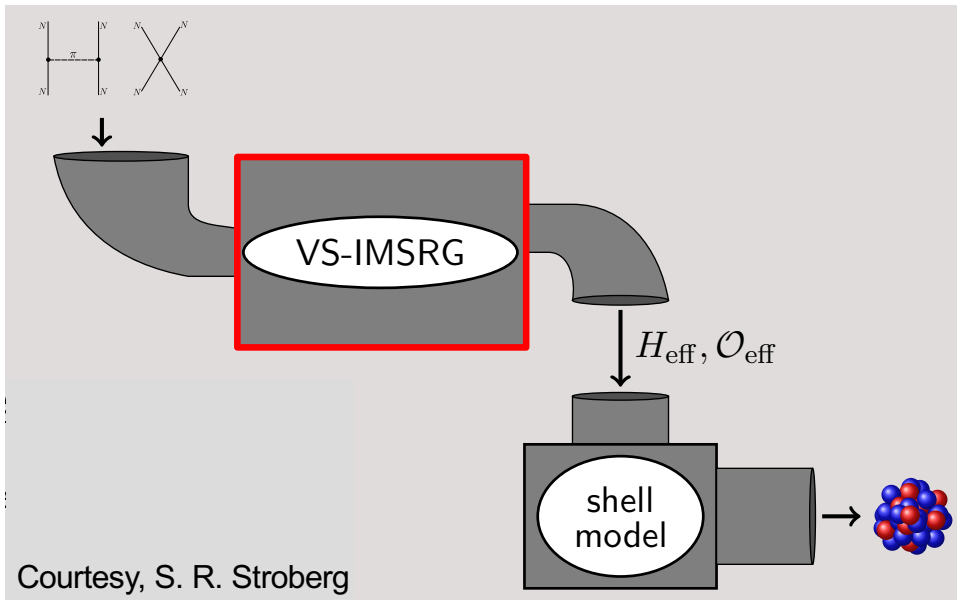
**SCGF:** Self-consistent Green's function



Aim of modern nuclear theory: develop unified *first-principles* picture of structure and reactions

(Approximately) solve nonrelativistic Schrödinger equation

$$H\psi_n = E_n\psi_n$$



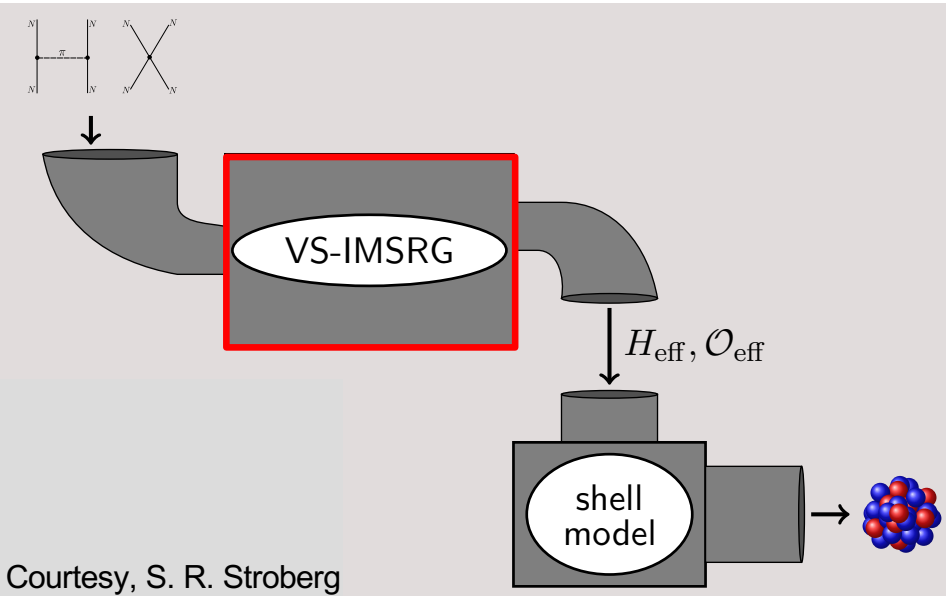
**Extends ab initio to scope of traditional nuclear shell model**



Aim of modern nuclear theory: develop unified *first-principles* picture of structure and reactions

(Approximately) solve nonrelativistic Schrödinger equation

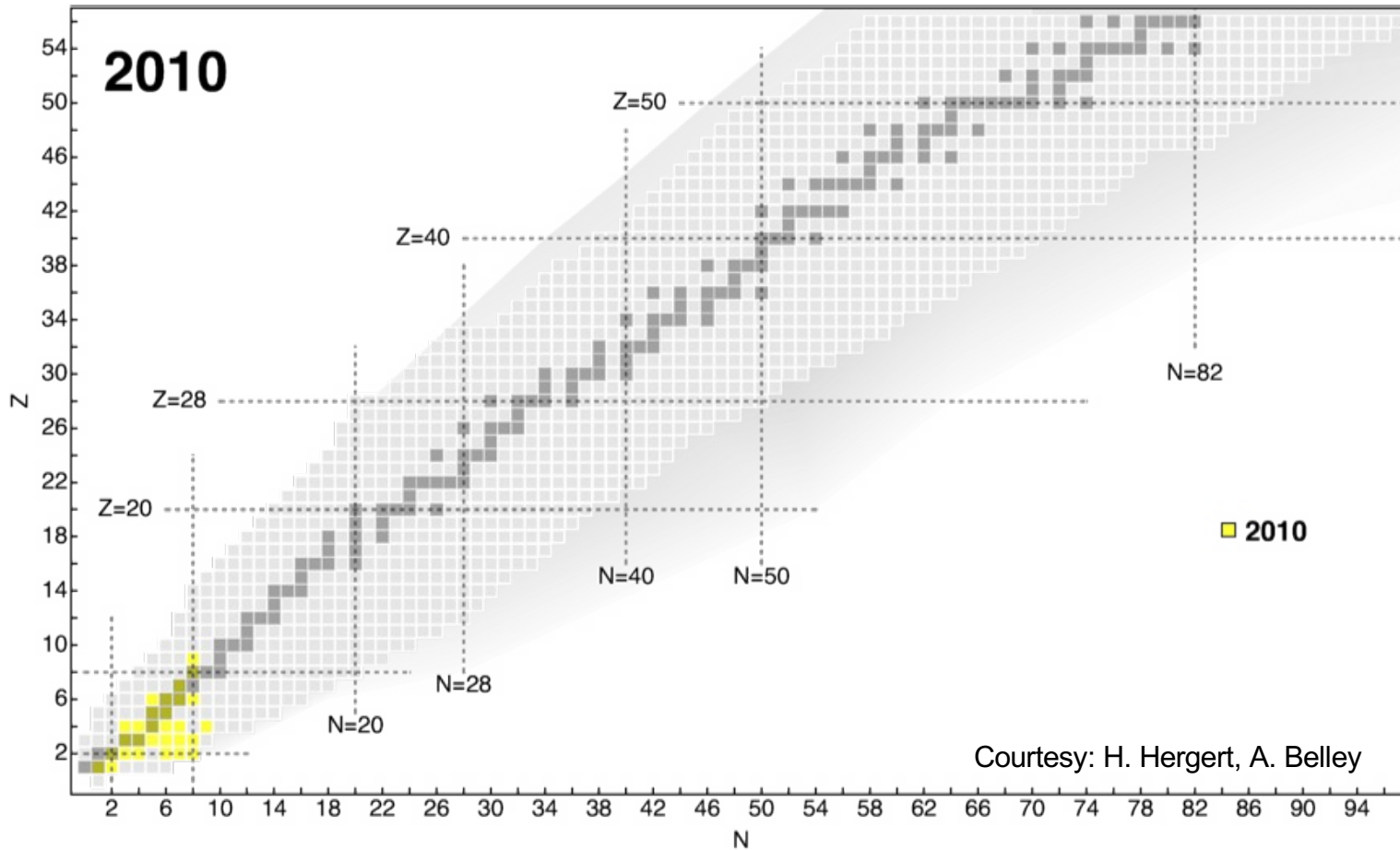
$$H\psi_n = E_n\psi_n$$



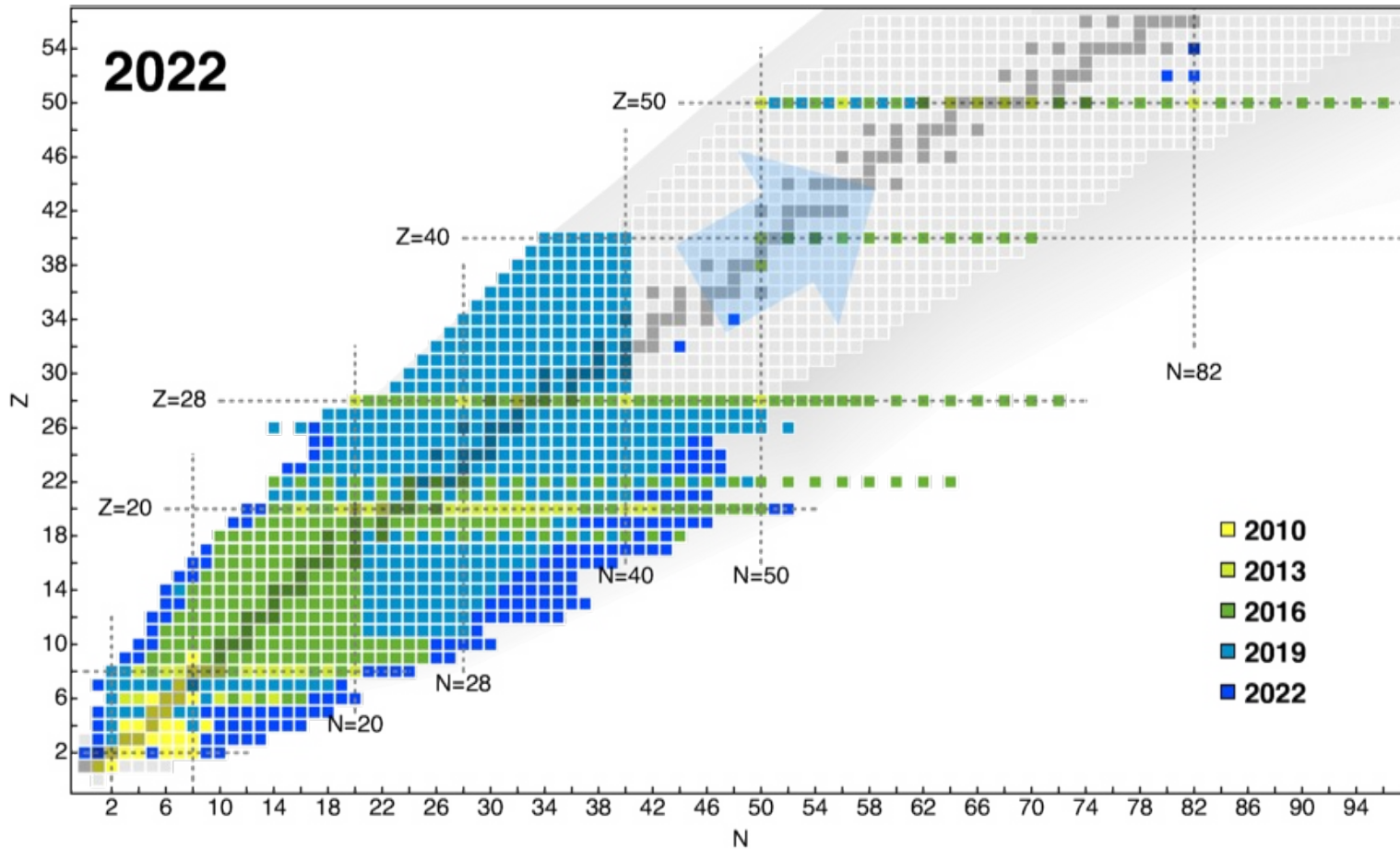
## Methods Exact up to Truncations

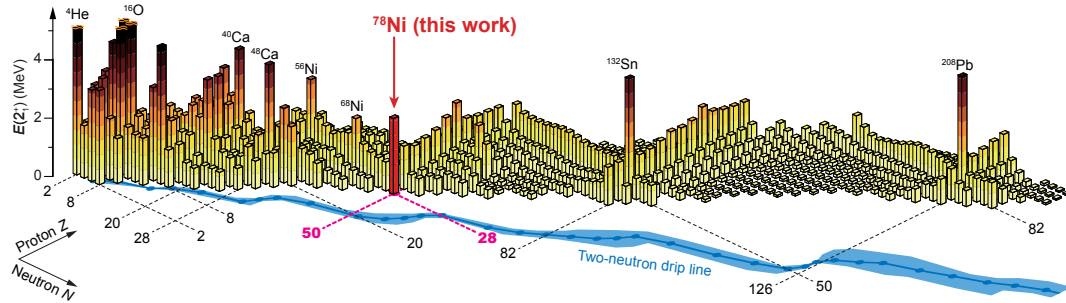
- ✓ Single-particle basis  $e_{\max} = 2n + l$
- ✓ Storage limits of 3N forces  $e_1 + e_2 + e_3 \leq E_{3\max}$
- 👤 Many-body operators: e.g., CCSD(T), IMSRG(2)

2010: Limited capabilities for 3N forces;  $^{16}\text{O}$  heaviest

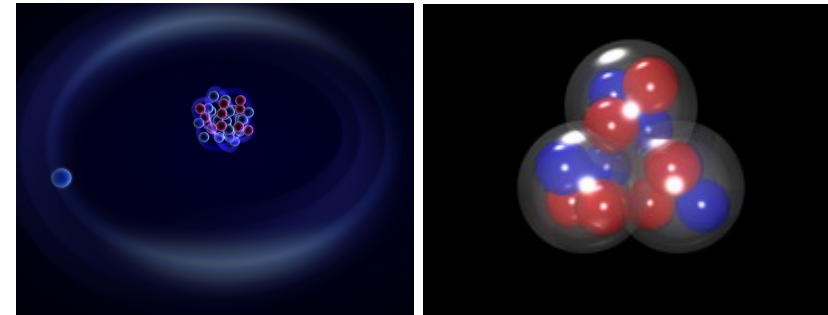


Tremendous progress in ab initio reach, largely due to polynomially scaling methods!

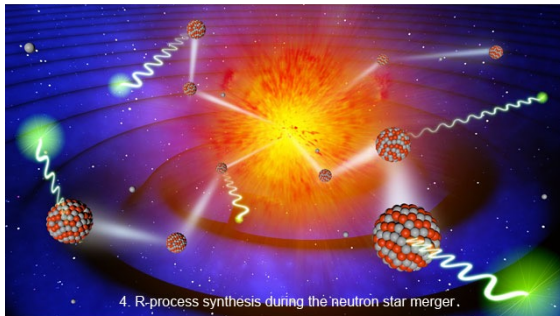




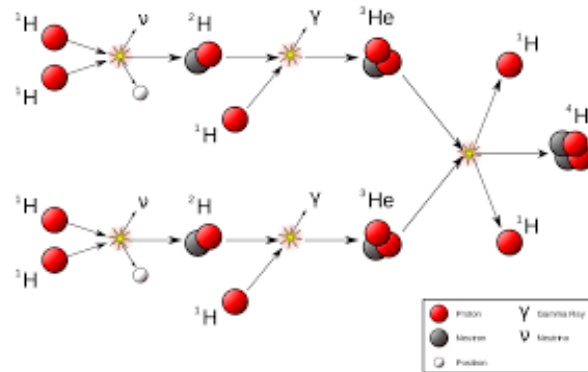
Limits of existence + formation/evolution of magic numbers



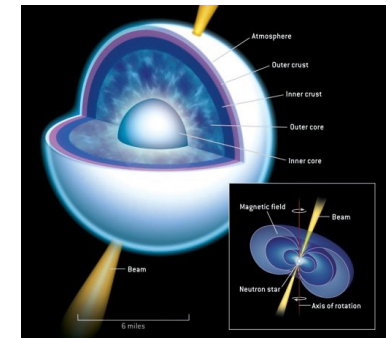
Nuclear skins/halos/clusters



Heavy Nuclei + r-process



Continuum and nuclear reactions



Infinite matter/Neutron stars

# Global Ab Initio Calculations: Proton/Neutron Driplines



Featured in Physics

Editors' Suggestion

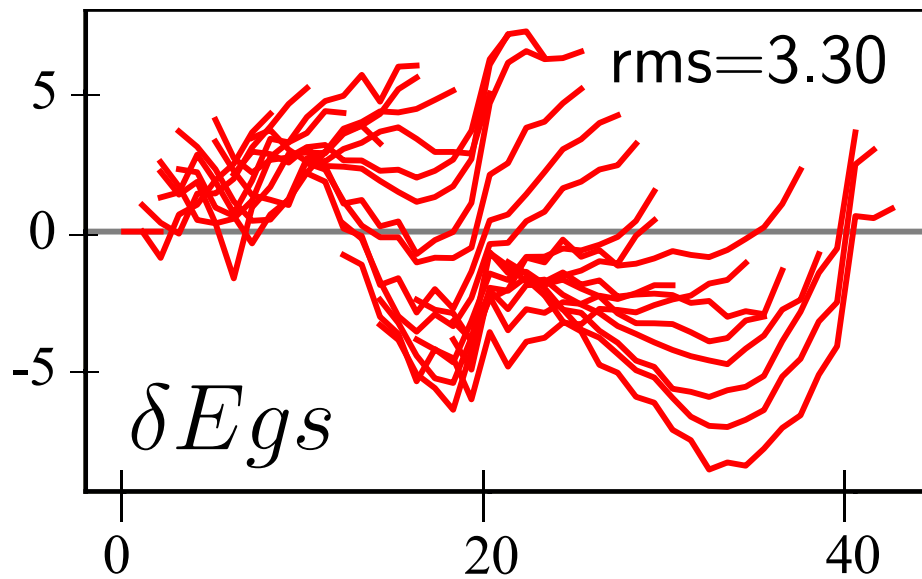
## *Ab Initio* Limits of Atomic Nuclei

S. R. Stroberg, J. D. Holt, A. Schwenk, and J. Simonis  
Phys. Rev. Lett. **126**, 022501 – Published 12 January 2021

Physics See synopsis: [Predicting the Limits of Atomic Nuclei](#)

Long considered the domain of DFT or shell model

**Ab initio calculations of ~700 nuclei from He to Fe!**



$$\delta\mathcal{O} \equiv \mathcal{O}^{(th)} - \mathcal{O}^{(exp)}$$

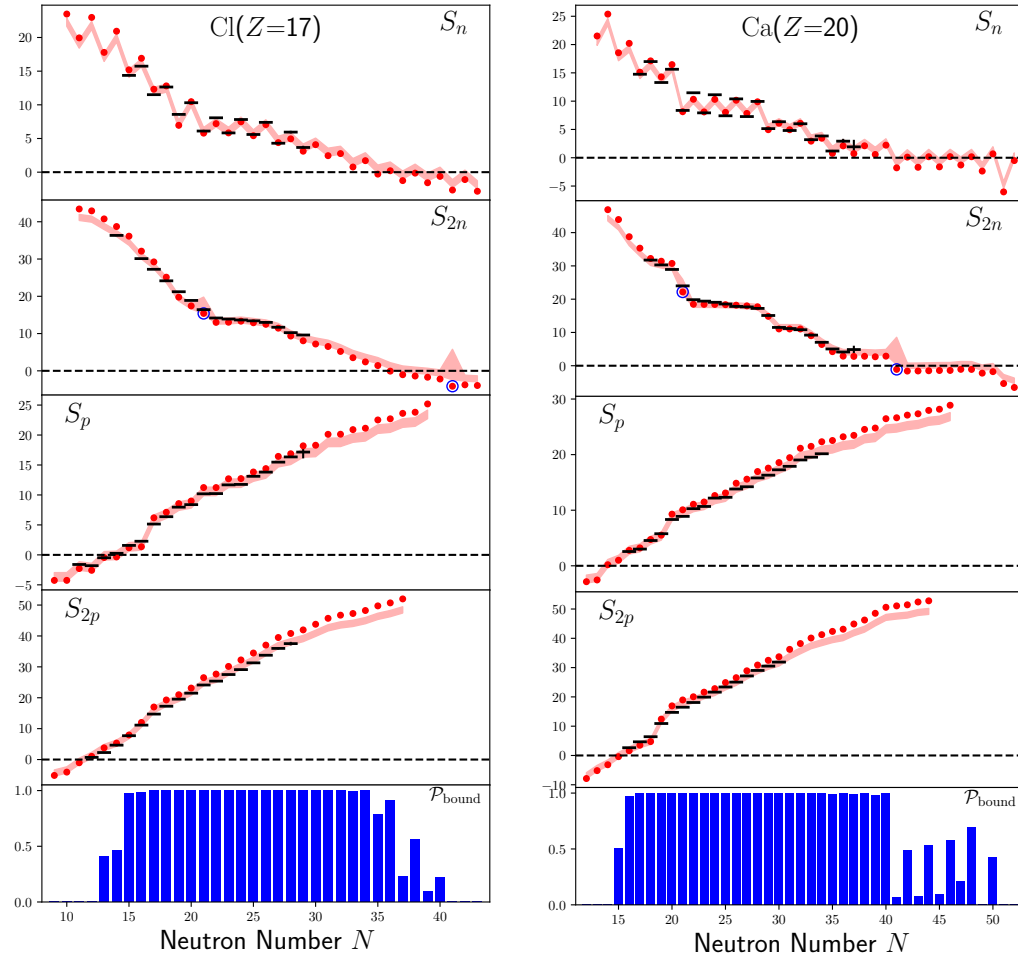
B-W Mass formula: ~3.5MeV (Z<28)

DFT: 0.6-2.0 MeV

Input Hamiltonians fit to A=2,3,4 – **not biased towards known data**

**Apply to proton/neutron driplines** separation energies?

rms deviation from experiment → model for theoretical uncertainties



rms = 0.7-1.4 MeV

Obtain PPD for separation energies

$$p(\tilde{S}^{\text{exp}} | \tilde{S}^{\text{th}}, S^{\text{th}}, S^{\text{exp}})$$

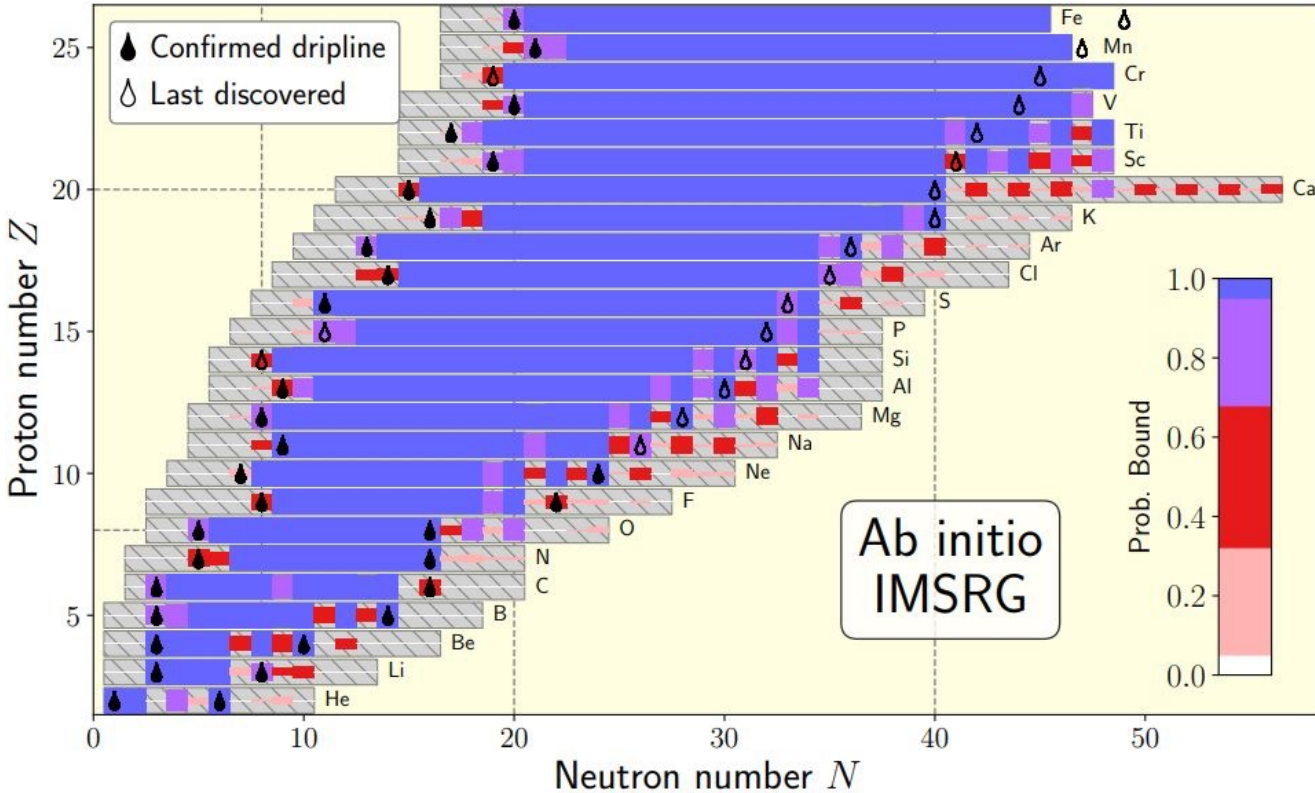
Total probability to be bound

$$\mathcal{P}_{\text{bound}} = \prod_{\alpha} \int_0^{\infty} d\tilde{S}_{\alpha}^{\text{exp}} p(\tilde{S}_{\alpha}^{\text{exp}} | \tilde{S}^{\text{th}}, S^{\text{th}}, S^{\text{exp}})$$

$$\alpha \in \{n, p, 2n, 2p\}$$

Determine probabilities for each nucleus

## Predictions of proton and neutron driplines from first principles



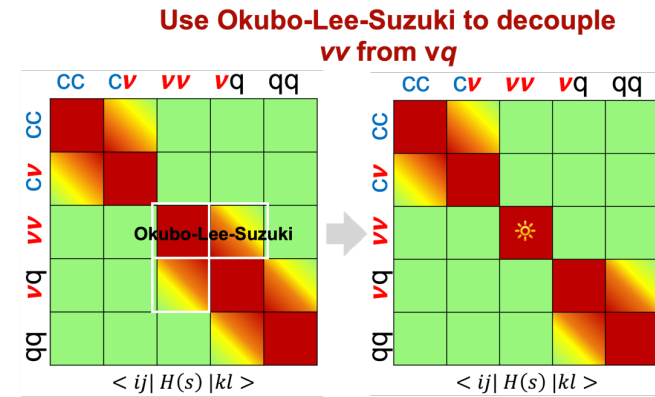
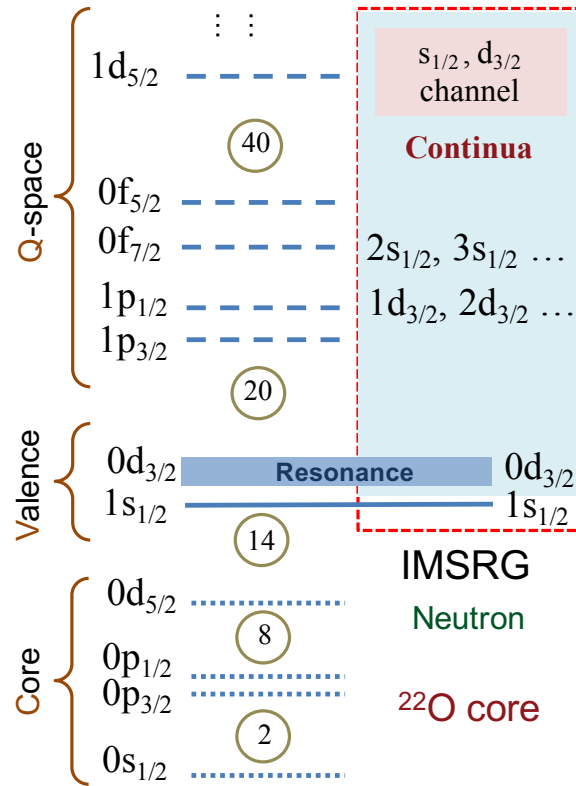
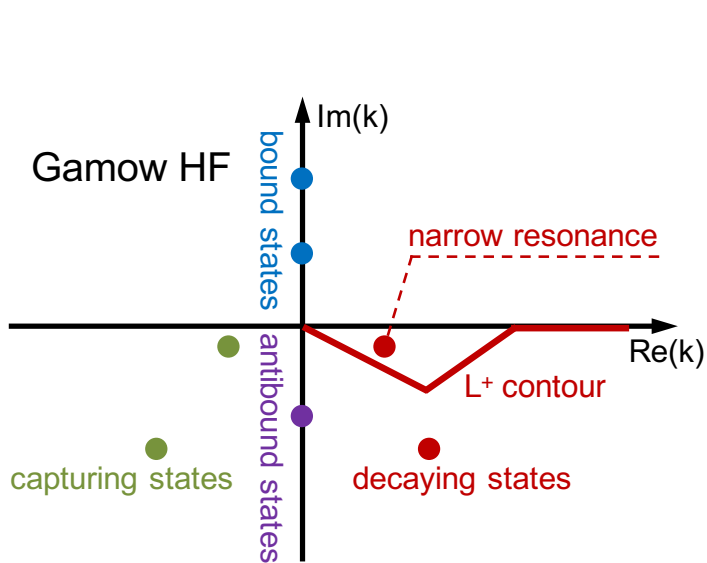
Known drip lines predicted within uncertainties (artifacts at shell closures)

