Theory Alliance
FACILITY FOR RARE ISOTOPE BEAMS

FRIB-TA Topical Program:
Theoretical Justifications and Motivations for Early High-Profile FRIB Experiments

# The equation of state of dense nuclear matter from heavy-ion collisions

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University of Washington

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# The EOS = key to understanding fundamental properties of QCD matter

- density-dependence of the EOS = information about strong interactions at different scales (long- vs. short-distance)
- probing different densities = probing different distances
- what are the options?:

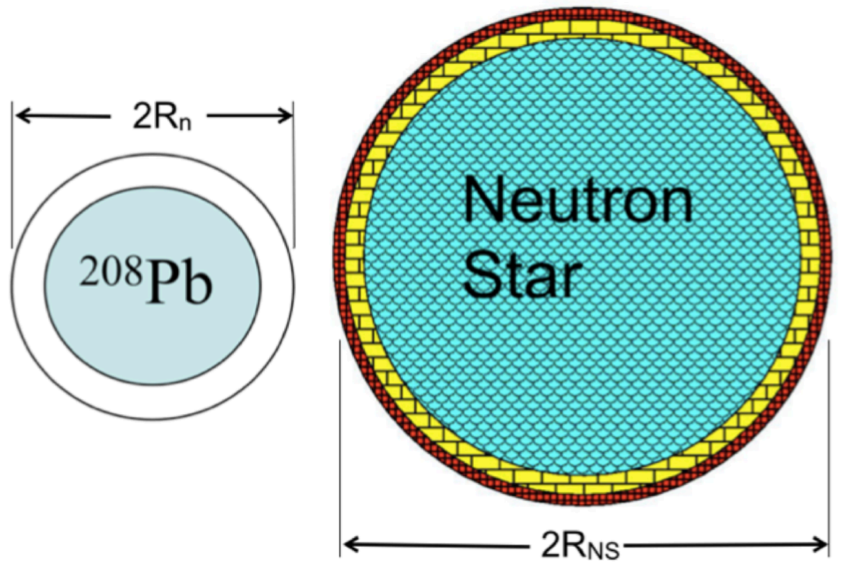
  - nuclei cores: ~$n_0$

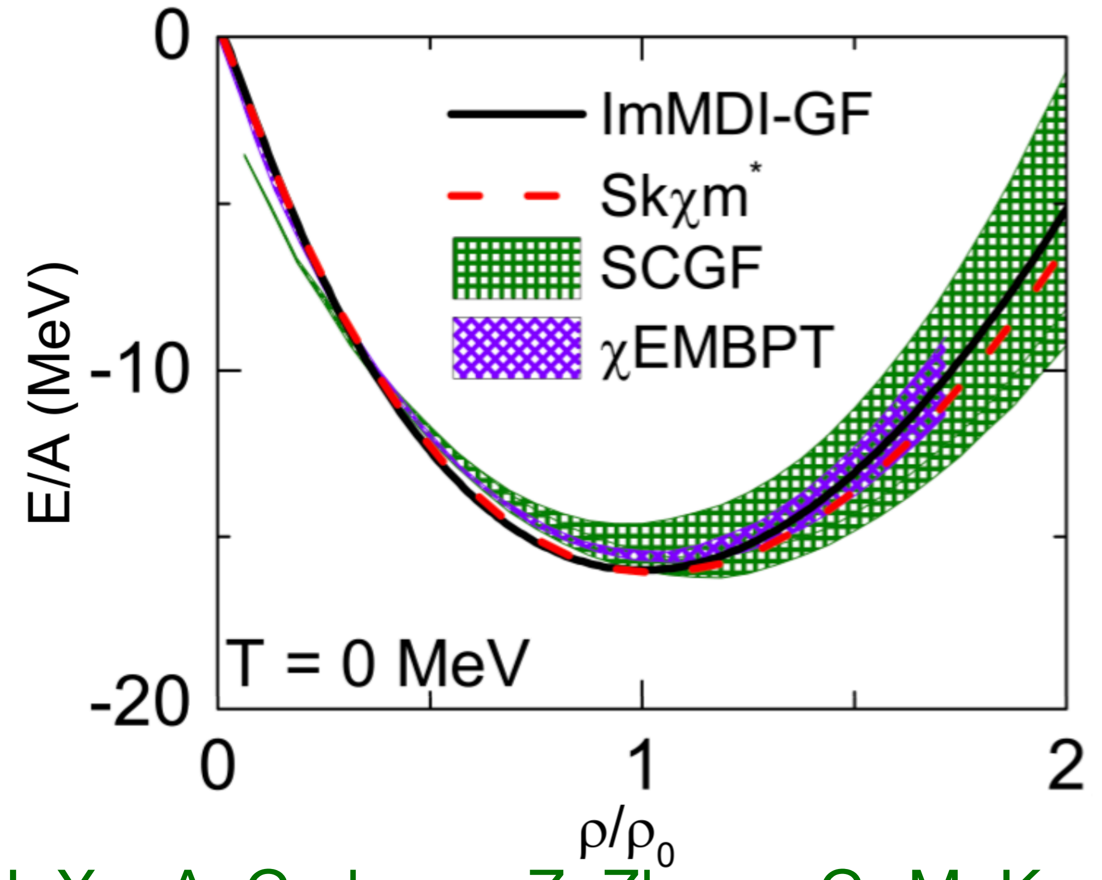
  - surface phenomena (neutron skins etc.): ~$^2/_3 n_0$

  - neutron stars: up to whatever the maximum core density is ($3n_0$? $5n_0$?...)

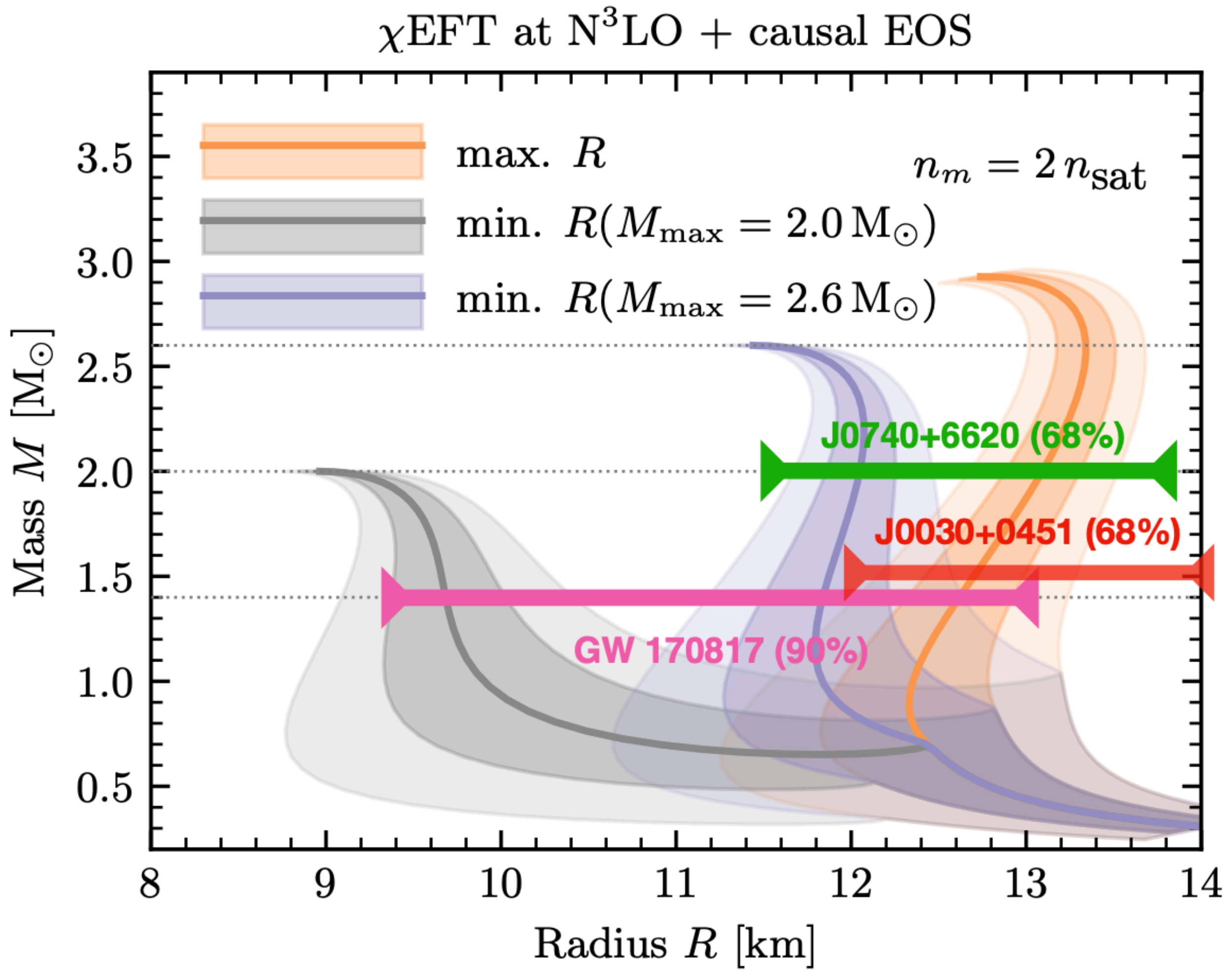
  - neutron star mergers: finite $T$ ~50 MeV

  - heavy-ion collisions from ~50 MeV/u to ~30 GeV/u (FXT frame): finite $T$, from ~$^1/_4 n_0$ up to ~$5n_0$




J. Xu, A. Carbone, Z. Zhang, C.-M. Ko, Phys. Rev. C **100**, 2, 024618 (2019) arXiv:1904.09669




* from S. Reddy's slides;
M-R results: C. Drischler, S. Han, J. M. Lattimer, M. Prakash, S. Reddy, T. Zhao, Phys. Rev. C **103** 4, 045808 (2021), arXiv:2009.06441

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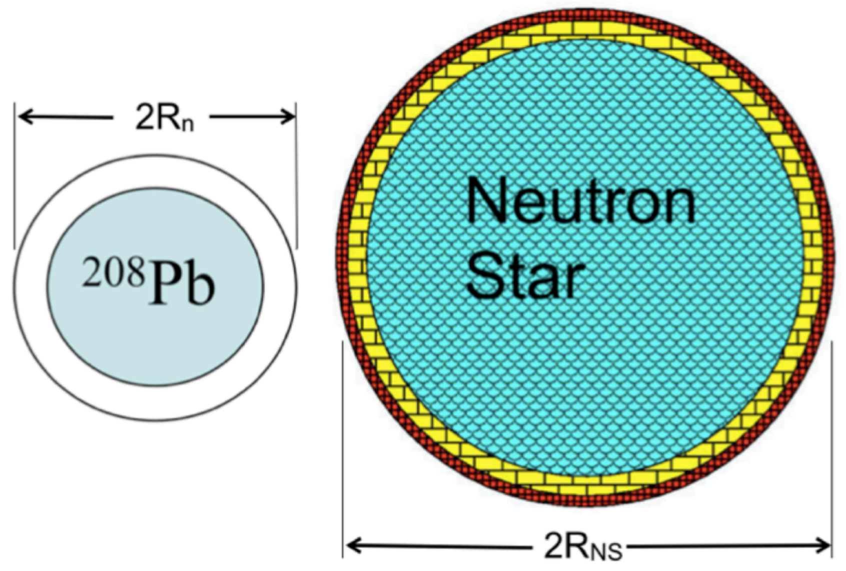


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**DYNAMIC EVOLUTION**

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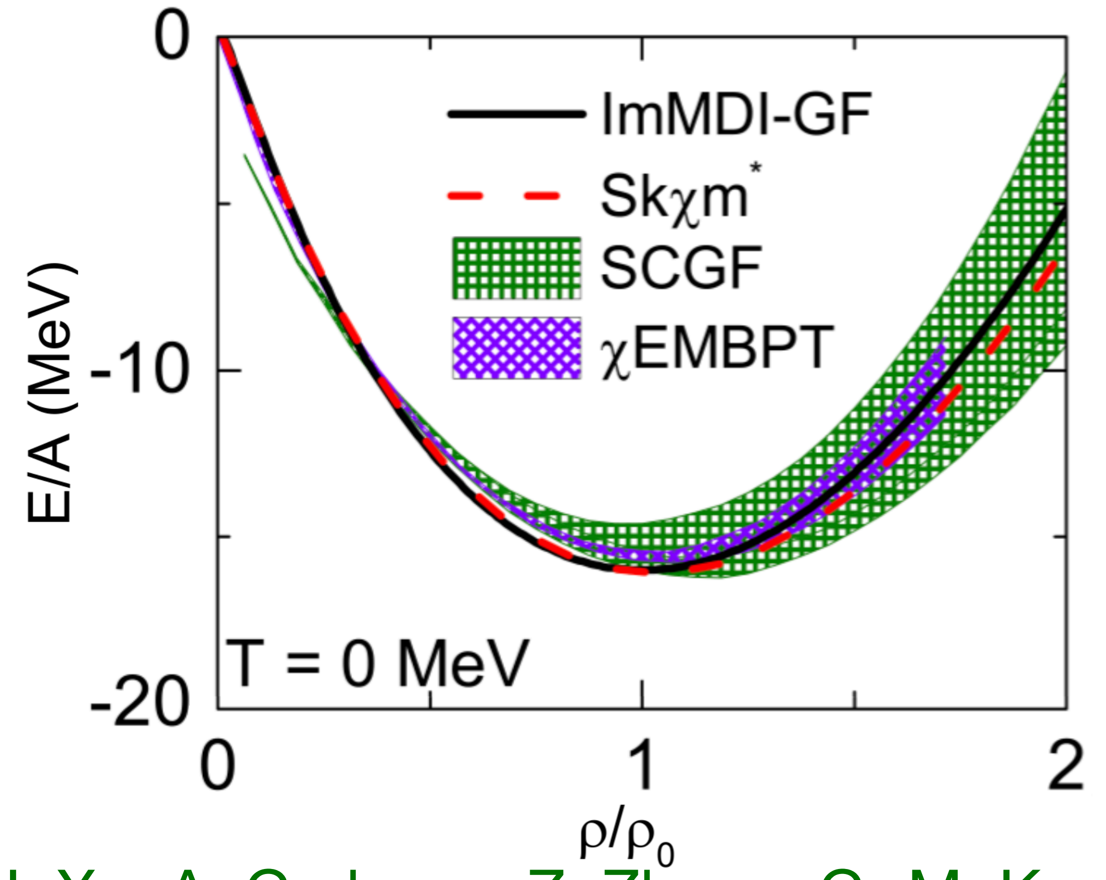


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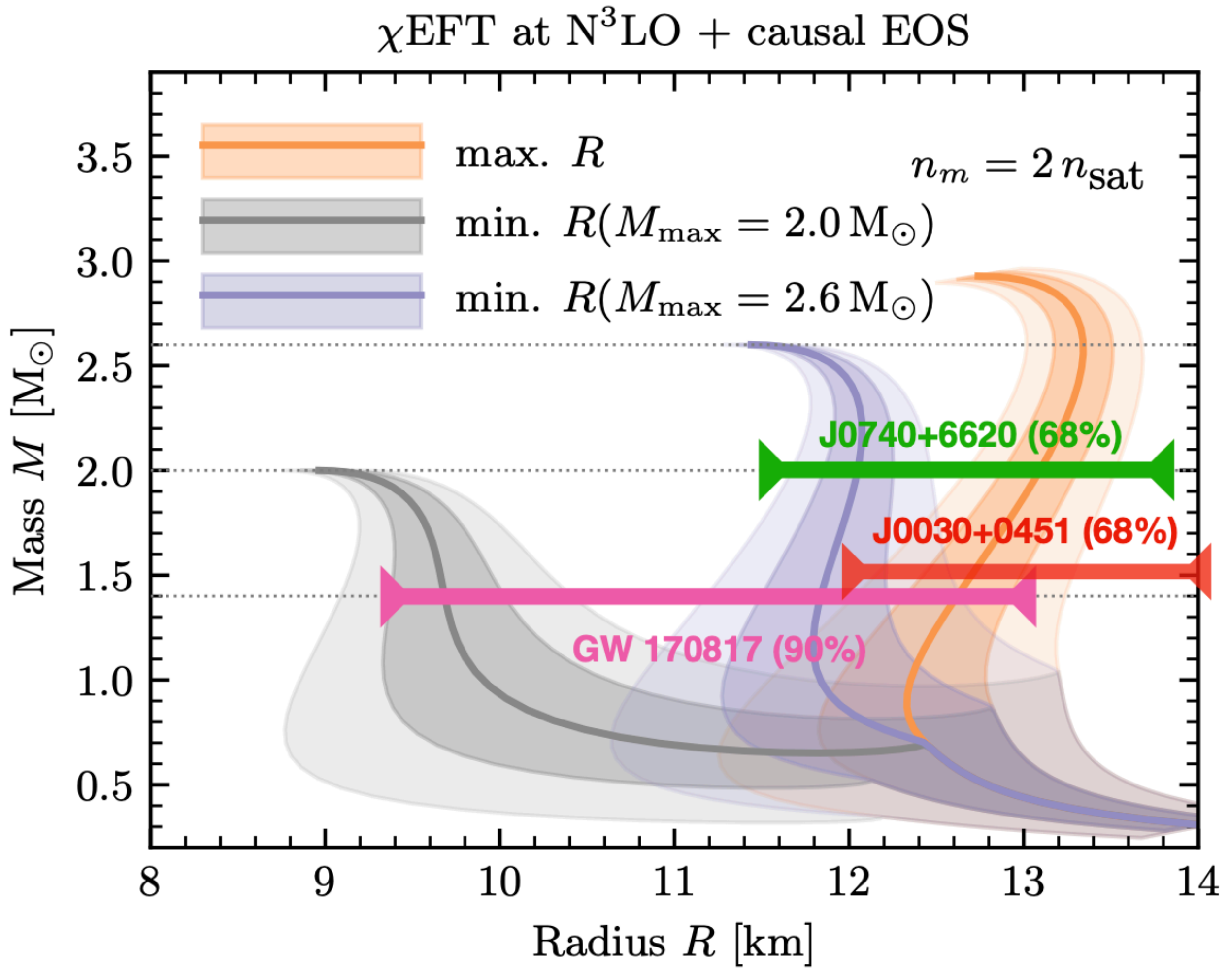


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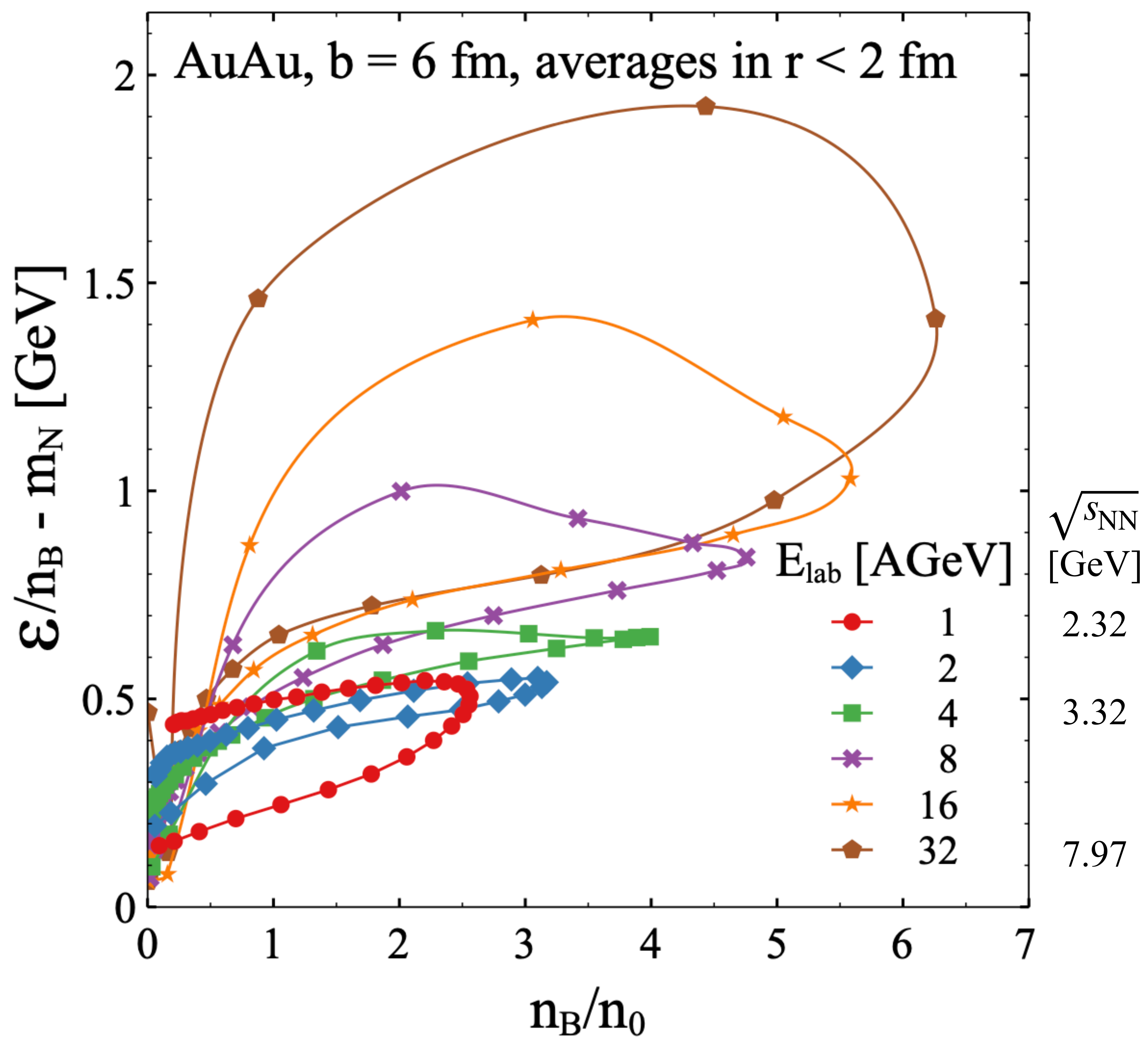
# Intermediate-energy heavy-ion collisions probe wide ranges of density and temperature

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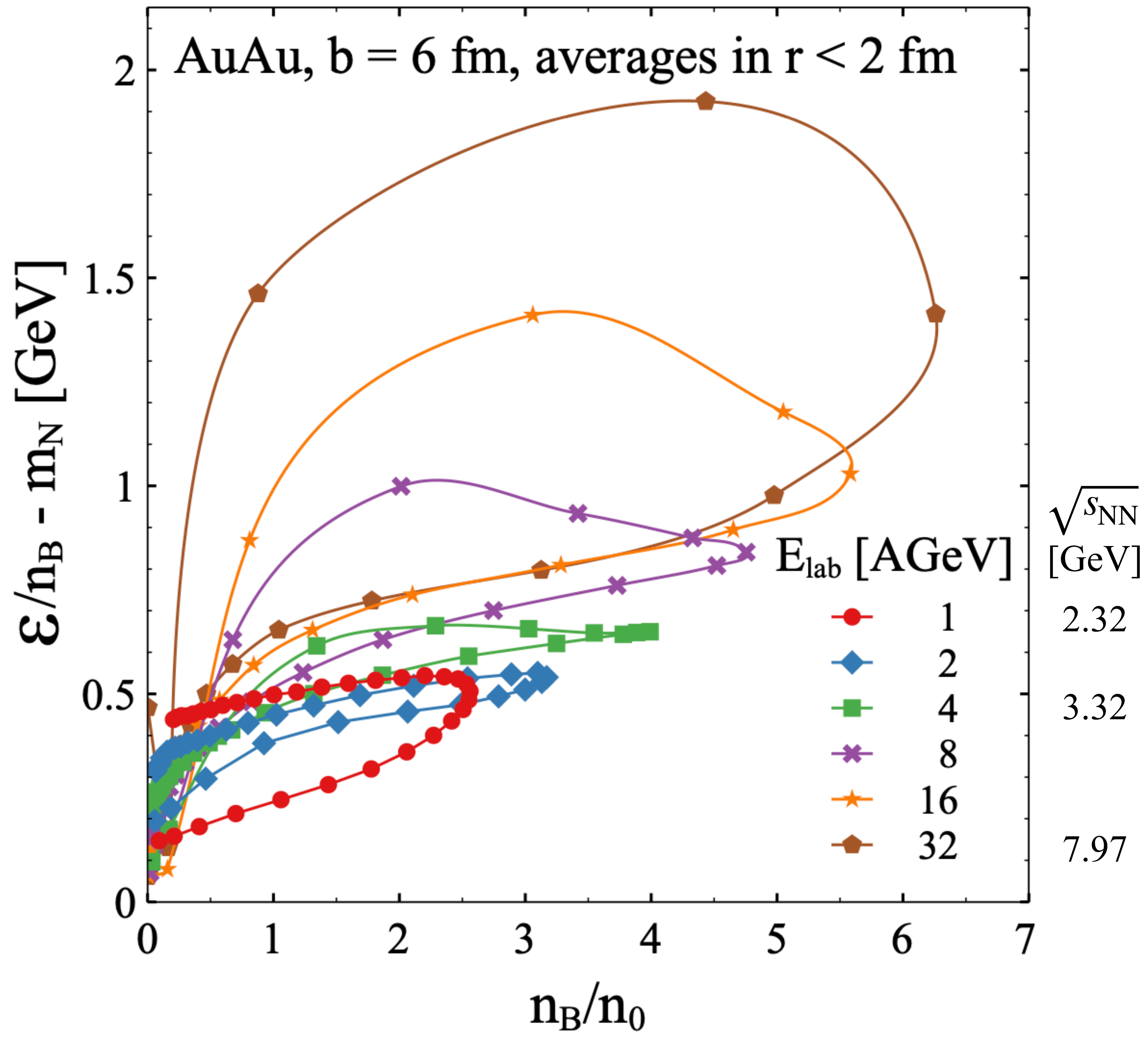