**Ab initio guide for neutron-rich driplines**



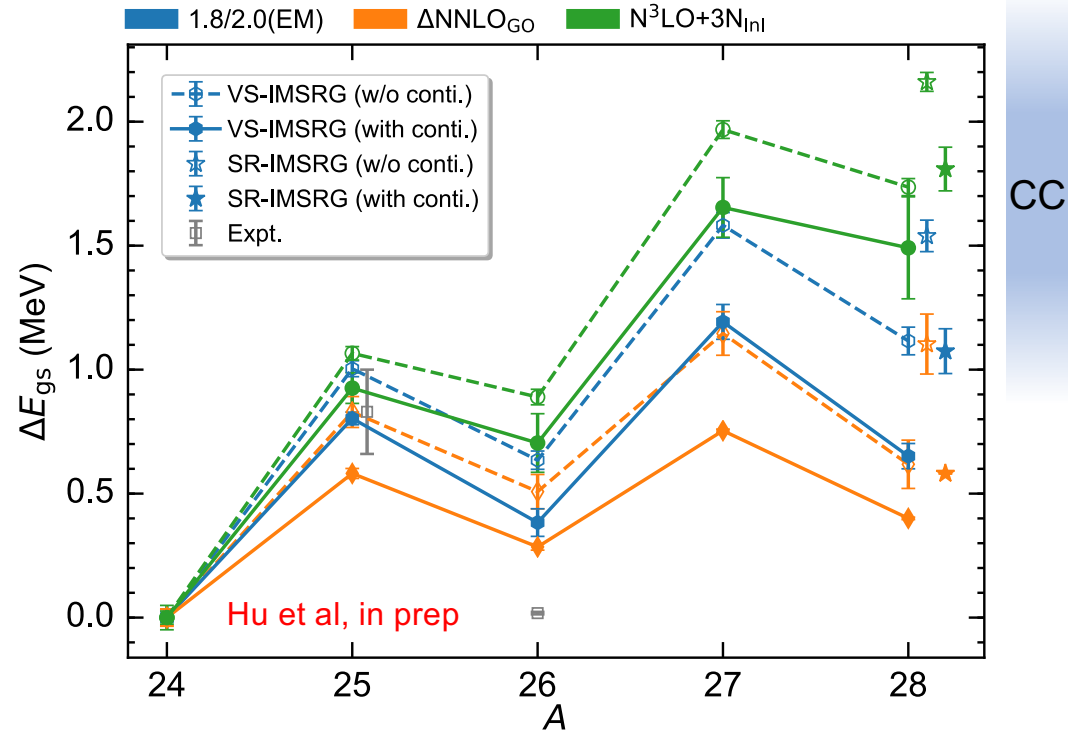
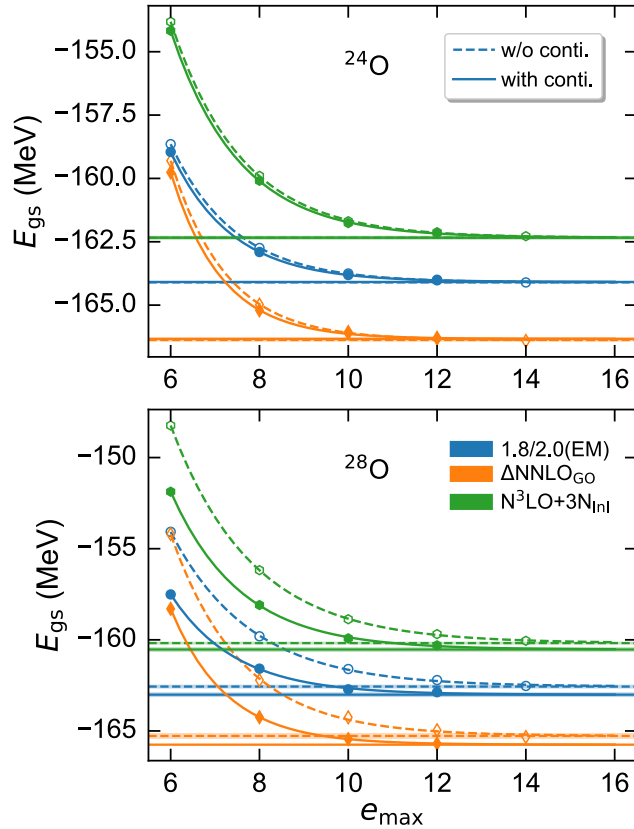
Use Gamow-Bergren basis for VS-IMSRG calculation

# VS-IMSRG with Continuum



Continuum states complicate IMSRG - Solution similar to multi-shell

New measurement at RIKEN of existence of energy in  $^{28}\text{O}$  – apply VS-IMSRG w/ continuum

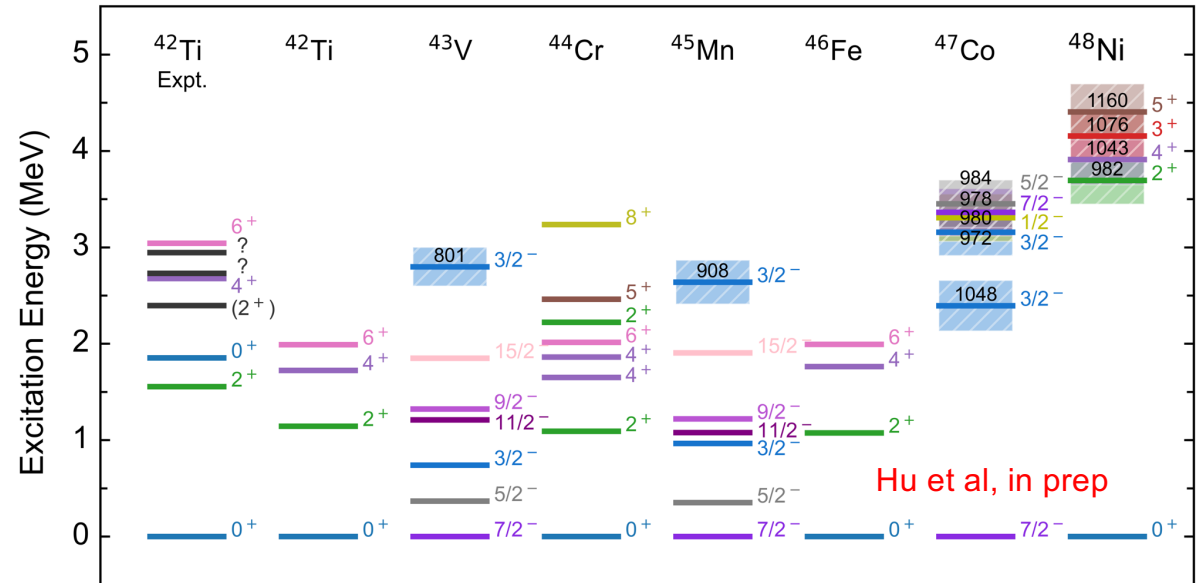
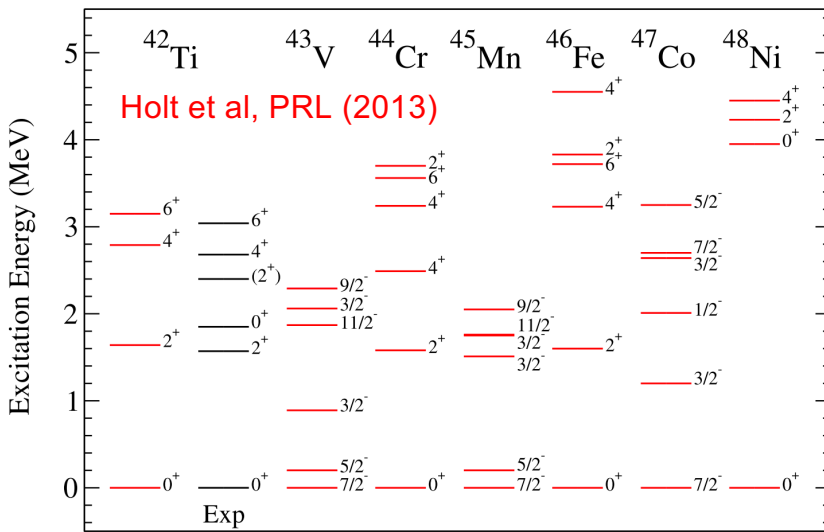


Chalmers + ORNL collab.

Continuum lowers calculated g.s. energy by  $\sim 300\text{keV}$

In all cases  $^{28}\text{O}$  predicted to be unbound... consistent with CC emulator predictions

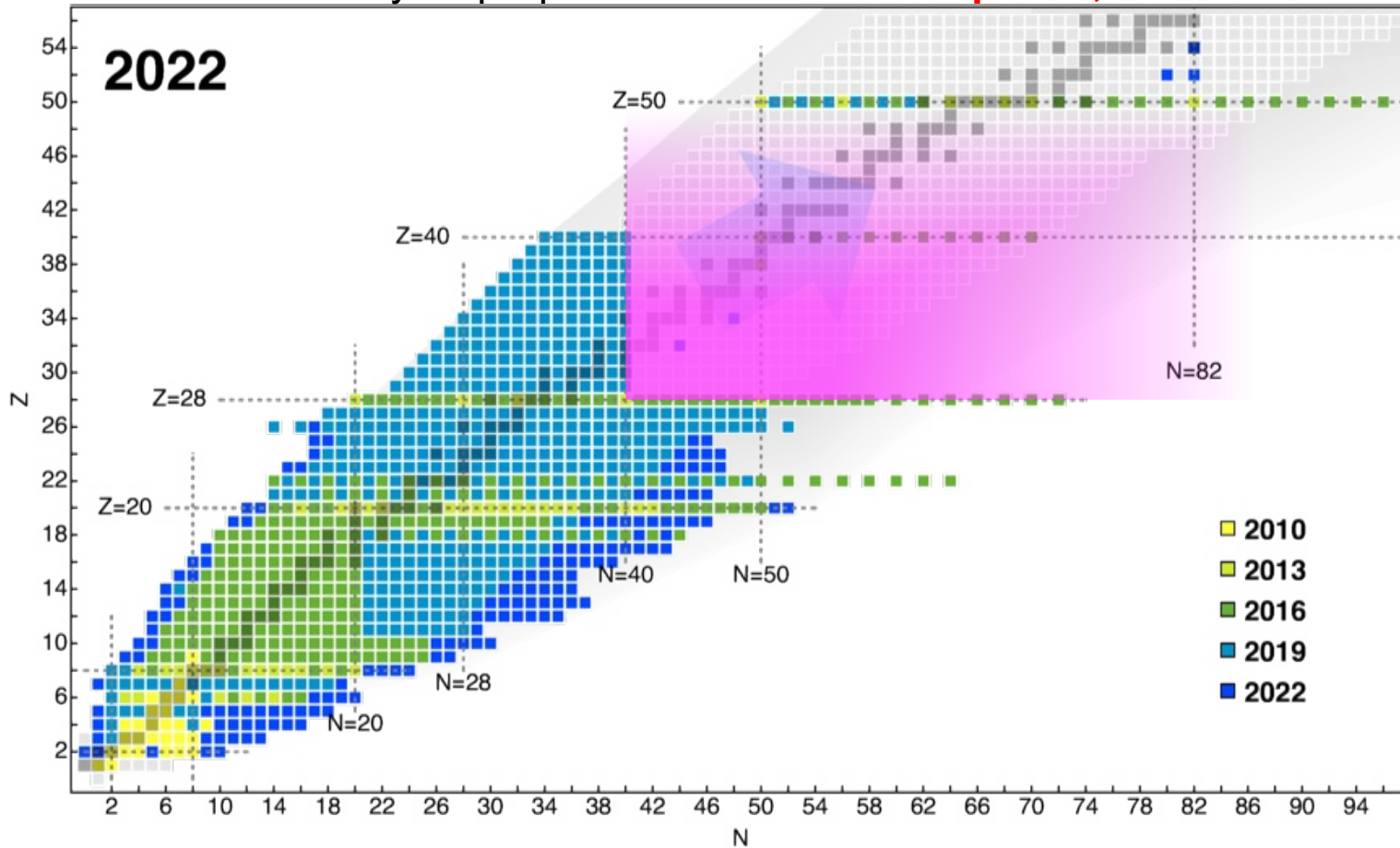
Probe limits of existence at proton dripline  $^{48}\text{Ni}$  – VS-IMSRG w/ continuum also necessary



Minor effect from continuum

Tremendous progress in ab initio reach, largely due to polynomially scaling methods!

Calculate essentially all properties all of nuclei... **up to N, Z ~ 50**



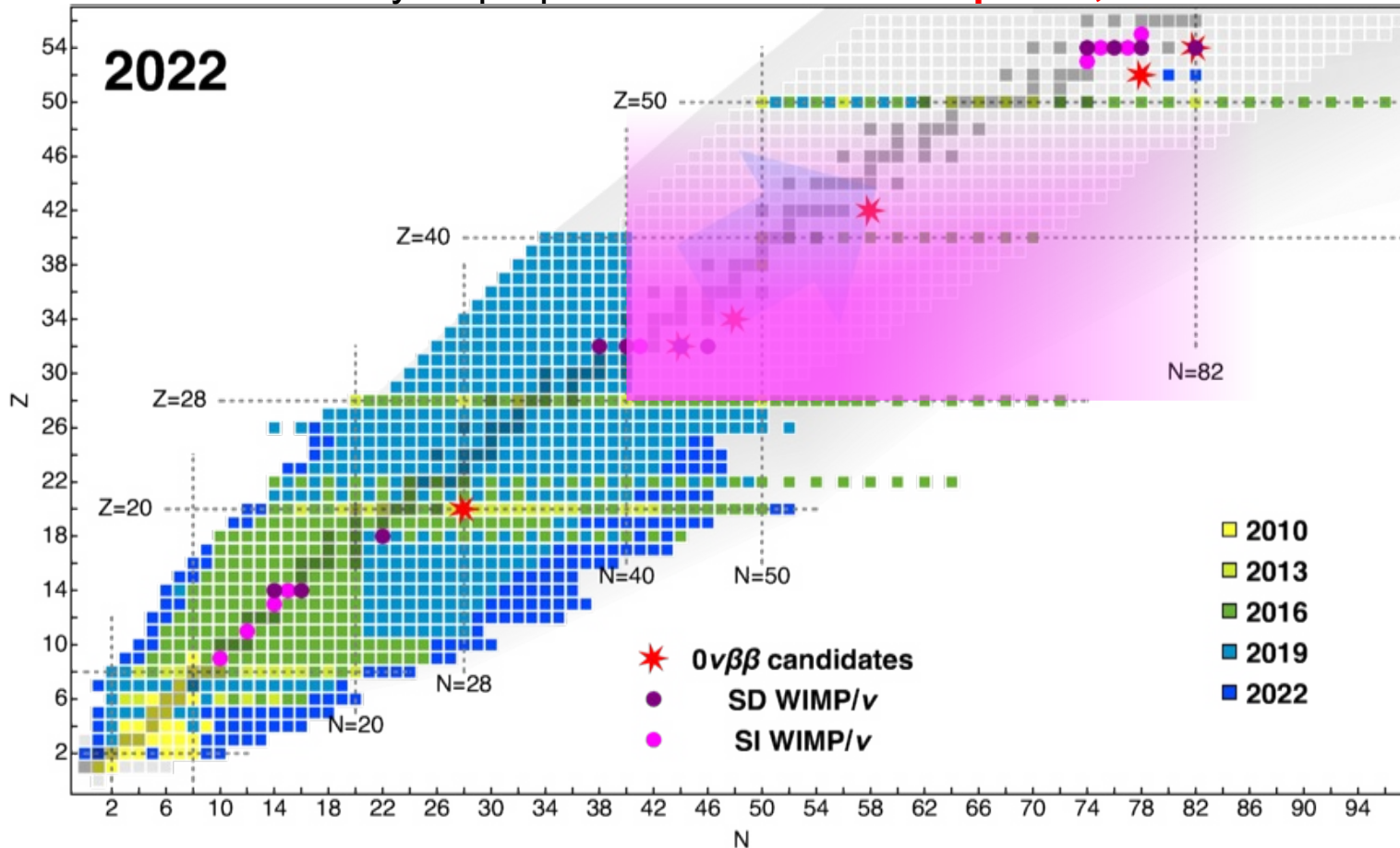
Key Limitation

**3NF matrix element storage**

$$e_1 + e_2 + e_3 \leq E_{3\max}$$

Tremendous progress in ab initio reach, largely due to polynomially scaling methods!

Calculate essentially all properties all of nuclei... **up to N, Z ~ 50**



Key Limitation

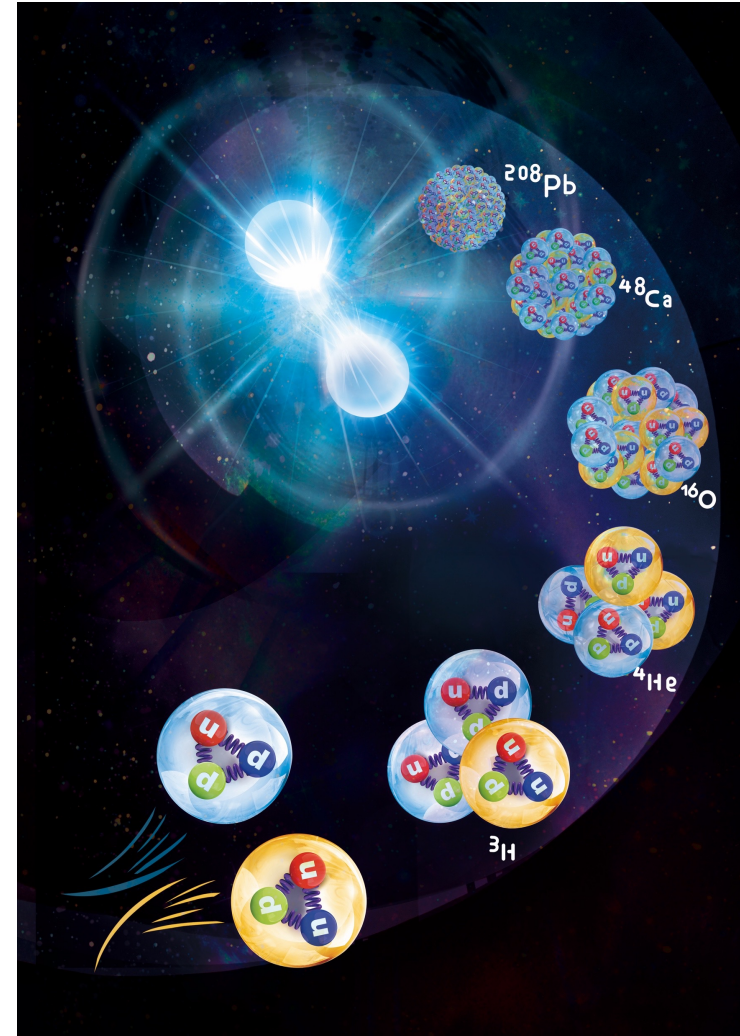
**3NF matrix element storage**

$$e_1 + e_2 + e_3 \leq E_{3\max}$$

# Converged Calculations in Heavy Nuclei

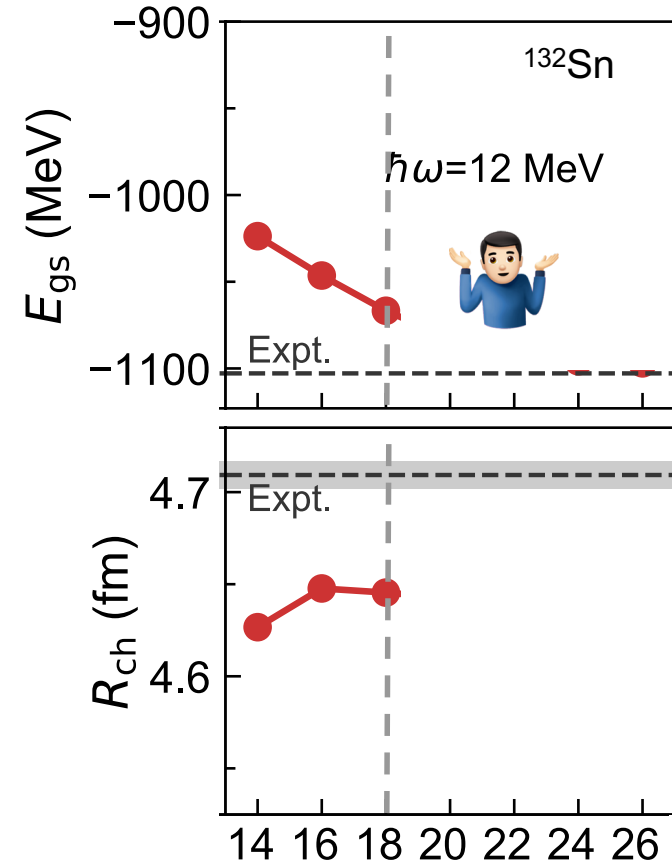
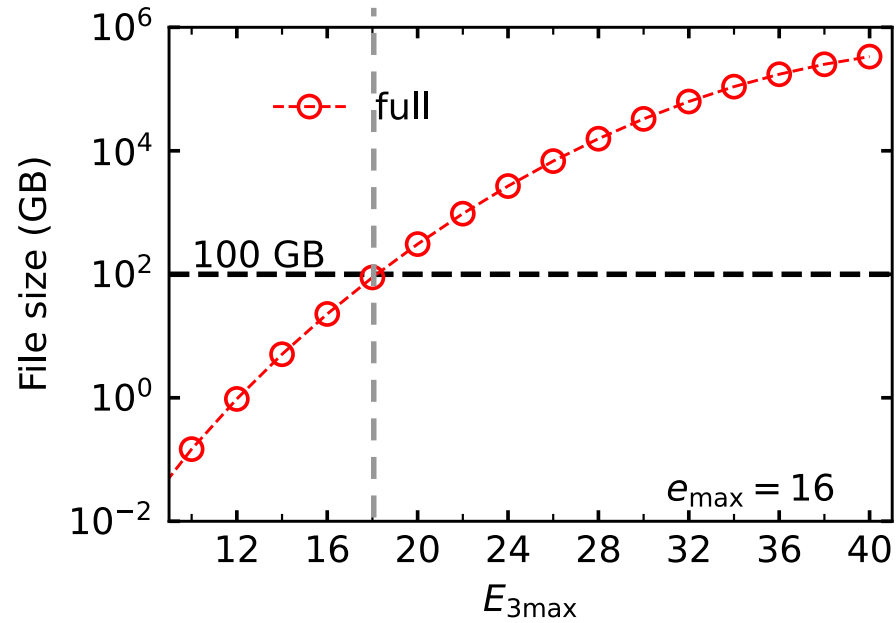
Converged *ab initio* calculations of heavy nuclei

T. Miyagi, S. R. Stroberg, P. Navrátil, K. Hebeler, and J. D. Holt  
Phys. Rev. C **105**, 014302 – Published 3 January 2022



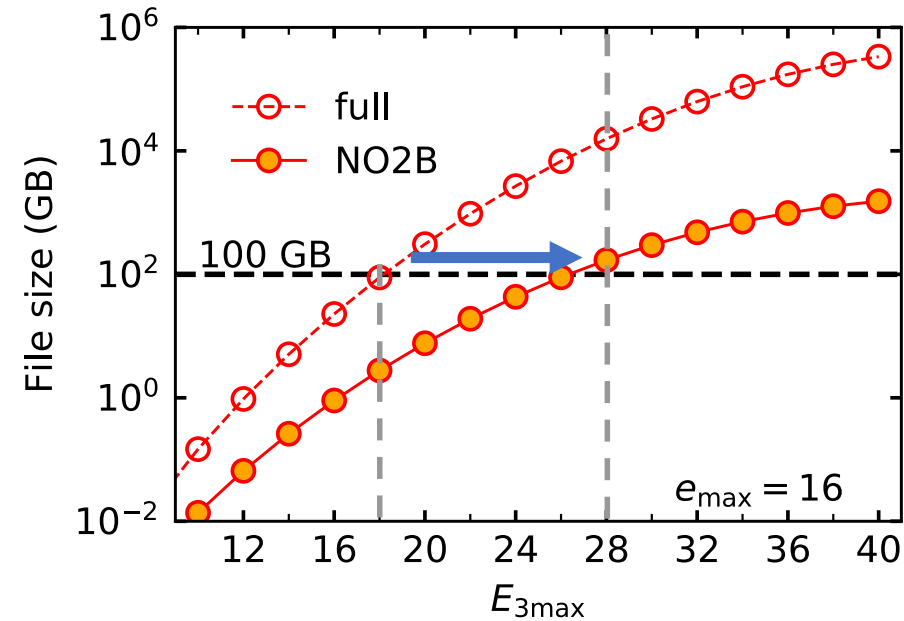
Limited by typical memory/node:  $e_1 + e_2 + e_3 \leq E_{3\max} = 18$

No sign of convergence in  $^{132}\text{Sn}$  -  $E_{\text{gs}}$  or  $R_{\text{ch}}$



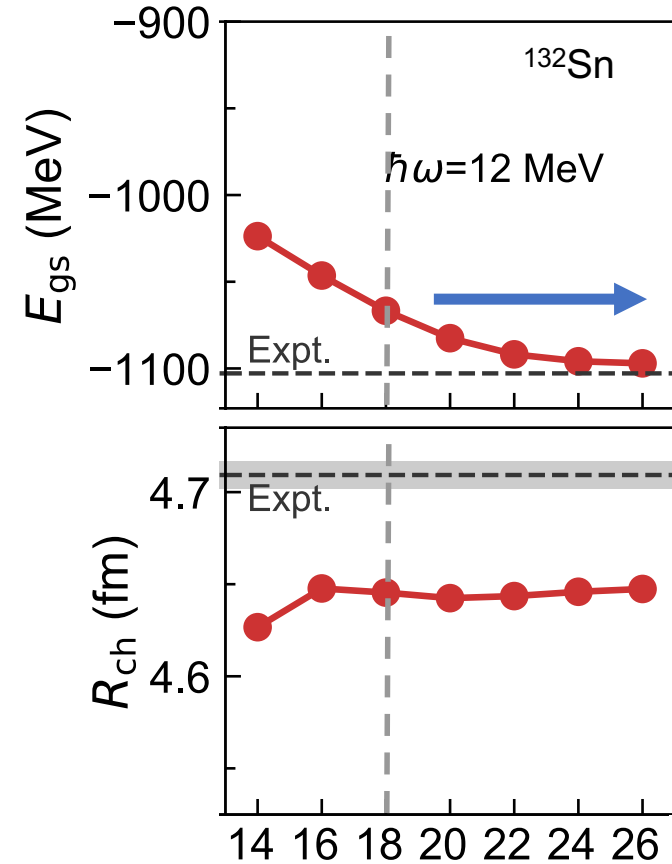
Limited by typical memory/node:  $e_1 + e_2 + e_3 \leq E_{3\max} = 18$

**Clever storage reduces needs by factor of 100!**



First converged ground-state properties of  $^{132}\text{Sn}$ !