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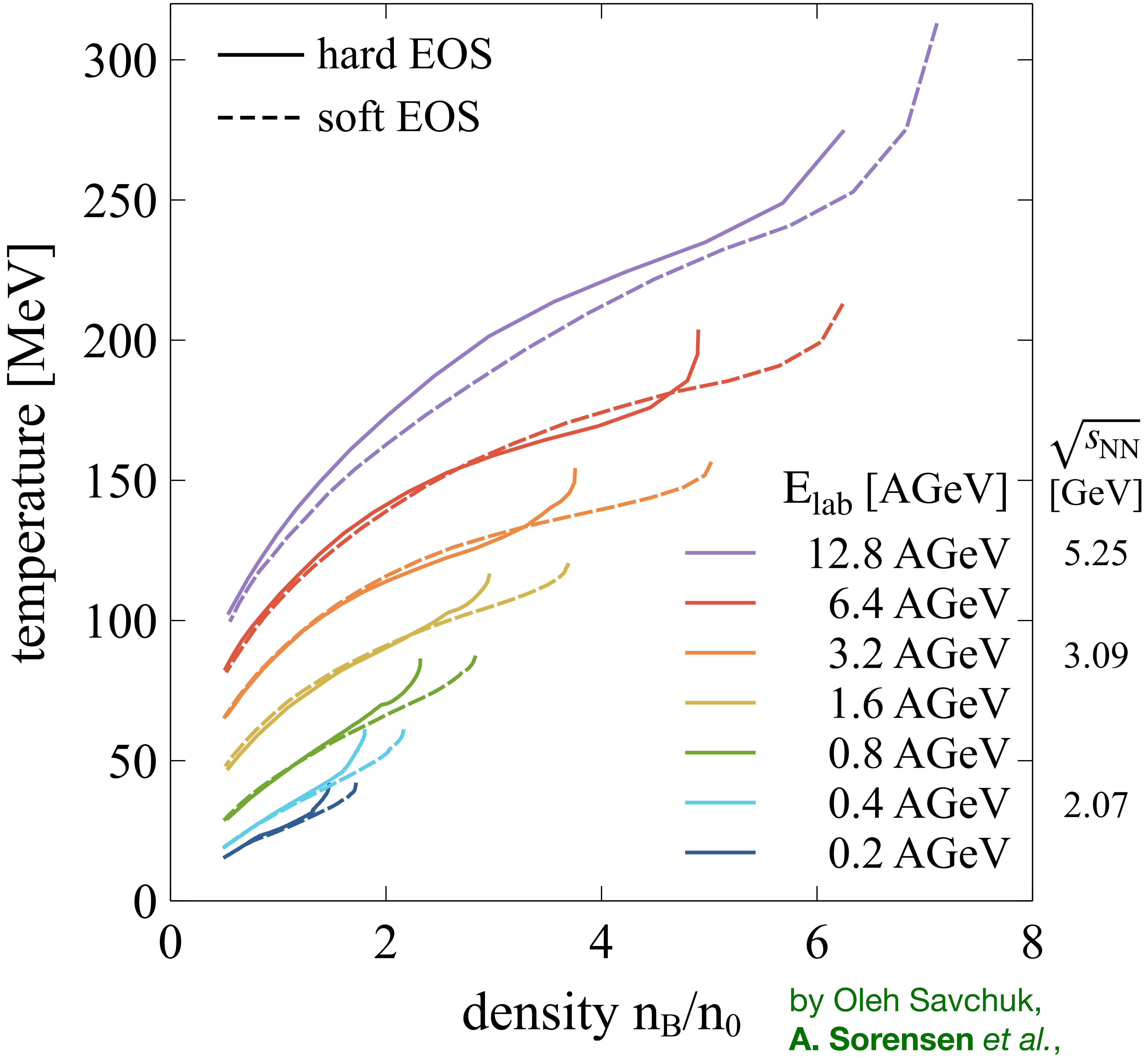


by Oleh Savchuk, **A. Sorensen** *et al.*, arXiv:2301.13253

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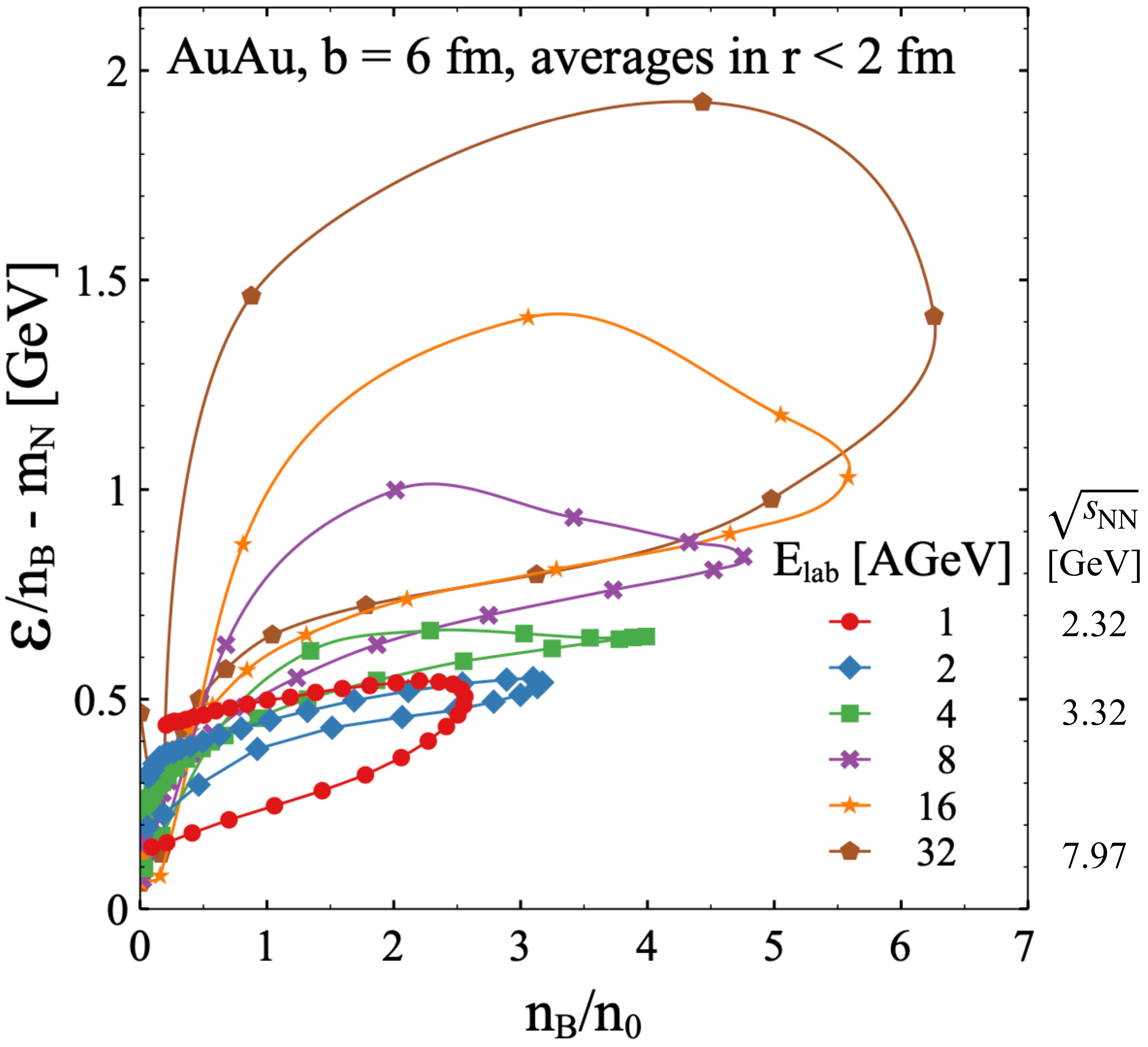


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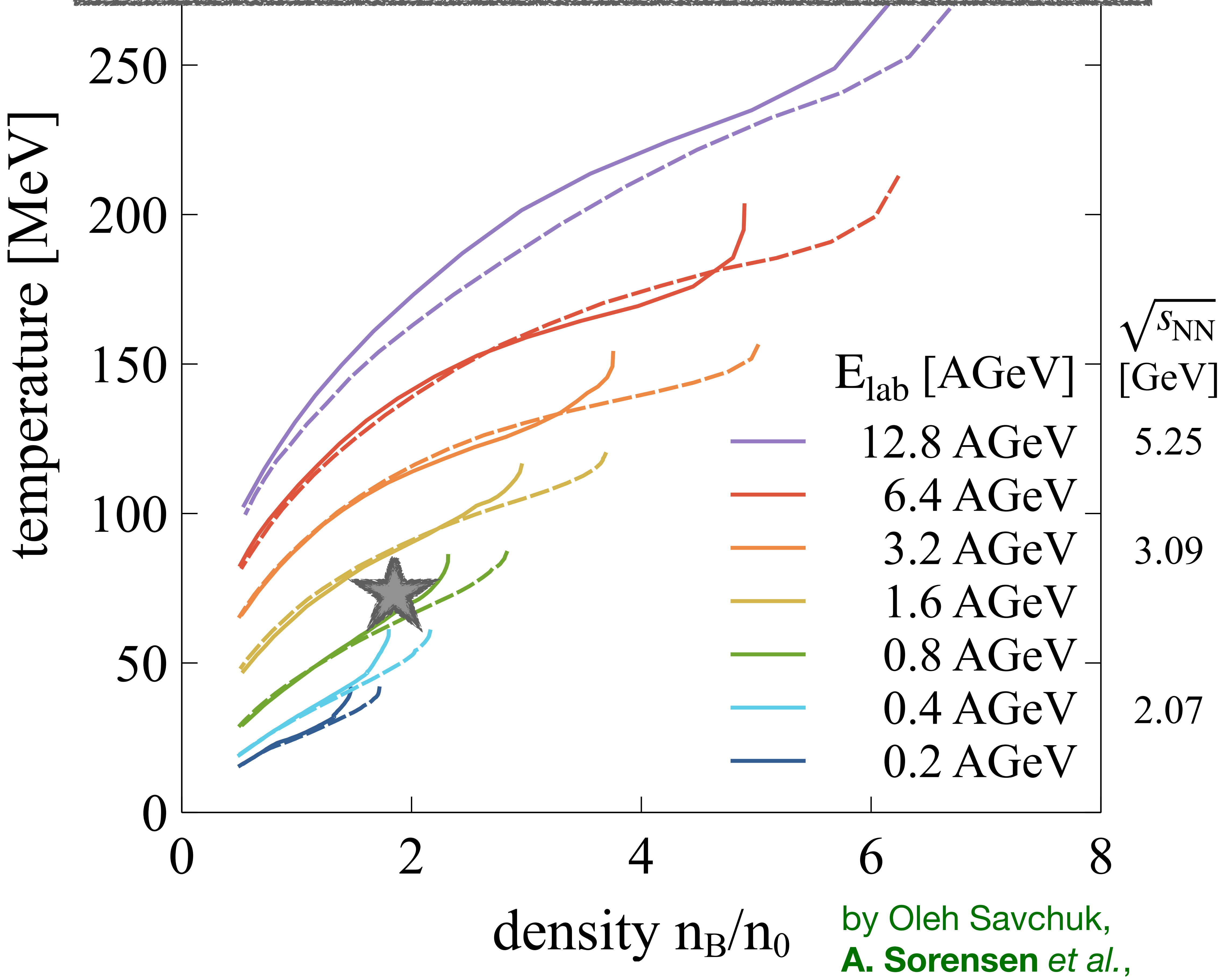
HADES ($E_{lab}$ = 1.23 $A$GeV) dilepton analysis:
$\langle T \rangle = 71.8 \pm 2.1$ MeV

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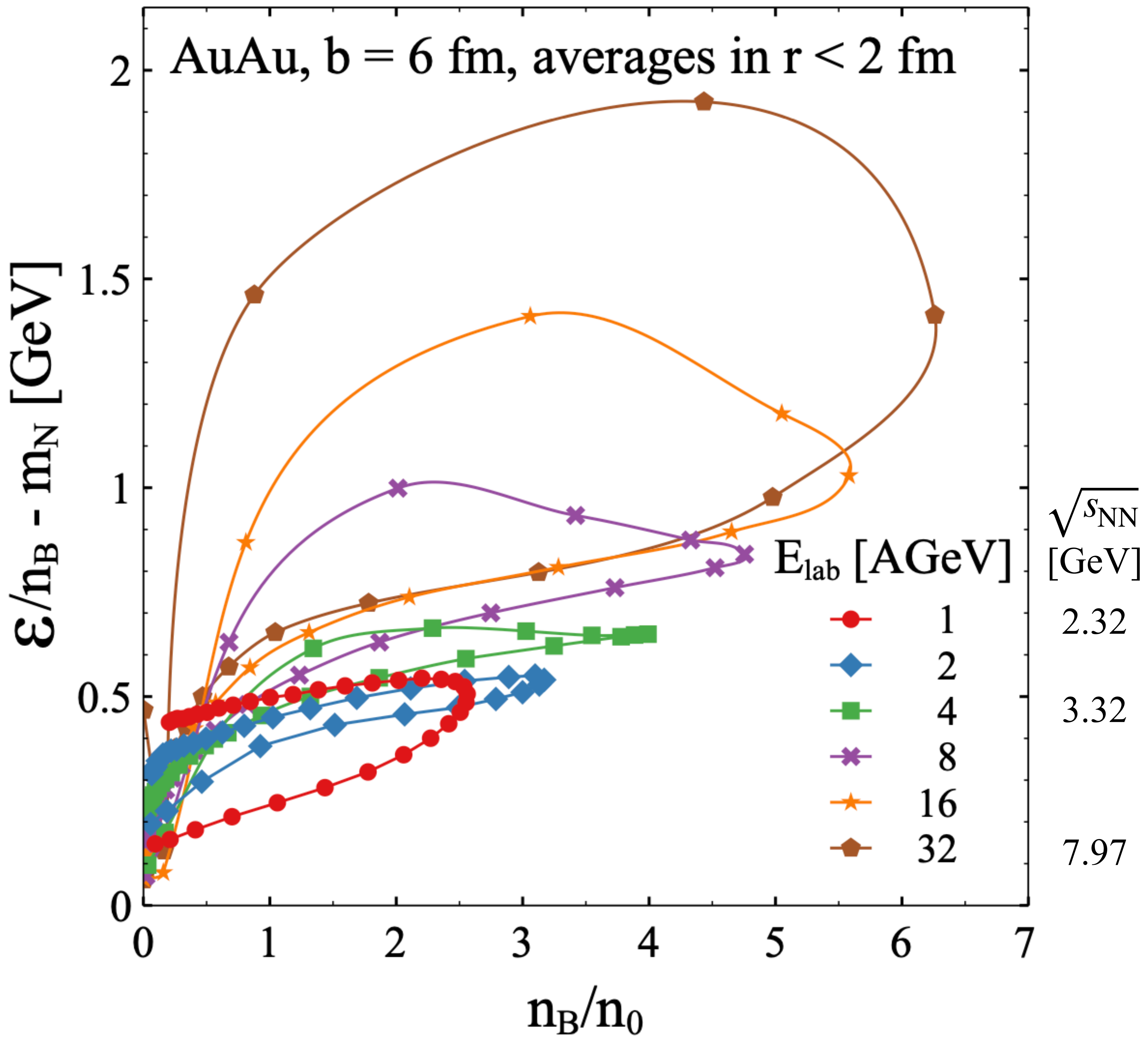


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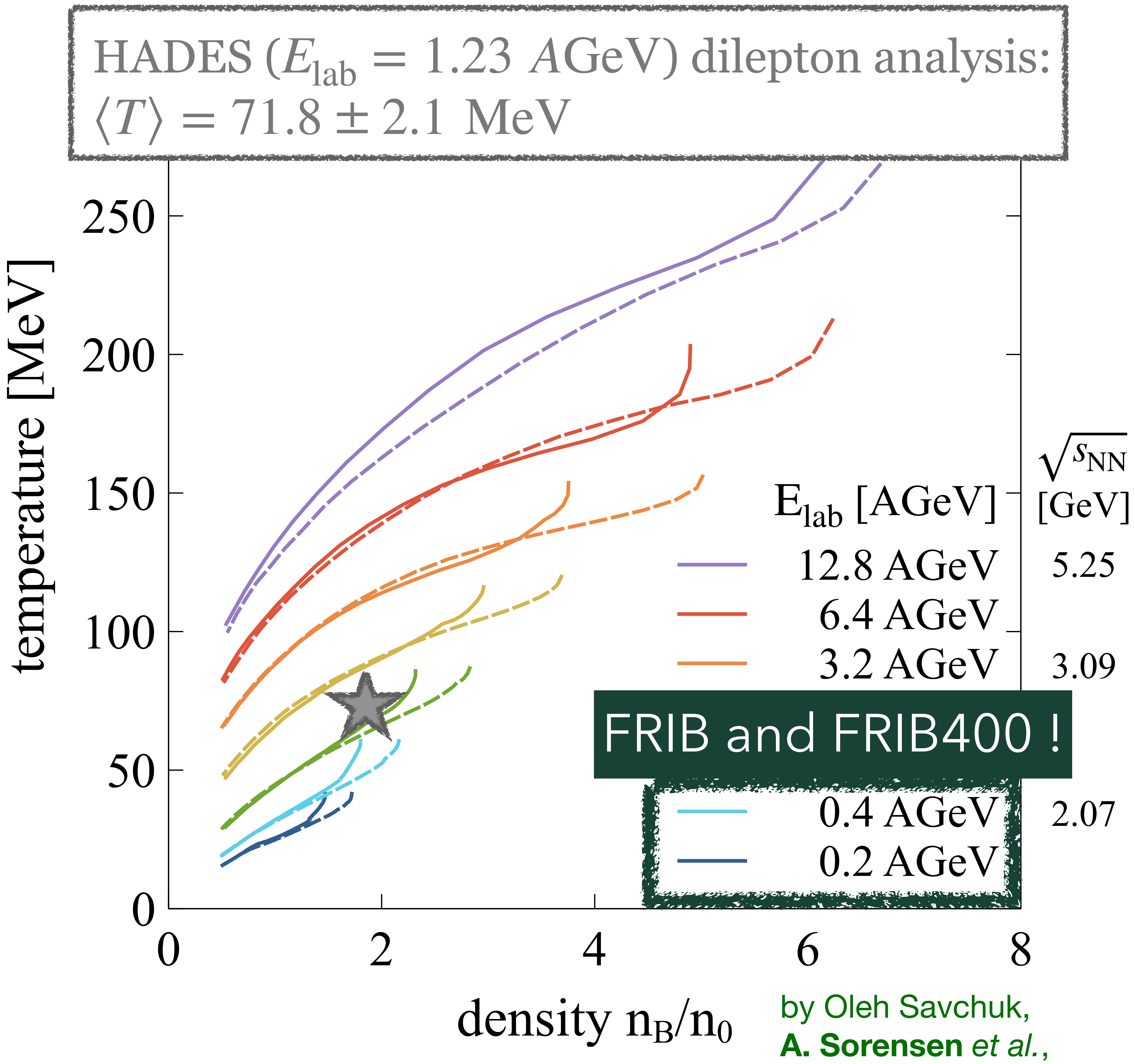


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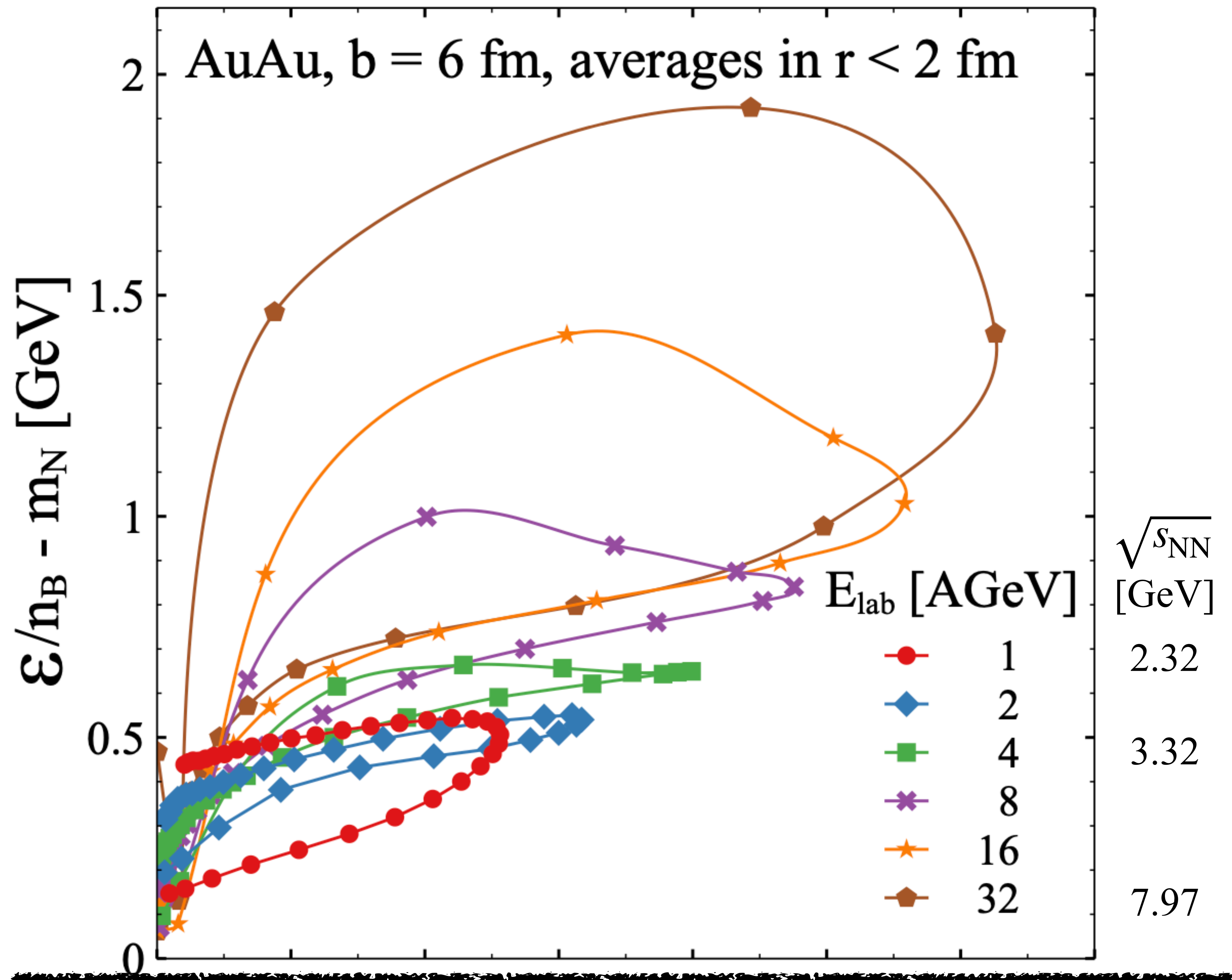


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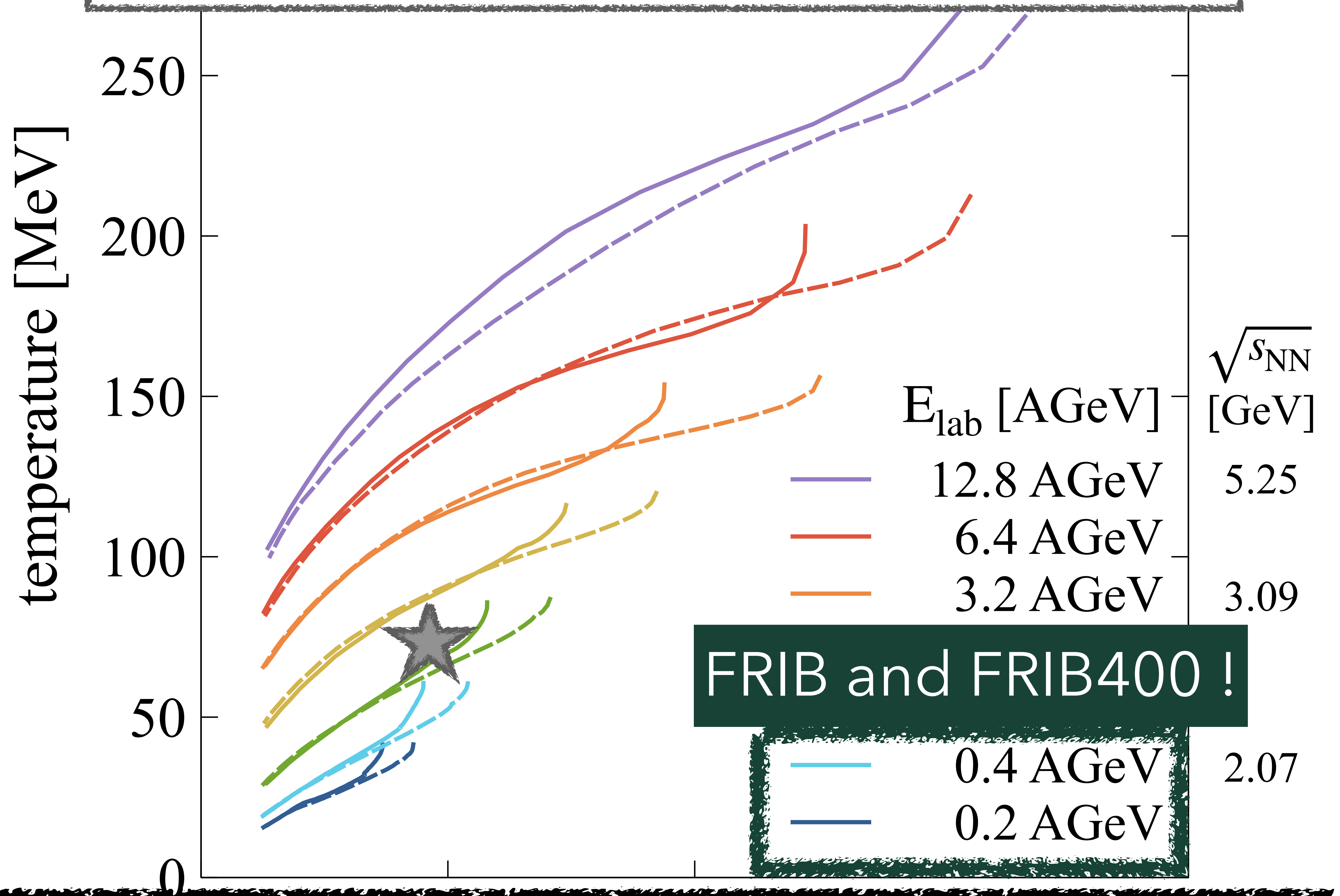


- HICs = the only means to probe densities away from $n_0$ in controlled terrestrial experiments
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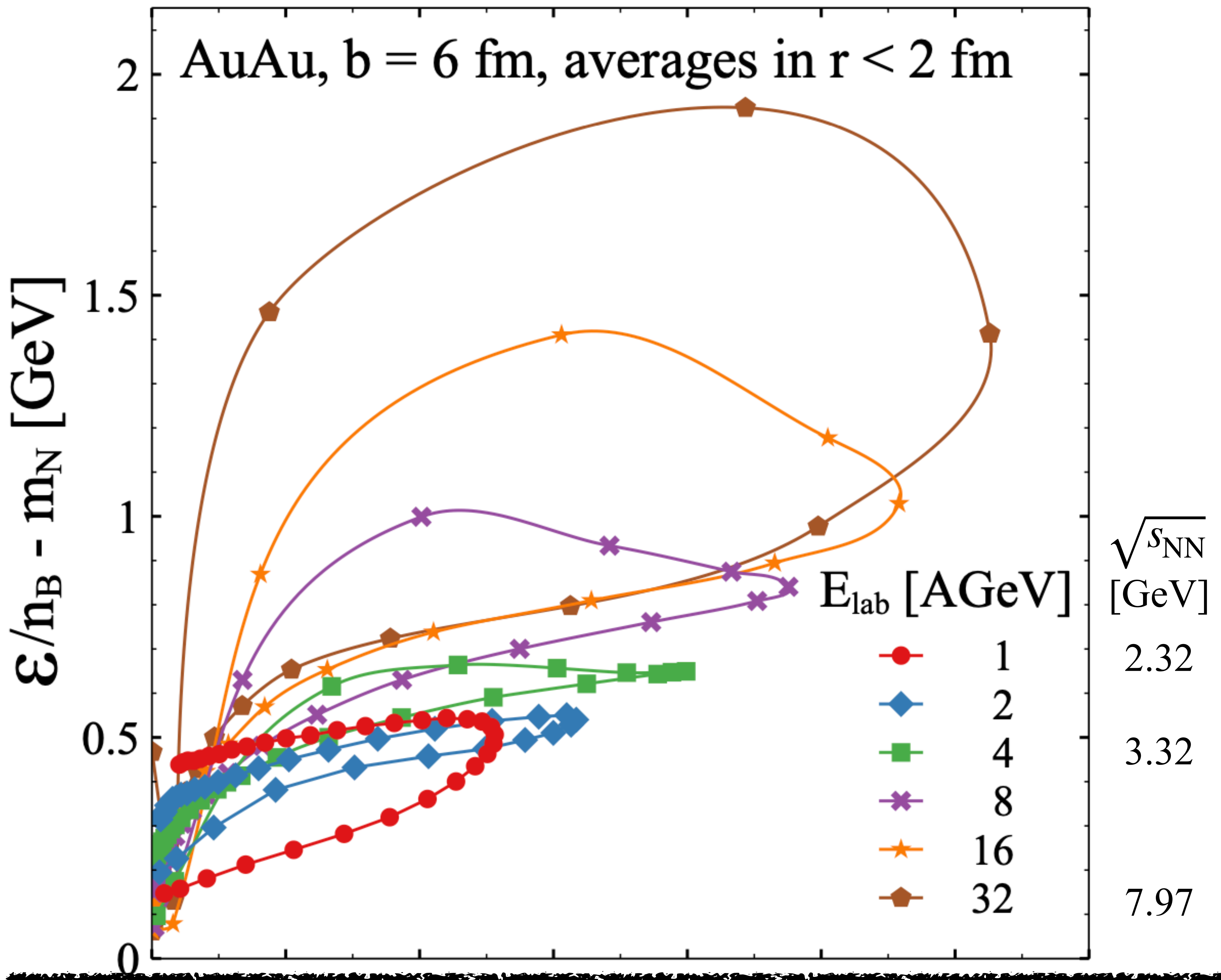


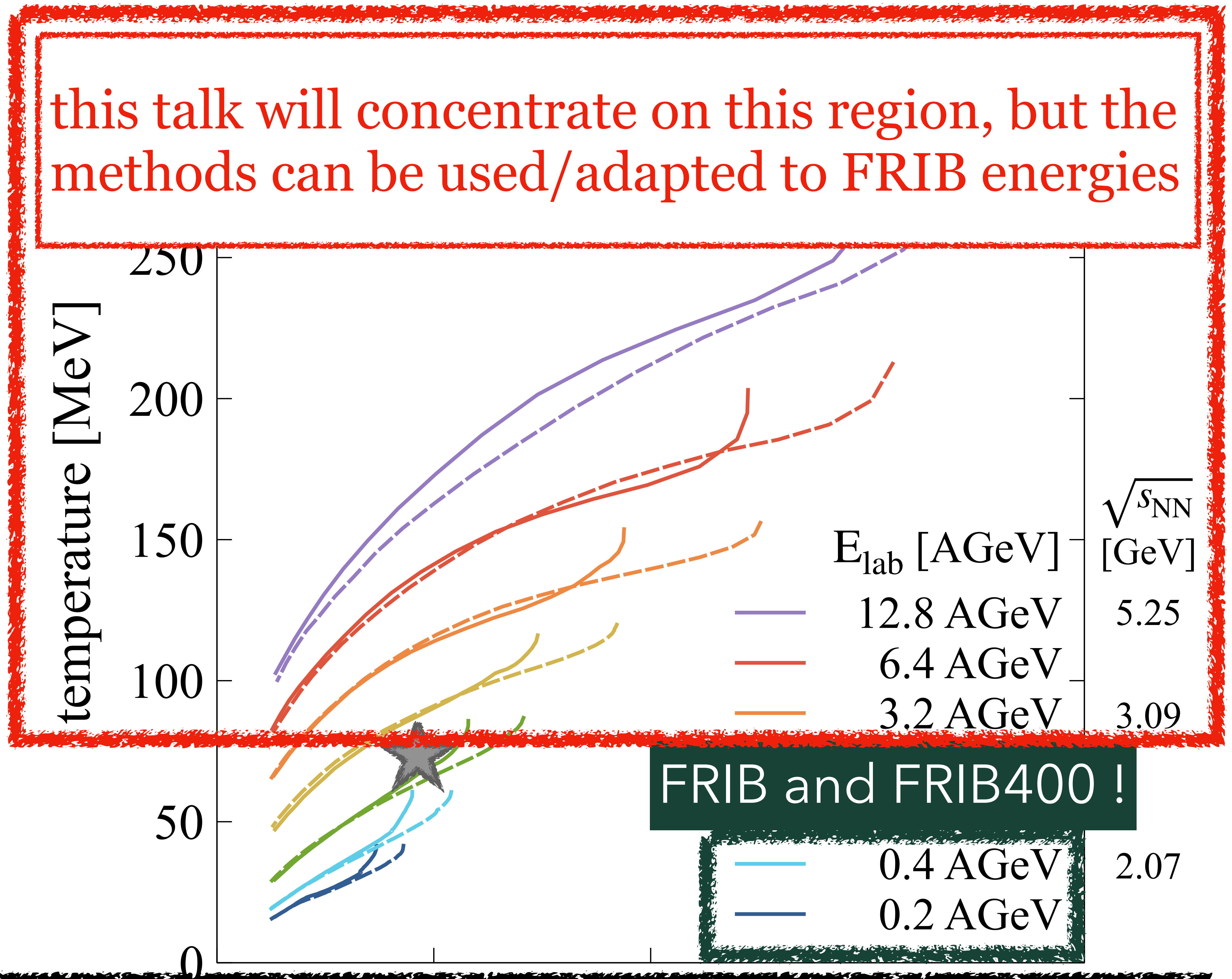


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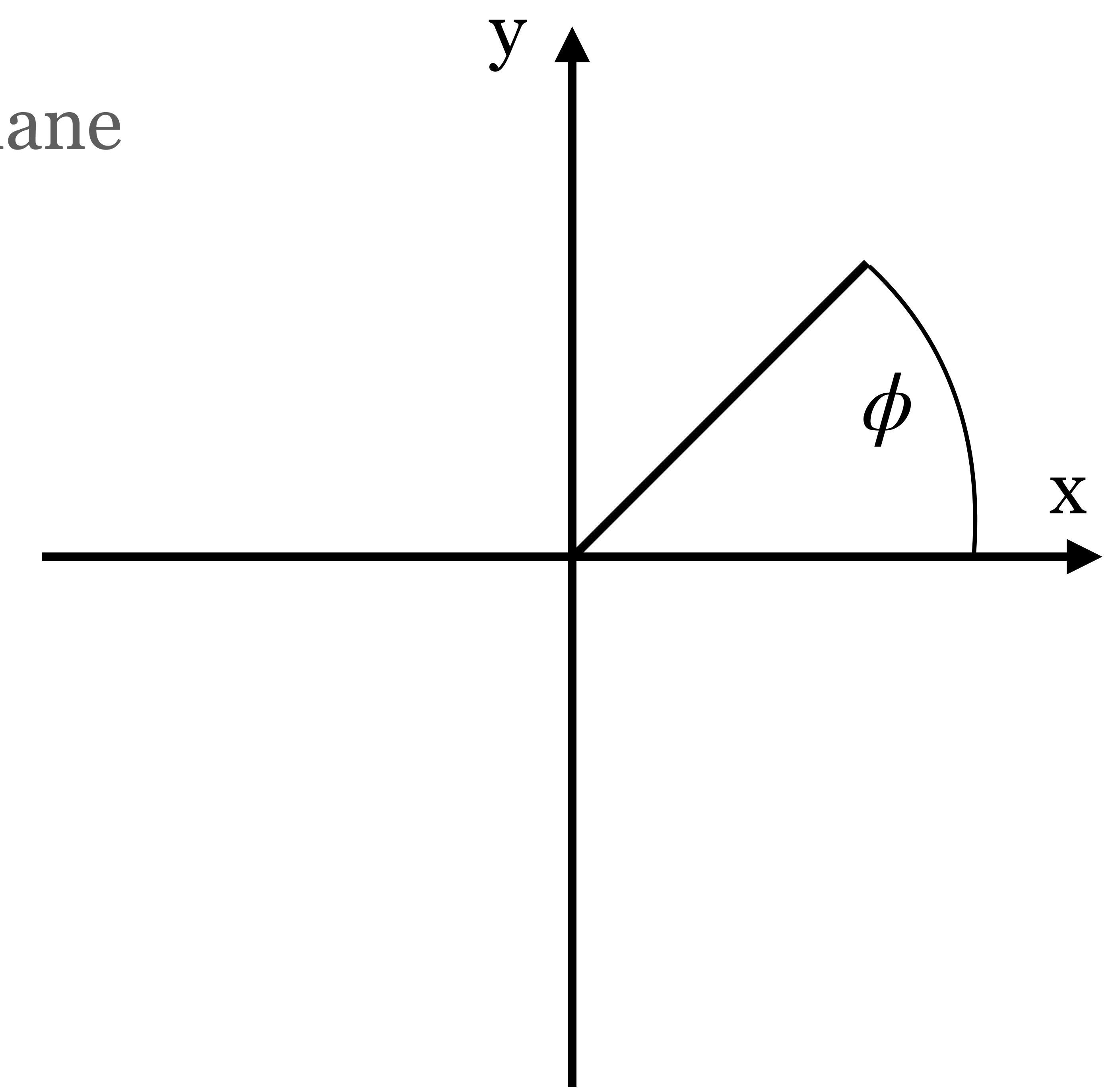
# EOS from flow observables in heavy-ion collisions

Flow $v_n \equiv \langle \cos(n\phi) \rangle$

transverse plane



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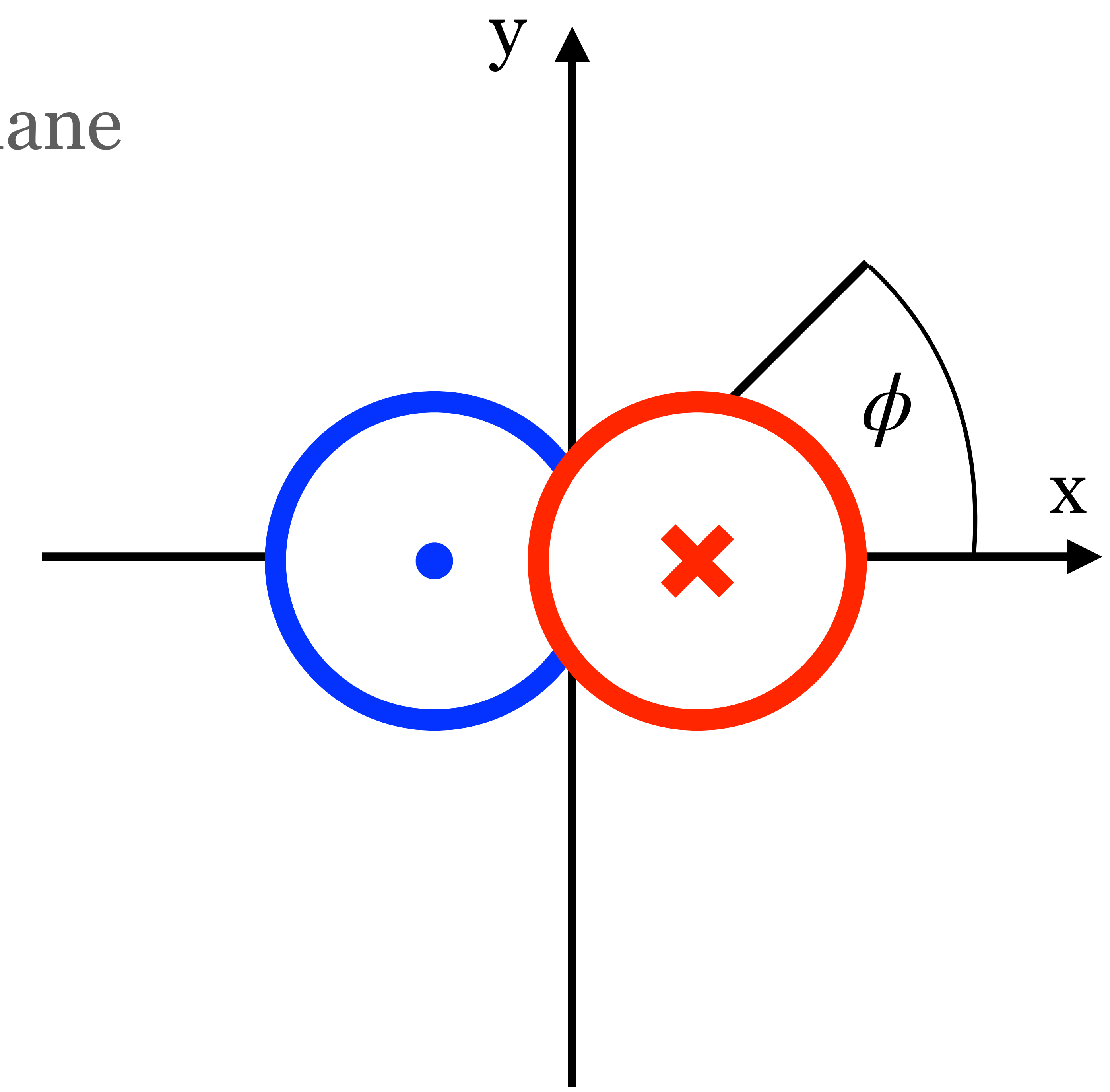

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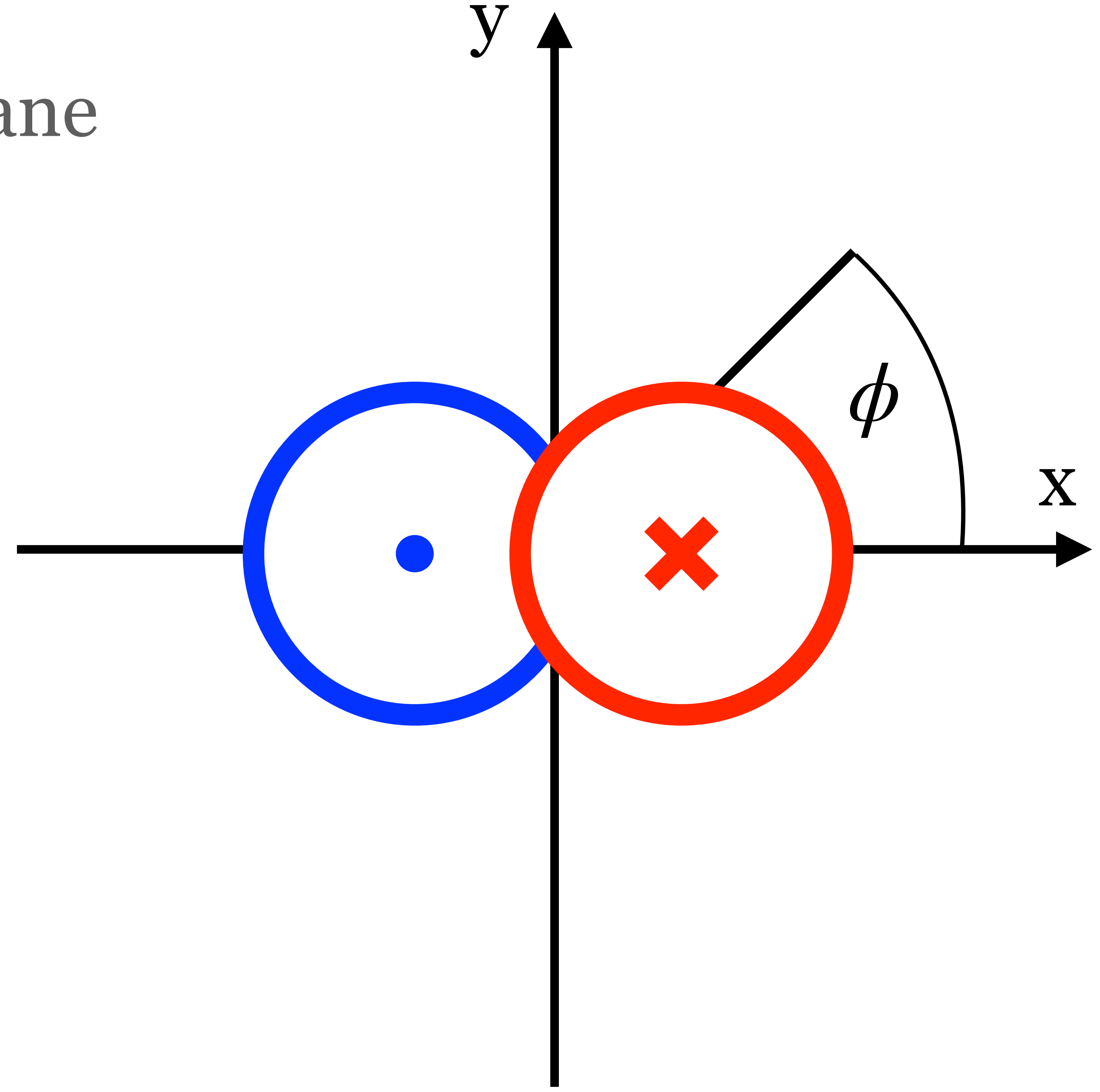

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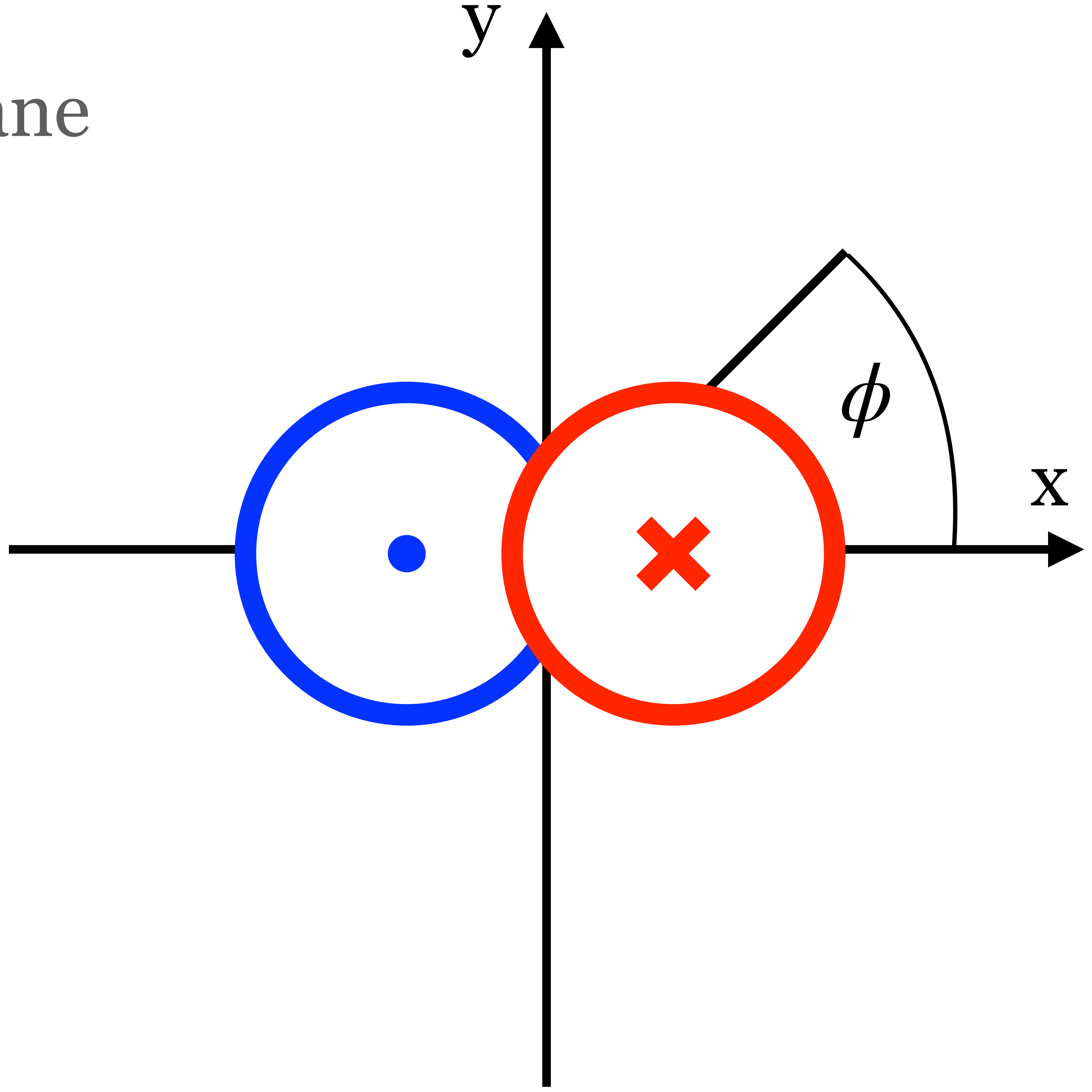