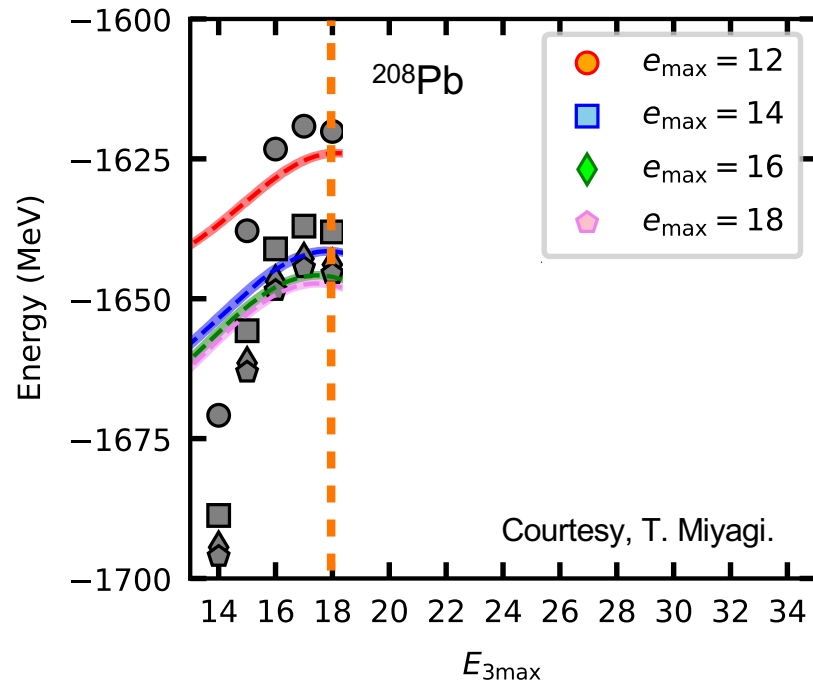
**Opens heavy region to ab initio...**





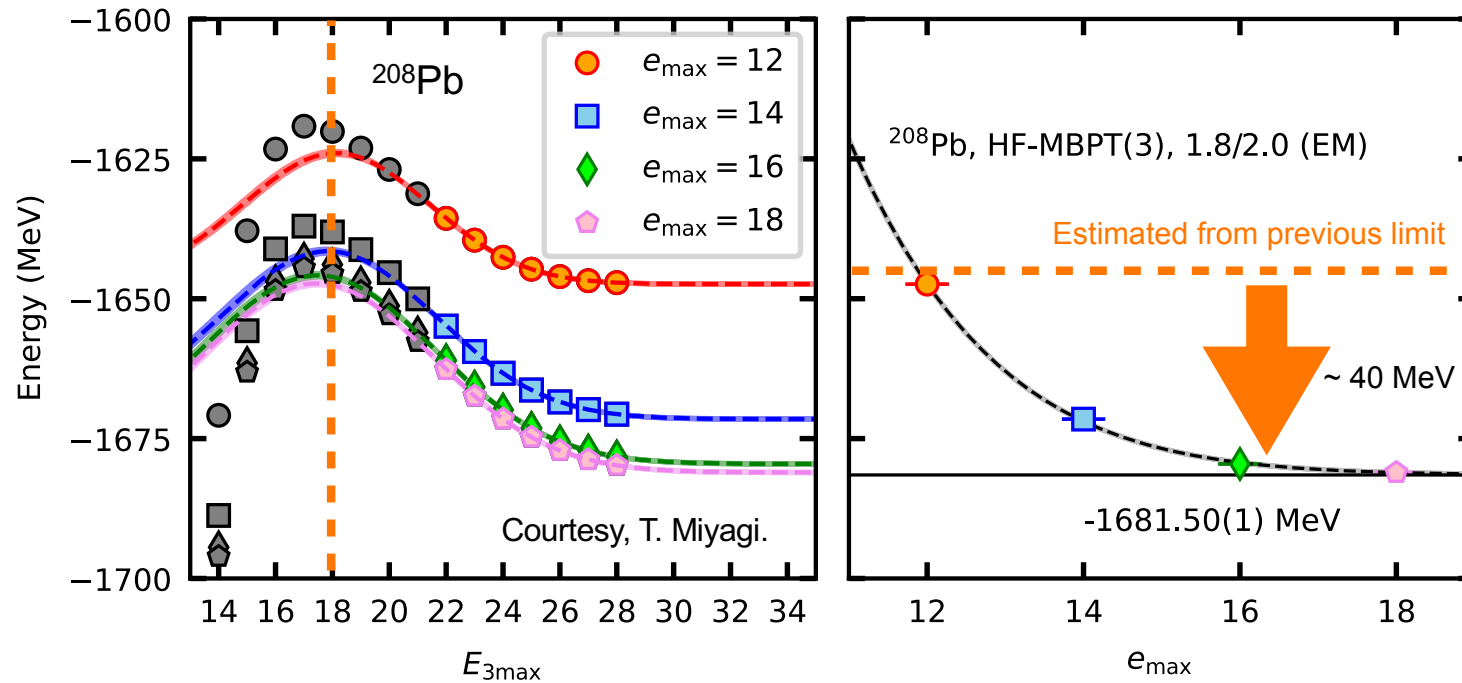


Previous limit, no hope of convergence in  $^{208}\text{Pb}$  g.s. energy...



Previous limit, no hope of convergence in  $^{208}\text{Pb}$  g.s. energy

Improved  $E_{3\text{max}} = 18 \rightarrow 28$  clear convergence



8

**First converged ab initio calculation of  $^{208}\text{Pb}$ !**

# Ab Initio Analysis: Neutron Skin of $^{208}\text{Pb}$ Linked with neutron star properties

nature  
physics

ARTICLES

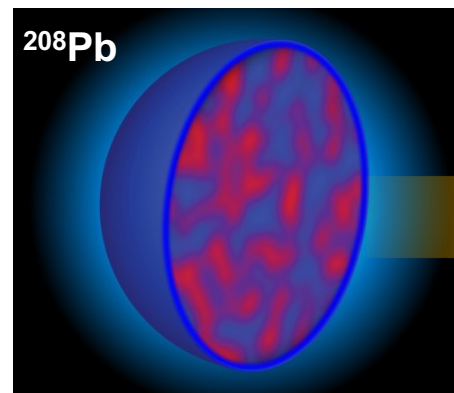
<https://doi.org/10.1038/s41567-022-01715-8>

 Check for updates

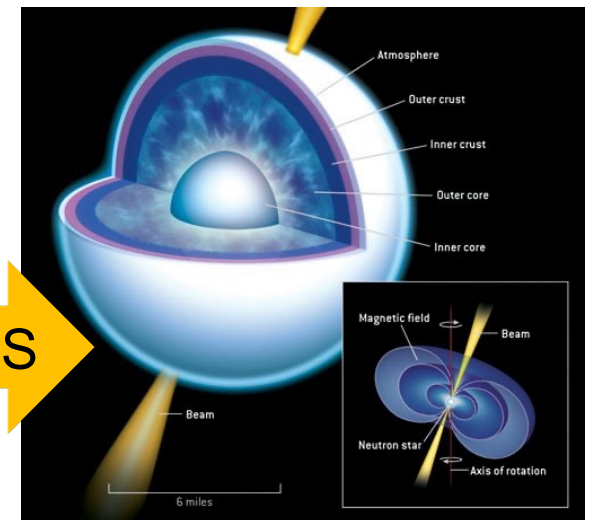
OPEN

## Ab initio predictions link the neutron skin of $^{208}\text{Pb}$ to nuclear forces

Baishan Hu<sup>1,11</sup>, Weiguang Jiang<sup>2,11</sup>, Takayuki Miyagi<sup>1,3,4,11</sup>, Zhonghao Sun<sup>5,6,11</sup>, Andreas Ekström<sup>2</sup>, Christian Forssén<sup>2</sup>, Gaute Hagen<sup>1,5,6</sup>, Jason D. Holt<sup>1,7</sup>, Thomas Papenbrock<sup>5,6</sup>, S. Ragnar Stroberg<sup>8,9</sup> and Ian Vernon<sup>10</sup>



Nuclear EOS





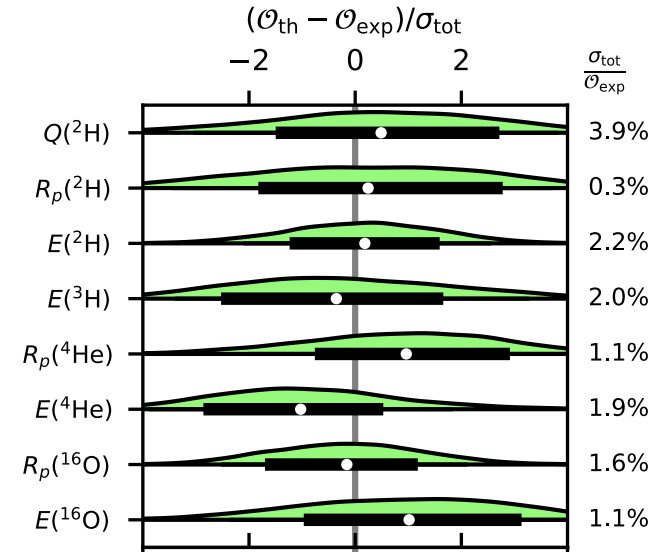
# Neutron Skin of $^{208}\text{Pb}$

Combine TRIUMF/ORNL/Chalmers advances!

**I: History Matching** confronted with  $A=2,3,4$  data +  $^{16}\text{O}$

$10^9$  calculations spanning EFT parameter space at  $\text{N}^2\text{LO}$

**34 non-implausible interactions**





# Neutron Skin of $^{208}\text{Pb}$

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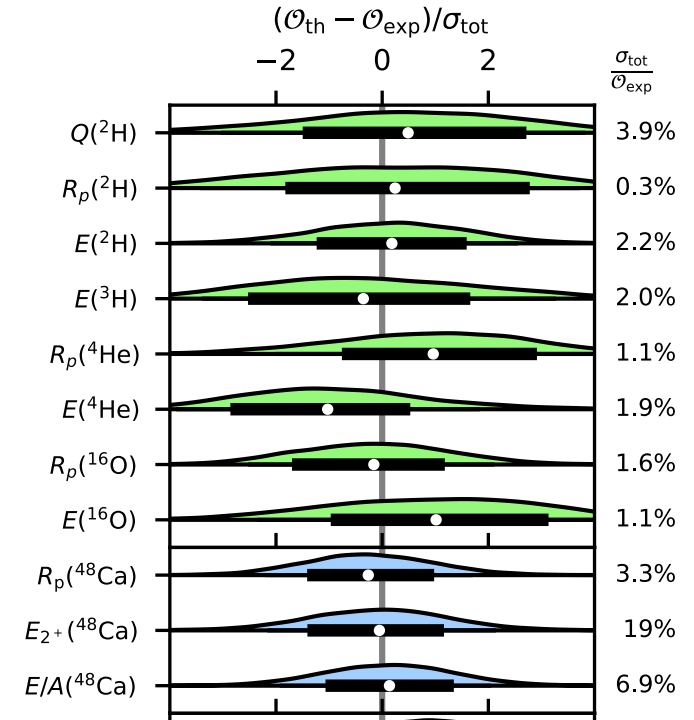
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II: **Calibration** use  $^{48}\text{Ca}$   $E/A$ ,  $E(2^+)$ ,  $R_p$ , dipole polarizability

**Importance resampling – statistically weight interactions**





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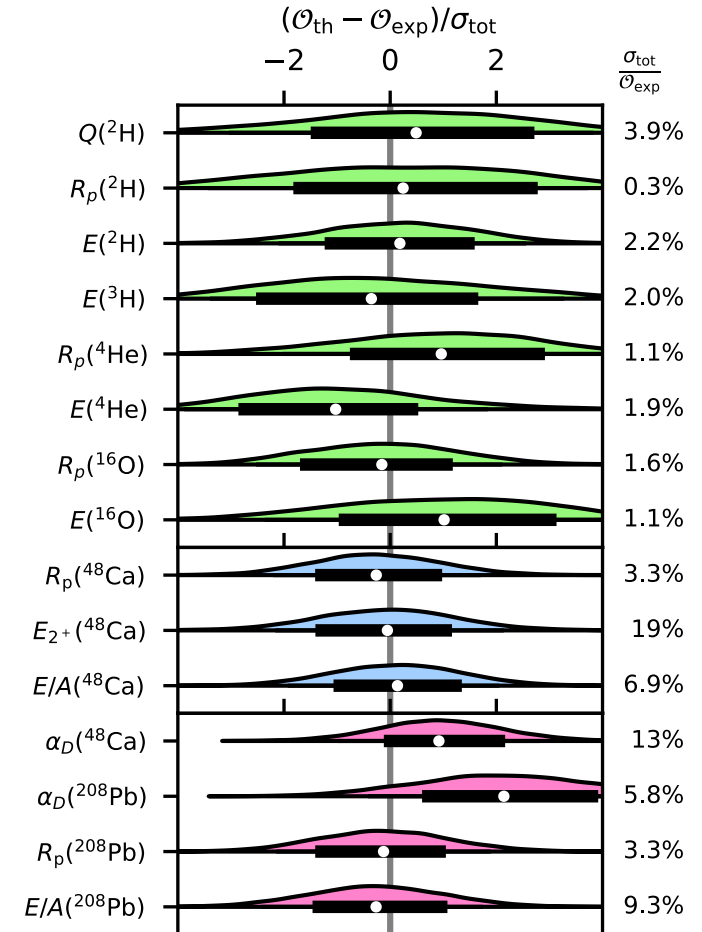
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**III: Validation**  $^{208}\text{Pb}$   $E/A$ ,  $R_p$  +  $^{48}\text{Ca}/^{208}\text{Pb}$  DP from ab initio

Clear quality description of data





# Neutron Skin of $^{208}\text{Pb}$

Combine TRIUMF/ORNL/Chalmers advances!

I: **History Matching** confronted with  $A=2,3,4$  data +  $^{16}\text{O}$

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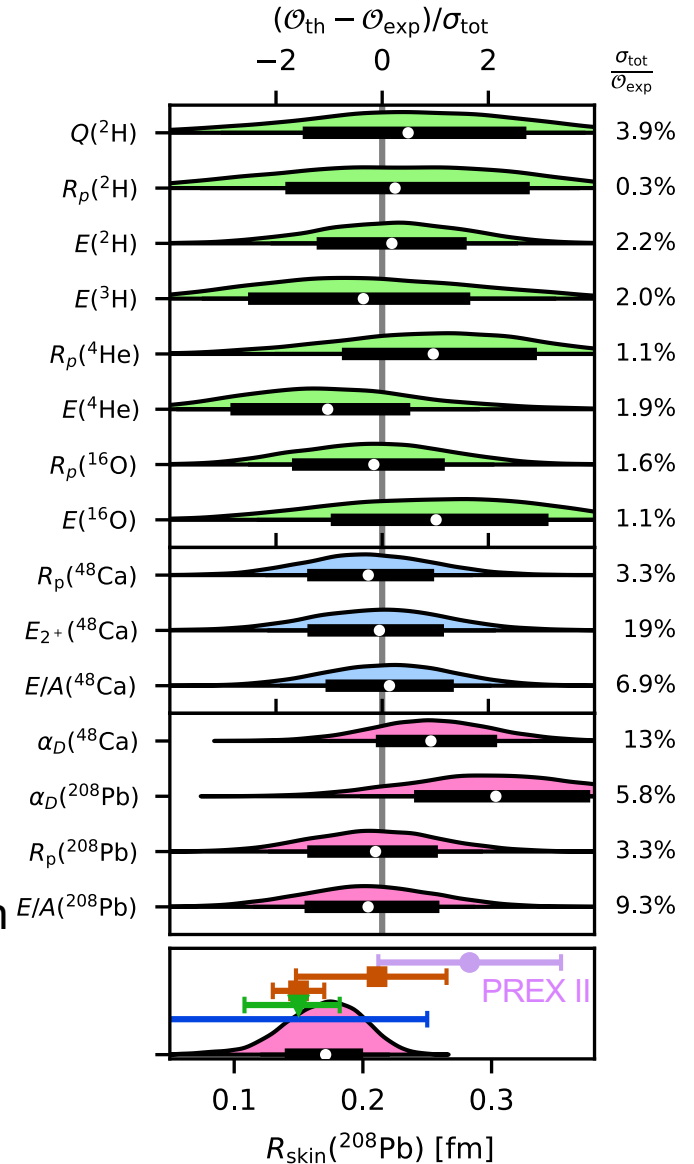
III: **Validation**  $^{208}\text{Pb}$   $E/A$ ,  $R_p$  +  $^{48}\text{Ca}/^{208}\text{Pb}$  DP from ab initio

Clear quality description of data

IV: **Prediction** - posterior predictive distribution for neutron skin  $E/A(^{208}\text{Pb})$

$R_{\text{skin}}(^{208}\text{Pb}) = 0.14\text{-}0.20\text{fm}$  (68% credible level)

Consistent(ish) with extracted **PREXII result**





## Explore correlations between finite nuclei and nuclear EOS

Use same 34 non-implausible interactions

Reveals correlation as seen in mean field models

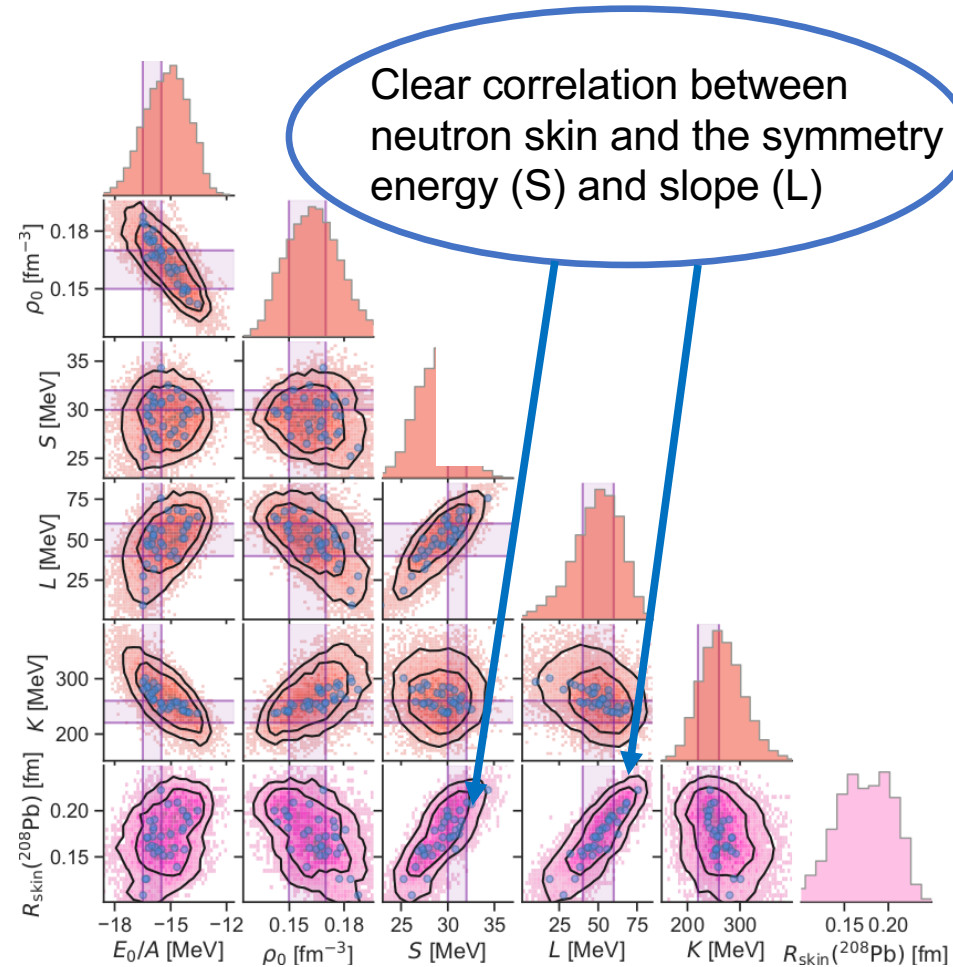
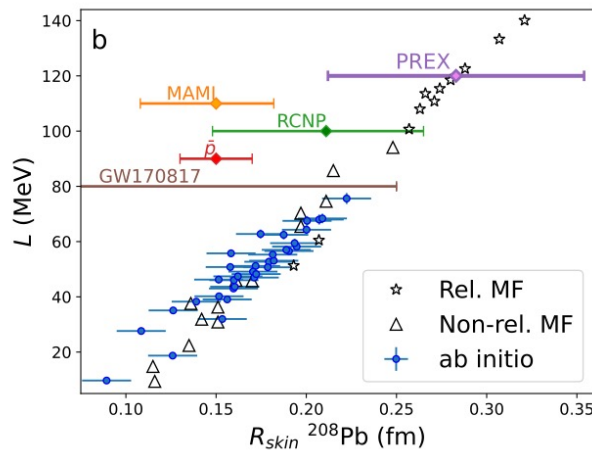
**L = 37-63 MeV**

Constrain forces potentially from:

Neutron star radii/mergers

Mean field accommodates large range of skins

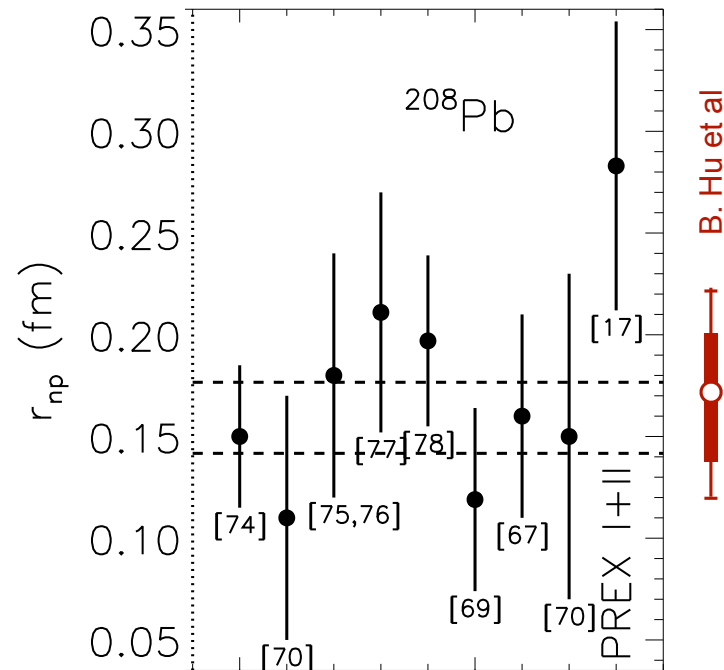
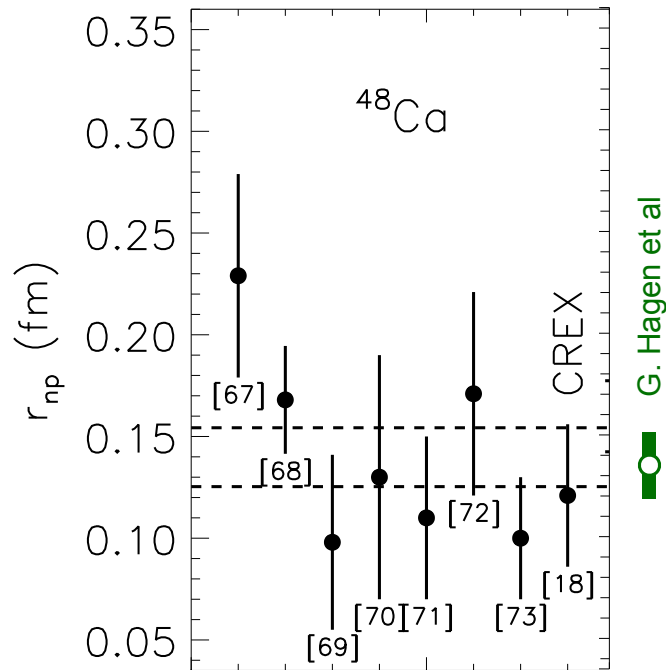
Tighter range from  
ab initio calculations



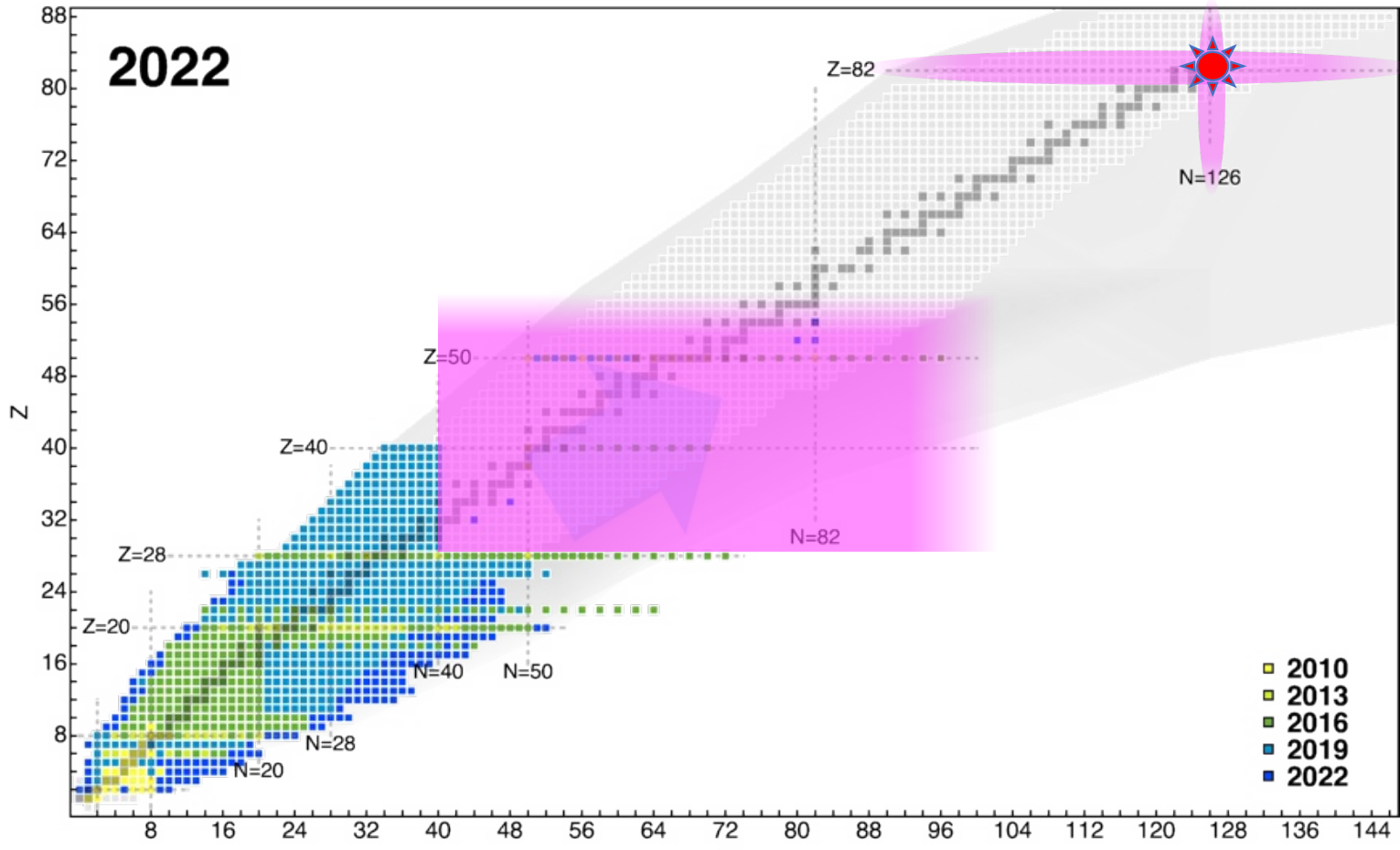
## Newly extracted neutron skin in $^{48}\text{Ca}$

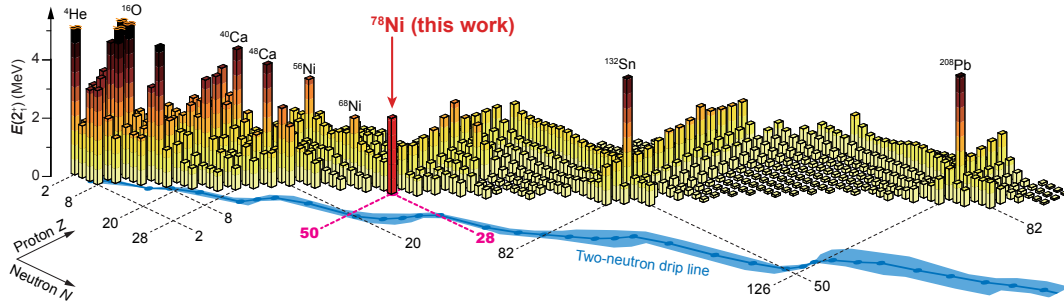
Use same 34 interactions – predictions in good agreement with CREX result

Constraints on Nuclear Symmetry Energy Parameters  
J. Lattimer (2023)

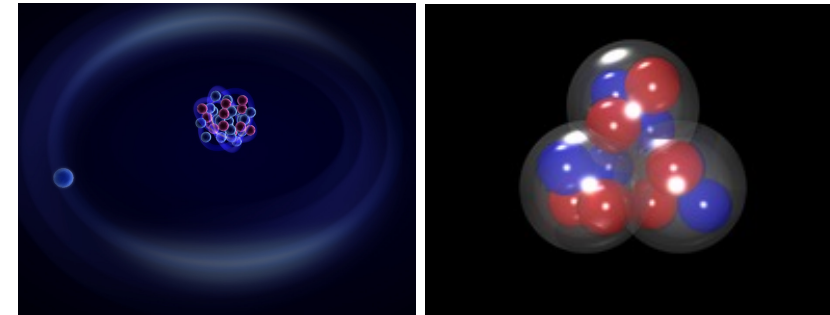


Rapid progress in ab initio reach, due to valence-space approach... up to...