$\cos(\phi) = +1$
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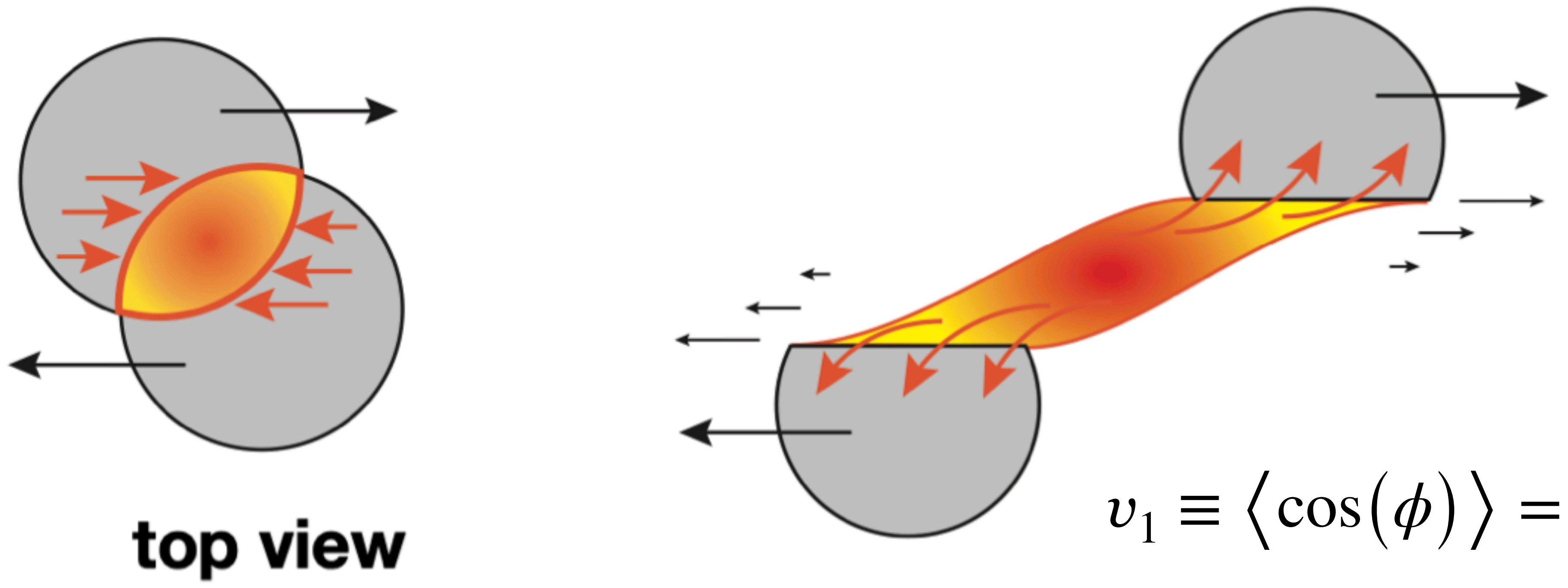
directed flow $v_1$ ($dv_1/dy$ ~ longitudinal expansion)




$v_1 \equiv \langle \cos(\phi) \rangle = \langle \frac{p_x}{p_T} \rangle$

illustrations from a presentation
by B. Kardan (HADES)

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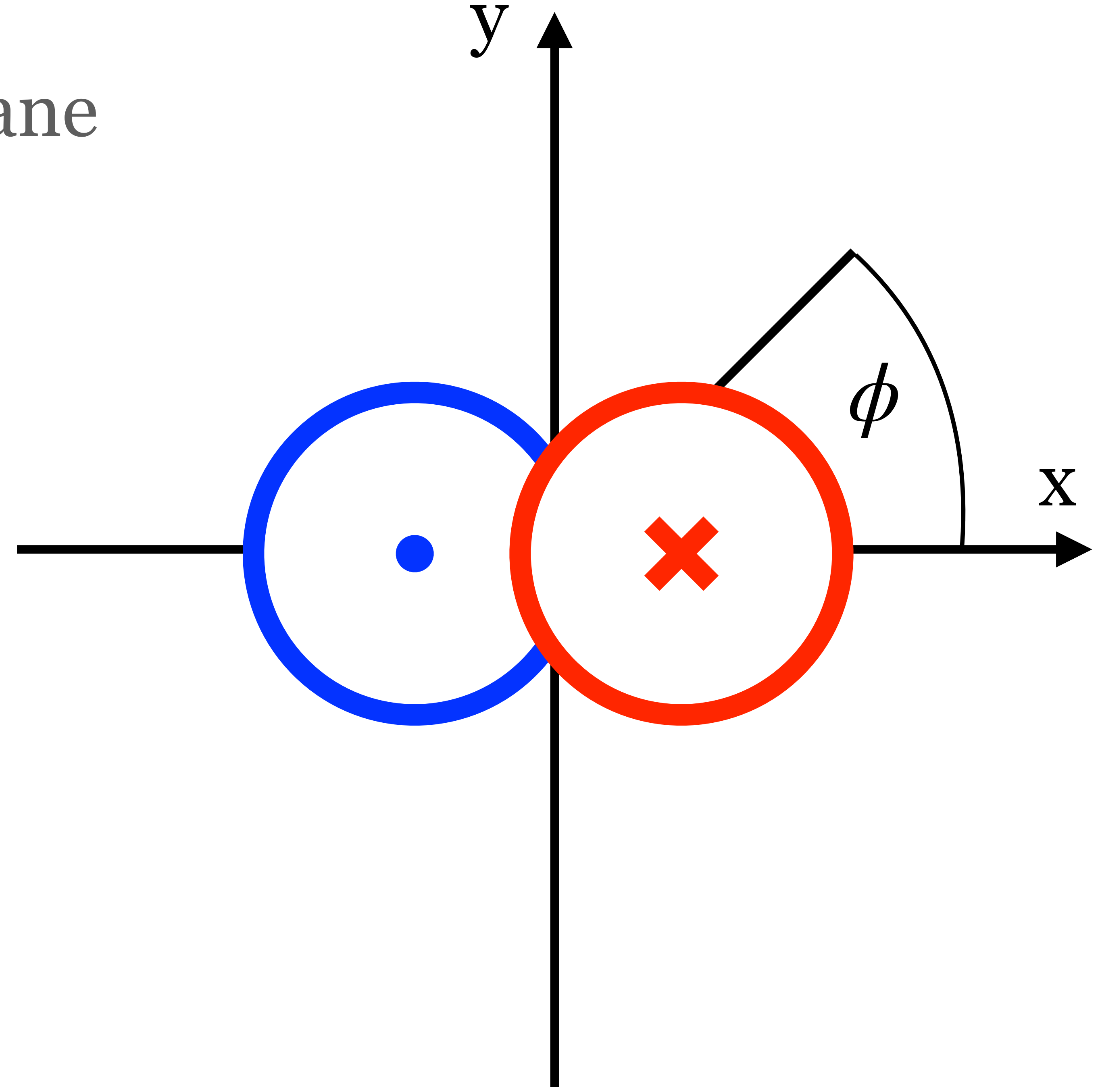


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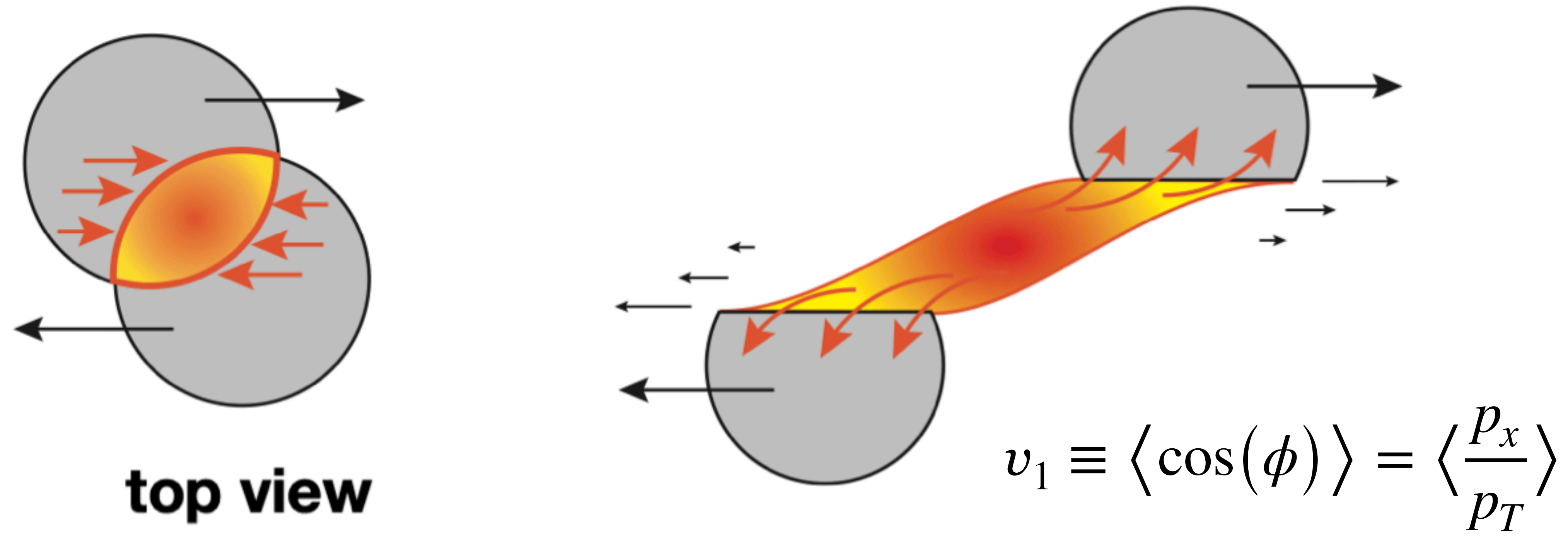
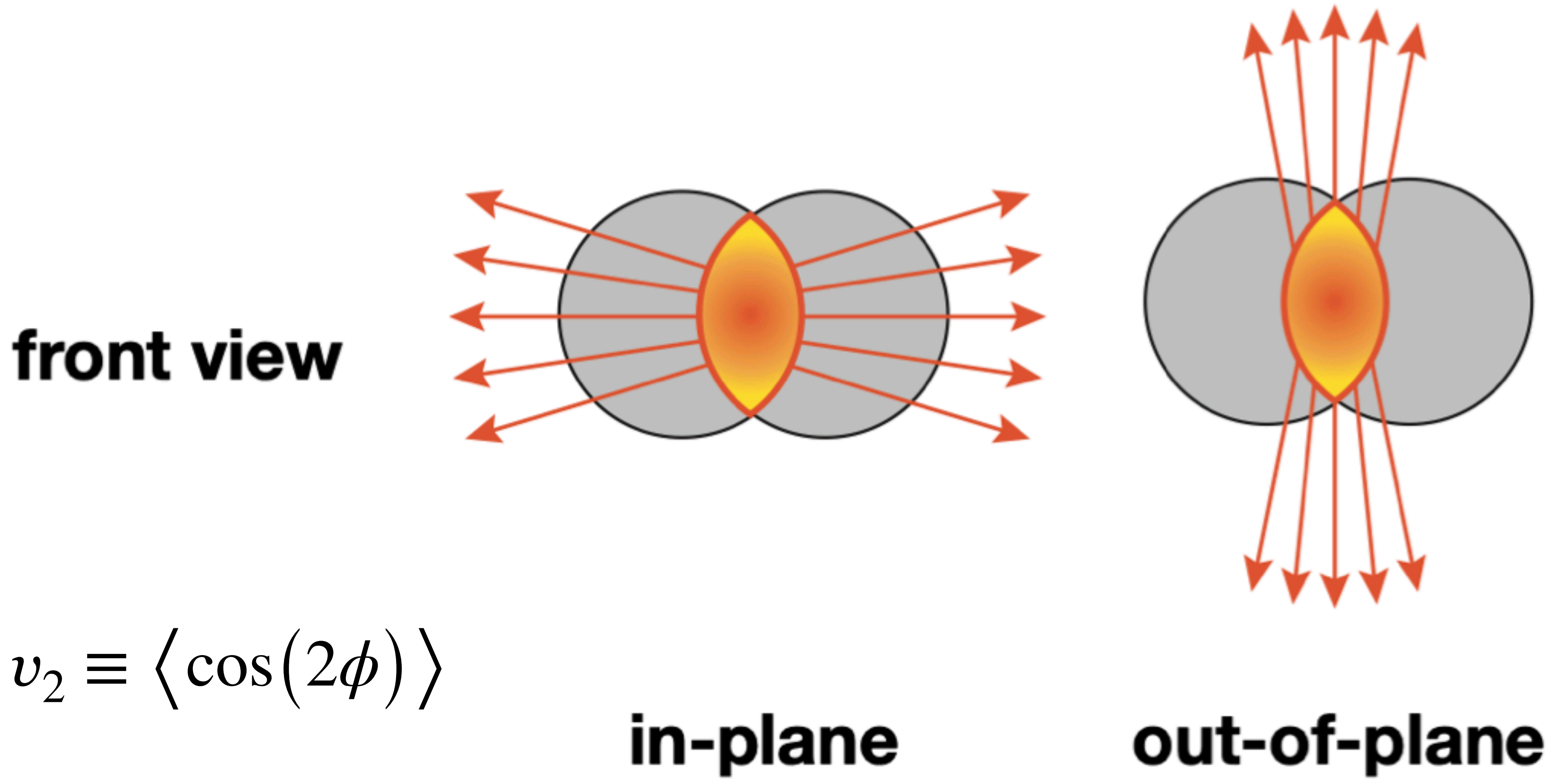



elliptic flow $v_2$ ($v_2(y \approx 0)$ ~ midrapidity)

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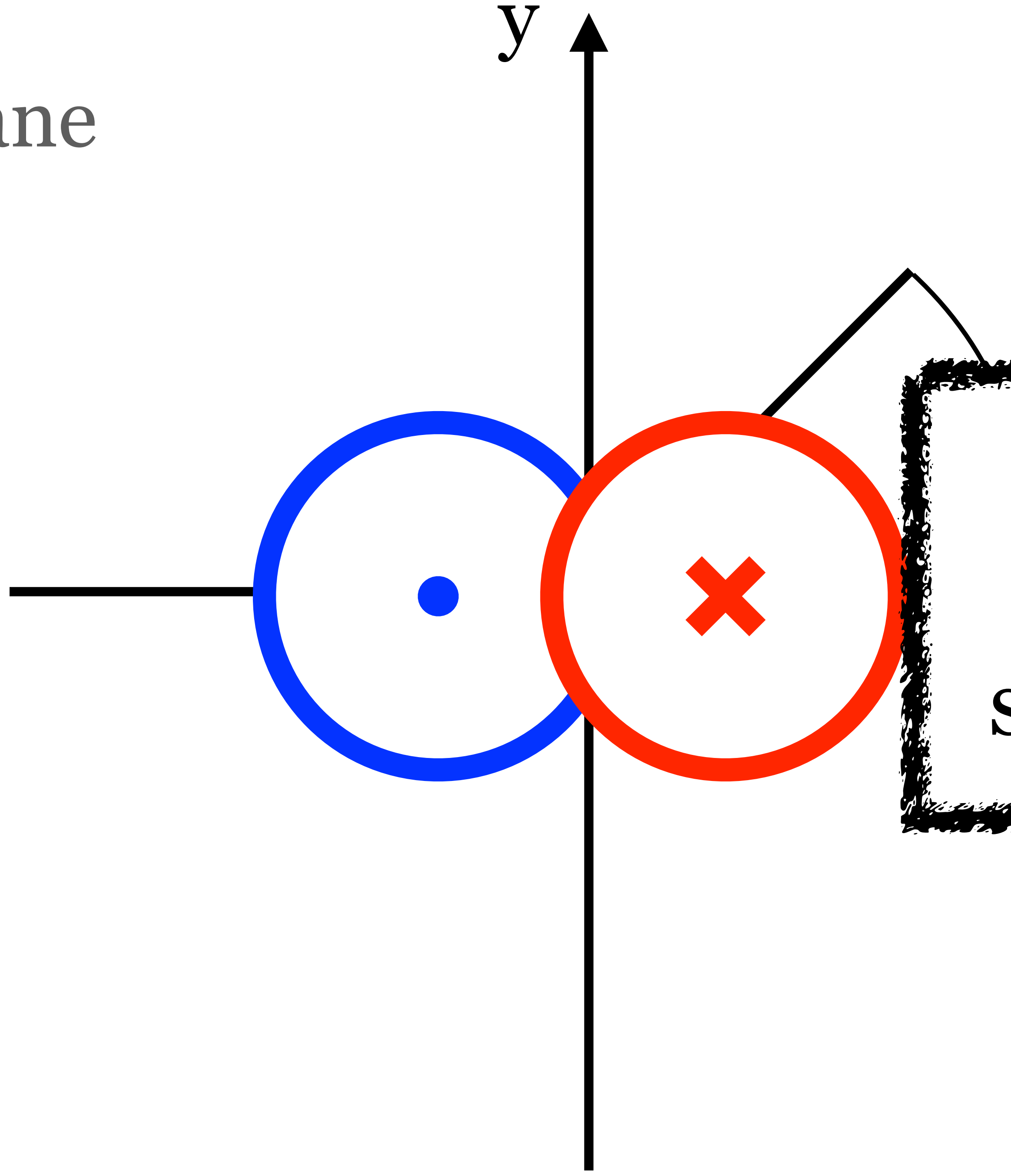


These observables are extremely sensitive to the EOS

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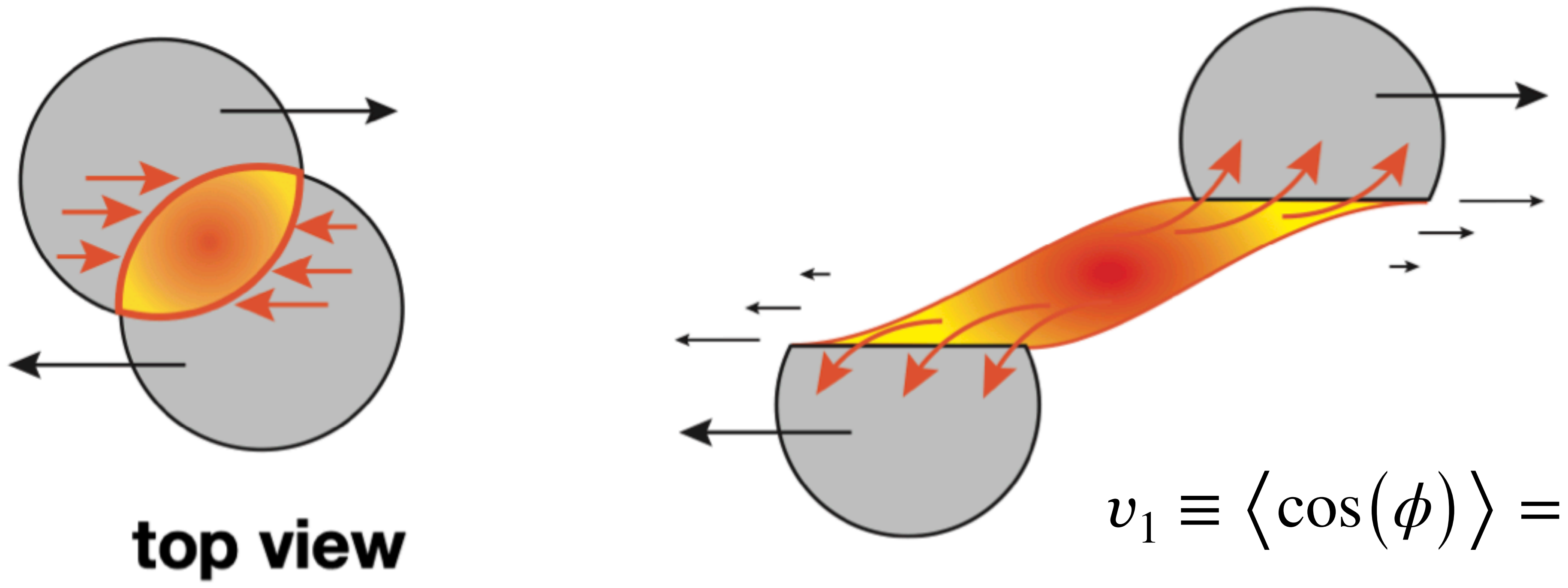


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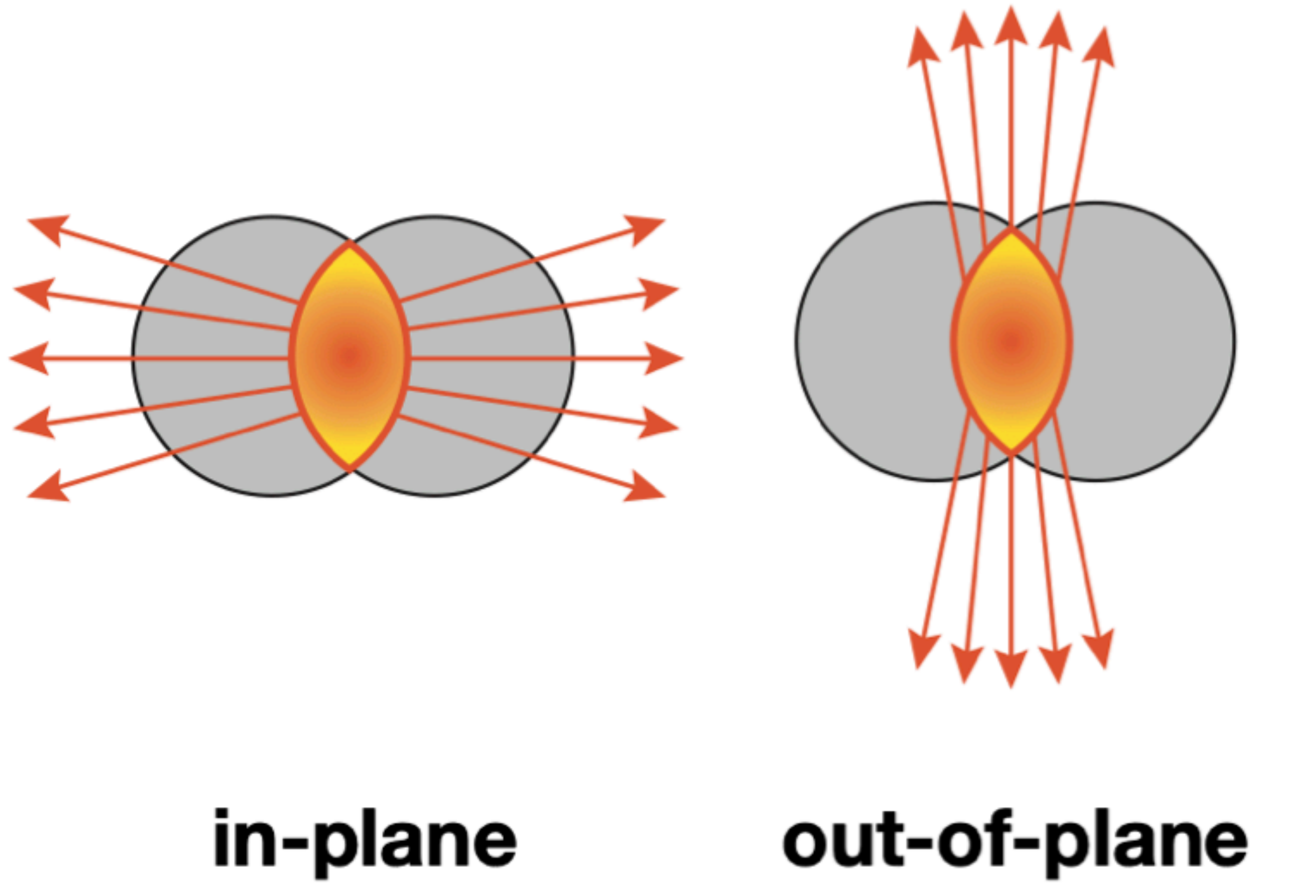
front view

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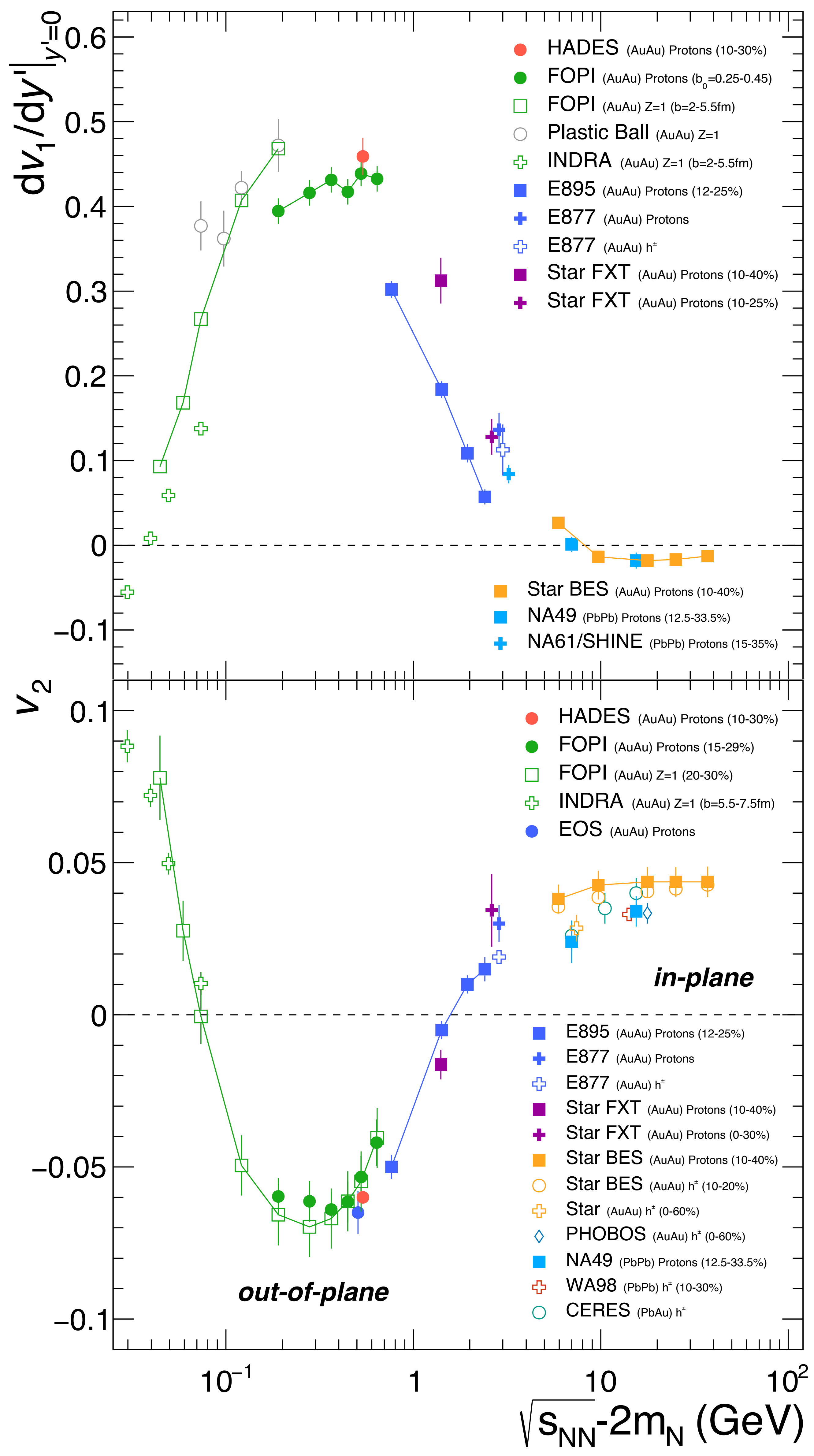
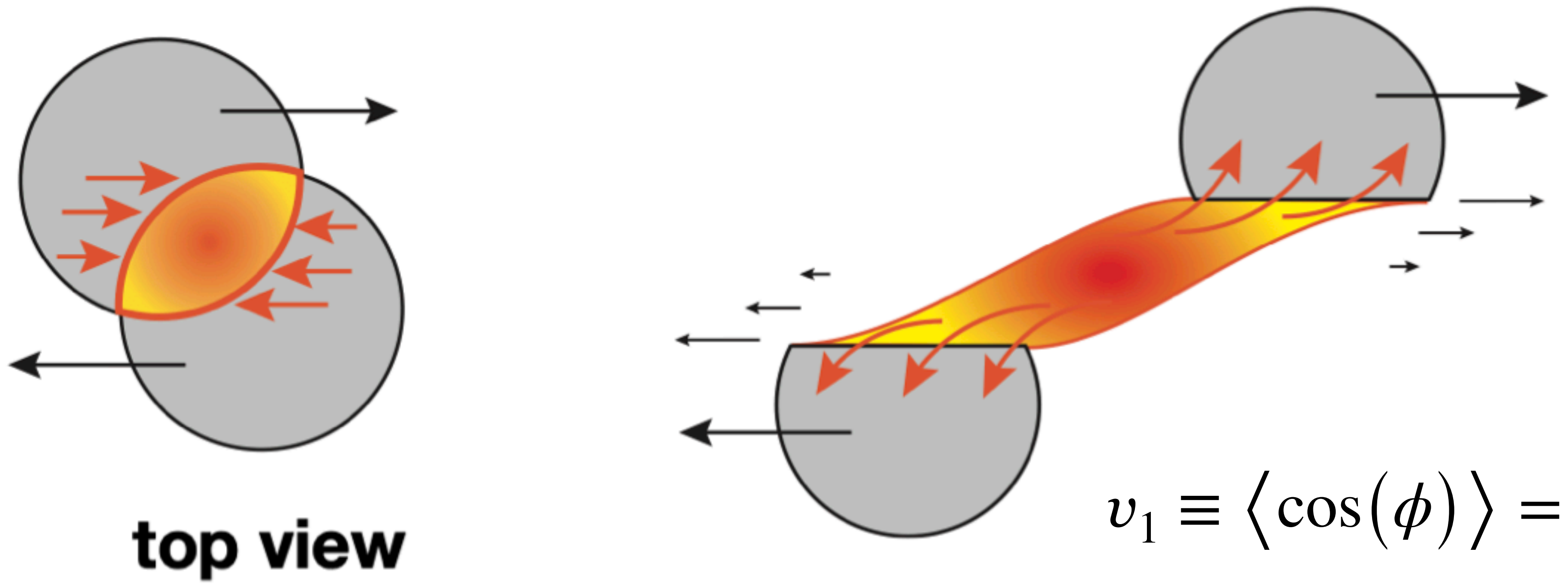



J. Adamczewski-Musch *et al.* (HADES), Eur.Phys.J.A 59 (2023) 4, 80, arXiv:2208.02740

**These observables are extremely sensitive to the EOS**

directed flow $v_1$ ($dv_1/dy$ ~ longitudinal expansion)

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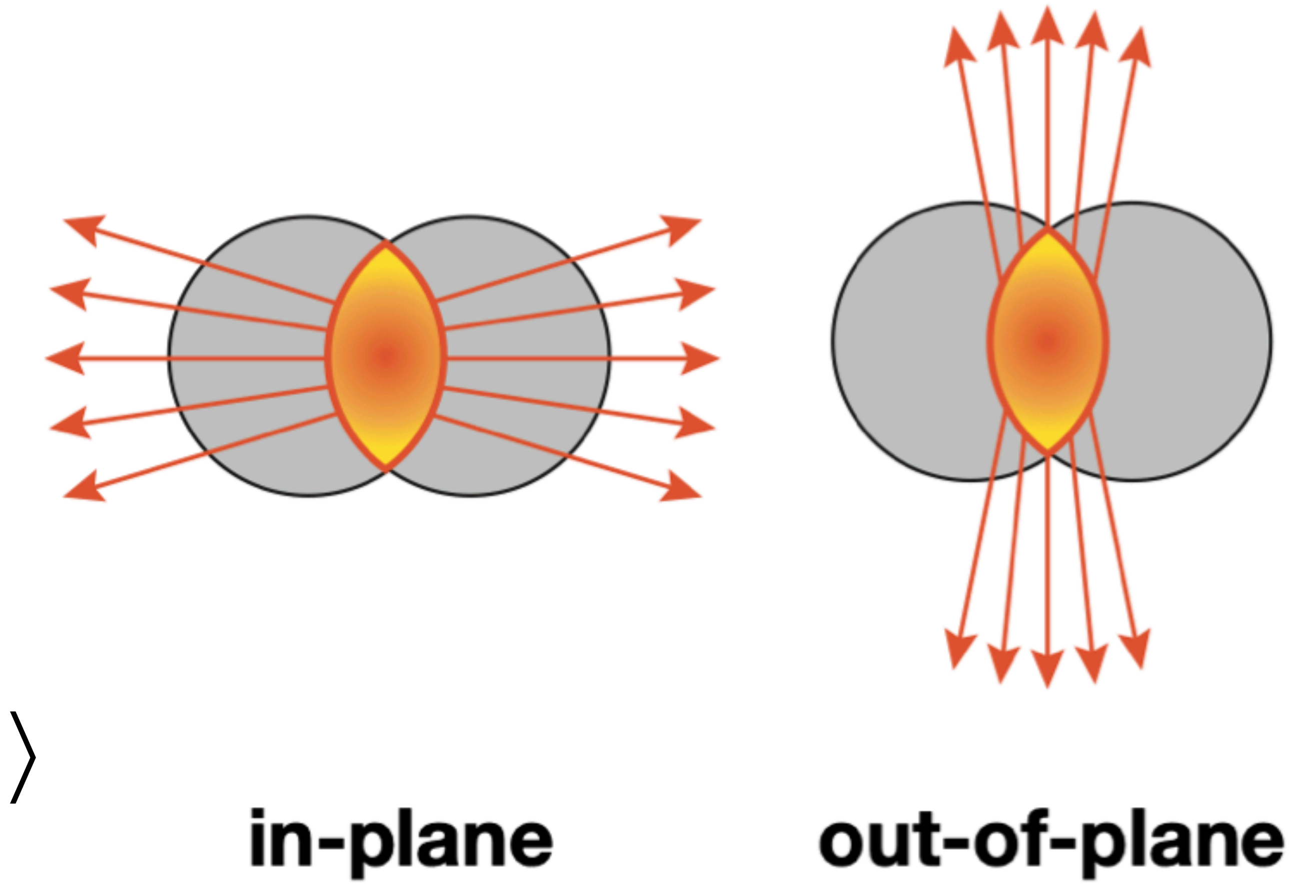


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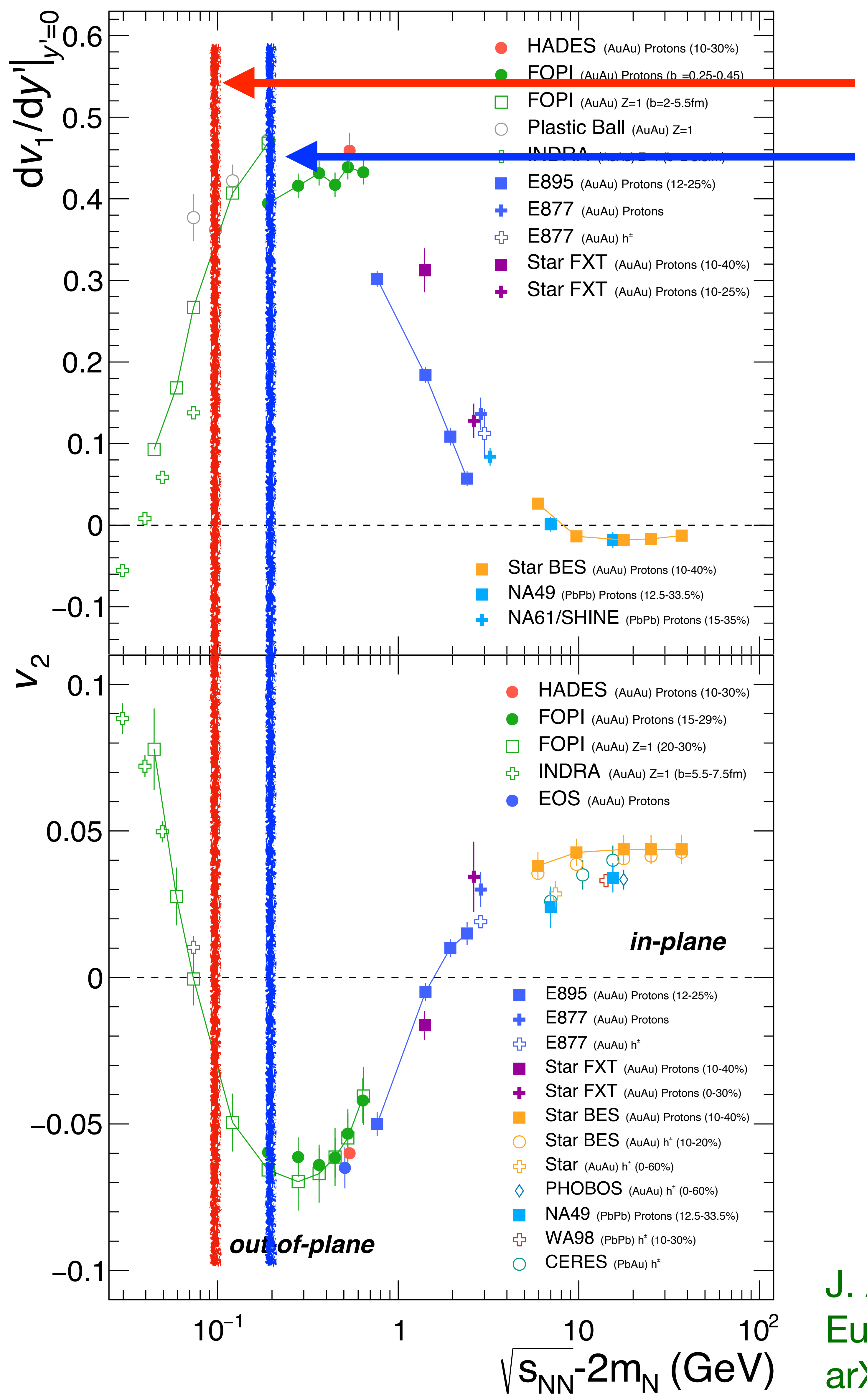


200 MeV/u

400 MeV/u

These observables are extremely sensitive to the EOS

Both observables are large at FRIB energies!

J. Adamczewski-Musch *et al.* (HADES), Eur.Phys.J.A 59 (2023) 4, 80, arXiv:2208.02740

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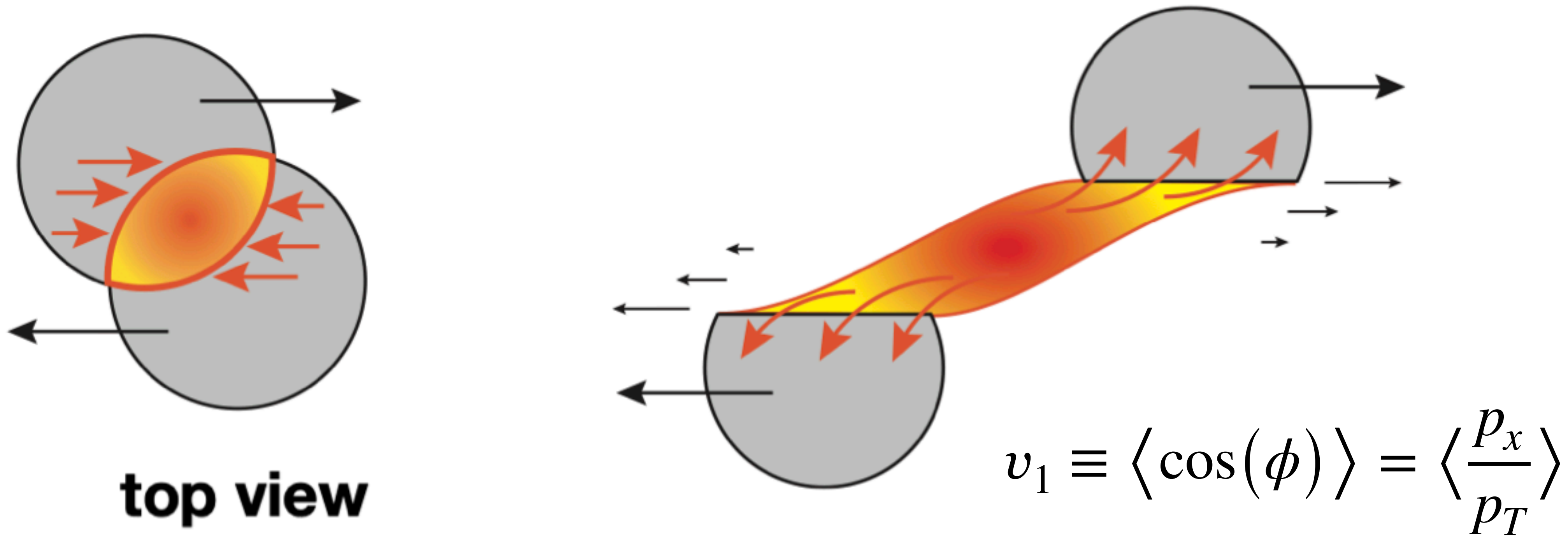


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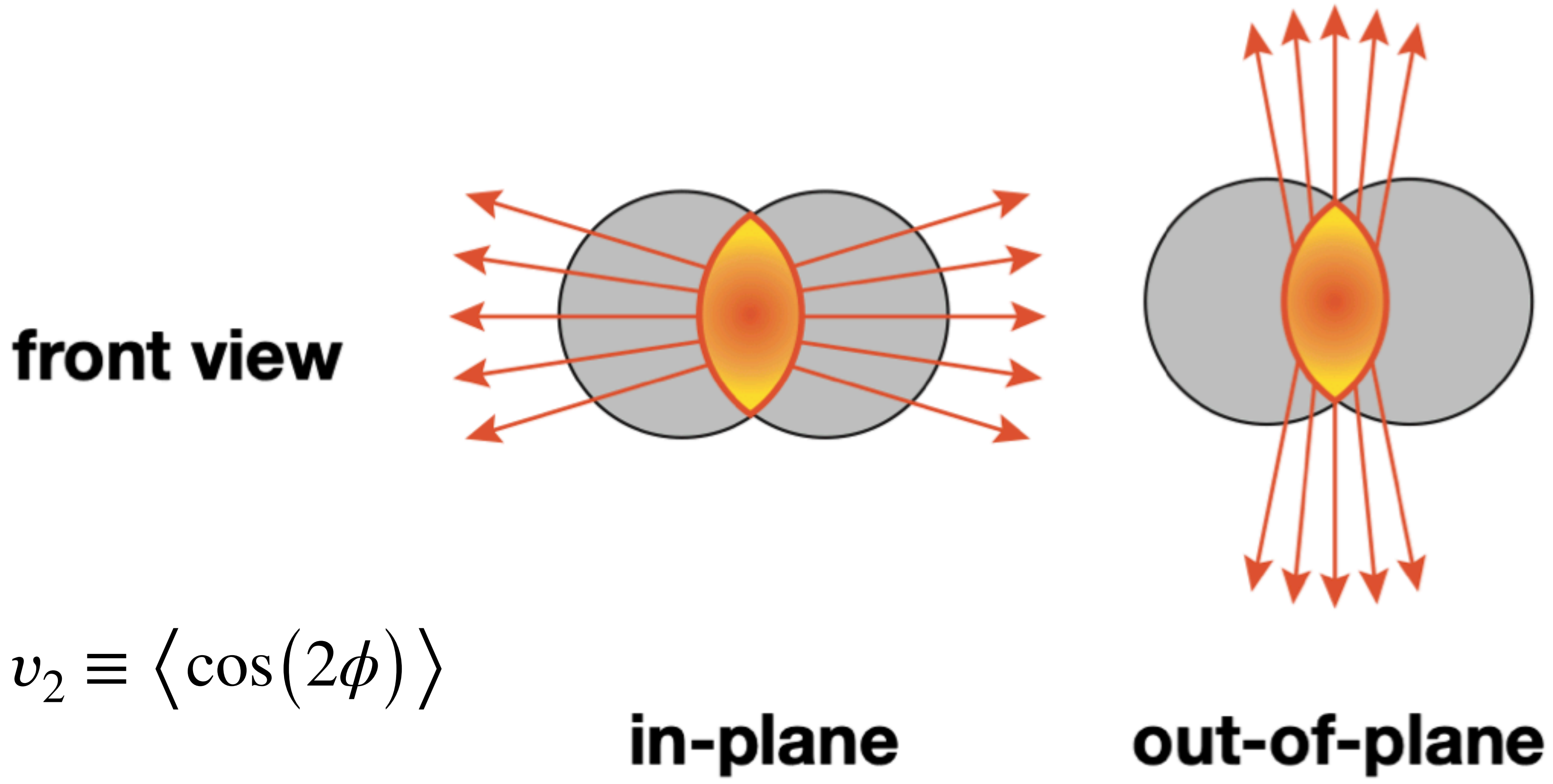
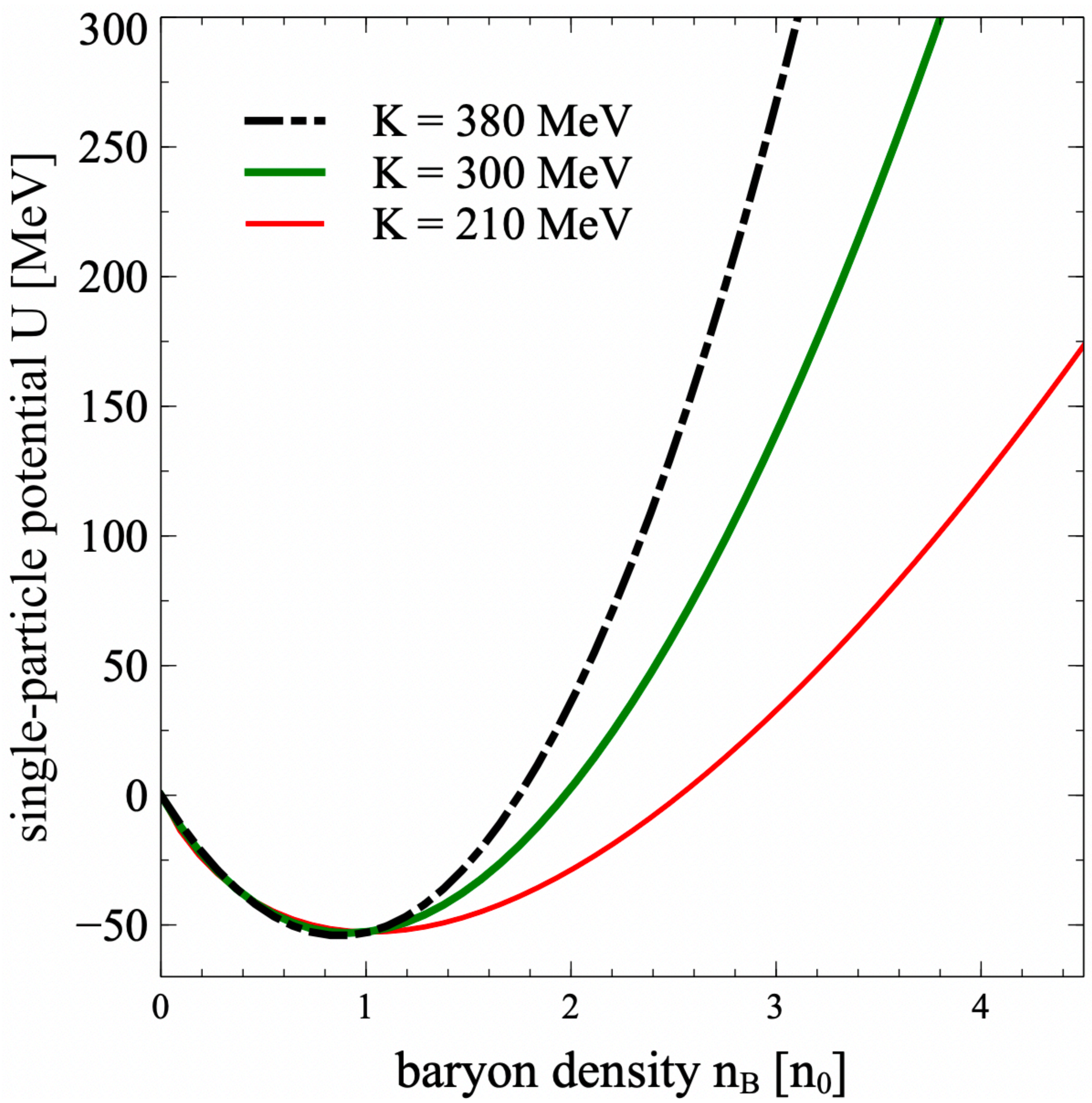
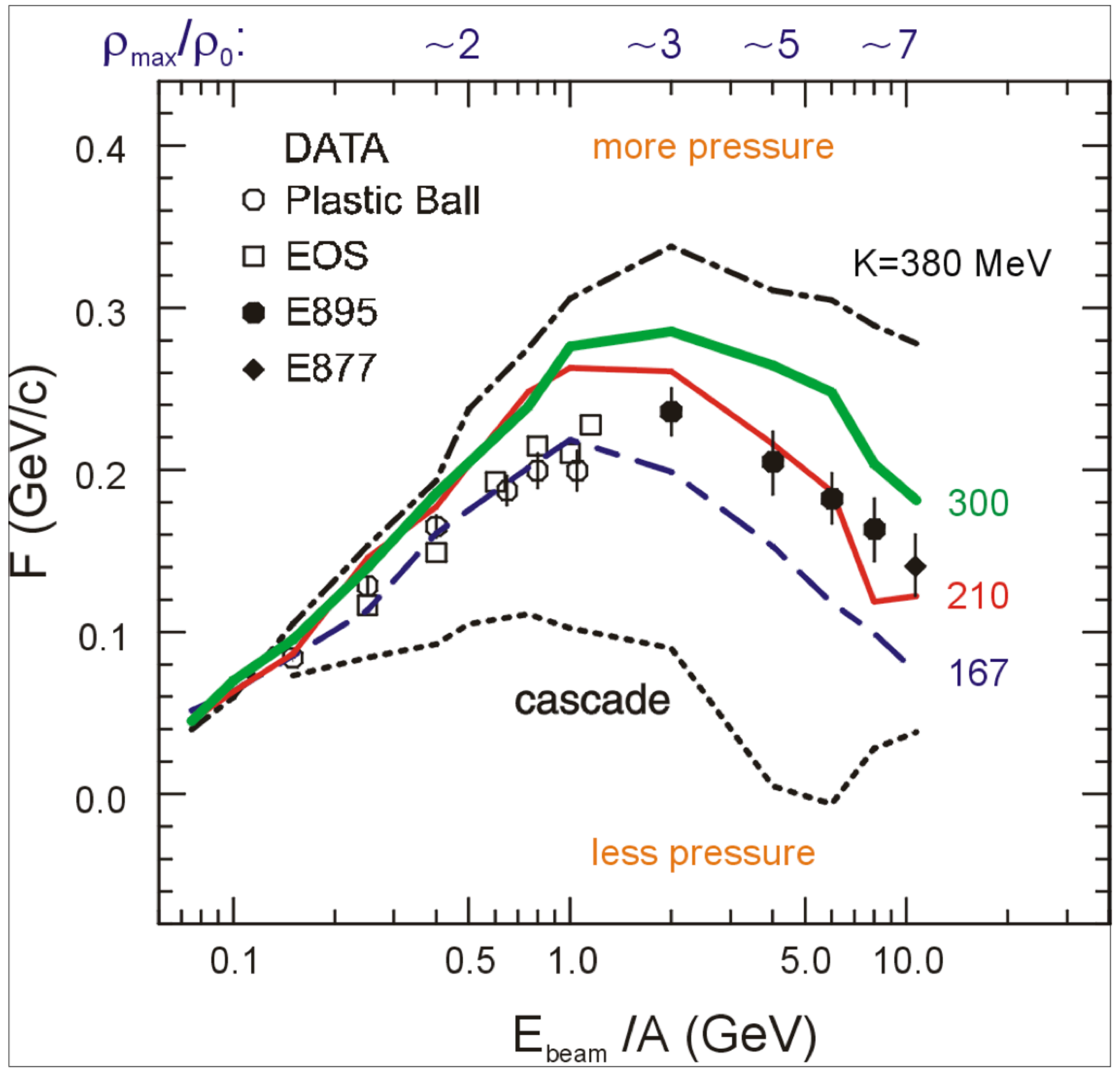



illustrations from a presentation by B. Kardan (HADES)