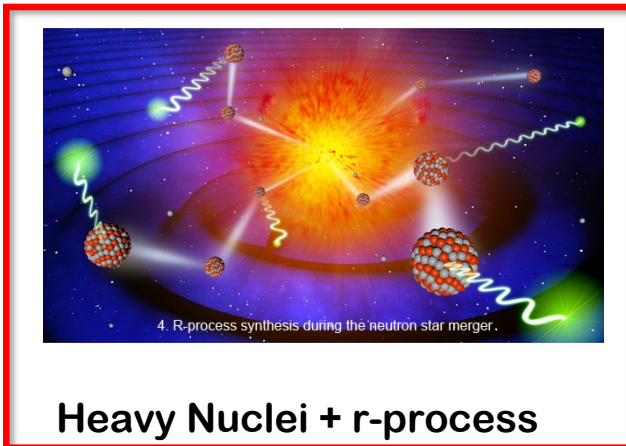




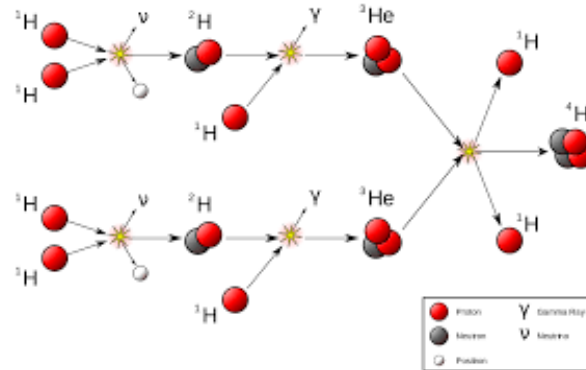
Limits of existence + formation/evolution of magic numbers



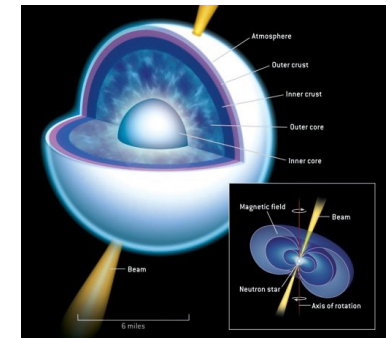
Nuclear radii/skins/halos/clusters



Heavy Nuclei + r-process



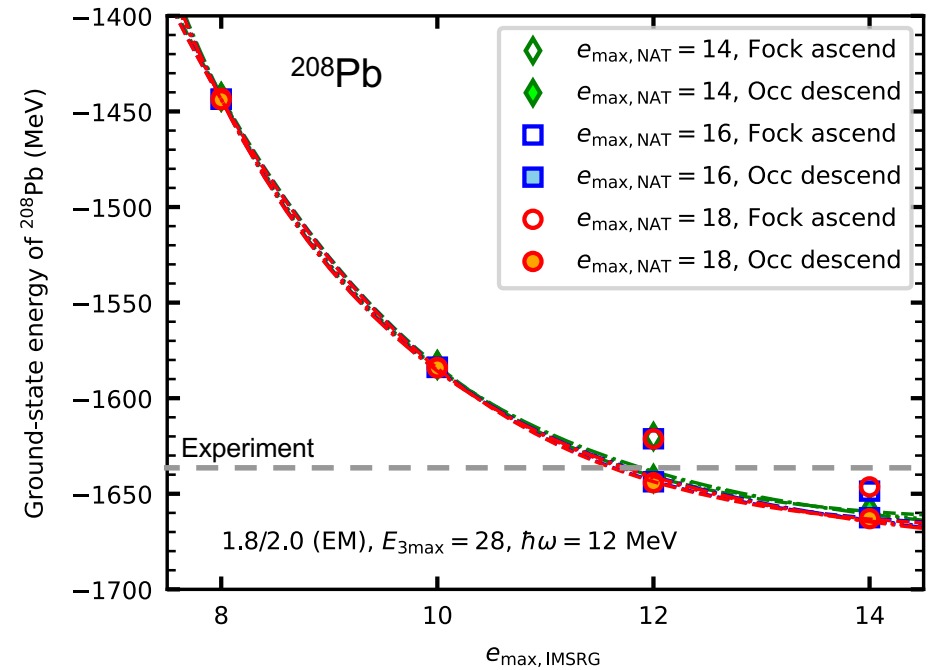
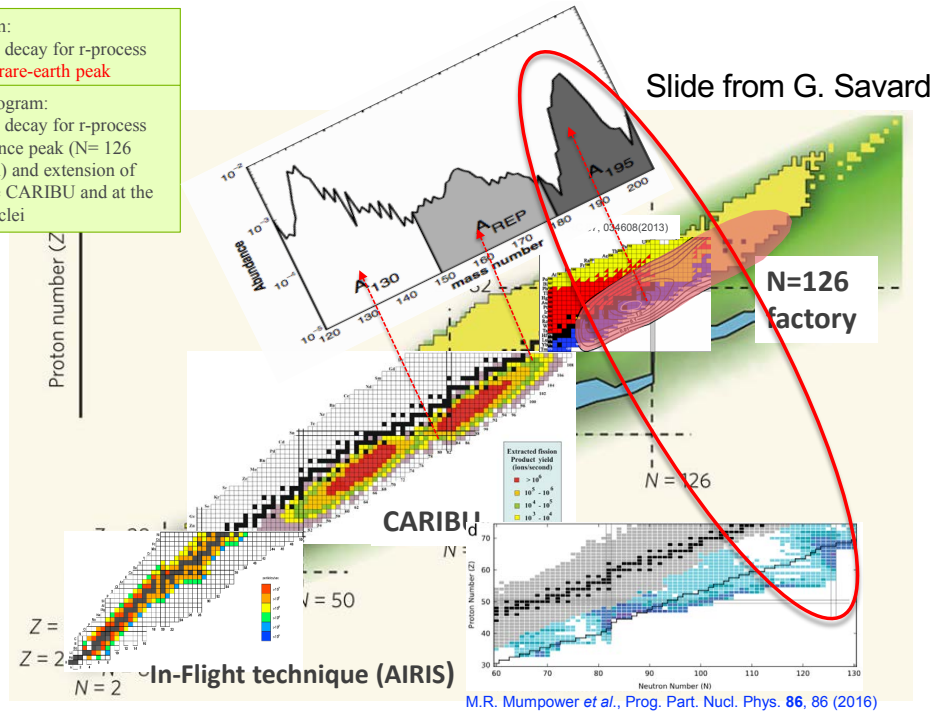
Continuum and nuclear reactions



Infinite matter/Neutron stars

Information for nuclei along N=126 necessary for third r-process abundance peak

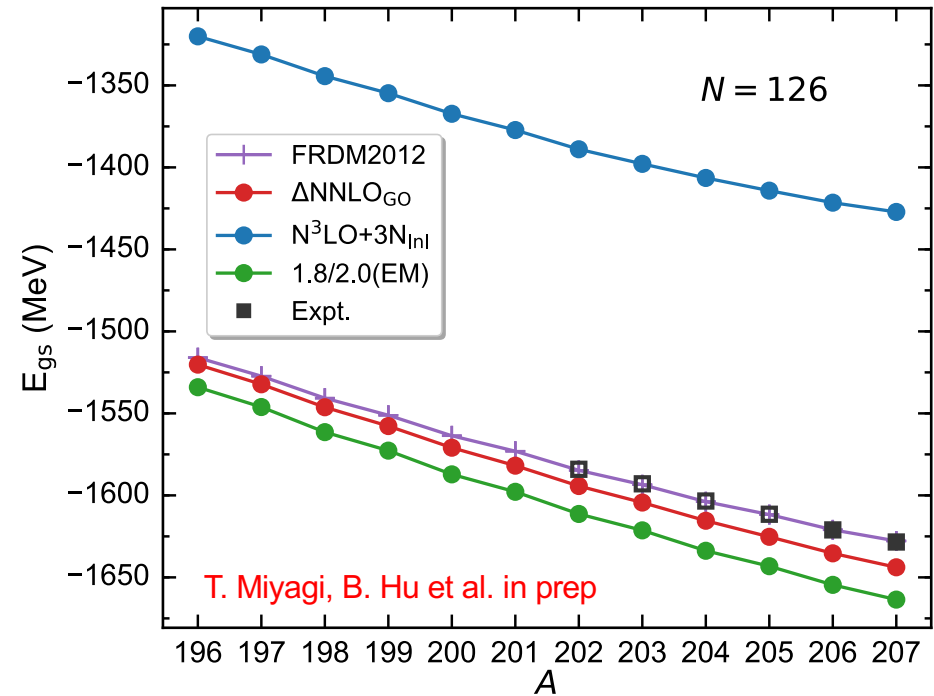
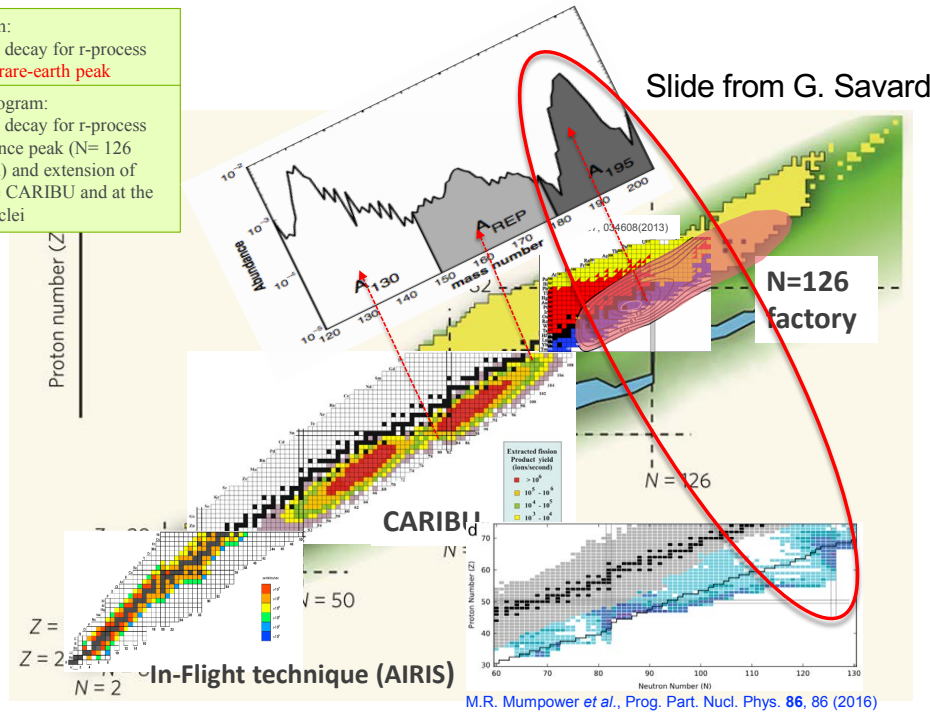
- Current program:
- Masses and decay for r-process (N=82 and **rare-earth peak**)
- Next 5 years program:
- Masses and decay for r-process last abundance peak (N=126 peak region) and extension of work above CARIBU and at the heaviest nuclei



Natural Orbital Basis (NAT) allows for rapid convergence

Information for nuclei along  $N=126$  necessary for third r-process abundance peak

- Current program:
- Masses and decay for r-process ( $N=82$  and rare-earth peak)
- Next 5 years program:
- Masses and decay for r-process last abundance peak ( $N=126$  peak region) and extension of work above CARIBU and at the heaviest nuclei

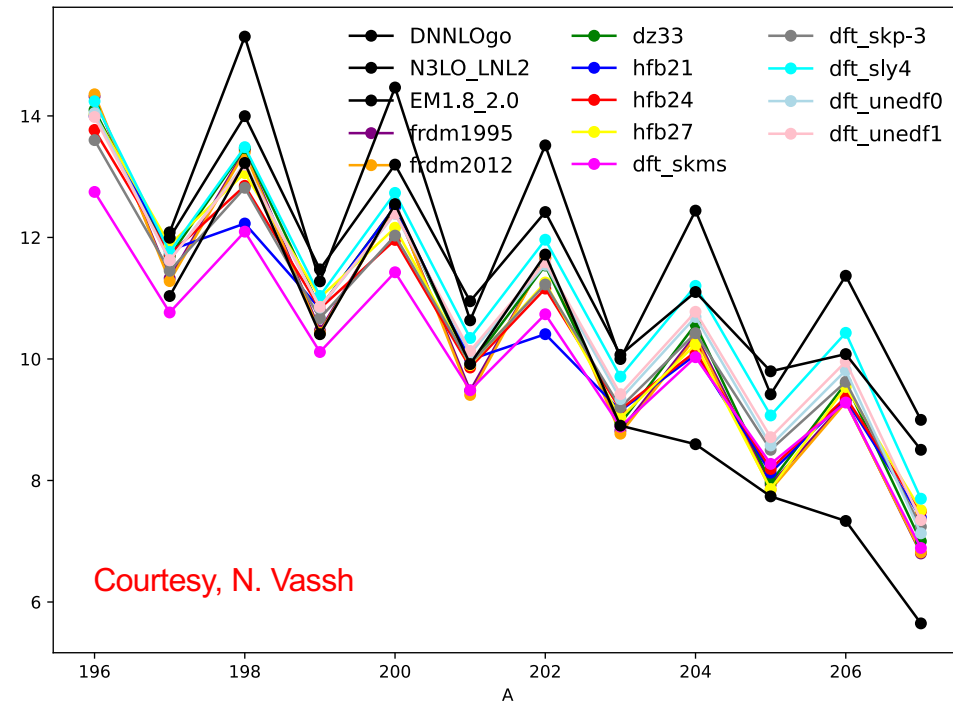
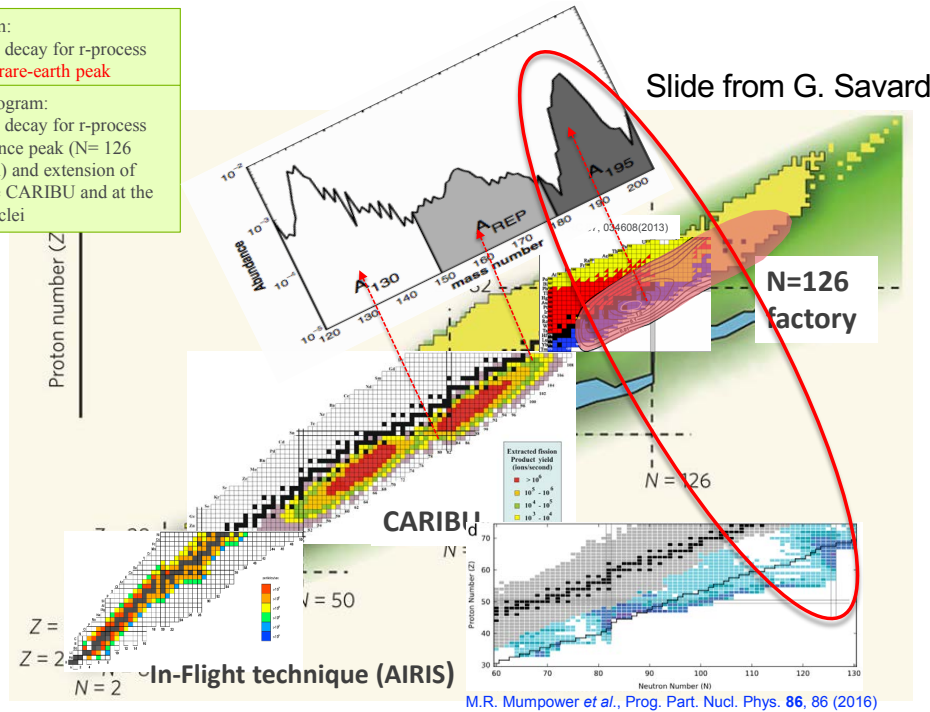


Natural Orbital Basis (NAT) allows for rapid convergence

Converged ground-state energies for  $Z=69-82$

Information for nuclei along  $N=126$  necessary for third r-process abundance peak

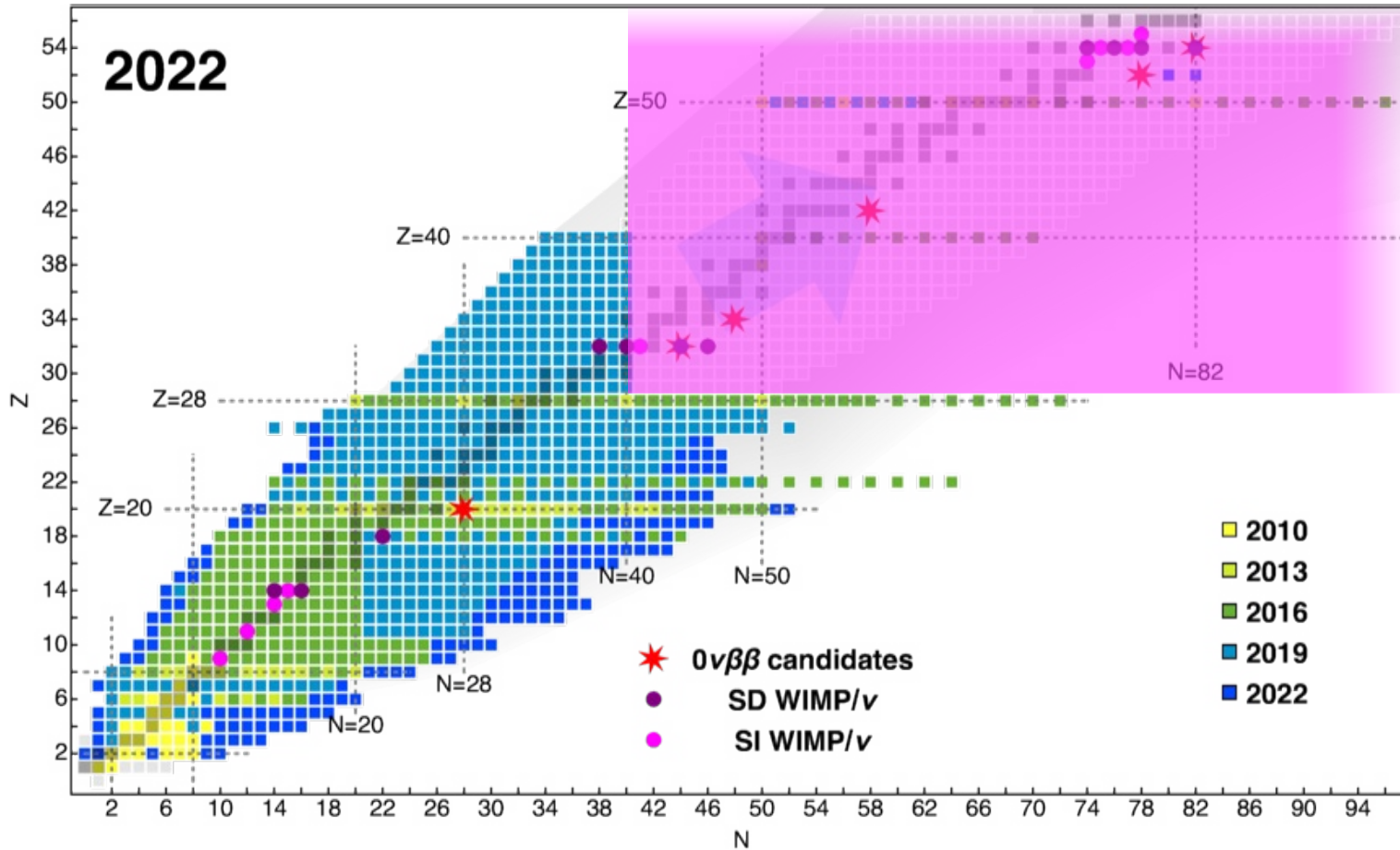
- Current program:
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- Next 5 years program:
- Masses and decay for r-process last abundance peak ( $N=126$  peak region) and extension of work above CARIBU and at the heaviest nuclei



Natural Orbital Basis (NAT) allows for rapid convergence

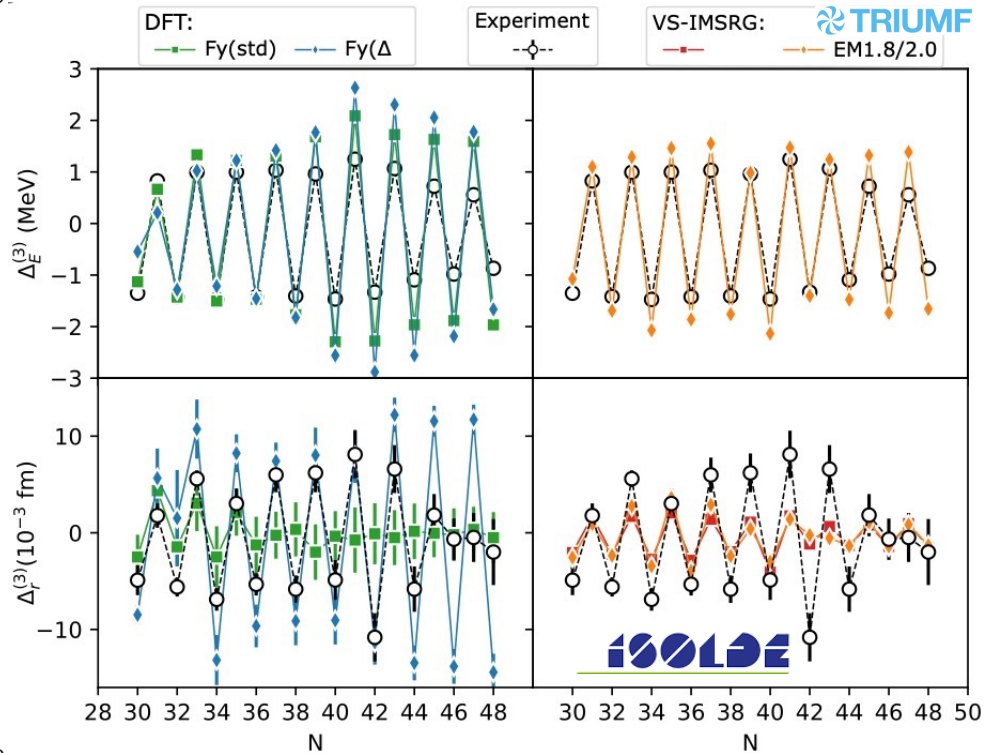
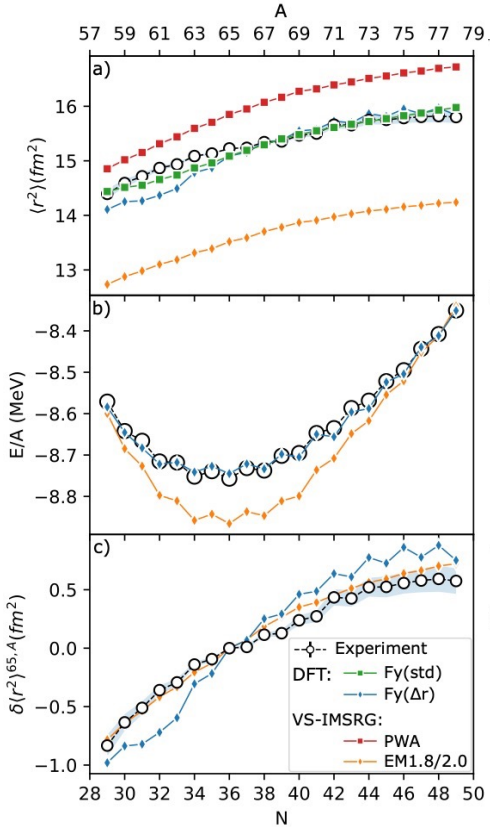
Significant systematic differences from mass models for  $S_p$

Rapid progress in ab initio reach, due to valence-space approach... up to...





## Odd-even staggering of charge radii across Cu chain



LETTERS  
<https://doi.org/10.1038/441567-020-0868-y>  
 nature physics  
 Check for updates

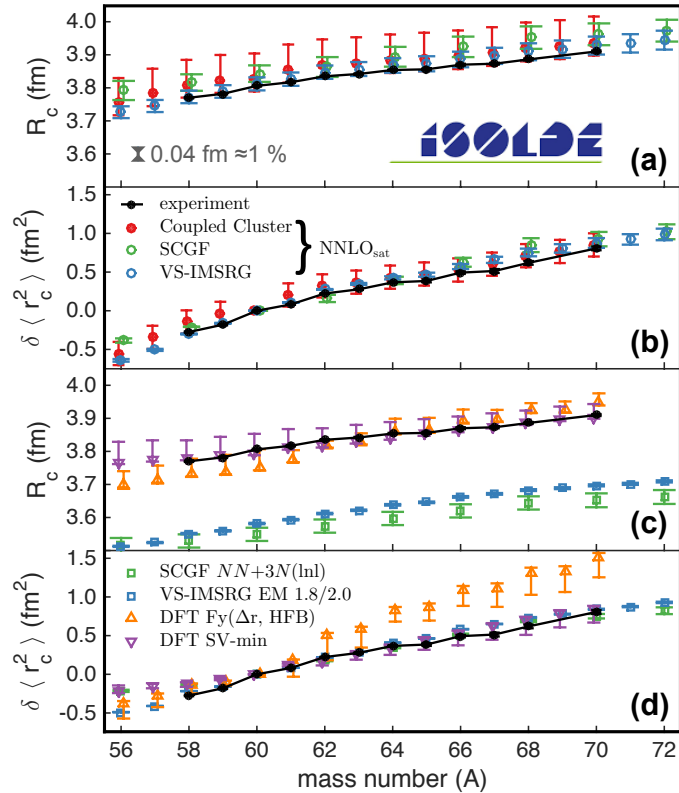
**OPEN**  
**Measurement and microscopic description of odd-even staggering of charge radii of exotic copper isotopes**

R. P. de Groote<sup>1,2,23</sup>, J. Billowes<sup>3</sup>, C. L. Binnersley<sup>3</sup>, M. L. Bissell<sup>3</sup>, T. E. Cocolios<sup>1</sup>, T. Day Goodacre<sup>4,5</sup>, G. J. Farooq-Smith<sup>1</sup>, D. V. Fedorov<sup>6</sup>, K. T. Flanagan<sup>3</sup>, S. Franchoo<sup>7</sup>, R. F. Garcia Ruiz<sup>3,8,9</sup>, W. Gins<sup>12</sup>, J. D. Holt<sup>5,10</sup>, Á. Koszorús<sup>1</sup>, K. M. Lynch<sup>9</sup>, T. Miyagi<sup>1</sup>, W. Nazarewicz<sup>11</sup>, G. Neyens<sup>13</sup>, P.-G. Reinhard<sup>12</sup>, S. Rothe<sup>3,4</sup>, H. H. Stroke<sup>13</sup>, A. R. Vernon<sup>1,3</sup>, K. D. A. Wendt<sup>14</sup>, S. G. Wilkins<sup>3,4</sup>, Z. Y. Xu<sup>1</sup> and X. F. Yang<sup>1,15</sup>

Cu isotopes, odd-even staggering well reproduced

Ab initio competitive with DFT (fit to reproduce odd-even staggering)

Study charge radii systematics across Ni isotopic chain



Nuclear Charge Radii of the Nickel Isotopes <sup>58–68,70</sup>Ni

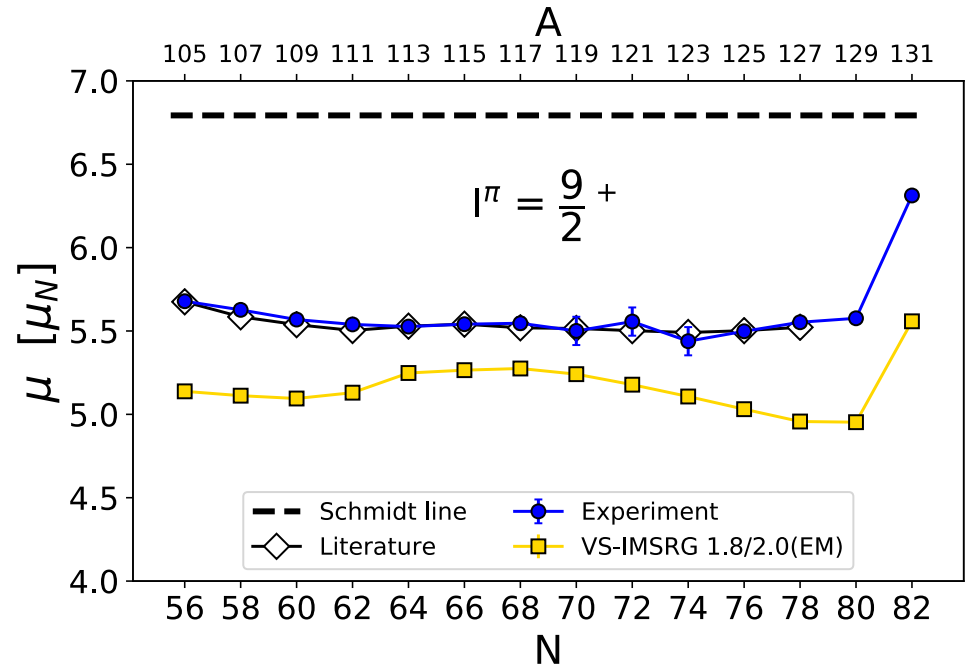
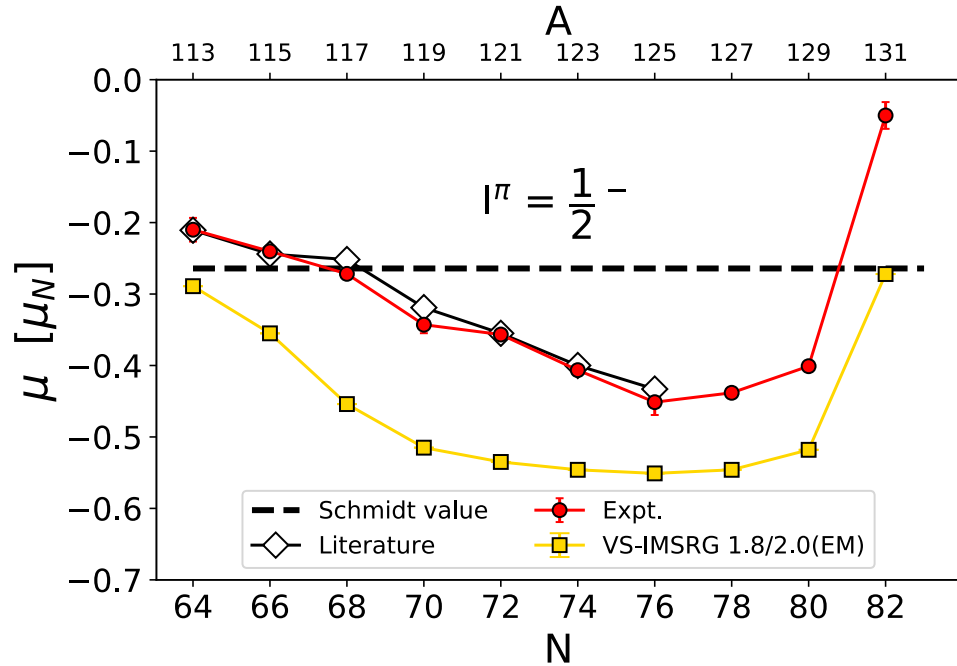
S. Malbrunot-Ettenauer *et al.*

Phys. Rev. Lett. **128**, 022502 – Published 14 January 2022

Multiple **ab-initio methods** largely agree within uncertainties

**Ab initio (again) competitive/complementary with DFT**

Electromagnetic moments of entire In chain – sharp increase at N=82



Ab initio reproduces trends of new measurements

Neglected physics: two-body meson-exchange currents

## Article

### Nuclear moments of indium isotopes reveal abrupt change at magic number 82

<https://doi.org/10.1038/s41586-022-04818-7>

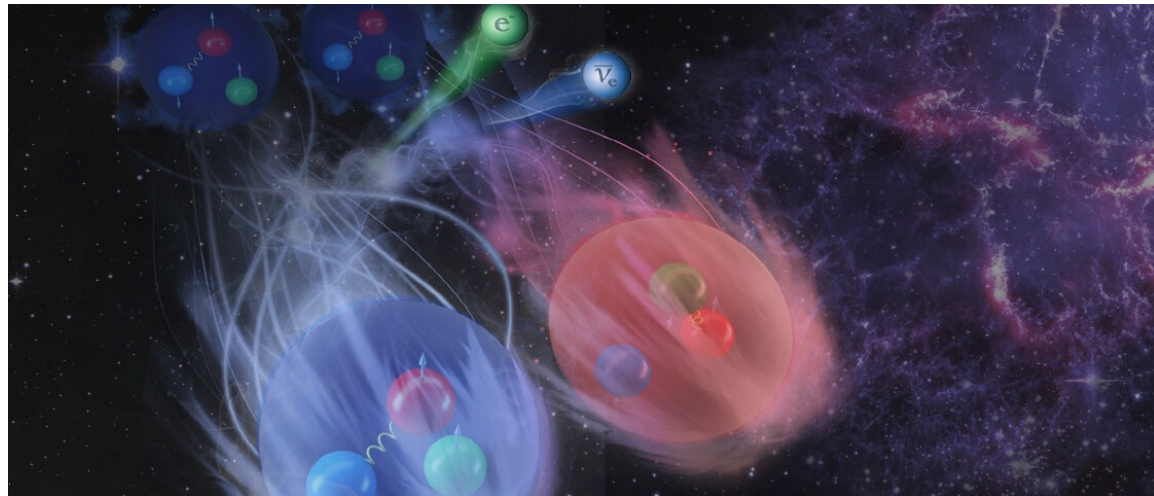
Received: 10 June 2021

Accepted: 28 April 2022

Published online: 13 July 2022

A. R. Vernon<sup>1,2,3,5</sup>, R. F. Garcia Ruiz<sup>2,4,5</sup>, T. Miyagi<sup>5</sup>, C. L. Binnersley<sup>1</sup>, J. Billowes<sup>1</sup>, M. L. Bissell<sup>1</sup>, J. Bonnard<sup>6</sup>, T. E. Cocolios<sup>7</sup>, J. Dobaczewski<sup>6,7</sup>, G. J. Farooq-Smith<sup>3</sup>, K. T. Flanagan<sup>1,8</sup>, G. Georgiev<sup>9</sup>, W. Gins<sup>3,10</sup>, R. P. de Groot<sup>3,10</sup>, R. Heinke<sup>4,11</sup>, J. D. Holt<sup>5,12</sup>, J. Hustings<sup>3</sup>, Á. Kozzorús<sup>3</sup>, D. Leimbach<sup>11,13,14</sup>, K. M. Lynch<sup>4</sup>, G. Neyens<sup>3,4</sup>, S. R. Stroberg<sup>15</sup>, S. G. Wilkins<sup>1,2</sup>, X. F. Yang<sup>3,16</sup> & D. T. Yordanov<sup>4,9</sup>

# Two-Body Currents for Gamow-Teller Transitions and $g_A$ Quenching



LETTERS

<https://doi.org/10.1038/s41567-019-0450-7>

nature  
physics

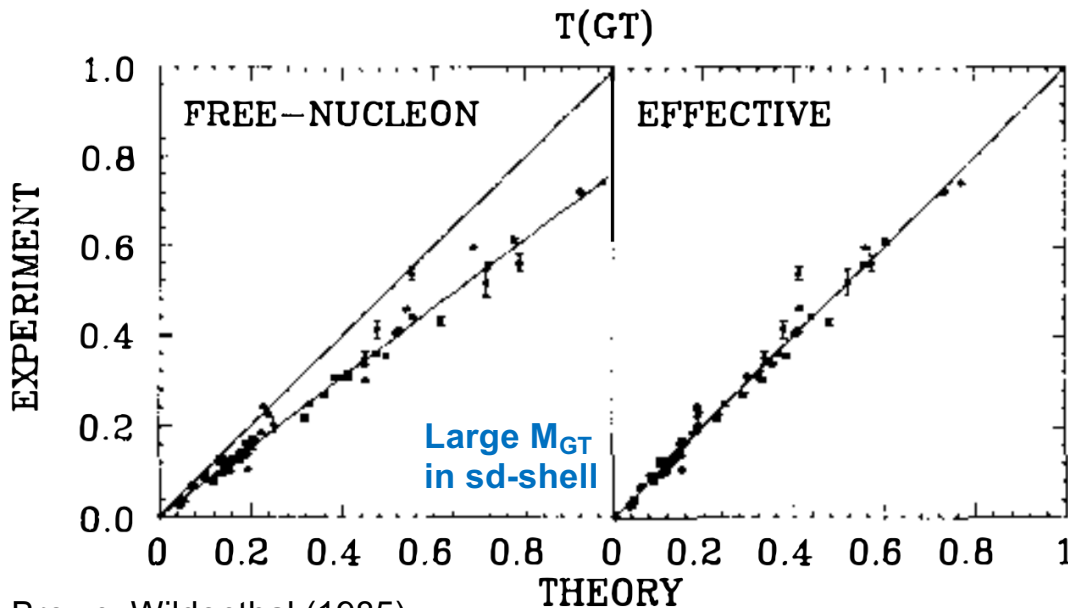
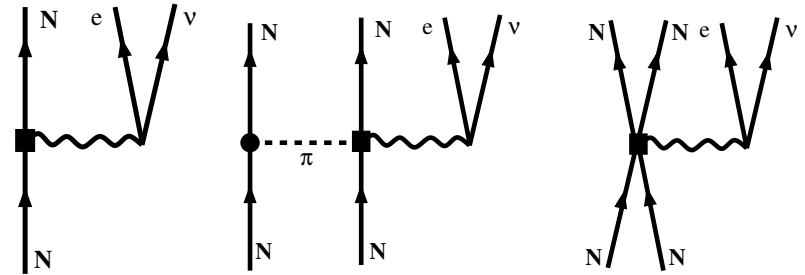
**Discrepancy between experimental and theoretical  $\beta$ -decay rates resolved from first principles**

P. Gysbers<sup>1,2</sup>, G. Hagen<sup>3,4\*</sup>, J. D. Holt<sup>5</sup>, G. R. Jansen<sup>3,5</sup>, T. D. Morris<sup>3,4,6</sup>, P. Navrátil<sup>6</sup>, T. Papenbrock<sup>3,4</sup>, S. Quaglioni<sup>7</sup>, A. Schwenk<sup>8,9,10</sup>, S. R. Stroberg<sup>1,11,12</sup> and K. A. Wendt<sup>7</sup>

Long-standing problem in weak decays: **experimental values systematically smaller than theory**

$$M_{GT} = g_A \langle f | \mathcal{O}_{GT} | i \rangle \quad \mathcal{O}_{GT} = \mathcal{O}_{\sigma\tau}^{1b} + \mathcal{O}_{2BC}^{2b}$$

Using  $g_A^{\text{eff}} \approx 0.77 \times g_A^{\text{free}}$  agrees with data



Brown, Wildenthal (1985)

- Missing wavefunction correlations
- Renormalized VS operator?
- Neglected two-body currents?
- Model-space truncations?

**Explore in ab initio framework**

Calculate **large GT matrix elements**

$$M_{GT} = g_A \langle f | \mathcal{O}_{GT} | i \rangle$$

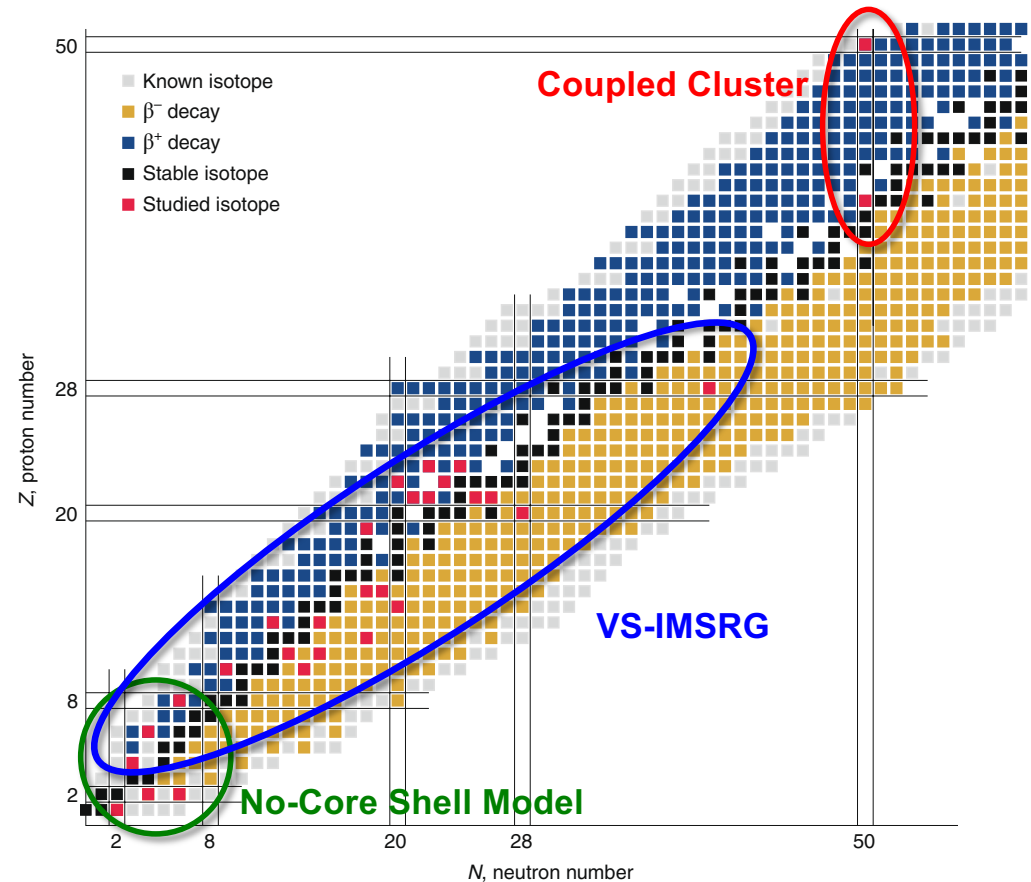
$$\mathcal{O}_{GT} = \mathcal{O}_{\sigma\tau}^{1b} + \mathcal{O}_{2BC}^{2b}$$

- Light, medium, and heavy regions
- Benchmark different ab initio methods
- Range of NN+3N forces
- Consistent inclusion of 2BC

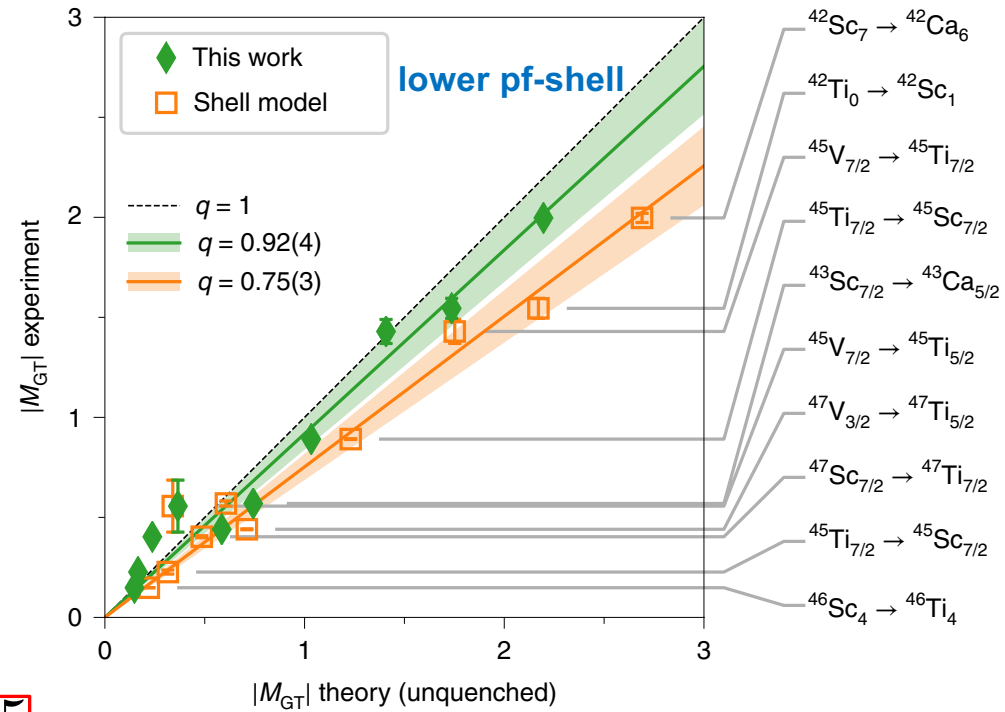
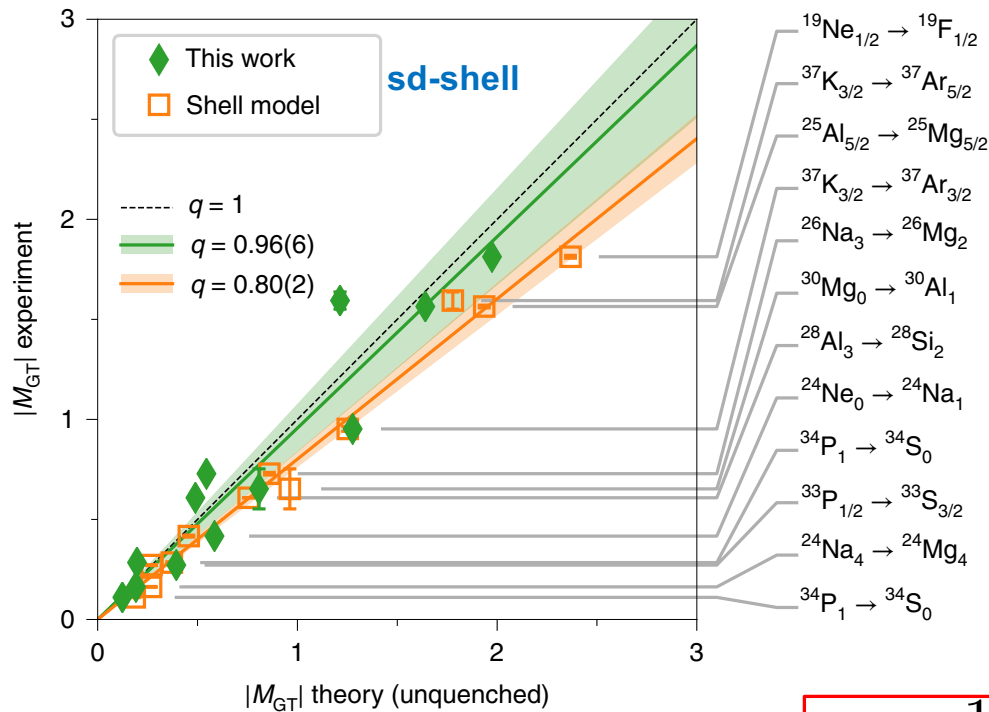
## NUCLEAR PHYSICS

### Beta decay gets the ab initio treatment

One of the fundamental radioactive decay modes of nuclei is  $\beta$  decay. Now, nuclear theorists have used first-principles simulations to explain nuclear  $\beta$  decay properties across a range of light- to medium-mass isotopes, up to  $^{100}\text{Sn}$ .



## VS-IMSRG calculations throughout sd and pf shells

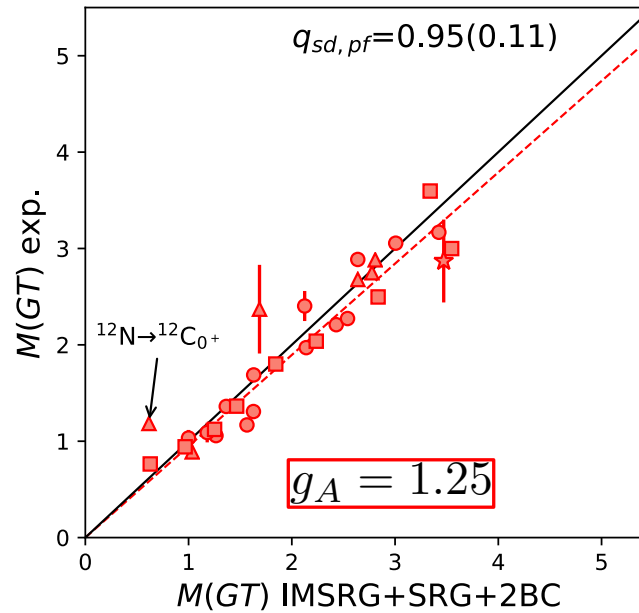
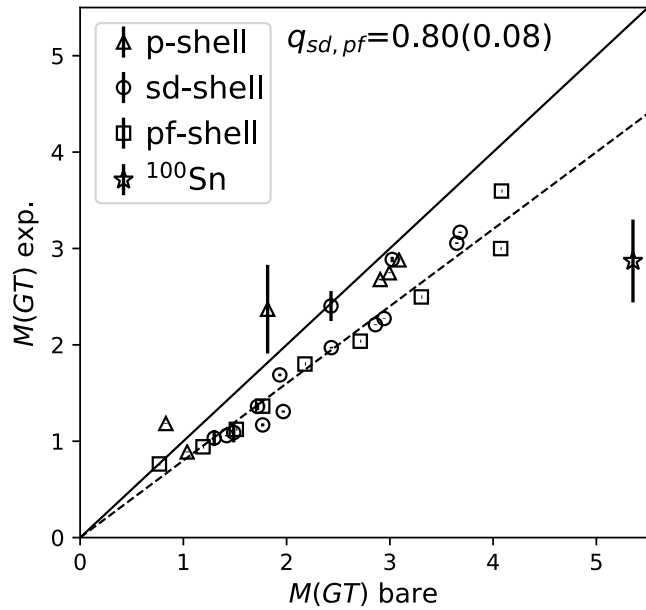


$$g_A = 1.25$$

**Ab initio calculations across the chart explain data with unquenched  $g_A$**

Refine results: improvements in forces and many-body methods

Ab initio calculations throughout sd and pf shells



Stroberg (2021)

Ab initio calculations across the chart explain data with unquenched  $g_A$

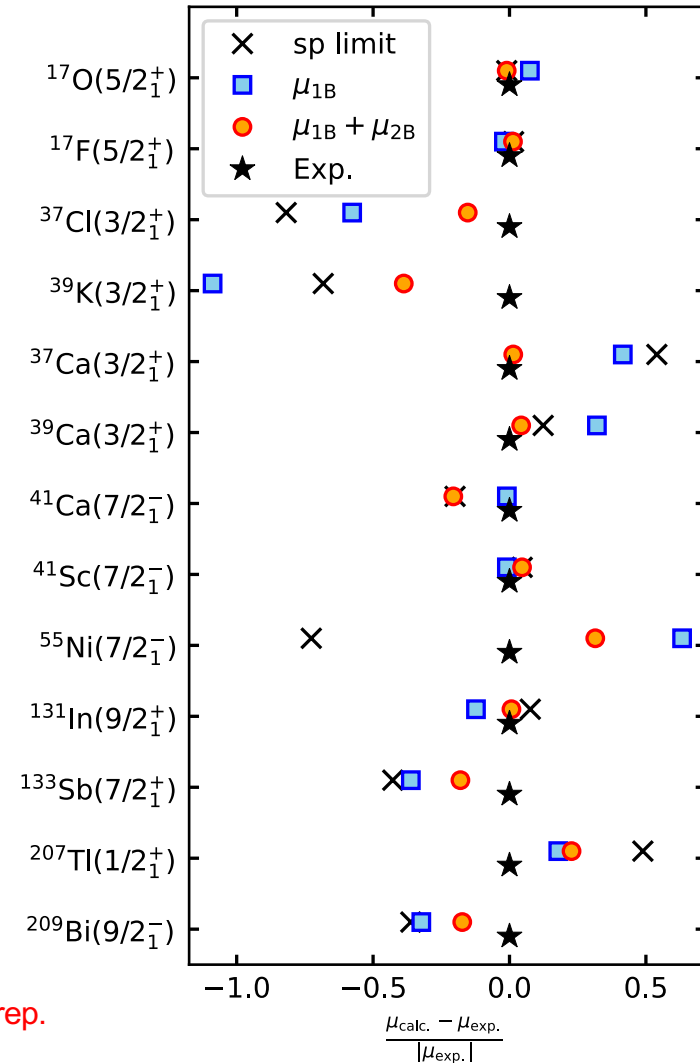
Including p-shell:  $q=0.99(21)$



Ab initio calculations throughout the nuclear chart

Including 2bc consistent with input forces

**Magnetic moments significantly improved**



T. Miyagi et al, in prep.



## Nuclear Structure/Astrophysics

Development of forces and currents

Ab initio to  $^{208}\text{Pb}$ : neutron skin, r-process

Dripline predictions to medium-masses

Evolution of magic numbers:

masses, radii, spectra, EM transitions

Multi-shell theory:

Islands of inversion, forbidden decays

Nuclear EOS/Neutron star properties

Atomic systems

\*T. Miyagi, B. S. Hu, L. Jokiniemi\*

A. Belley, I. Ginnett, C. G. Payne

M. Bruneault, J. Padua

S. Leutheusser

E. Love

K. Evidence, D. Kush

G. Tenkila, H. Patel, V. Chand

B. Wong, X. Cao

S. R. Stroberg N. Vassh



THE UNIVERSITY OF  
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COLUMBIA



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UNIVERSITY

# Present and Future for Ab Initio Theory

## Fundamental Symmetries/BSM Physics

EW operators: GT quenching, muon capture

$0\nu\beta\beta$  decay matrix elements + DGT/ECEC/Dg

WIMP-Nucleus scattering for dark matter detection

Coherent elastic neutrino-nucleus scattering

Superaligned Fermi transitions

Symmetry-violating moments: EDM, anapole...

## Work in progress

Higher-order many-body physics: IMSRG(3)

Monte Carlo shell model diagonalization

Extension to superheavy nuclei



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

A. Schwenk



THE UNIVERSITY OF  
TENNESSEE  
KNOXVILLE

G. Hagen  
T. Papenbrock



MICHIGAN STATE  
UNIVERSITY

J.M. Yao  
H. Hergert



COLORADO SCHOOL OF  
MINES

M. Martin  
K. G. Leach



Massachusetts  
Institute of  
Technology

R. F. Garcia-Ruiz



UNIVERSITAT DE  
BARCELONA

J. Menéndez



THE UNIVERSITY  
of NORTH CAROLINA  
at CHAPEL HILL

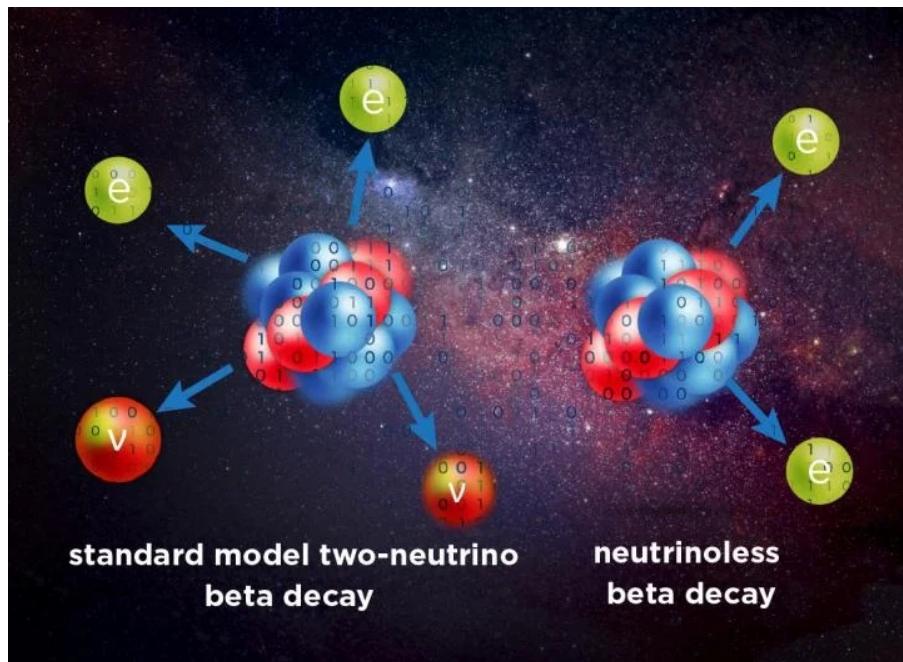
J. Engel



TEXAS A&M  
UNIVERSITY

J. W. Holt

# Neutrinoless Double Beta Decay NMEs for Major Players: $^{76}\text{Ge}$ , ( $^{100}\text{Mo}$ ), $^{130}\text{Te}$ , $^{136}\text{Xe}$



*Ab Initio* Treatment of Collective Correlations and the Neutrinoless Double Beta Decay of  $^{48}\text{Ca}$

J. M. Yao, B. Bally, J. Engel, R. Wirth, T. R. Rodríguez, and H. Hergert  
Phys. Rev. Lett. **124**, 232501 – Published 11 June 2020

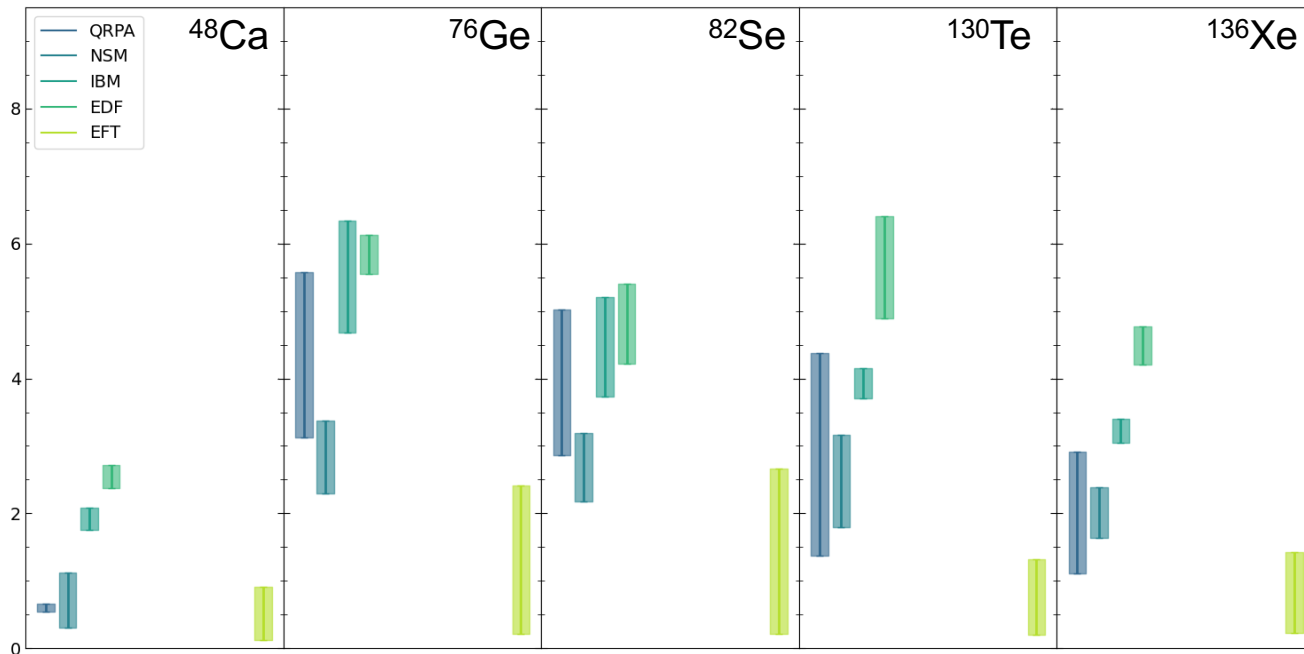
*Ab Initio* Neutrinoless Double-Beta Decay Matrix Elements for  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ , and  $^{82}\text{Se}$

A. Belley, C. G. Payne, S. R. Stroberg, T. Miyagi, and J. D. Holt  
Phys. Rev. Lett. **126**, 042502 – Published 29 January 2021