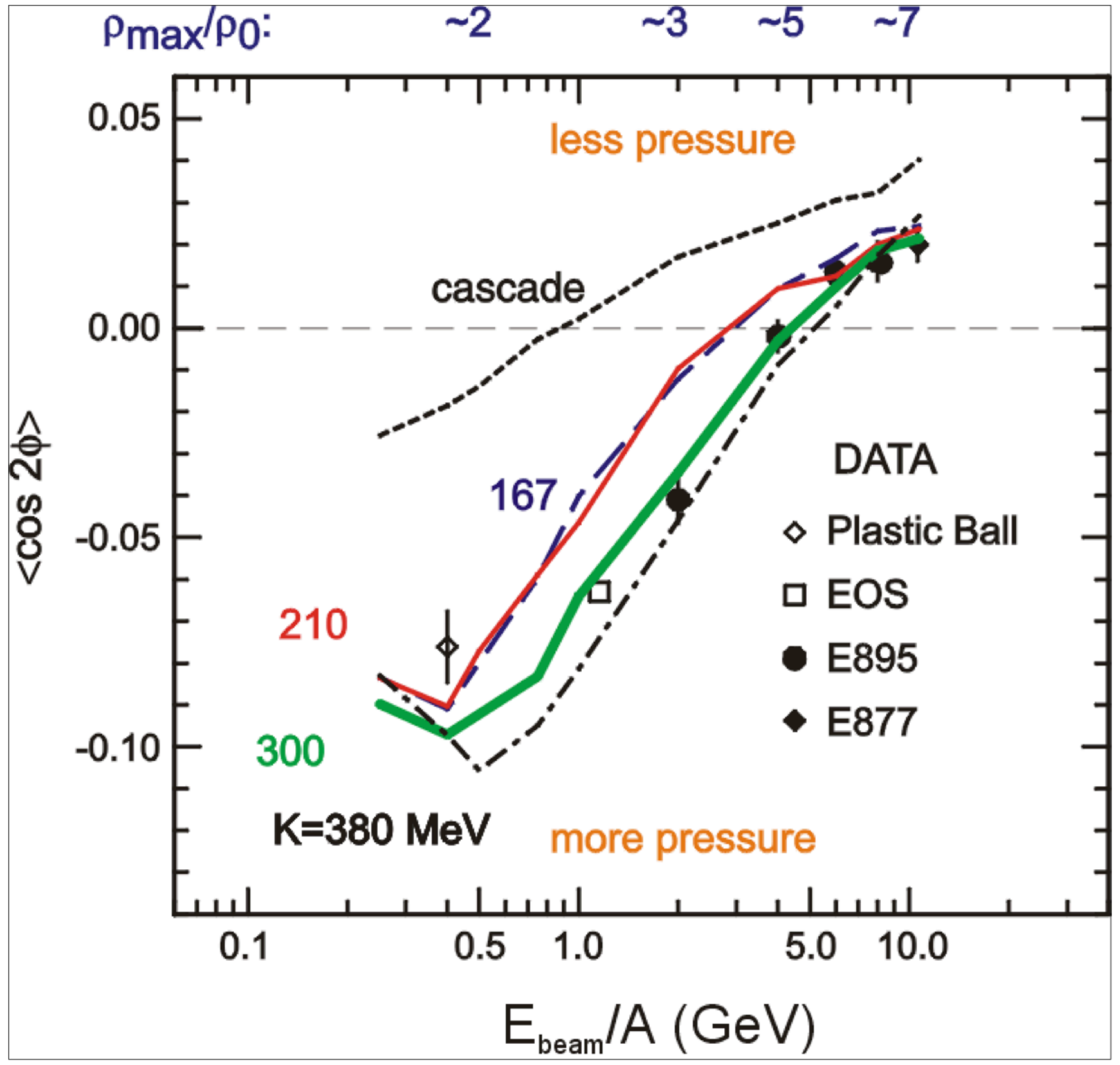
# Standard way of modeling the EOS in HICs: Skyrme potential

The most common form of the EOS in transport is the “Skyrme potential”: $U(n_B) = A\left(\frac{n_B}{n_0}\right) + B\left(\frac{n_B}{n_0}\right)^{\tau}$



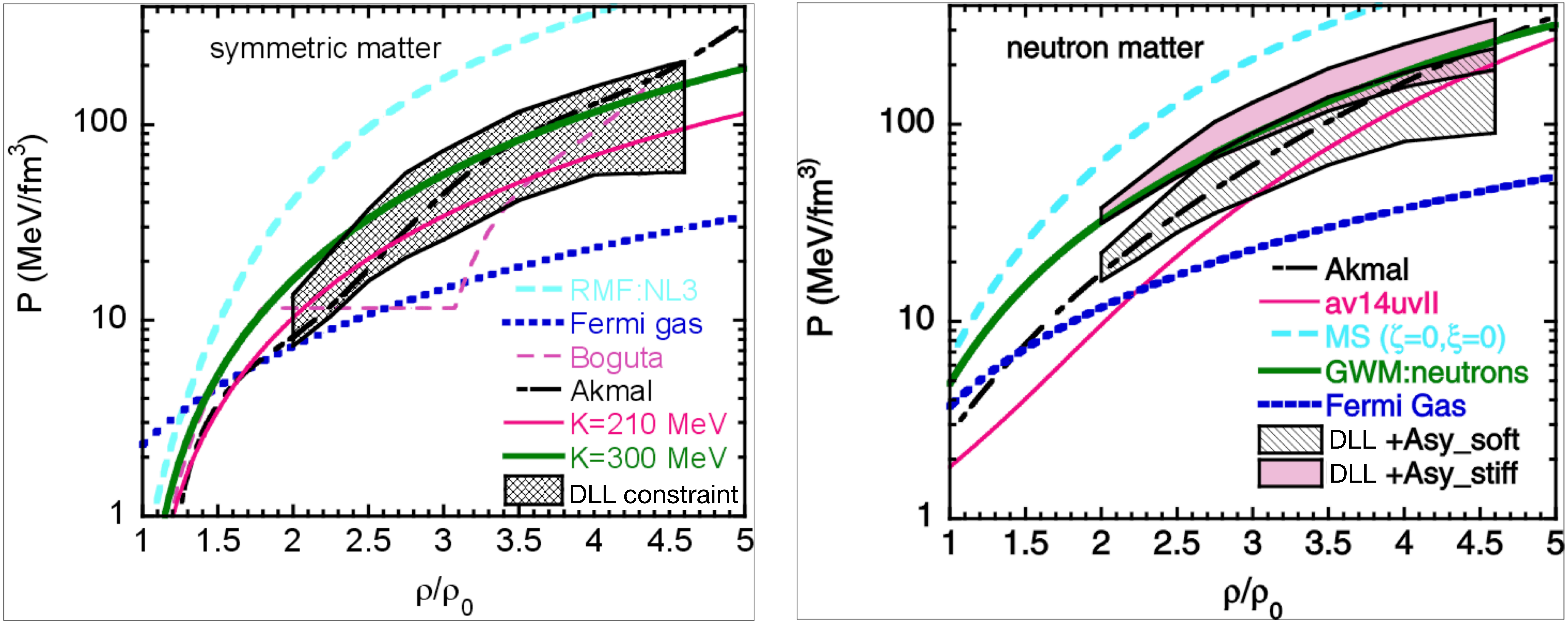

$$F = \left.\frac{d\langle p_x/A\rangle}{d(y/y_{cm})}\right|_{y/y_{cm}=1}$$




P. Danielewicz, R. Lacey, W. G. Lynch,
Science **298**, 1592–1596 (2002), arXiv:nucl-th/0208016

# Standard way of modeling the EOS in HICs: Skyrme potential

"the heavy-ion constraint"



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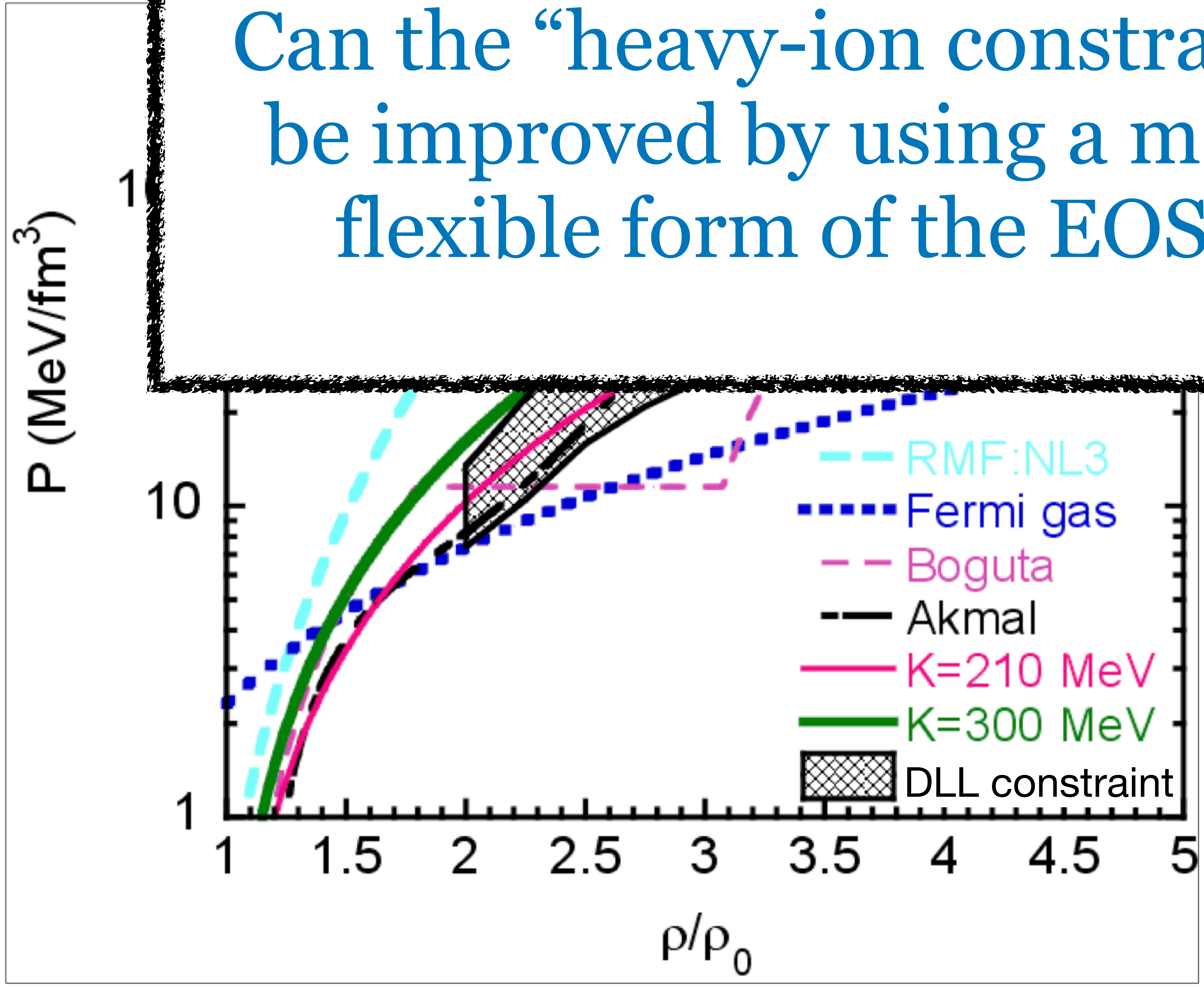
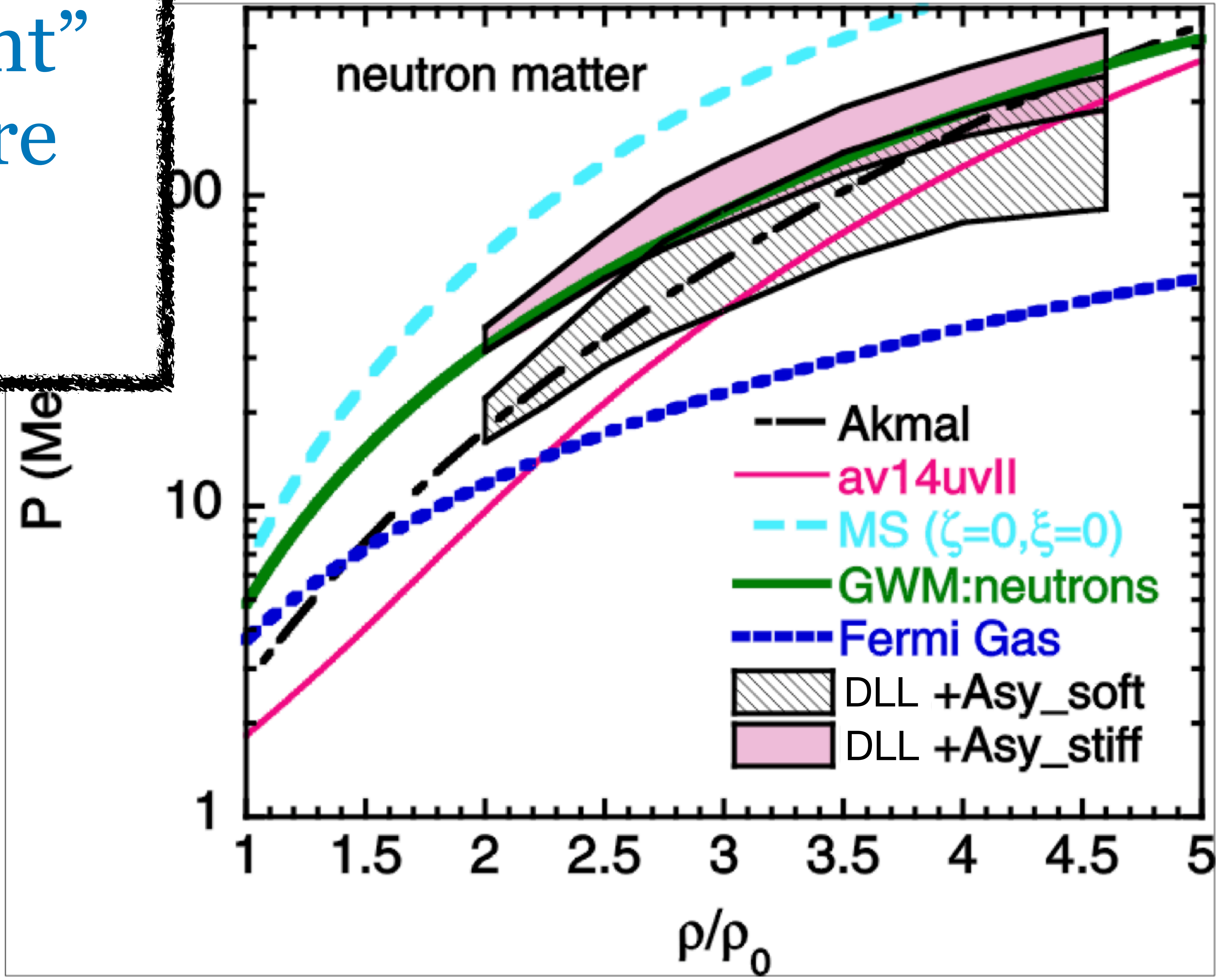



**Can the "heavy-ion constraint" be improved by using a more flexible form of the EOS?**

"the heavy-ion constraint"



# Relativistic vector density functional (VDF) model

**A. Sorensen**, V. Koch, Phys. Rev. C **104** (2021) 3, 034904, arXiv:2011.06635

inspired by relativistic Landau Fermi-liquid theory: G. Baym, S. A. Chin, Nucl. Phys. A **262**, 527 (1976)

1) Postulate the energy density of the system:

$$\mathcal{E}_N = \mathcal{E}_N[f_{\mathbf{p}}] = g \int \frac{d^3p}{(2\pi)^3}\, \epsilon_{\text{kin}}\, f_{\mathbf{p}} + \sum_{i=1}^{N} C_i \left(j_\mu j^\mu\right)^{\frac{b_i}{2}-1} \left[ j^0 j^0 - g^{00} \left(\frac{b_i - 1}{b_i}\right) j_\lambda j^\lambda \right]$$

⟵ Lorentz covariant

$j_\mu j^\mu = n_B^2$

$$\mathcal{E}_N \Big|_{\substack{\text{rest}\\\text{frame}}} = g \int \frac{d^3p}{(2\pi)^3} \sqrt{\vec{p}^{\,2} + m^2}\, f_{\mathbf{p}} + \sum_{i=1}^{N} \frac{C_i}{b_i} n_B^{b_i}$$

⟵ mean-field interactions parameterized by $C_i$ and $b_i$

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3) Get EOMs:

$$\frac{dx^i}{dt} \equiv -\frac{\partial \varepsilon_{\mathbf{p}}}{\partial p_i}\,, \qquad \frac{dp^i}{dt} \equiv \frac{\partial \varepsilon_{\mathbf{p}}}{\partial x_i}$$

⟵ input to transport code; use in Boltzmann eq. to obtain $T^{\mu\nu}$

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$$\mathcal{E}_N = \mathcal{E}_N[f_{\mathbf{p}}] = g \int \frac{d^3p}{(2\pi)^3}\, \epsilon_{\text{kin}}\, f_{\mathbf{p}} + \sum_{i=1}^{N} C_i \left(j_\mu j^\mu\right)^{\frac{b_i}{2}-1} \left[ j^0 j^0 - g^{00} \left( \frac{b_i - 1}{b_i} \right) j_\lambda j^\lambda \right]$$

⟵ Lorentz covariant $\qquad j_\mu j^\mu = n_B^2$

$$\mathcal{E}_N \Big|_{\substack{\text{rest}\\ \text{frame}}} = g \int \frac{d^3p}{(2\pi)^3} \sqrt{\vec{p}^{\,2} + m^2}\, f_{\mathbf{p}} + \sum_{i=1}^{N} \frac{C_i}{b_i} n_B^{b_i}$$

⟵ mean-field interactions parameterized by $C_i$ and $b_i$

2) Quasiparticle energy:

$$\varepsilon_{\mathbf{p}} \equiv \frac{\delta \mathcal{E}[f_{\mathbf{p}}]}{\delta f_{\mathbf{p}}} = \epsilon_{\text{kin}} + \sum_{i=1}^{N} C_i \left(j_\mu j^\mu\right)^{\frac{b_i}{2}-1} j^0$$

3) Get EOMs:

$$\frac{dx^i}{dt} \equiv -\frac{\partial \varepsilon_{\mathbf{p}}}{\partial p_i}, \qquad \frac{dp^i}{dt} \equiv \frac{\partial \varepsilon_{\mathbf{p}}}{\partial x_i}$$

⟵ input to transport code; use in Boltzmann eq. to obtain $T^{\mu\nu}$

4) Use $T^{\mu\nu}$ to get the pressure, *assuming equilibrium*:

$$P_N = \frac{1}{3} \sum_k T^{kk} \Big|_{\substack{\text{rest}\\ \text{frame}}} = g \int \frac{d^3p}{(2\pi)^3}\, T\, \ln\left[1 + e^{-\beta(\varepsilon_{\mathbf{p}} - \mu_B)}\right] + \sum_{i=1}^{N} C_i \frac{b_i - 1}{b_i} n_B^{b_i}$$

# Relativistic vector density functional (VDF) model

**A. Sorensen**, V. Koch, Phys. Rev. C **104** (2021) 3, 034904, arXiv:2011.06635

inspired by relativistic Landau Fermi-liquid theory: G. Baym, S. A. Chin, Nucl. Phys. A **262**, 527 (1976)

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⟵ mean-field interactions parameterized by $C_i$ and $b_i$

**thermodynamic consistency!**

2) Quasiparticle energy:

$$\varepsilon_{\mathbf{p}} \equiv \frac{\delta \mathcal{E}[f_{\mathbf{p}}]}{\delta f_{\mathbf{p}}} = \epsilon_{\text{kin}} + \sum_{i=1}^{N} C_i \left(j_\mu j^\mu\right)^{\frac{b_i}{2}-1} j^0$$

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# VDF model: two 1st order phase transitions

A. **Sorensen**, V. Koch, Phys. Rev. C **104** (2021) 3, 034904, arXiv:2011.06635

Systems with two 1st order phase transitions: nuclear and “quark/hadron”, or “QGP-like”

- degrees of freedom: nucleons
- “QGP-like” PT: “more dense” matter coexists with “less dense” matter
- minimal model: 4 interactions terms = 8 parameters to fix:

$$P = g \int \frac{d^3p}{(2\pi)^3}\, T \, \ln\left[1 + e^{-\beta(\varepsilon_{\mathbf{p}} - \mu_B)}\right] + \sum_{i=1}^{N=4} C_i \frac{b_i - 1}{b_i} n_B^{b_i}$$