Coupled-Cluster Calculations of Neutrinoless Double- $\beta$  Decay in  $^{48}\text{Ca}$

S. Novario, P. Gysbers, J. Engel, G. Hagen, G. R. Jansen, T. D. Morris, P. Navrátil, T. Papenbrock, and S. Quaglioni  
Phys. Rev. Lett. **126**, 182502 – Published 7 May 2021

Calculations to date from phenomenological models; large spread in results

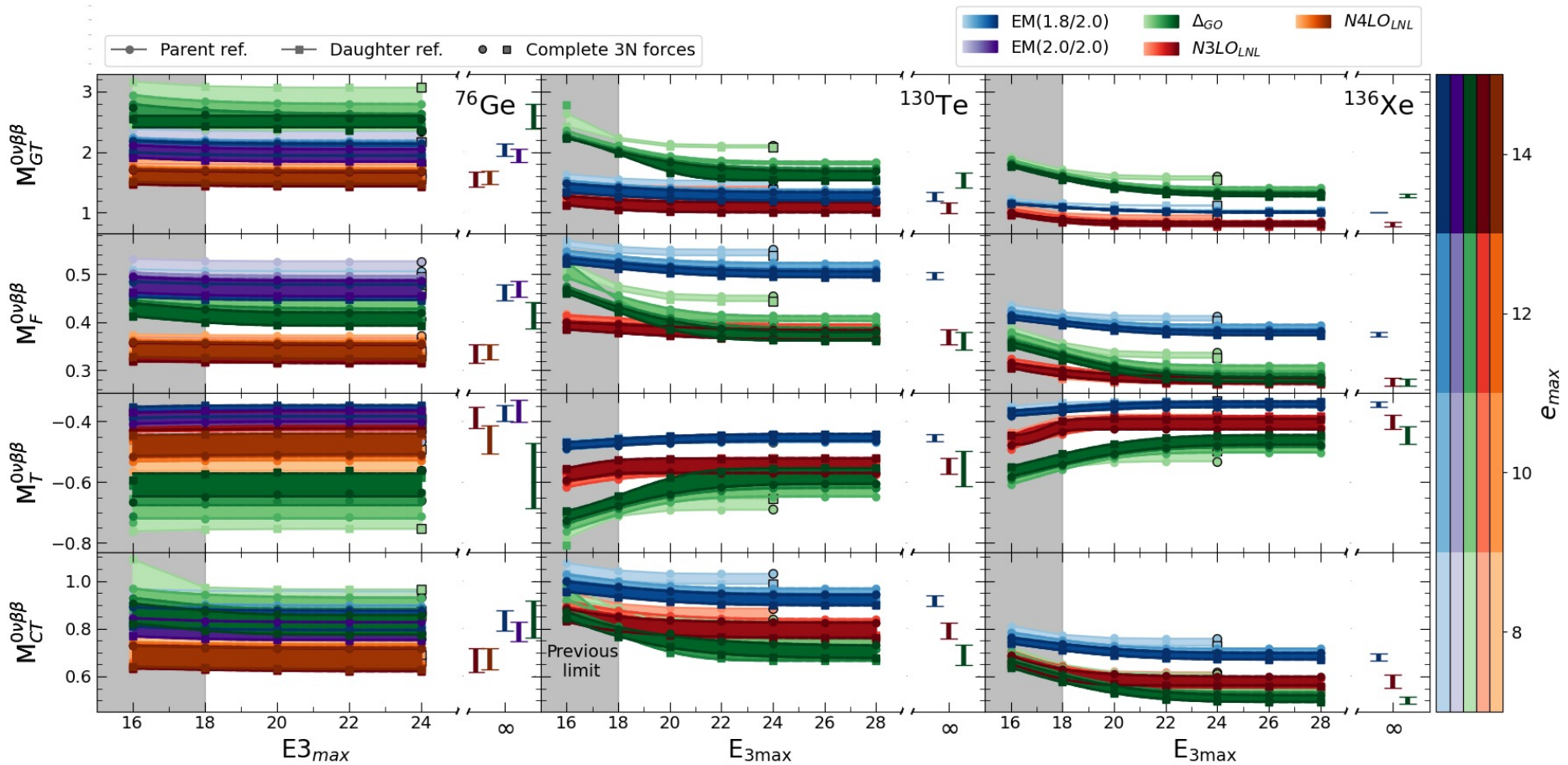


Compiled values from: Engel and Menéndez (2017); Brase et al, PRC (2022)

All models missing essential physics: correlations, single-particle levels, two-body currents

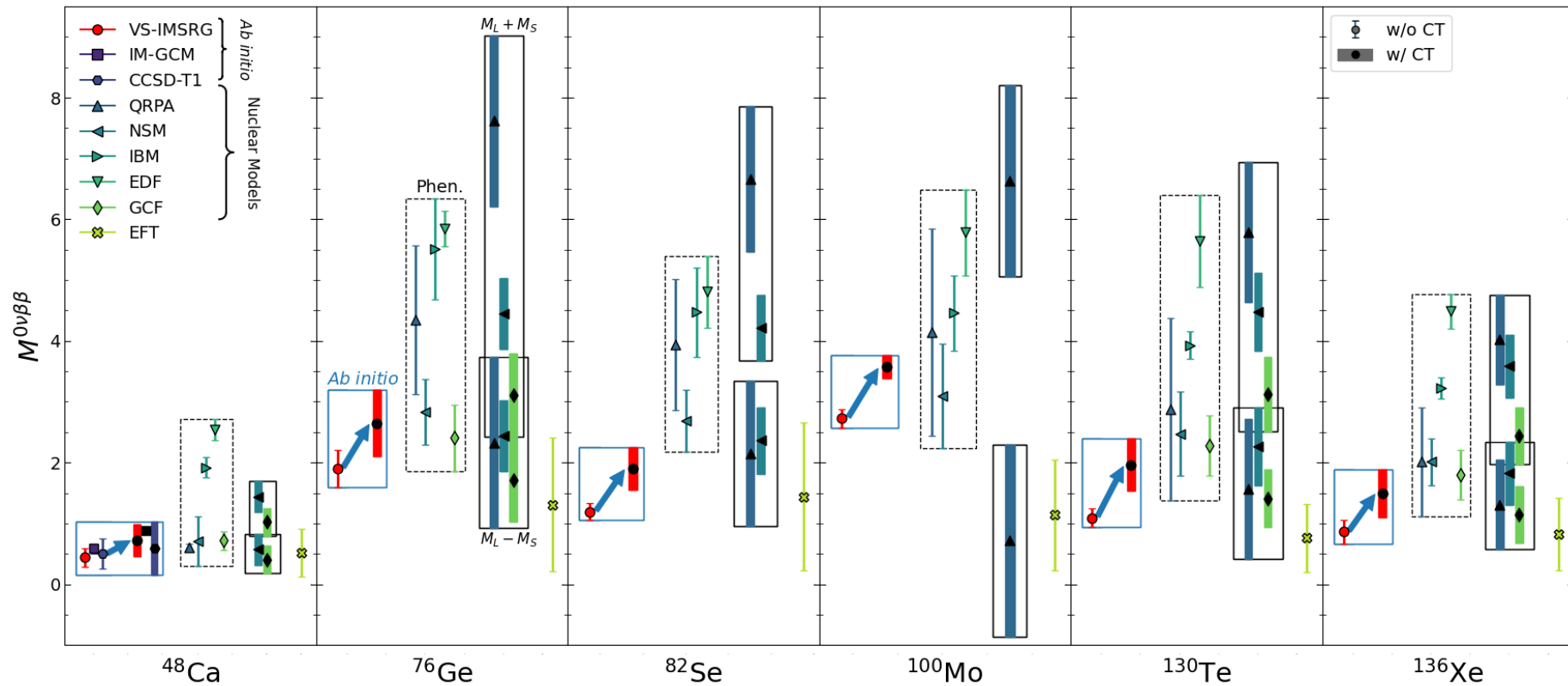
**Address with *ab initio* theory**

## Converged NMEs for major players in global searches: $^{76}\text{Ge}$ , $^{130}\text{Te}$ , $^{136}\text{Xe}$



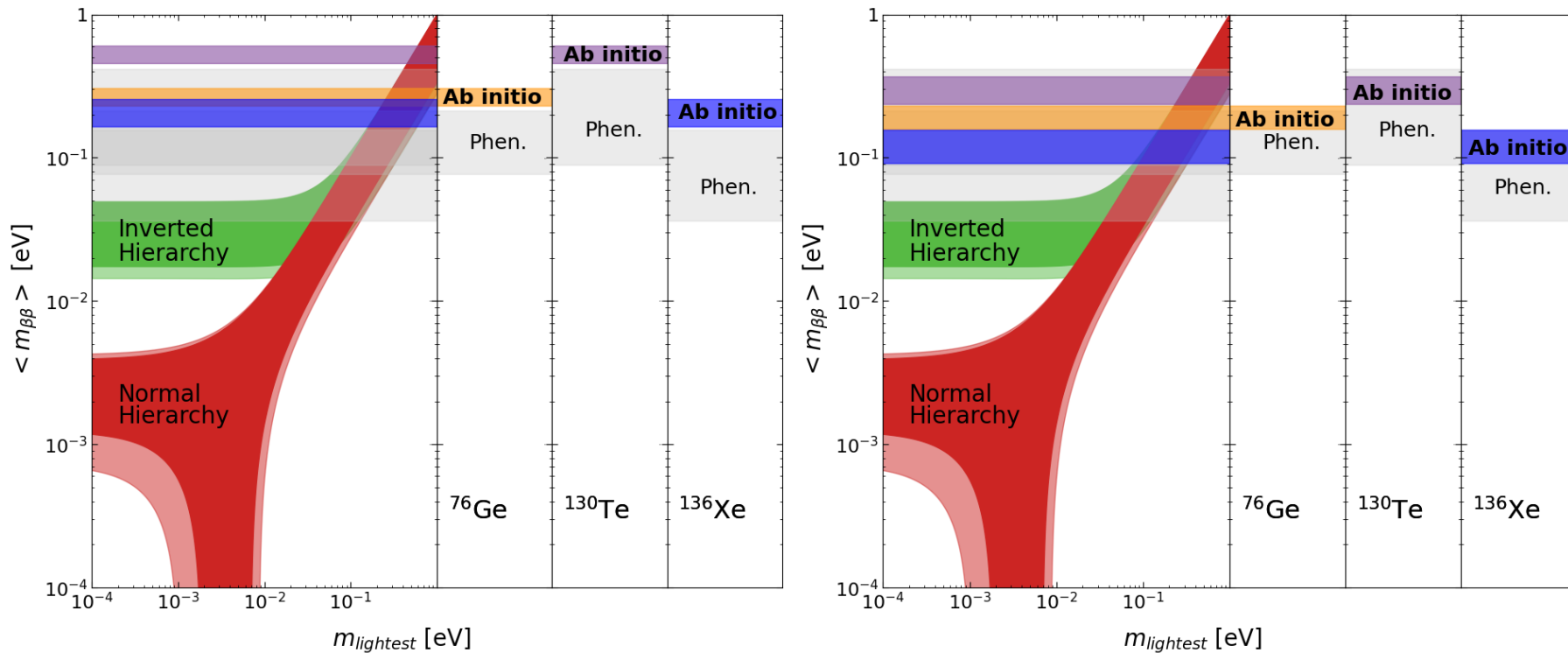
Converged NMEs for major players in global searches:  $^{76}\text{Ge}$ ,  $^{100}\text{Mo}$ ,  $^{130}\text{Te}$ ,  $^{136}\text{Xe}$

Ab initio results: differences from models; **large NMEs strongly disfavored**



Impact for next-generation searches: Large matrix elements disfavored, lowers expected rates

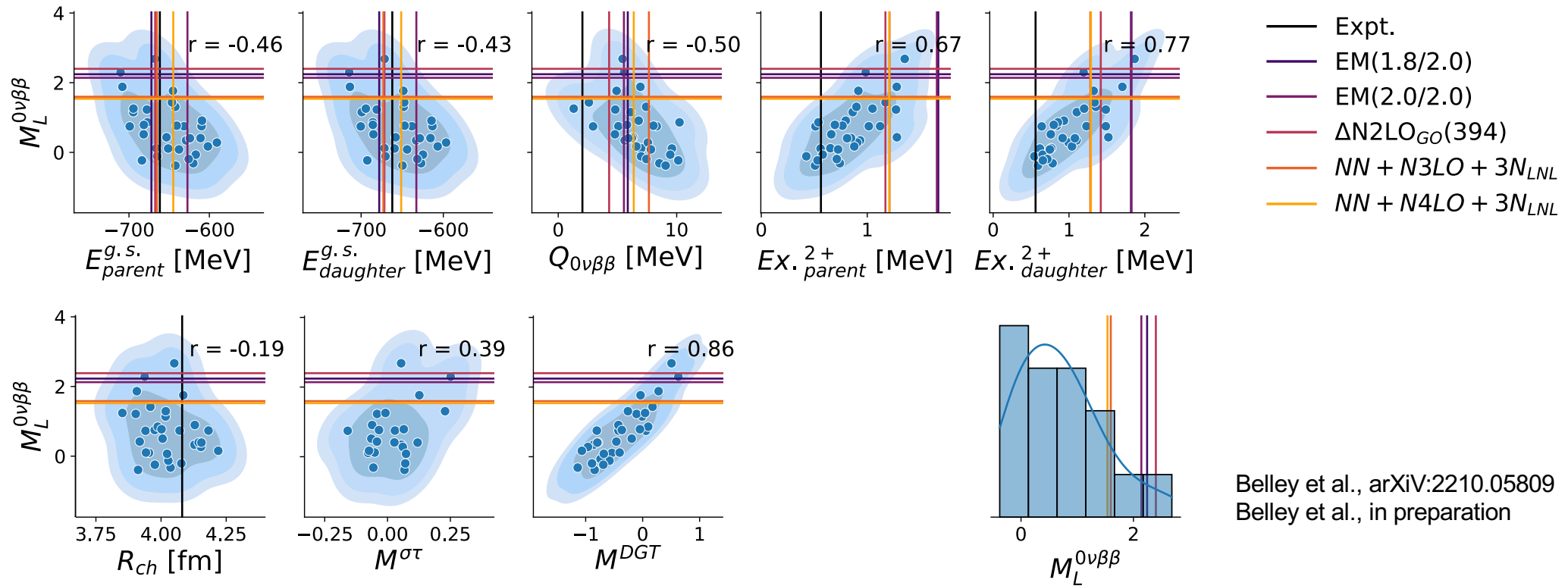
Current experimental reach – improved with effects of contact term,



**Not the end of the story: estimate three-body corrections + two-body currents**

## $^{76}\text{Ge}$ : Explore correlations with other observables from systematic analysis (34 interactions)

Few clear correlations, except DGT



Maybe with first excited  $2^+$  states?

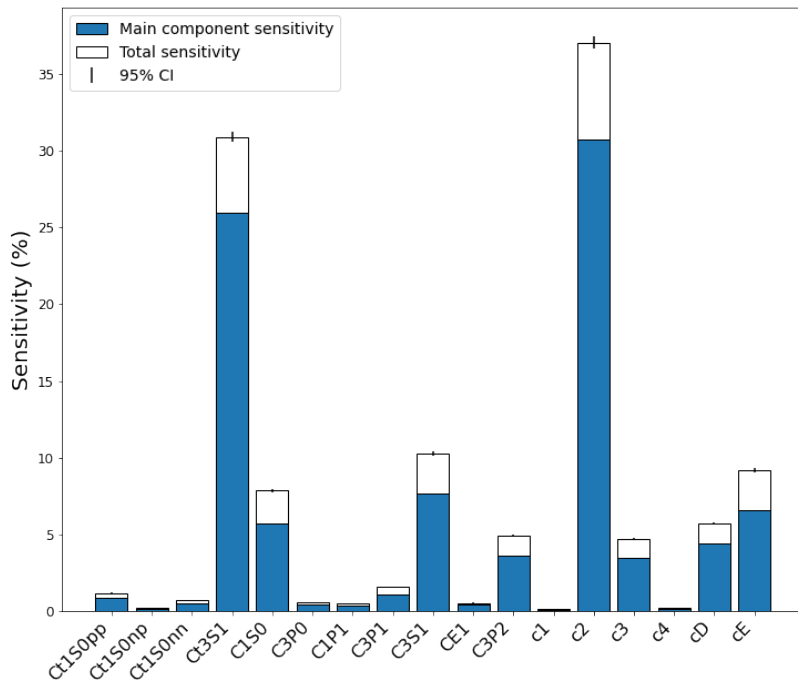


## Explore correlations with other observables from systematic analysis (34 interactions)

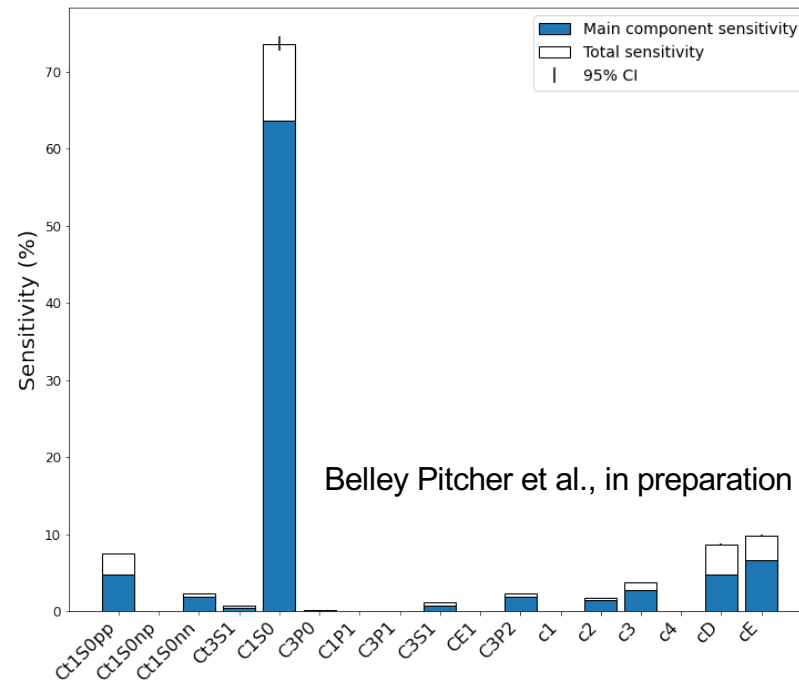
Similar sensitivity as found in  $^{208}\text{Pb}$  study!

26

Ground state energies



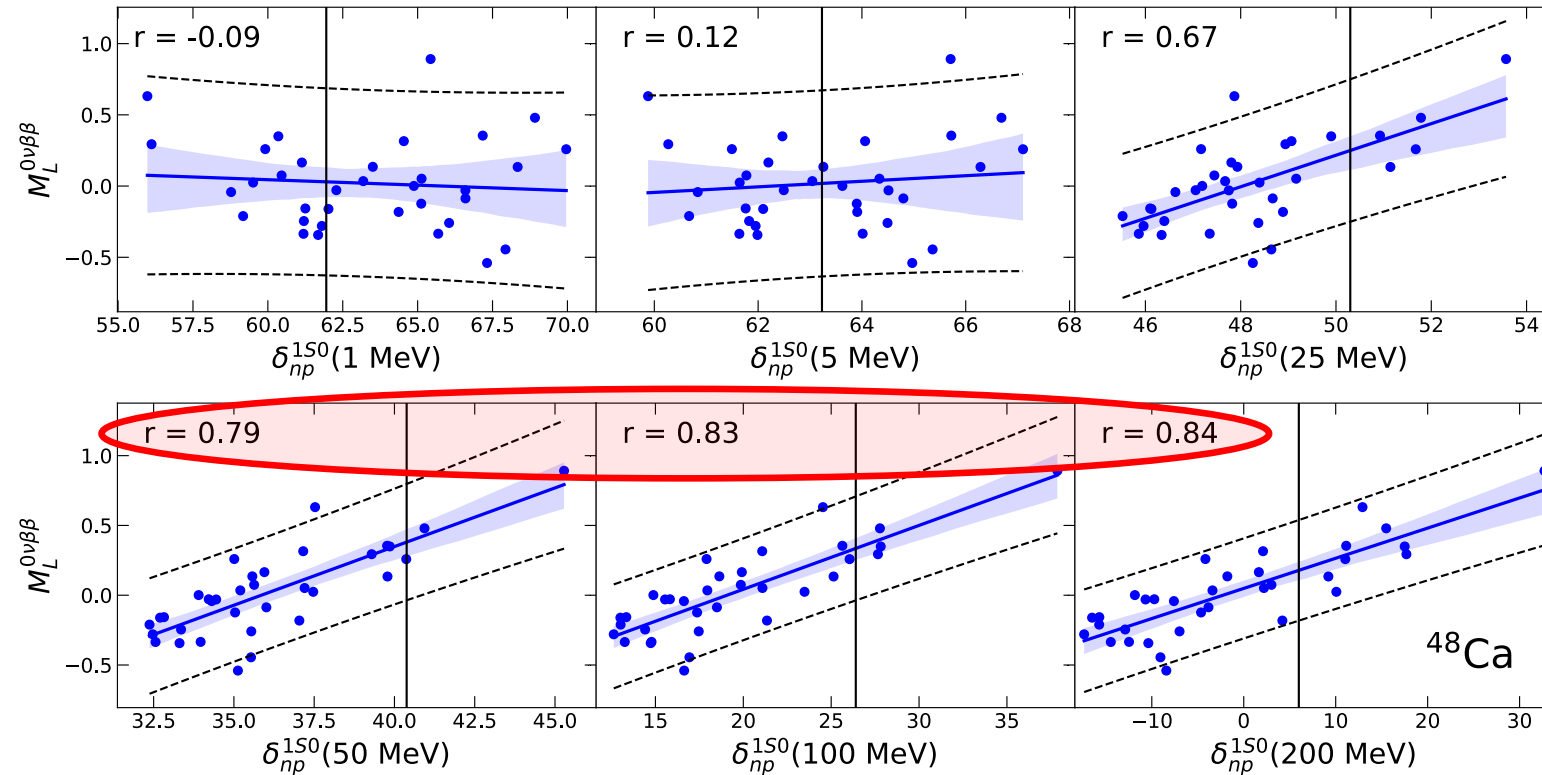
$M_L^{0\nu}$



**Highly sensitive to C1S0** – possible correlation with  $^1\text{S}_0$  phase shift (observable!)

Explore correlations with  $^1S_0$  phase shift from 34 non-implausible interactions

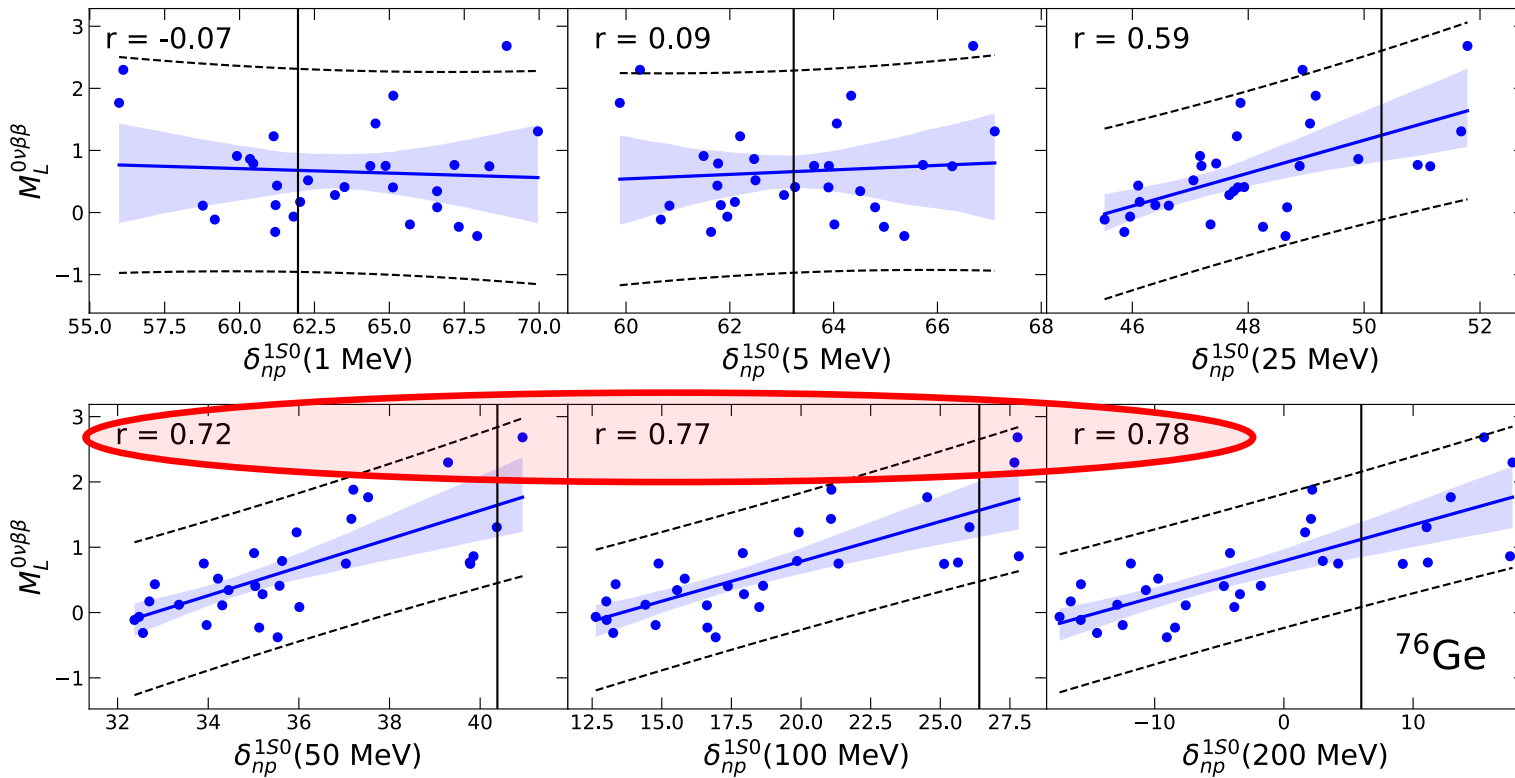
Long-range component in  $^{48}\text{Ca}$



**Clear correlation with (measured!)  $^1S_0$  phase shift at high scattering energies**

Explore correlations with  $^1S_0$  phase shift from 34 non-implausible interactions

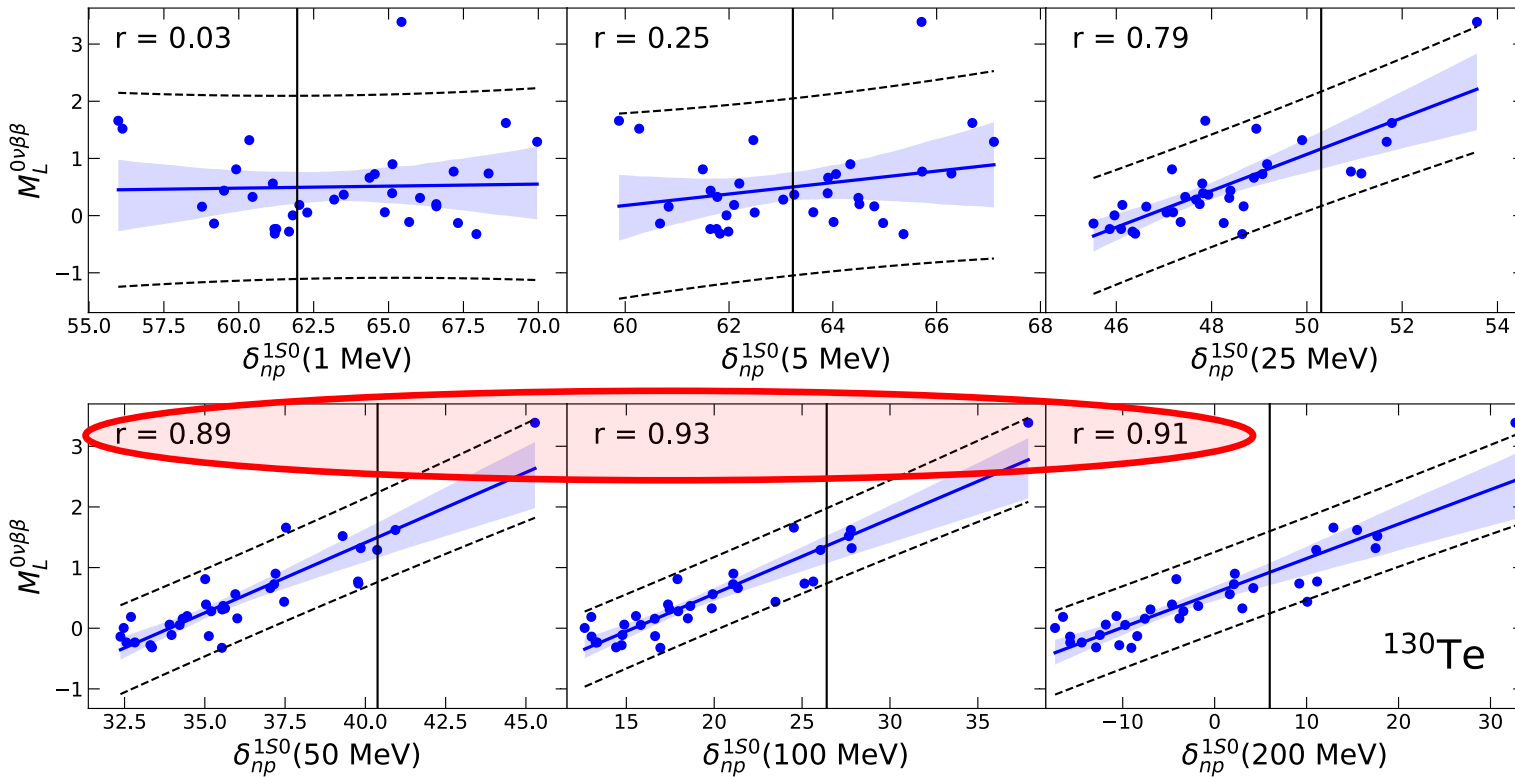
Long-range component in  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$



**Clear correlation with (measured!)  $^1S_0$  phase shift at high scattering energies**

Explore correlations with  $^1S_0$  phase shift from 34 non-implausible interactions

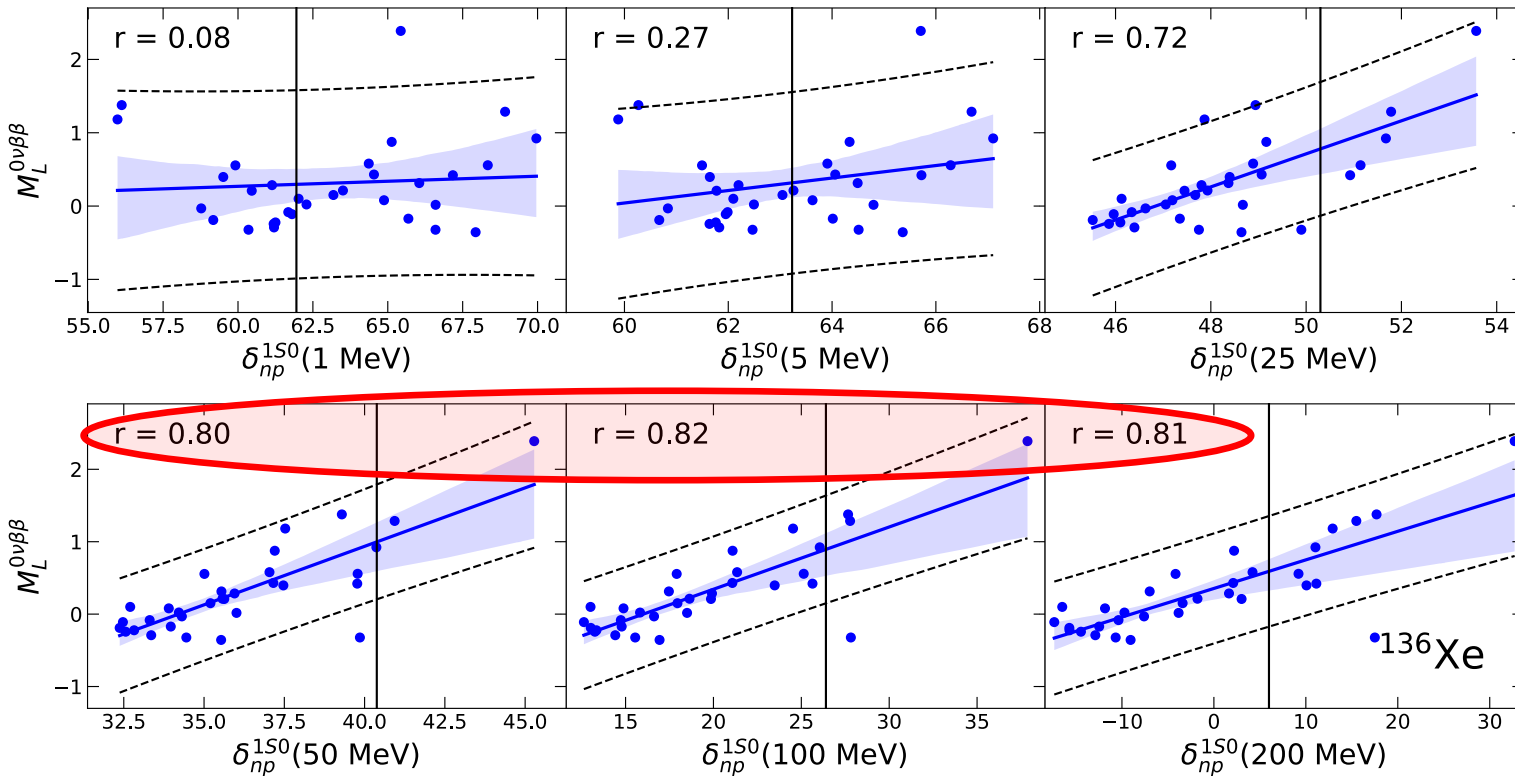
Long-range component in  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ ,  $^{130}\text{Te}$



**Clear correlation with (measured!)  $^1S_0$  phase shift at high scattering energies**

Explore correlations with  $^1S_0$  phase shift from 34 non-implausible interactions

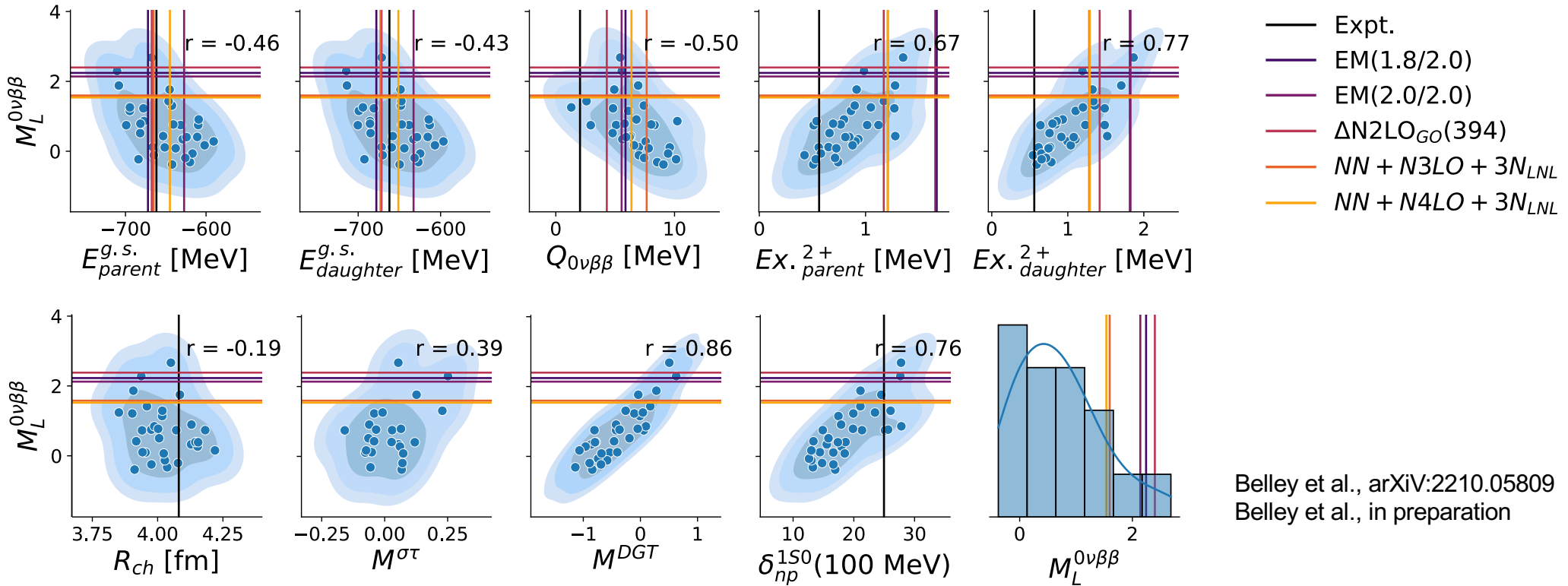
Long-range component in  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ ,  $^{130}\text{Te}$ ,  $^{136}\text{Xe}$



**Clear correlation with (measured!)  $^1S_0$  phase shift at high scattering energies**

## Explore correlations with other observables from systematic analysis (34 interactions)

Few clear correlations, except DGT in  $^{76}\text{Ge}$

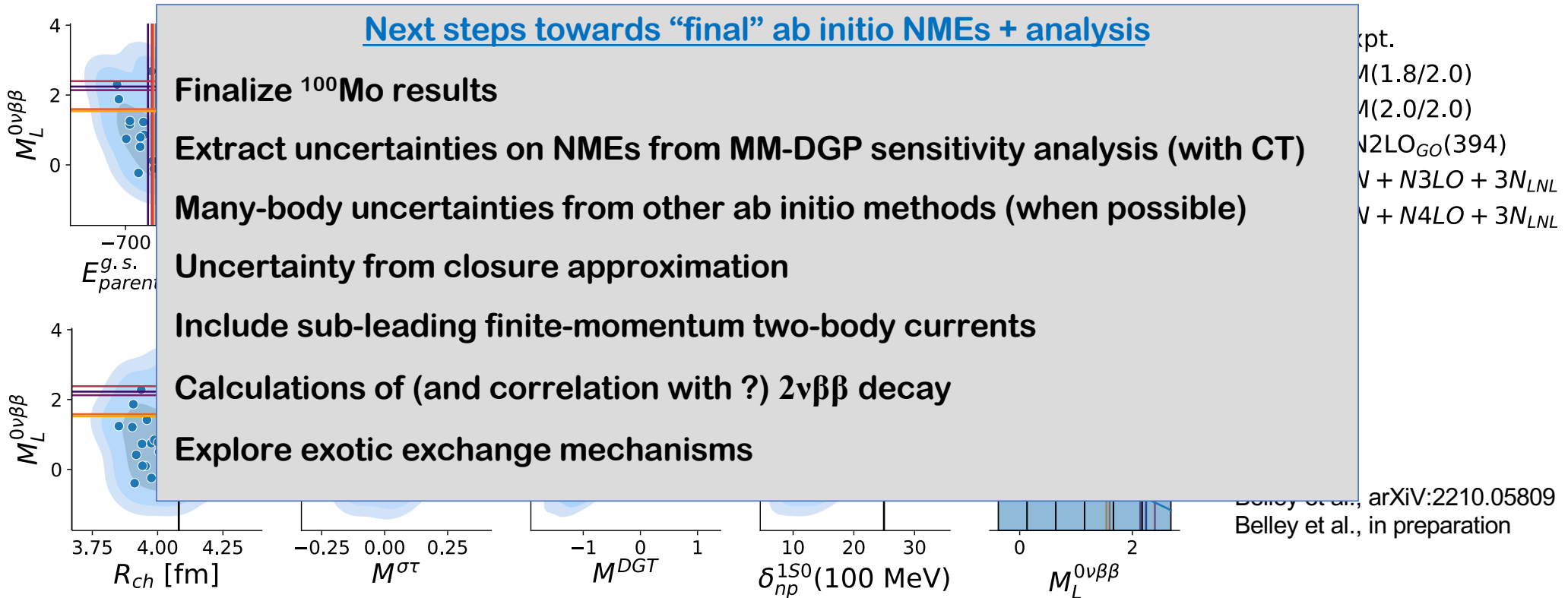


Belley et al., arXiv:2210.05809  
 Belley et al., in preparation

Now clear correlation with **measured**  $^1S_0$  phase shift!

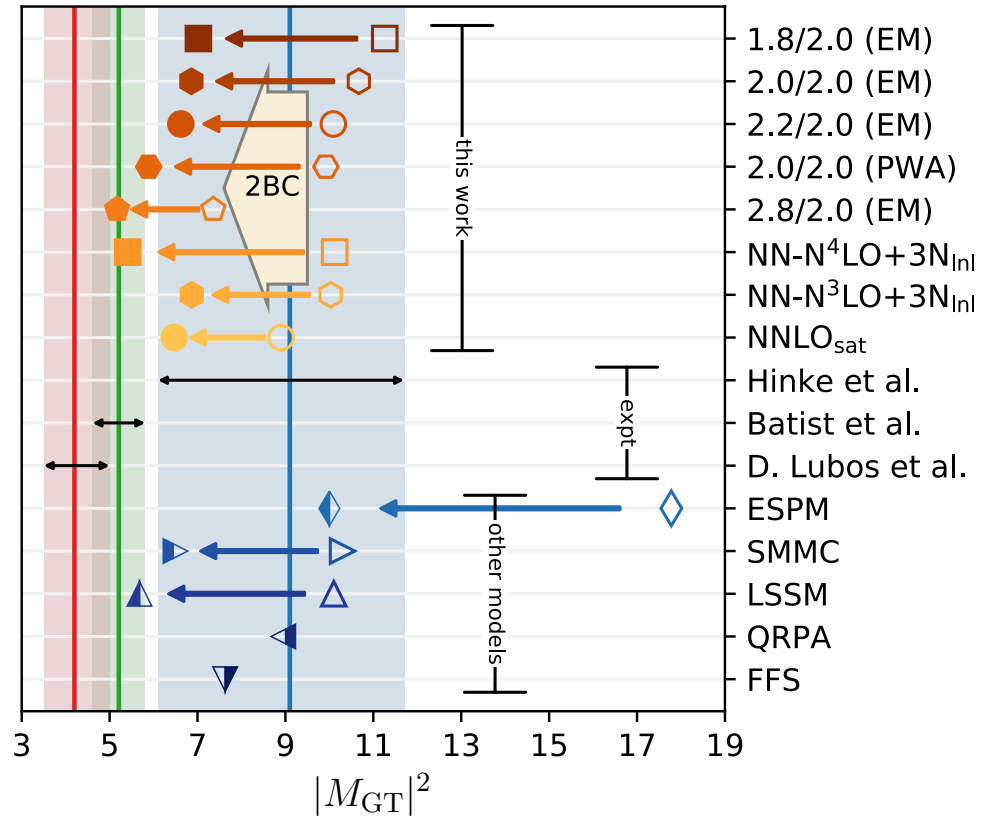
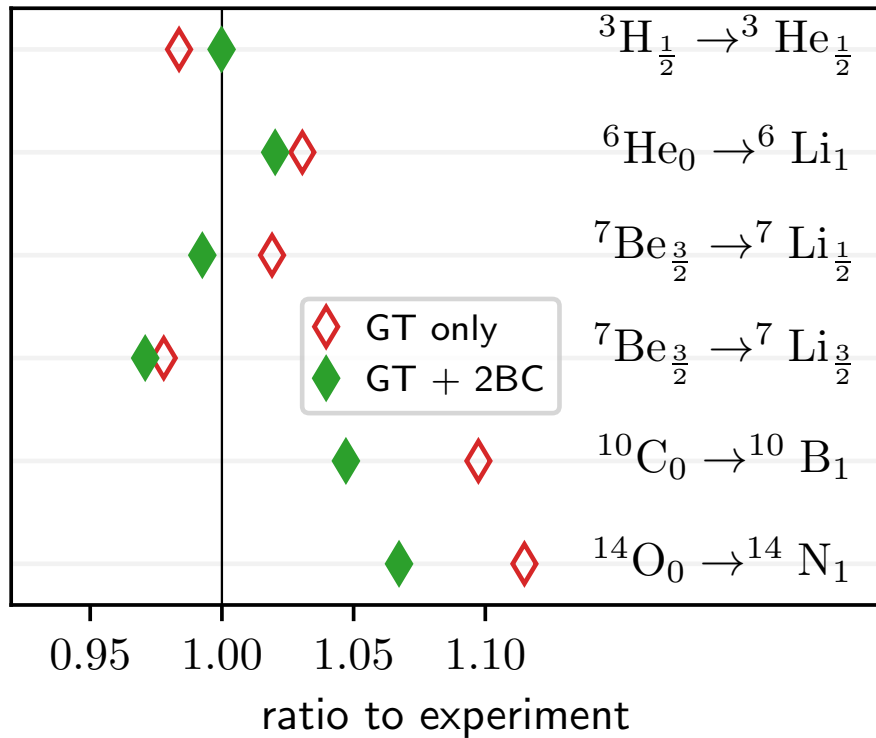
## Explore correlations with other observables from systematic analysis (34 interactions)

Few clear correlations, except DGT in  $^{76}\text{Ge}$



Now clear correlation with **measured**  $^1S_0$  phase shift!

NCSM in light nuclei, CC calculations of GT transition in  $^{100}\text{Sn}$  from different forces



Large quenching from correlations in  $^{100}\text{Sn}$

**Addition of 2BC further quenches; reduces spread in results**



Explicitly construct unitary transformation from sequence of rotations

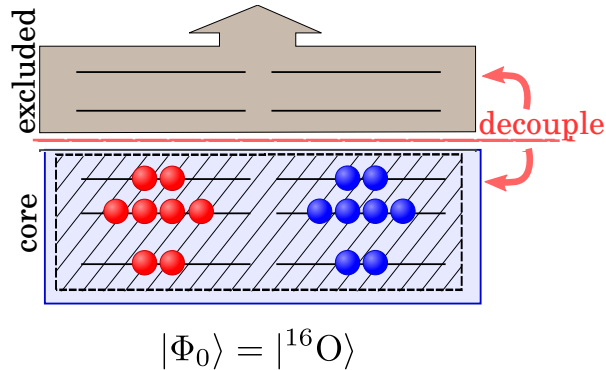
$$U = e^{\Omega} = e^{\eta_n} \dots e^{\eta_1} \quad \eta = \frac{1}{2} \arctan \left( \frac{2H_{\text{od}}}{\Delta} \right) - \text{h.c.}$$

$$\tilde{H} = e^{\Omega} H e^{-\Omega} = H + [\Omega, H] + \frac{1}{2} [\Omega, [\Omega, H]] + \dots$$

**All operators truncated at two-body level IMSRG(2)**  
**IMSRG(3) in progress**

Tsukiyama, Bogner, Schwenk, PRC 2012  
 Morris, Parzuchowski, Bogner, PRC 2015

## Step 1: Decouple core



Can we achieve accuracy  
of large-space methods?

$$\langle \tilde{\Psi}_n | P \tilde{H} P | \tilde{\Psi}_n \rangle \approx \langle \Psi_i | H | \Psi_i \rangle$$

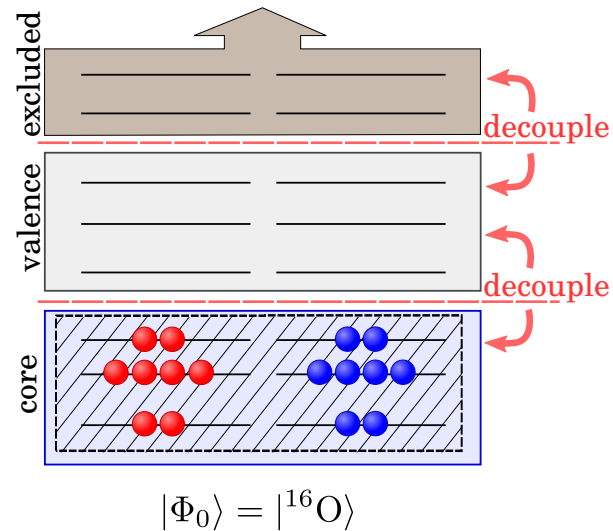
Explicitly construct unitary transformation from sequence of rotations

$$U = e^{\Omega} = e^{\eta_n} \dots e^{\eta_1} \quad \eta = \frac{1}{2} \arctan \left( \frac{2H_{\text{od}}}{\Delta} \right) - \text{h.c.}$$

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**IMSRG(3) in progress**

Tsukiyama, Bogner, Schwenk, PRC 2012  
 Morris, Parzuchowski, Bogner, PRC 2015



**Step 1: Decouple core**  
**Step 2: Decouple valence space**

Can we achieve accuracy  
 of large-space methods?

$$\langle \tilde{\Psi}_n | P \tilde{H} P | \tilde{\Psi}_n \rangle \approx \langle \Psi_i | H | \Psi_i \rangle$$

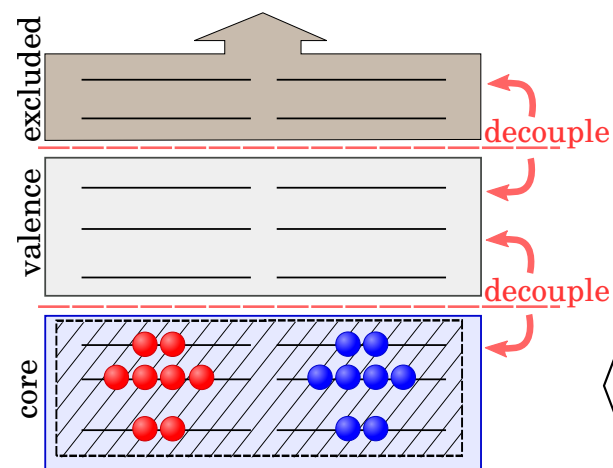
$\langle P   H   P \rangle$	$\langle P   H   Q \rangle \rightarrow 0$
$\langle Q   H   P \rangle \rightarrow 0$	$\langle Q   H   Q \rangle$

Explicitly construct unitary transformation from sequence of rotations

$$U = e^{\Omega} = e^{\eta_n} \dots e^{\eta_1} \quad \eta = \frac{1}{2} \arctan \left( \frac{2H_{\text{od}}}{\Delta} \right) - \text{h.c.}$$

$$\tilde{H} = e^{\Omega} H e^{-\Omega} = H + [\Omega, H] + \frac{1}{2} [\Omega, [\Omega, H]] + \dots$$

$$\tilde{\mathcal{O}} = e^{\Omega} \mathcal{O} e^{-\Omega} = \mathcal{O} + [\Omega, \mathcal{O}] + \frac{1}{2} [\Omega, [\Omega, \mathcal{O}]] + \dots$$



**Step 1: Decouple core**

**Step 2: Decouple valence space**

**Step 3: Decouple additional operators**

$$\langle \tilde{\Psi}_n | P \tilde{H} P | \tilde{\Psi}_n \rangle \approx \langle \Psi_i | H | \Psi_i \rangle$$

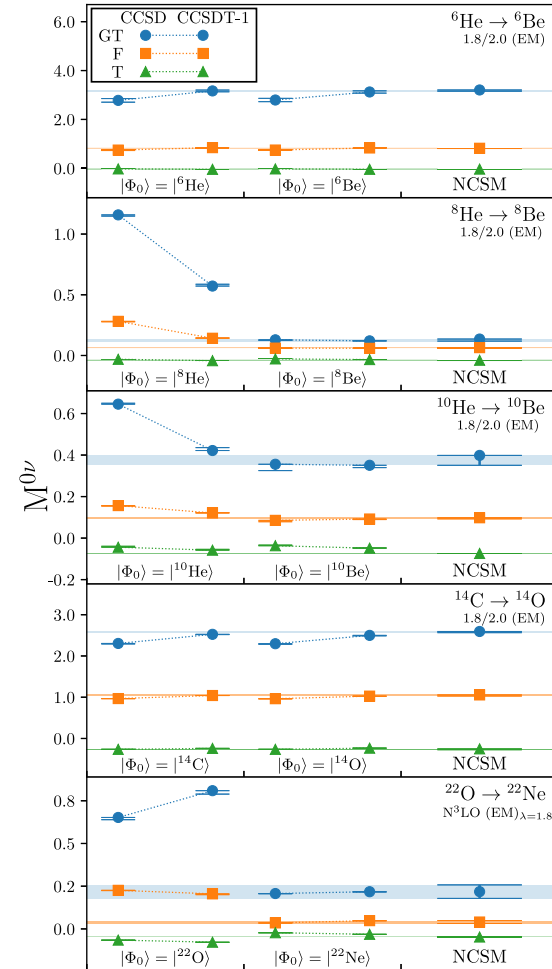
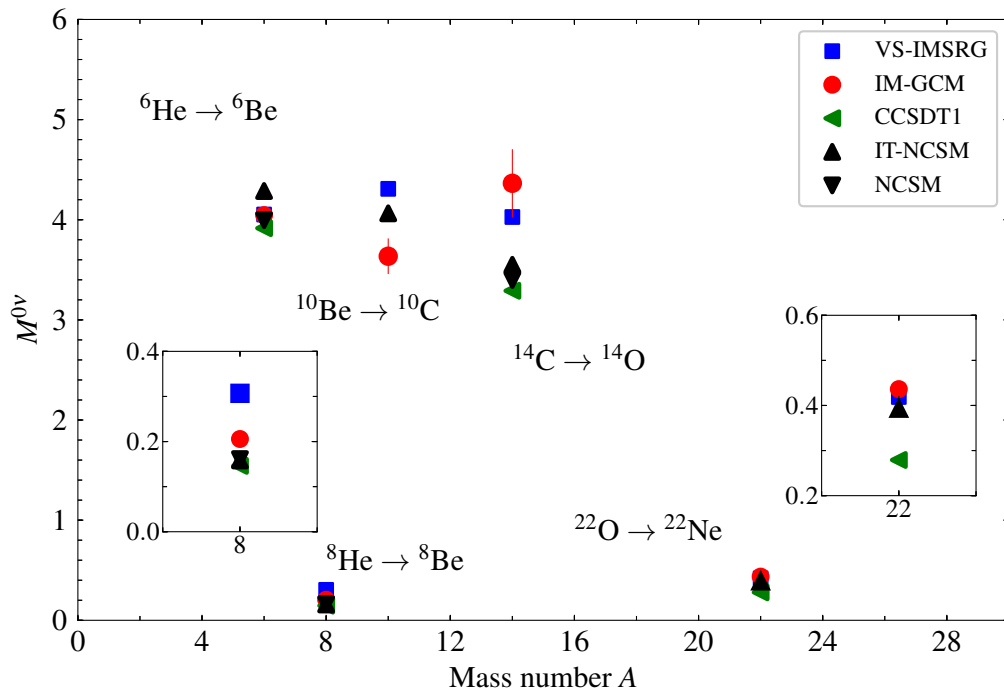
$$\langle \tilde{\Psi}_n | P \tilde{M}_{0\nu} P | \tilde{\Psi}_n \rangle \approx \langle \Psi_i | M_{0\nu} | \Psi_i \rangle$$

**Careful benchmarking essential**

$$|\Phi_0\rangle = |^{16}\text{O}\rangle$$

$\langle P   H   P \rangle$	$\langle P   H   Q \rangle \rightarrow 0$
$\langle Q   H   P \rangle \rightarrow 0$	$\langle Q   H   Q \rangle$

Benchmark with **quasi-exact NCSM**, IT-NCSM, IM-GCM, and CC in light systems:  $A=6-22$

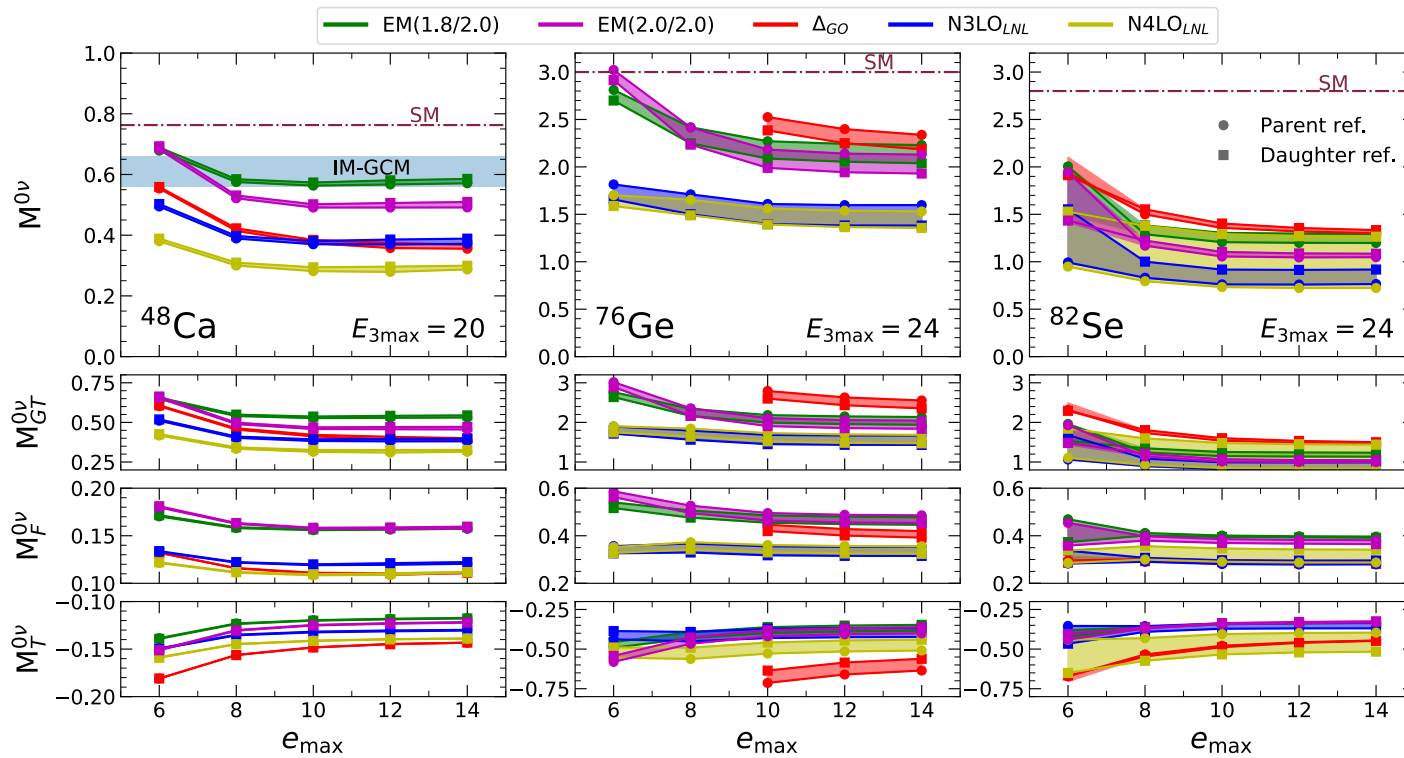


Reasonable to good agreement in all cases

Pursue true double-beta decay candidates!