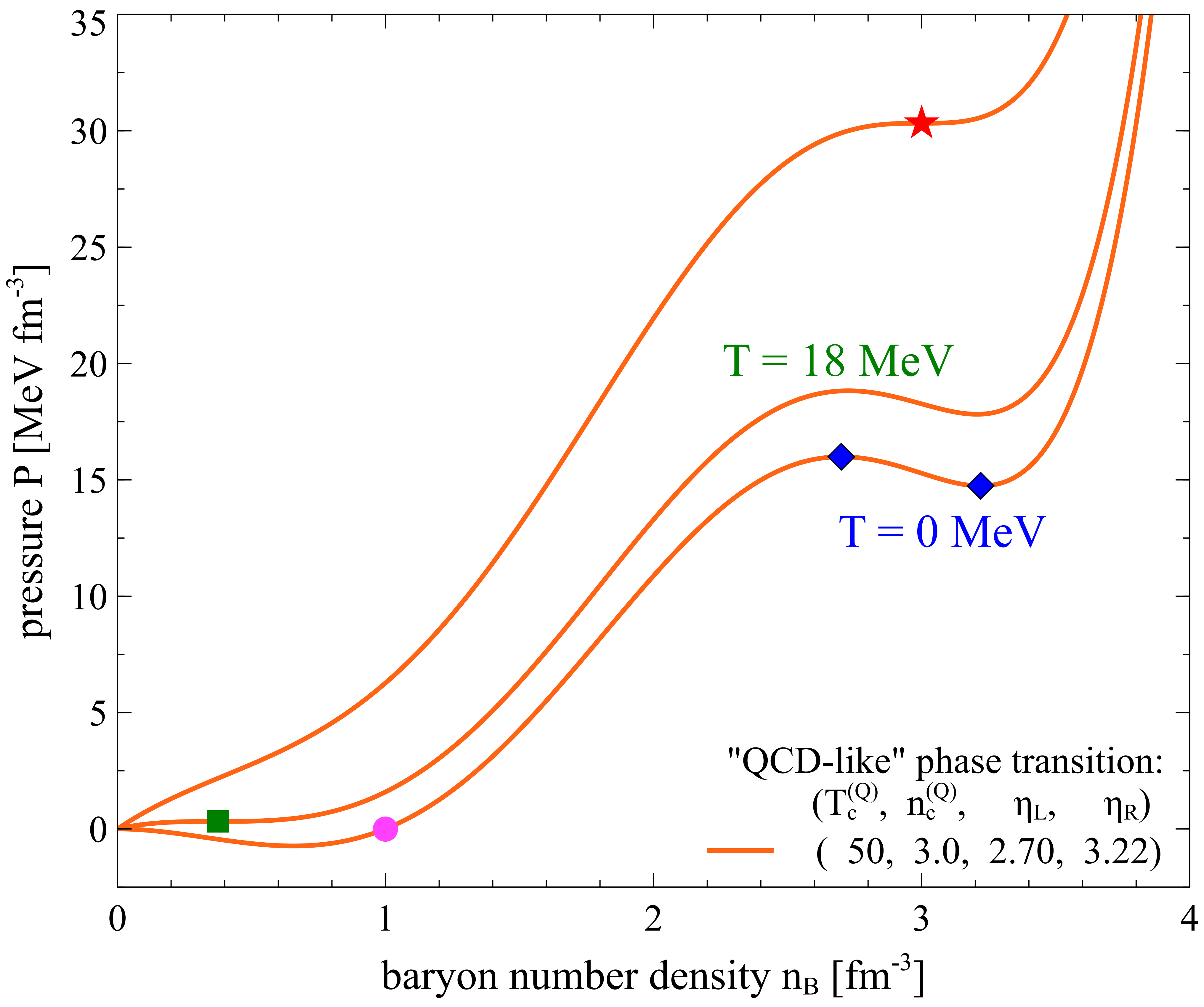
$C_i$ and $b_i$ are fitted to reproduce:

$n_0 = 0.160\ \mathrm{fm}^{-3}$, $E_B = -16.3$ MeV

$T_c^{(N)} = 18$ MeV, $n_c^{(N)} = 0.375 n_0$

$T_c^{(Q)} = ?$, $n_c^{(Q)} = ?$

$\eta_L = ?$, $\eta_R = ?$



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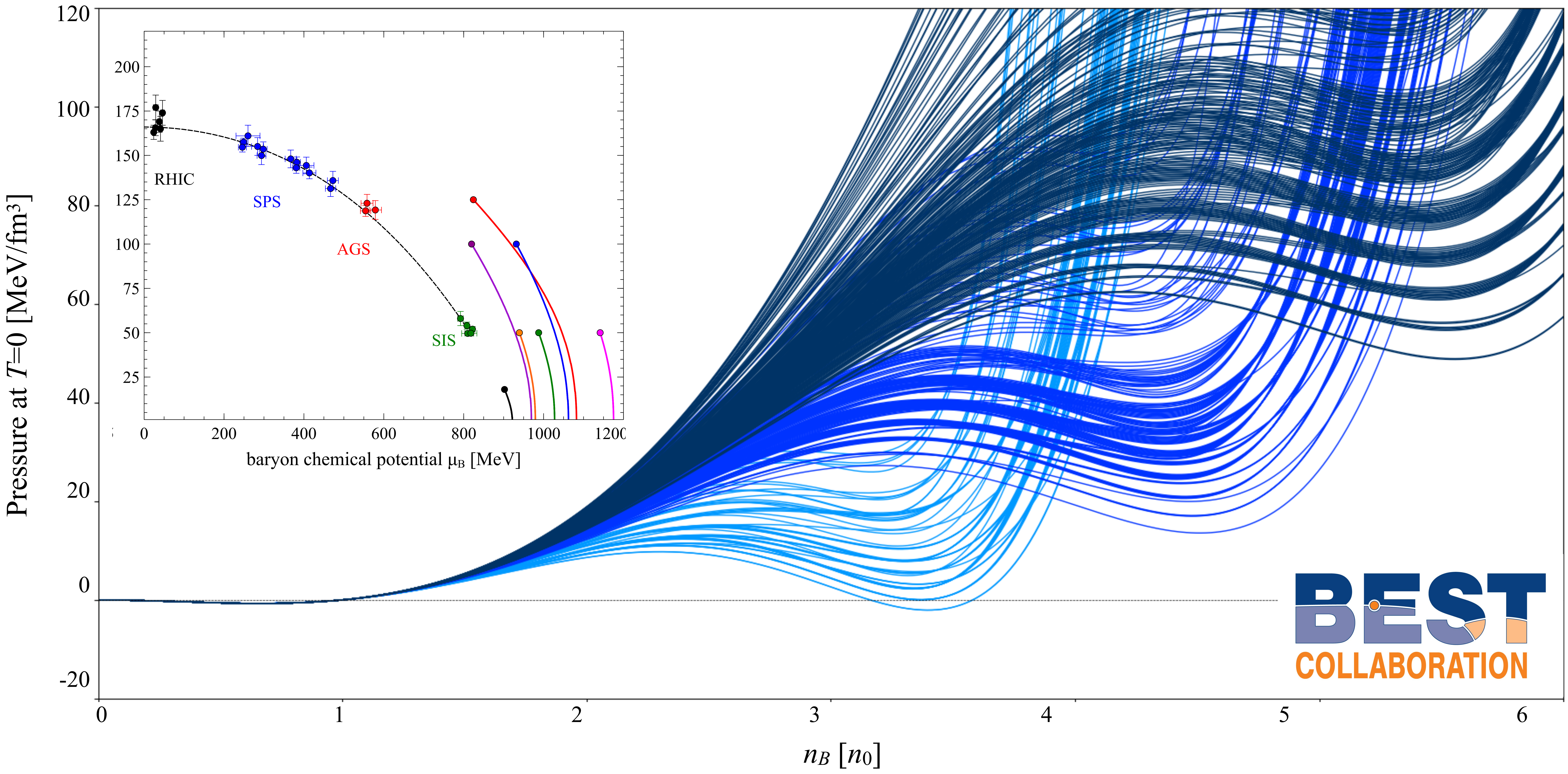
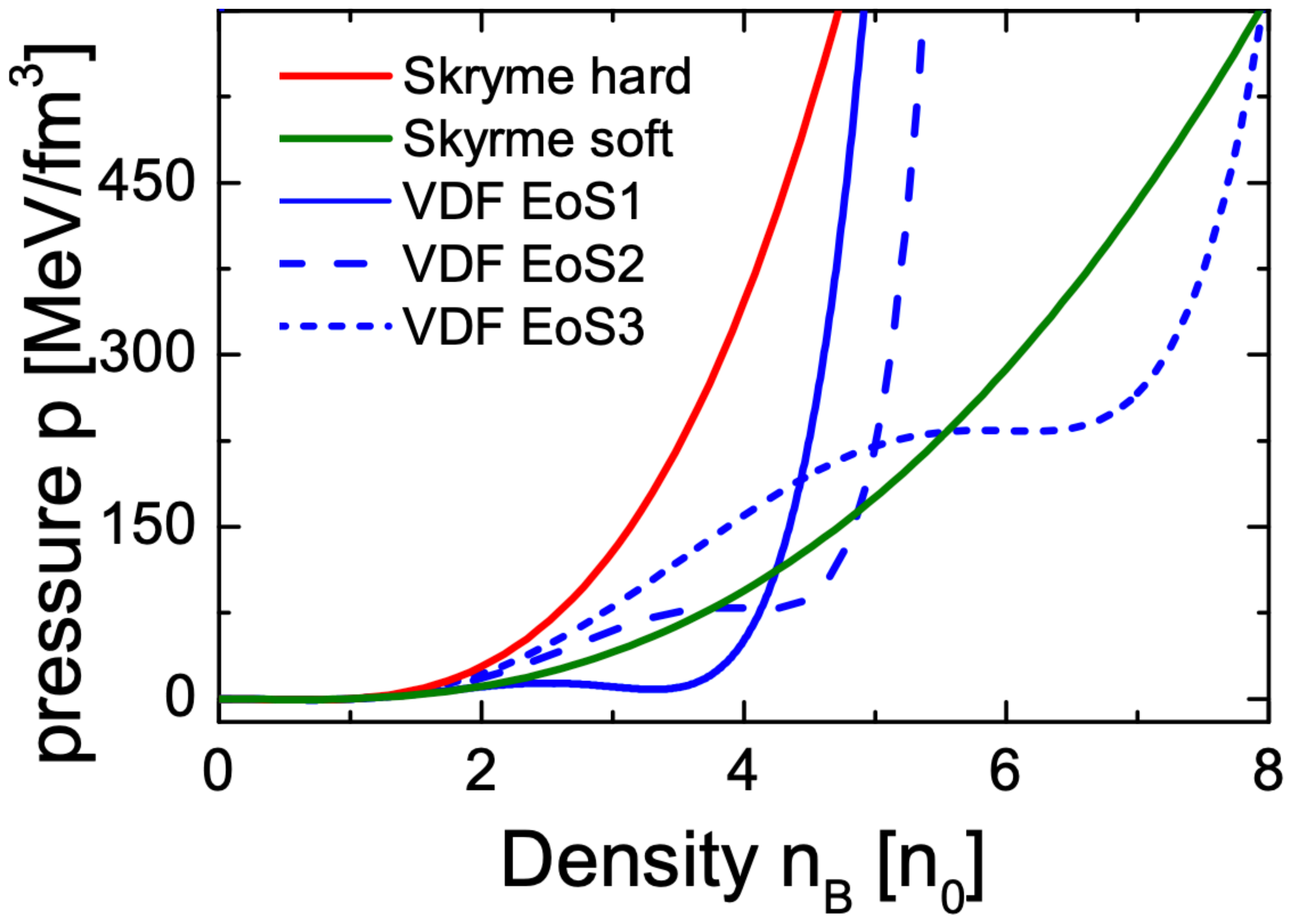
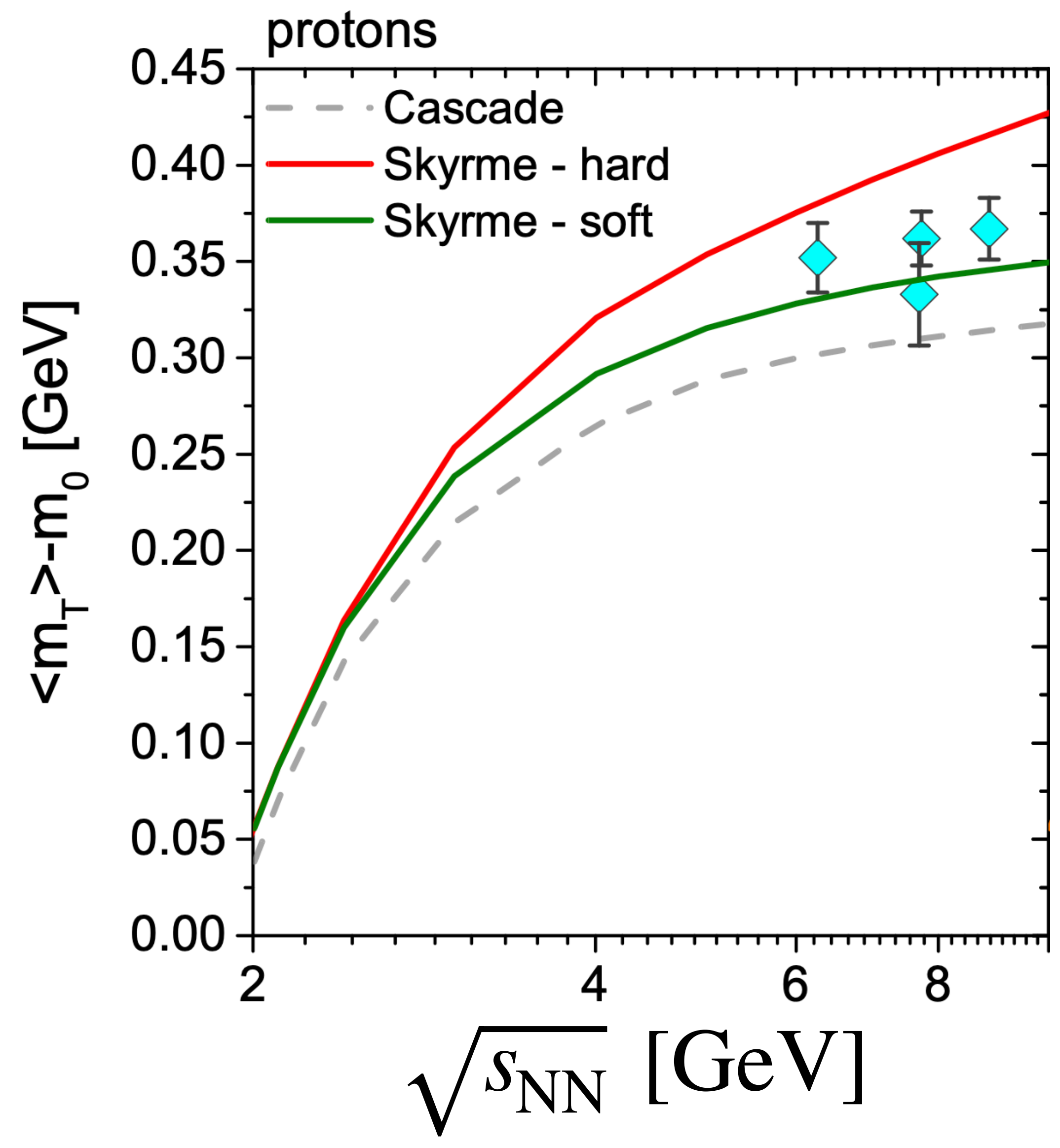
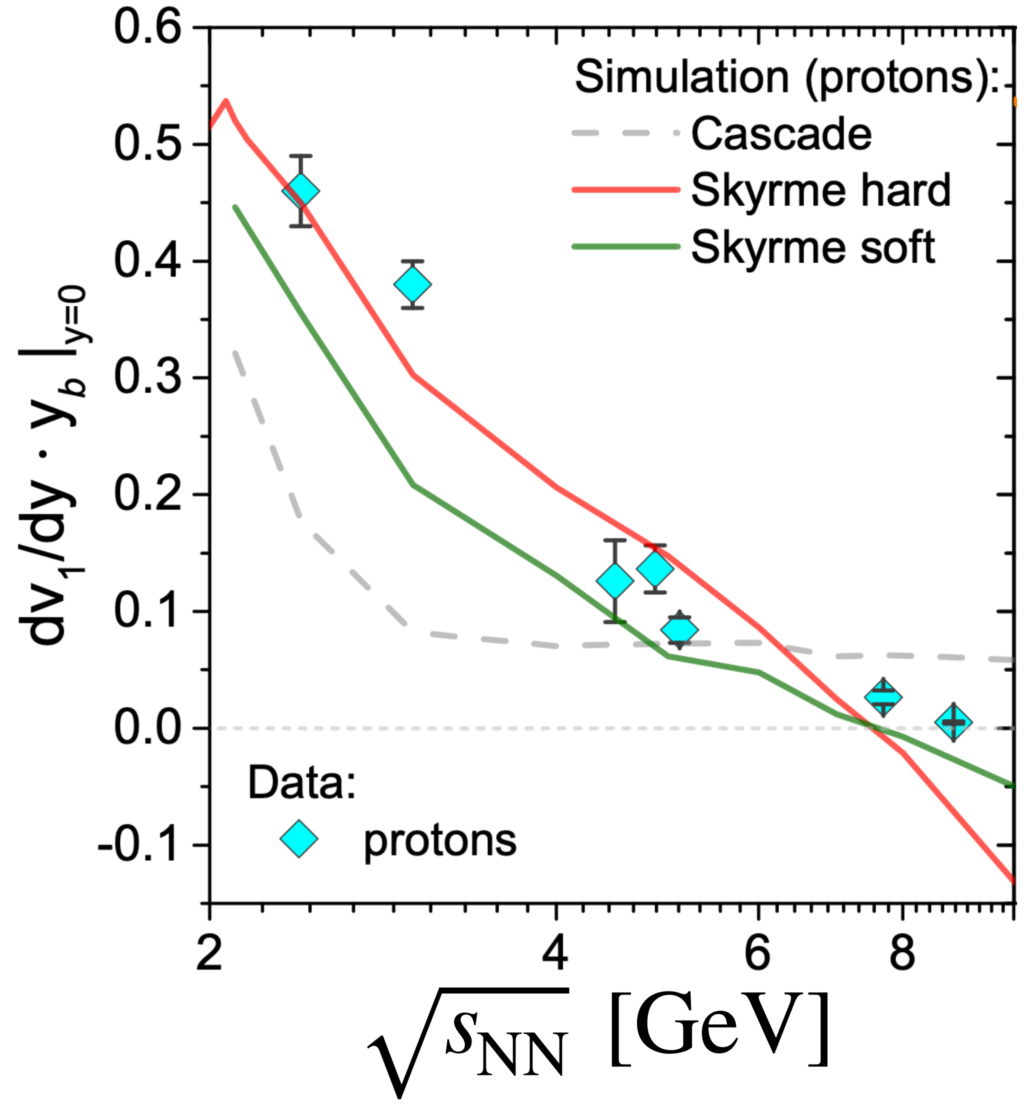


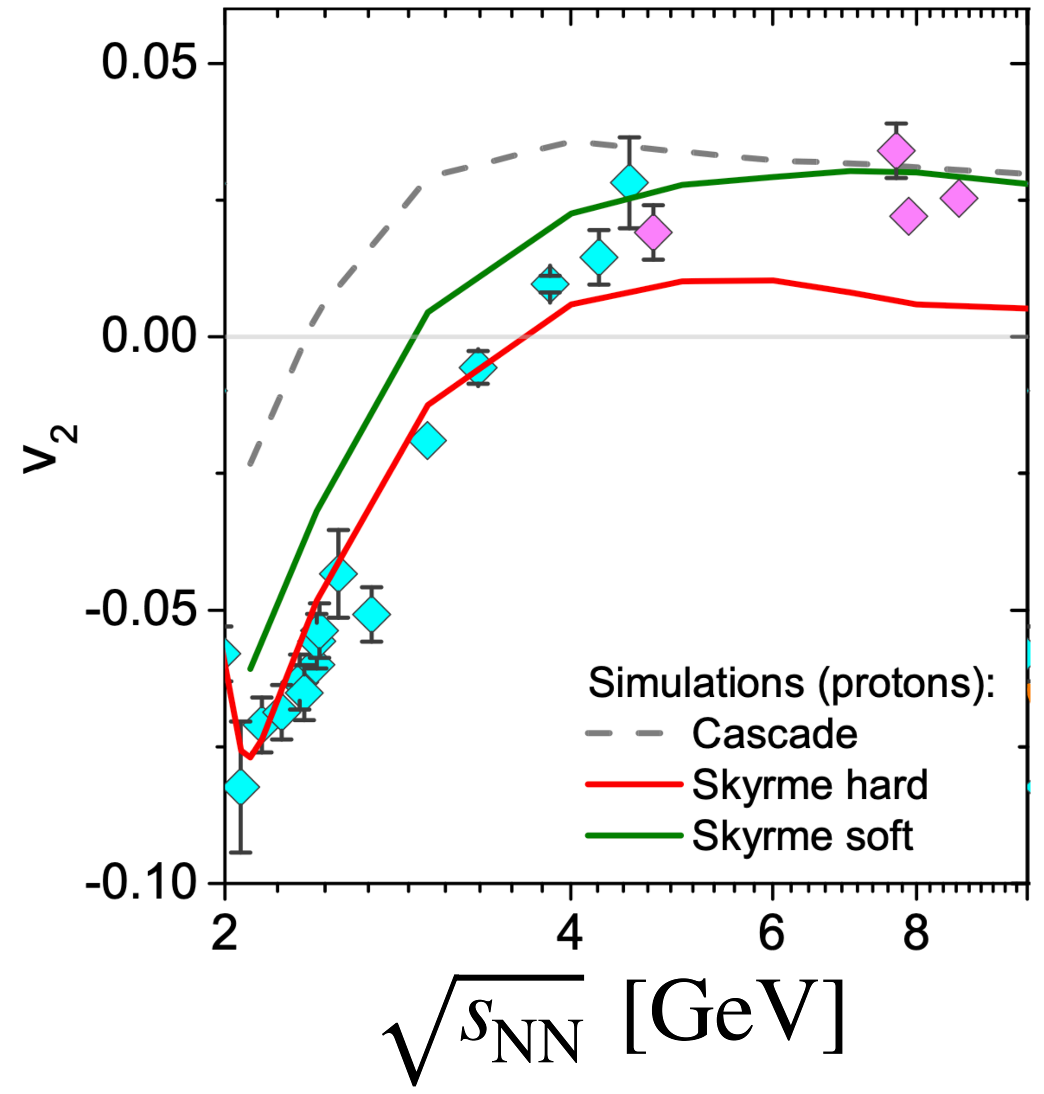
# Results from UrQMD with (non-relativistic) VDF

J. Steinheimer, A. Motornenko, **A. Sorensen**, Y. Nara, V. Koch,
M. Bleicher, Eur. Phys. J. C **82**, 10, 911 (2022) arXiv:2208.12091




| EoS | $T_c^{(N)}$[MeV] | $n_c^{(Q)}[n_0]$ | $T_c^{(Q)}$[MeV] | $K_0$[MeV] |
|---|---|---|---|---|
| VDF1 | 18 | 3.0 | 100 | 261 |
| VDF2 | 18 | 4.0 | 50 | 279 |
| VDF3 | 22 | 6.0 | 50 | 356 |

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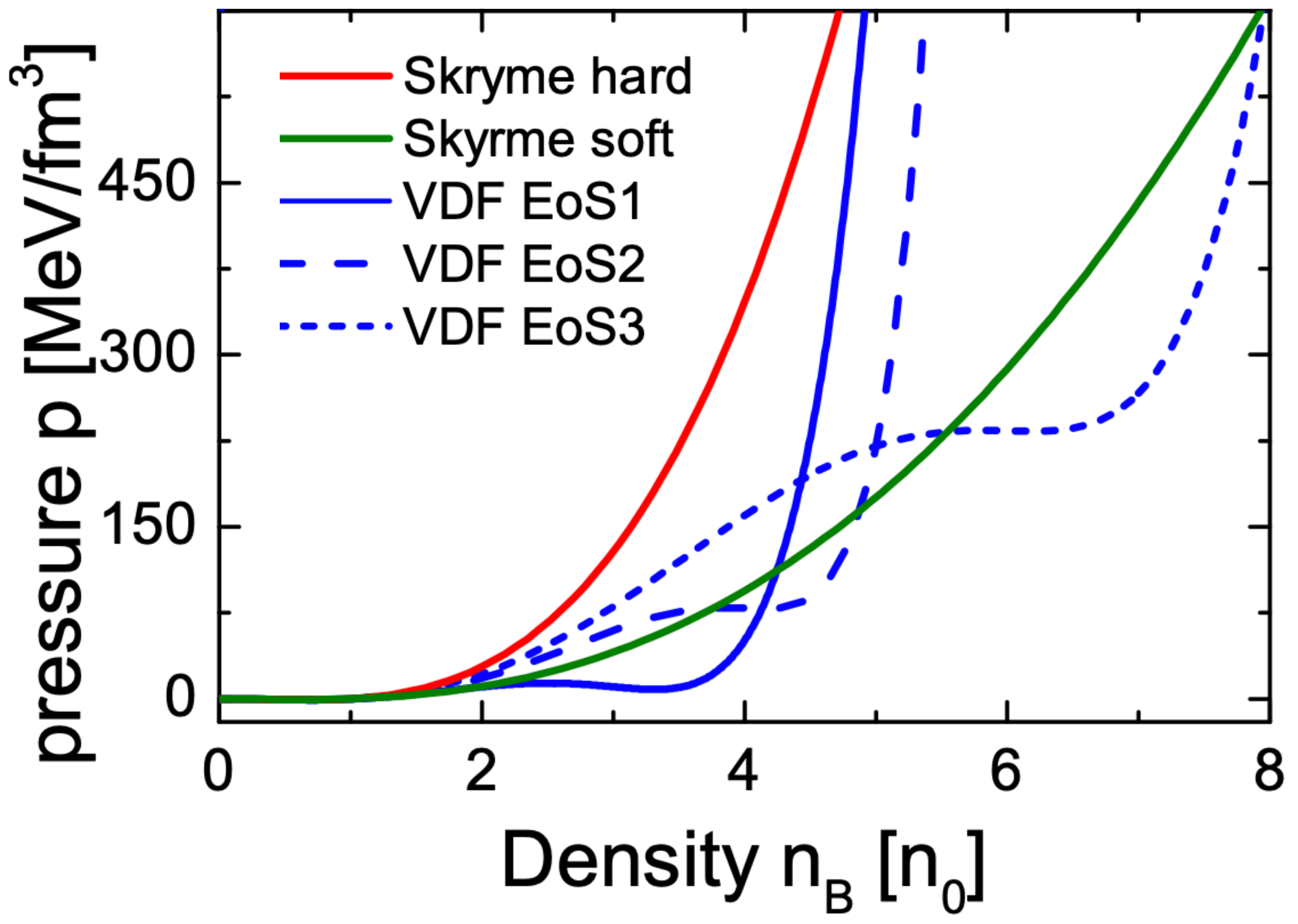