“Uncertainty” bands from input NN+3N forces with **5 chiral Hamiltonians**

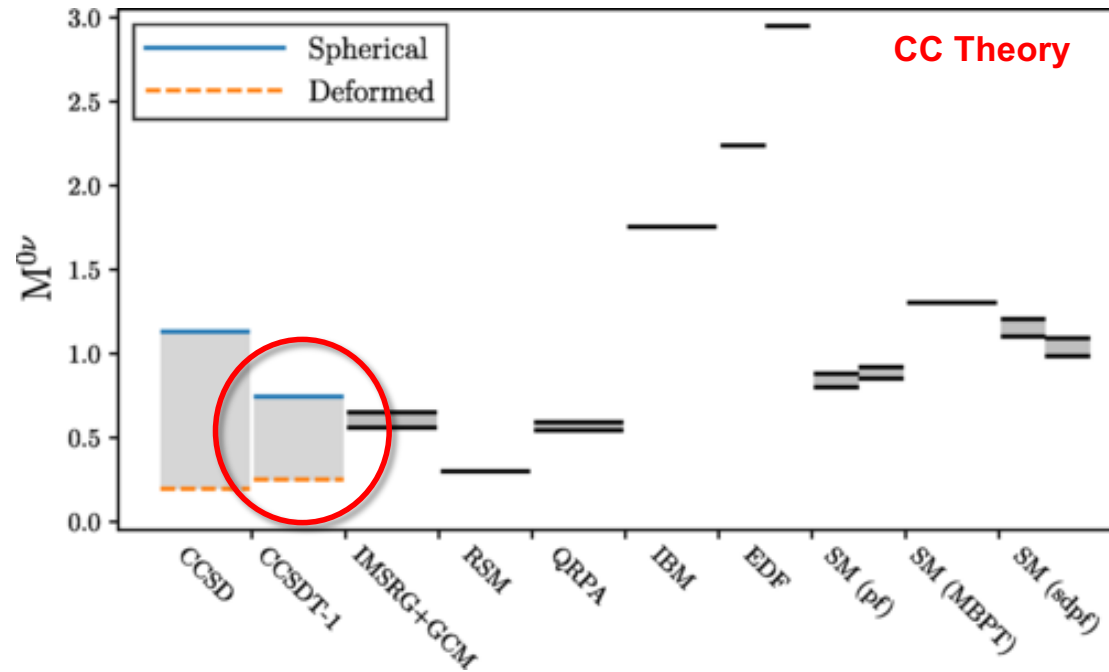
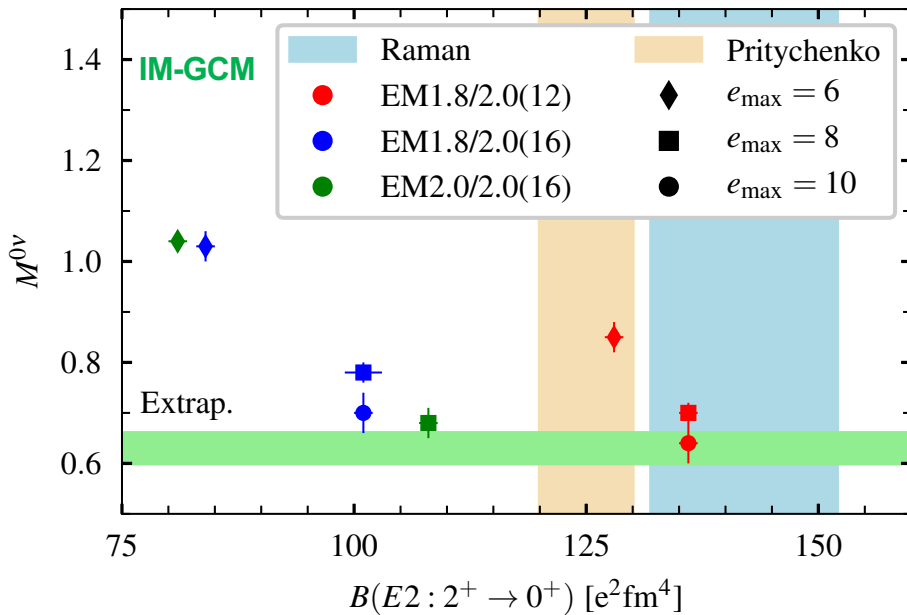
**VS-IMSRG**: clear convergence for  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ ,  $^{82}\text{Se}$



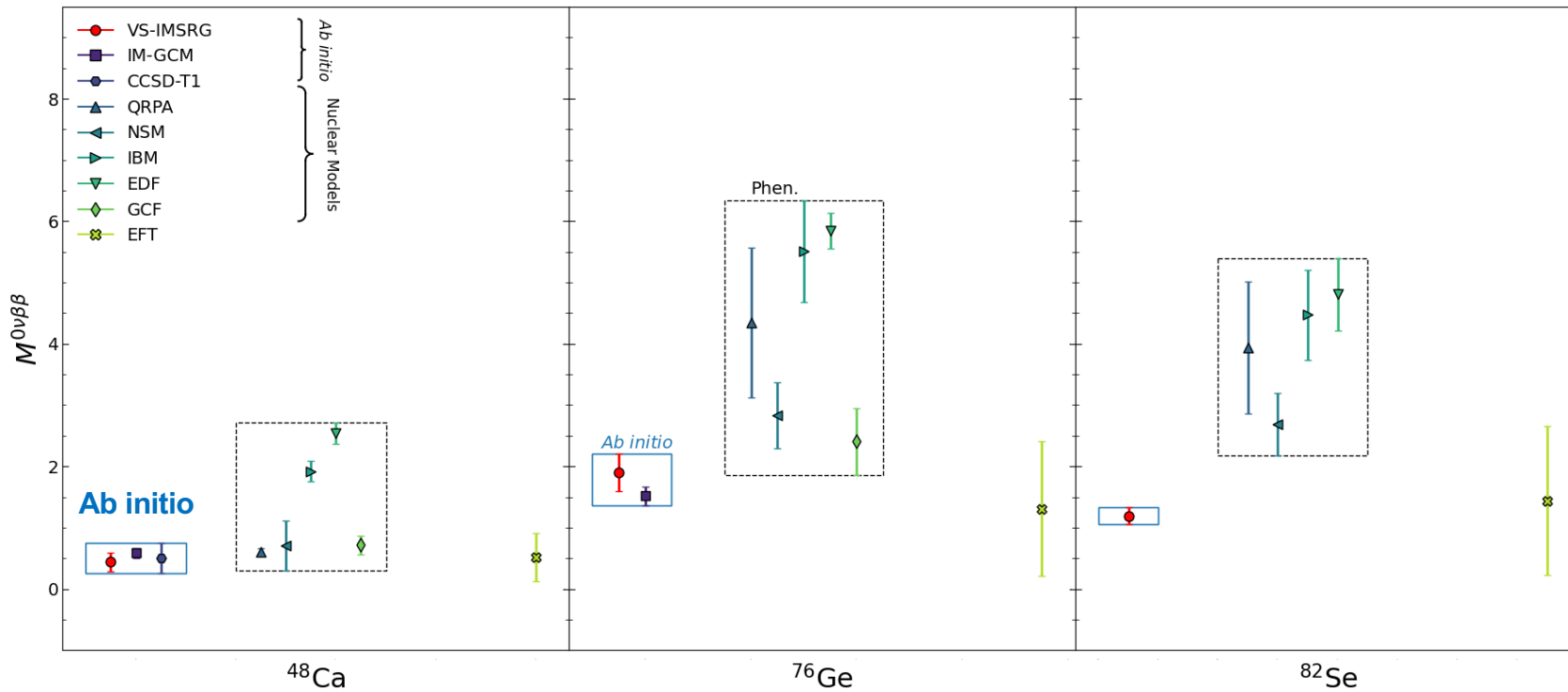
# TRIUMF Strategy II: “Uncertainties” from Many-Body Methods

Calculations in  $^{48}\text{Ca}$  from IM-GCM and CC theory using same interactions

Key development: **treatment of deformation** in **CC** and **IMSRG**



Ab initio NMEs generally smaller than phenomenology; less spread from uncertainties



**Ab initio results agree within uncertainties!**

Promising results, but...

## Proper renormalization requires short-range contact term at leading order

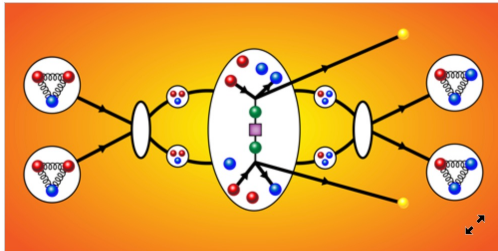
Physics

SYNOPSIS

### A Missing Piece in the Neutrinoless Beta-Decay Puzzle

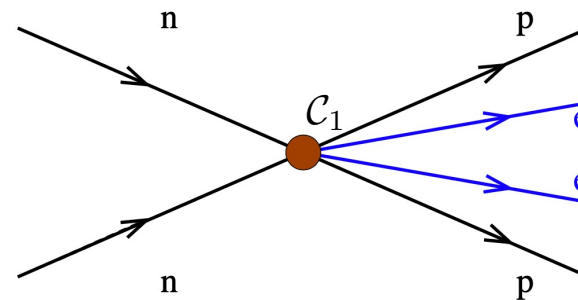
May 16, 2018 • *Physics* 11, s58

The inclusion of short-range interactions in models of neutrinoless double-beta decay could impact the interpretation of experimental searches for the elusive decay.



J. de Vries/Nikhef, adapted by APS/Alan Stonebraker

Cirigliano et al. PRL (2018)



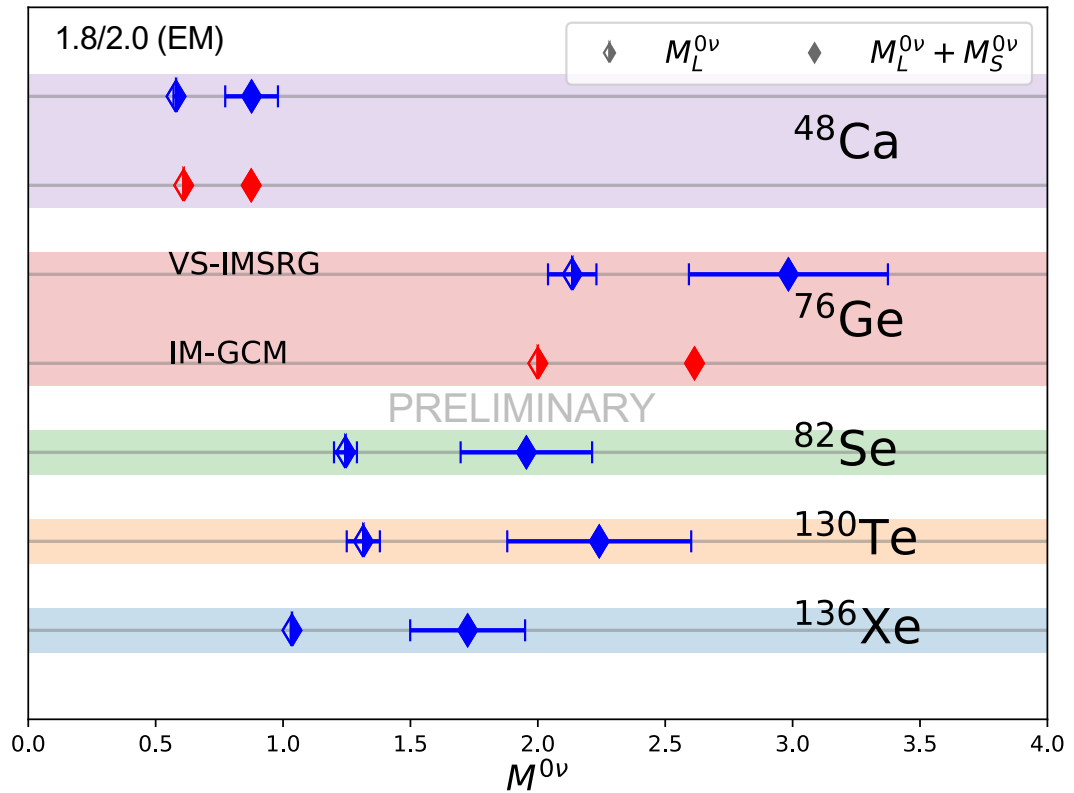
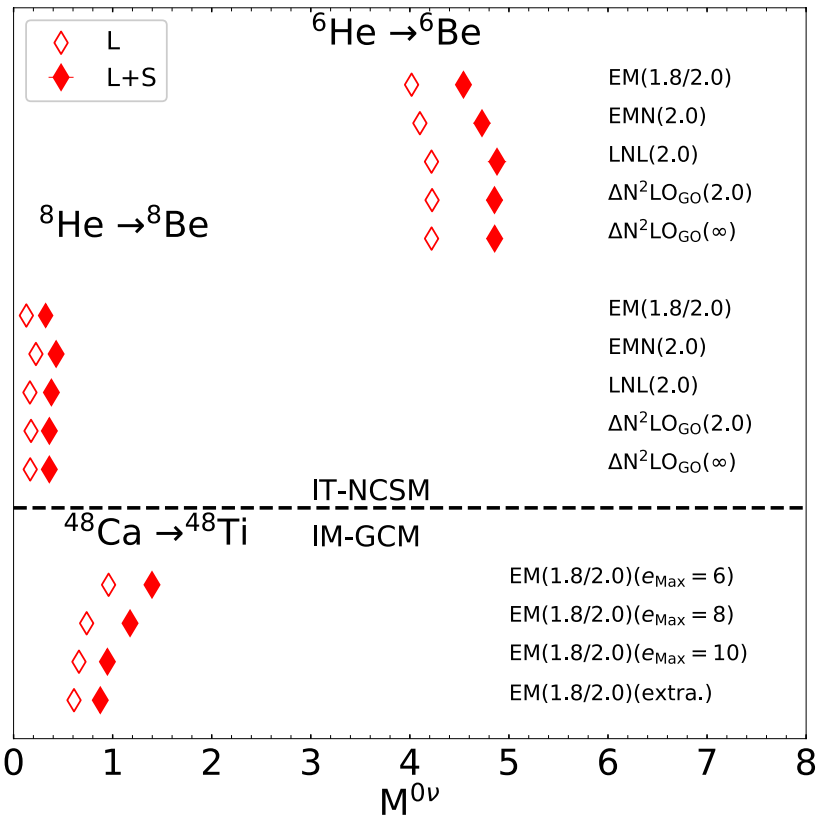
New physics inside blob:  
High-energy  $\nu$  exchange

**New paradigm for  $0\nu\beta\beta$  decay:** include long- and short-range terms

$$M^{0\nu} \rightarrow M_L + M_S = M_{GT} + \frac{M_F}{g_A^2} + M_T + M_{CT}$$



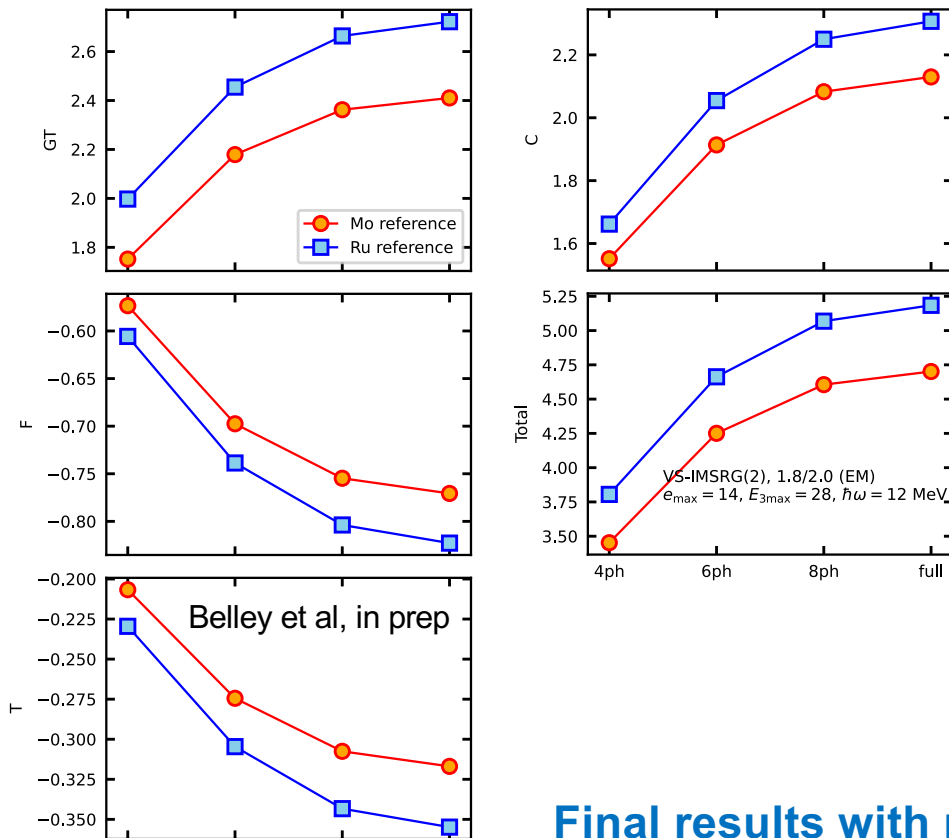
Match  $nn \rightarrow pp+ee$  amplitude from approximate QCD methods: **estimate contact term to 30%**



**Increase of 40% ( ${}^{76}\text{Ge}$ ) to 60% ( ${}^{130}\text{Te}/{}^{136}\text{Xe}$ )**

Final competitive candidate in worldwide searches: AMoRE, NEMO 3, CUORE...

Highly mid-shell, difficult for SM - access with p-h truncations in KSHELL

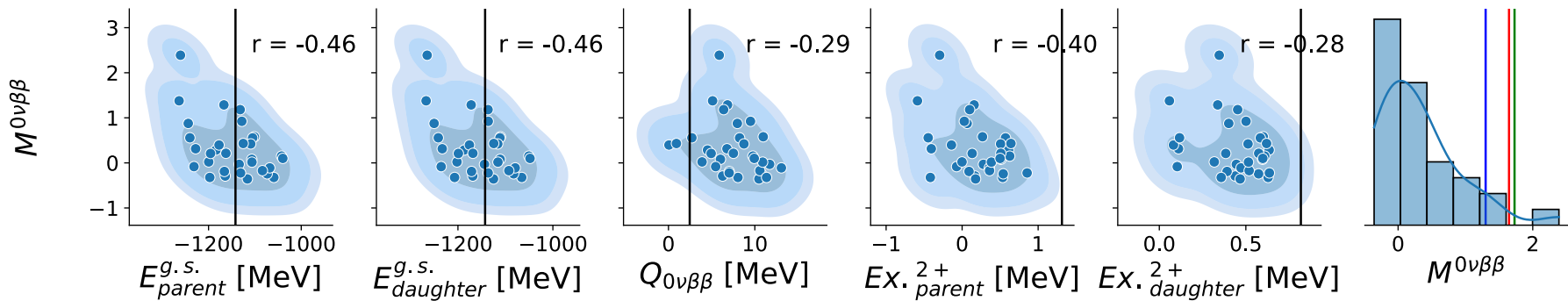


**Final results with multiple NN+3N forces coming soon!**

Explore correlations with other observables from systematic analysis (34 interactions)

Few clear correlations, except DGT

Similar picture in  $^{136}\text{Xe}$ ... BUT no correlation with  $2^+$



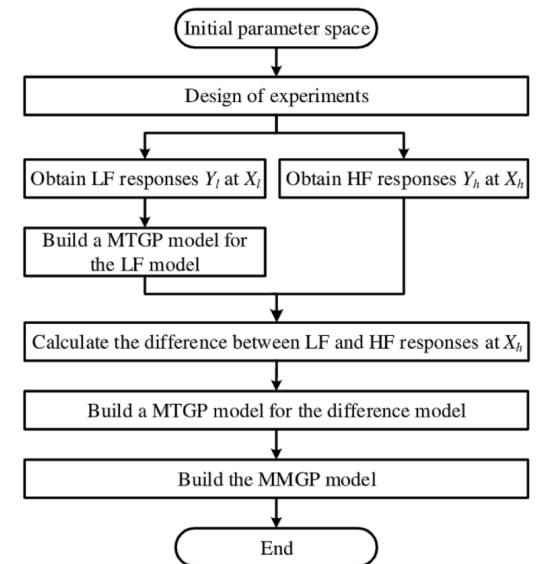
Explore dependence on chiral EFT LECs: requires many samples (as in  $^{208}\text{Pb}$ )

Use gaussian processes as an emulator

**Multi-Fidelity Gaussian Process:** connects few (complicated) high-fidelity data points (eg, full IMSRG) w/ many low-fidelity data points (HF, low  $e_{\text{max}}$ , etc)

Difference function fit with Gaussian process: predict HF from LF

When relation between LF and HF is complicated, MFGP fails



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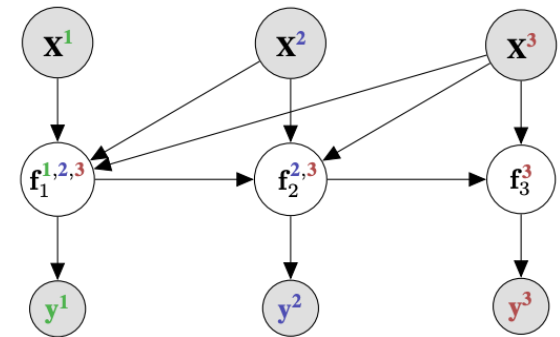
Difference function fit with Gaussian process: predict HF from LF

**Deep Gaussian Process:** Neural network links multiple GP

Include outputs of previous fidelity as new HF point:  
Improves modeling of difference between LF and HF

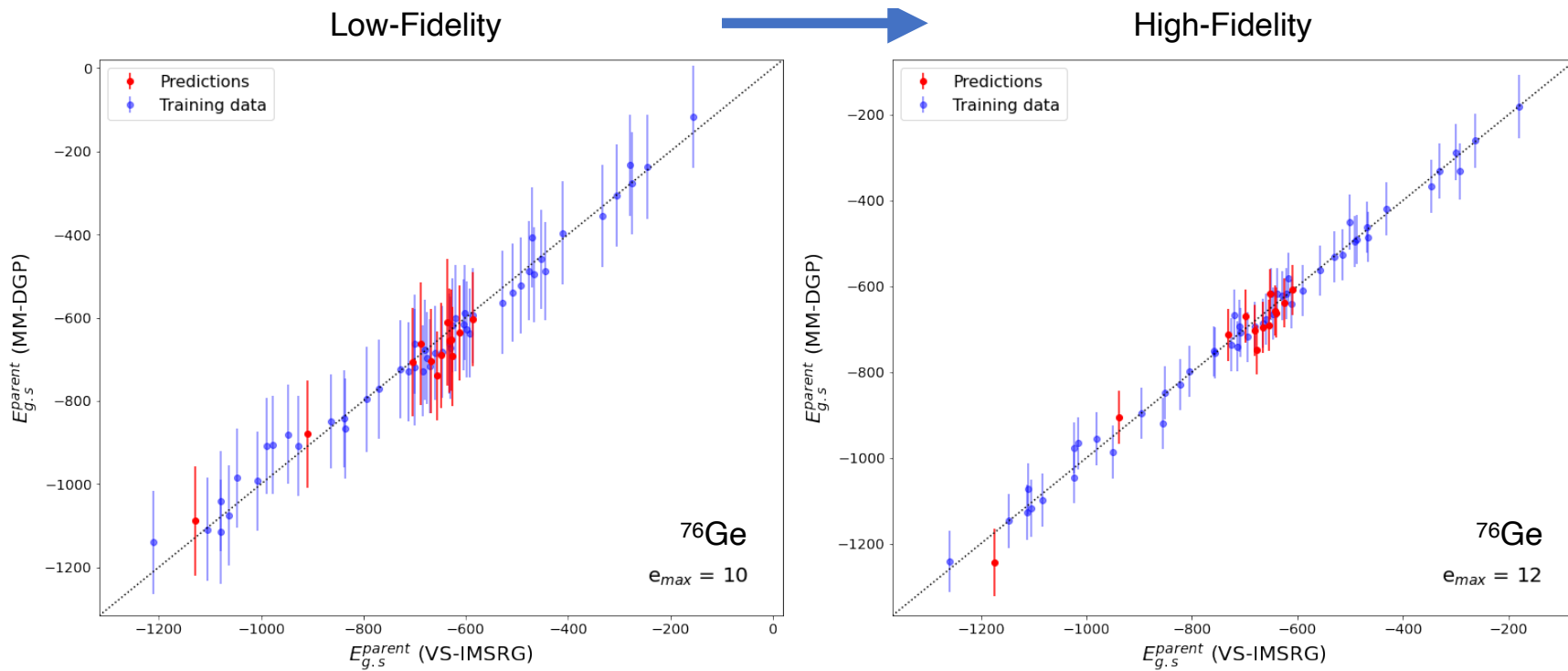
Adapted for multi output:

**Multi-Output Multi-Fidelity Deep Gaussian Process (MM-DGP)**



## Testing MM-DGP: use delta-full chiral EFT at N2LO

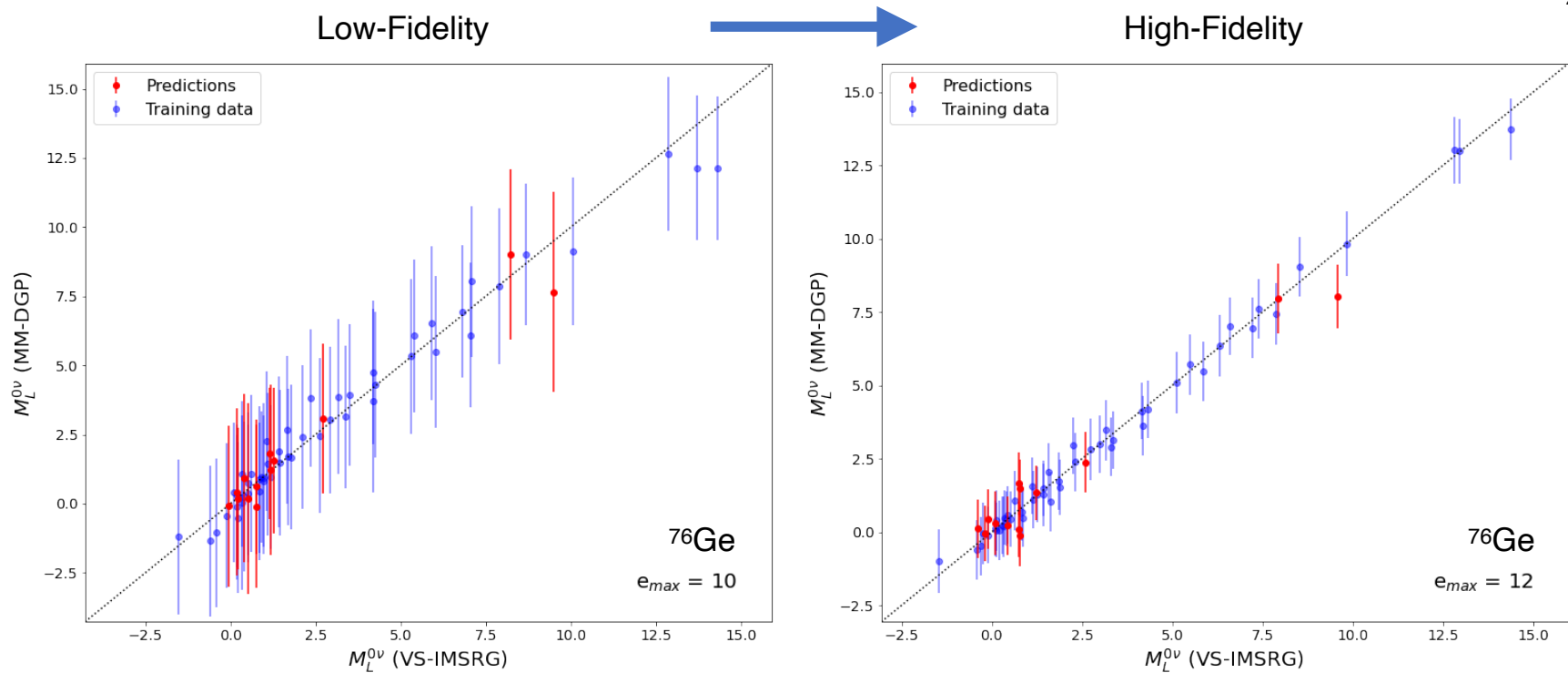
Improved energy predictions with high-fidelity training points



Belley, Pitcher et al. in prep.

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