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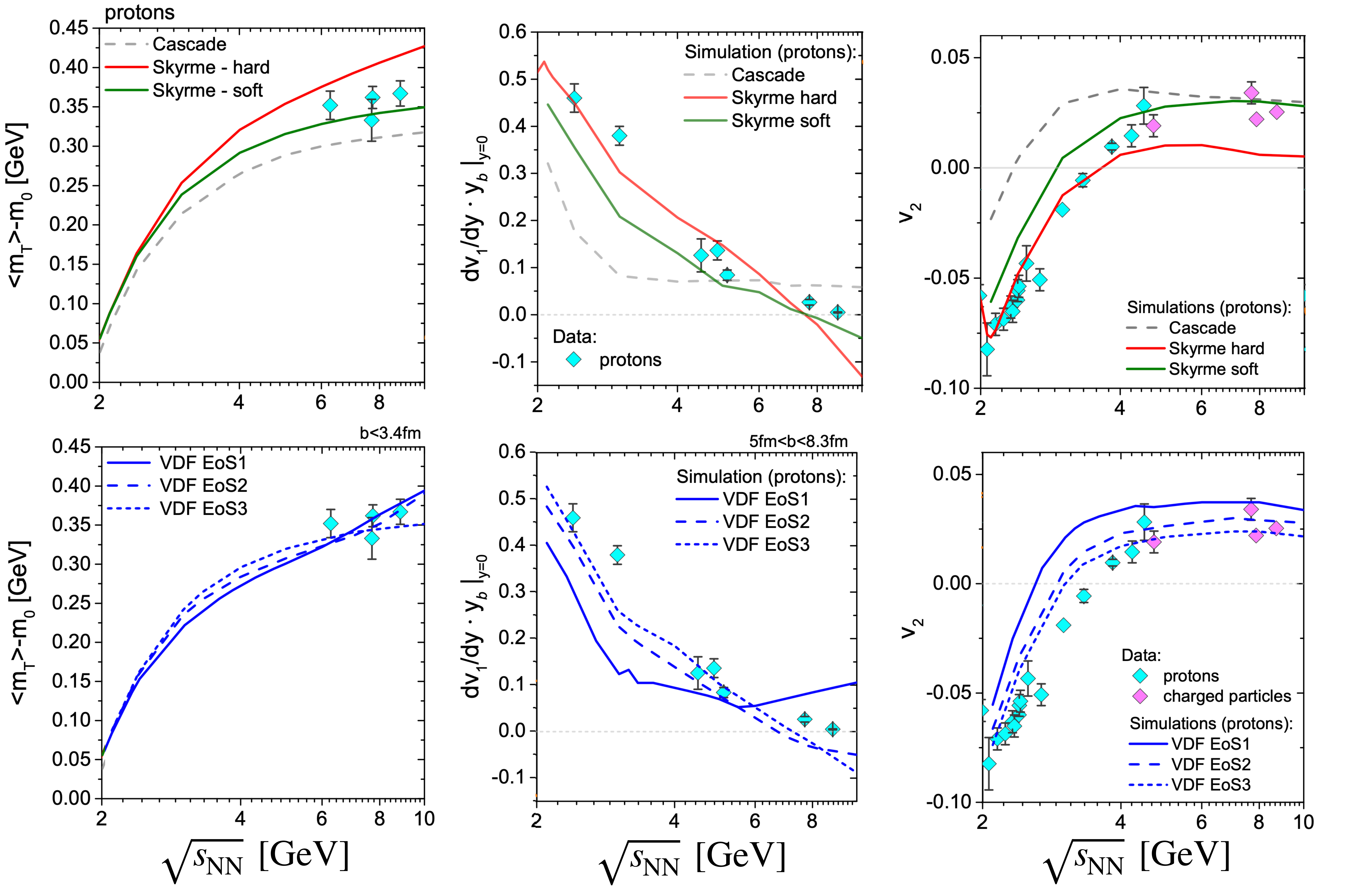

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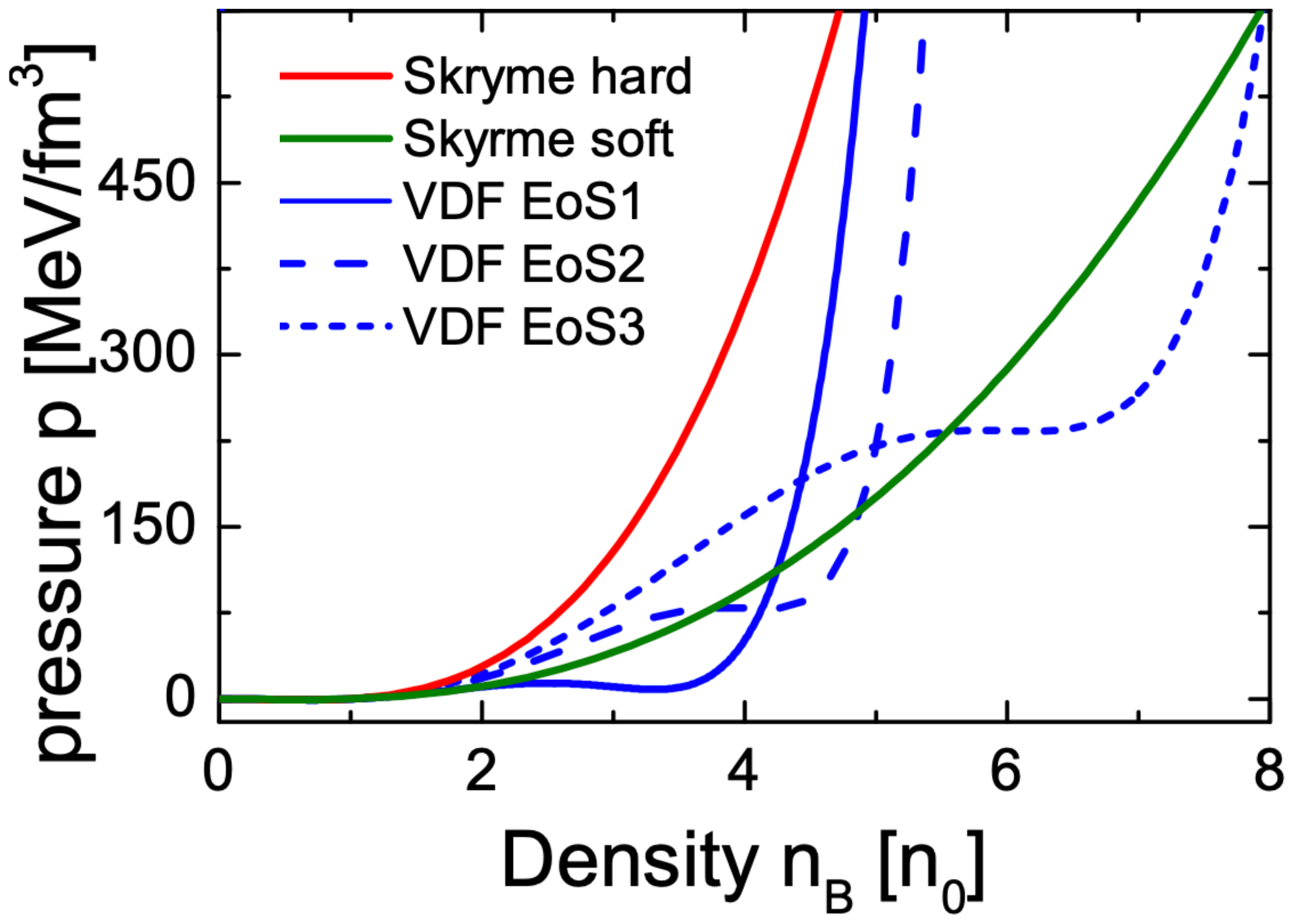


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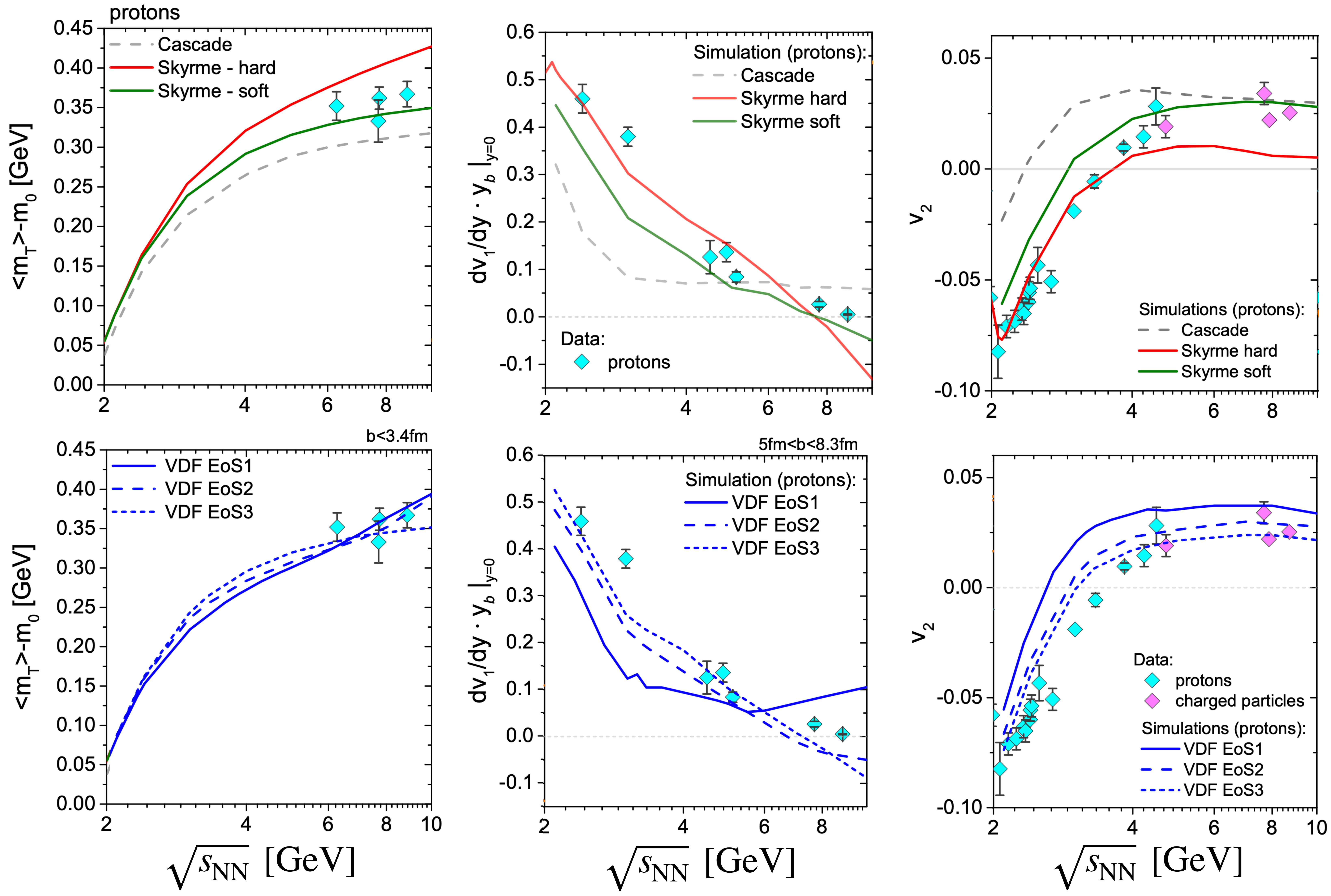
Very soft EOS at $n_B \in (2,3)n_0$ not supported in VDF+UrQMD



# Generalized VDF model: custom $c_s^2$

$$\mathcal{E}_{\mathrm{VDF}}\Big|_{\substack{\text{rest}\\\text{frame}}} = g\int \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^{\,2}+m^2}\,f_{\mathbf{p}} + \sum_{i=1}^{N}\frac{C_i}{b_i}n_B^{b_i}$$

Assume arbitrary vector interactions: $A^\mu = \alpha(n_B)j^\mu$ ($A^\mu$ is the single-particle potential)

Connect $\alpha(n_B)$ to $c_s^2(n_B)$: $\alpha(n_B) = \frac{1}{n_B}\left[\mu_B\left(n_B^{(0)}\right)\exp\left(\int_{n_B^{(0)}}^{n_B} d\ln n\ c_s^2(n)\right) - \sqrt{m^2 + \left(\frac{6\pi n_B}{g}\right)^{2/3}}\right]$

These interactions, parametrized with a chosen shape of $c_s^2(n_B)$, can be used in simulations!

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

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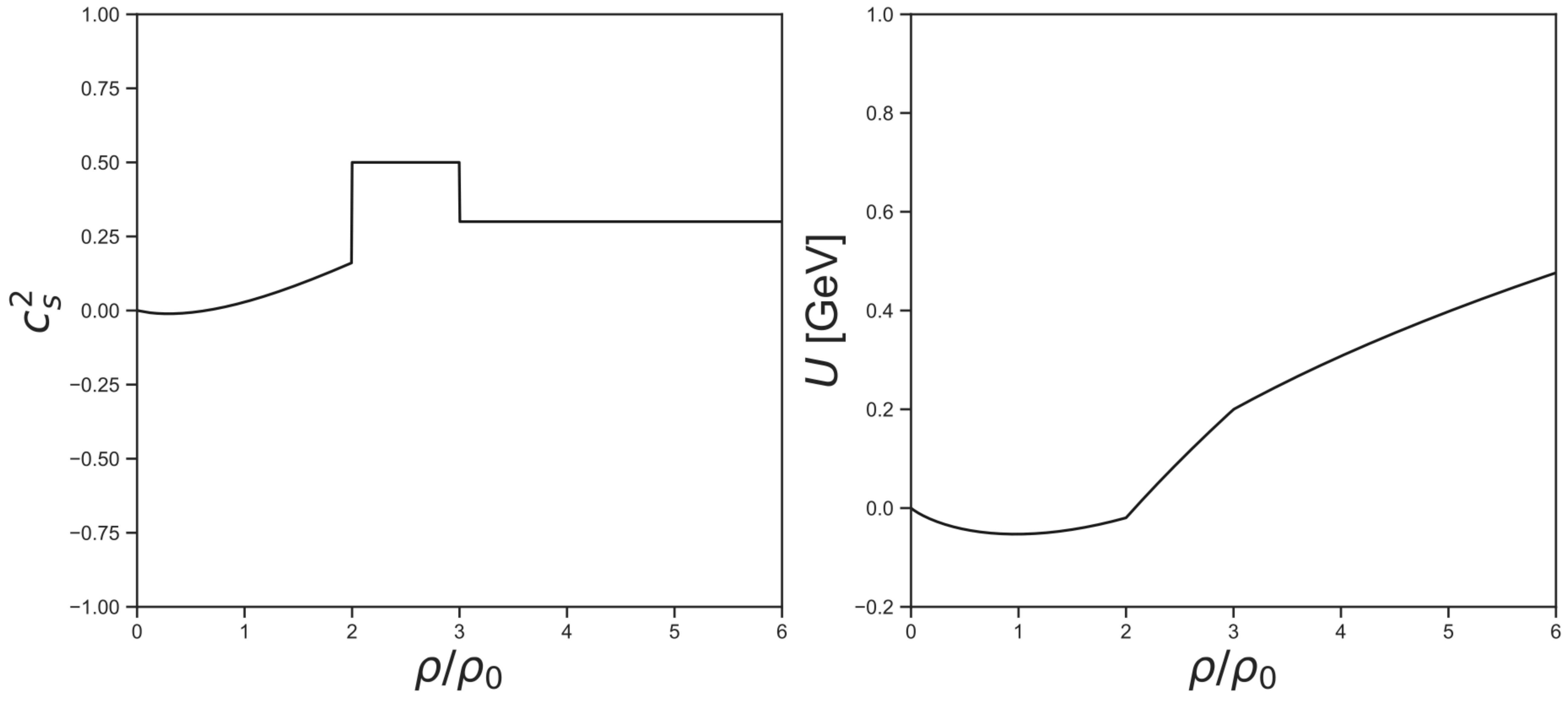
Piecewise parametrization of $c_s^2(n_B)$:

$$c_s^2(n_B) = \begin{cases} c_s^2(\text{Skyrme}), & n_B < n_1 = 2n_0 \\ c_1^2, & n_1 < n_B < n_2 \\ c_2^2, & n_2 < n_B < n_3 \\ \dots & \\ c_m^2, & n_m < n_B \end{cases}$$

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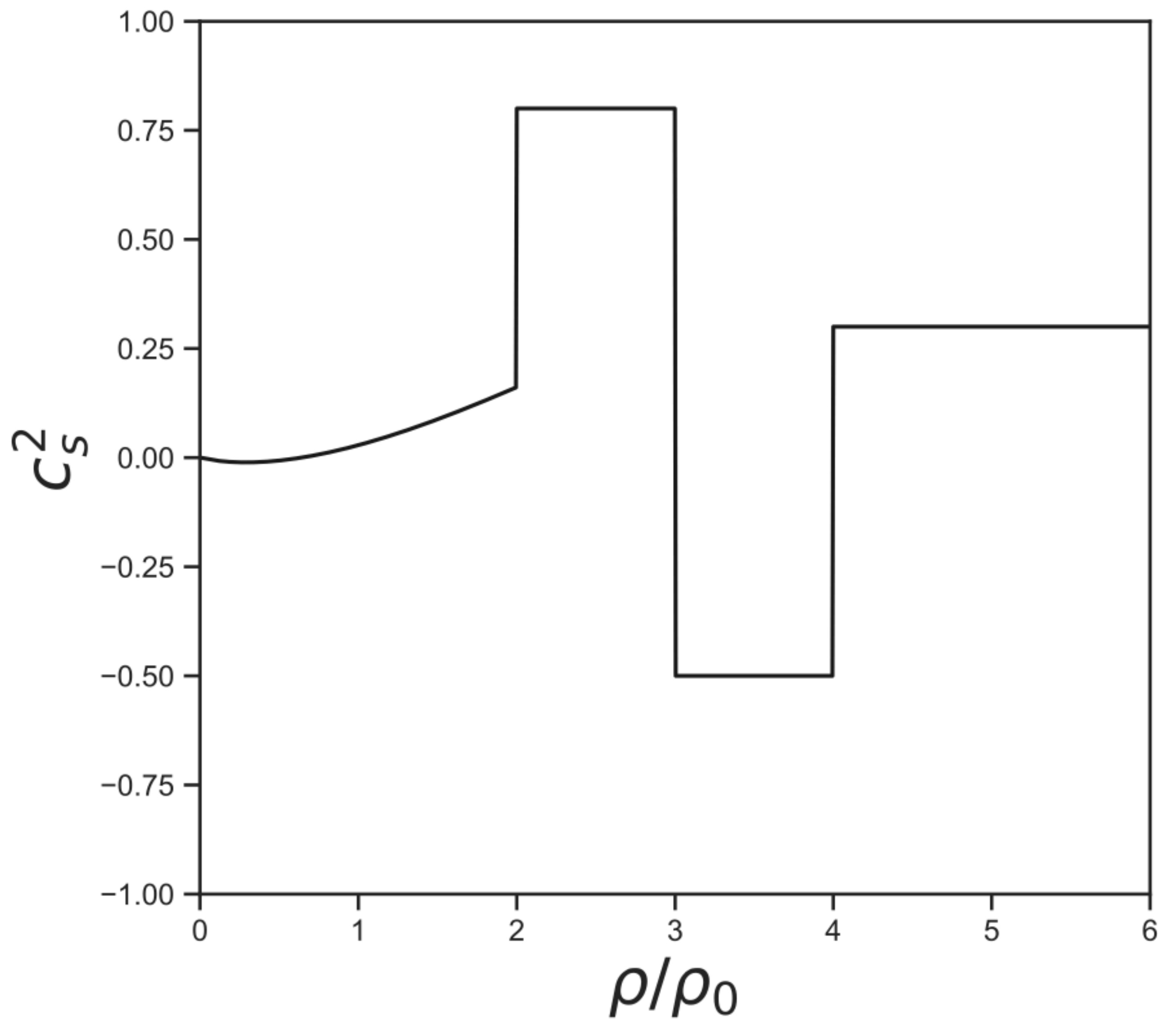


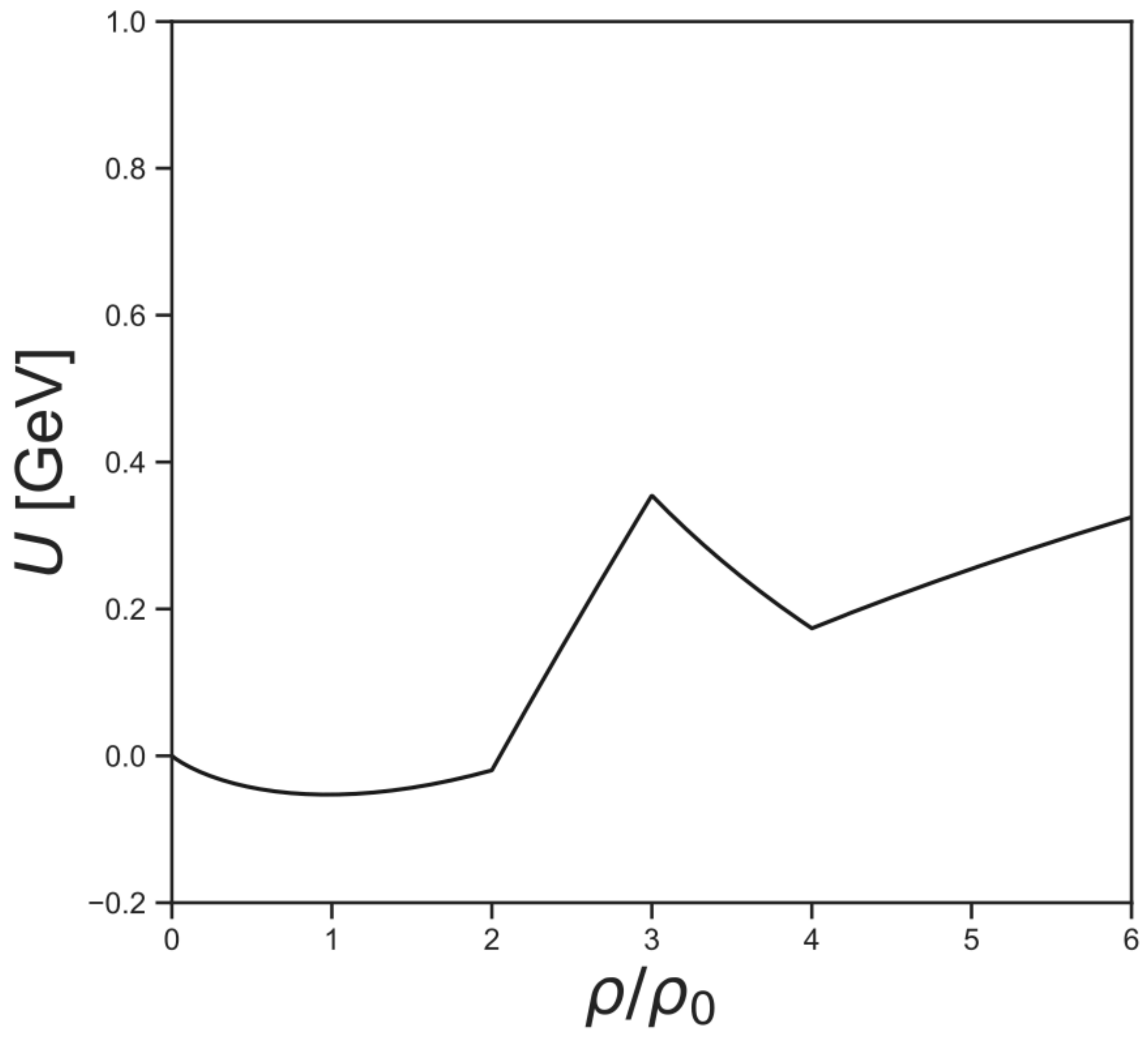


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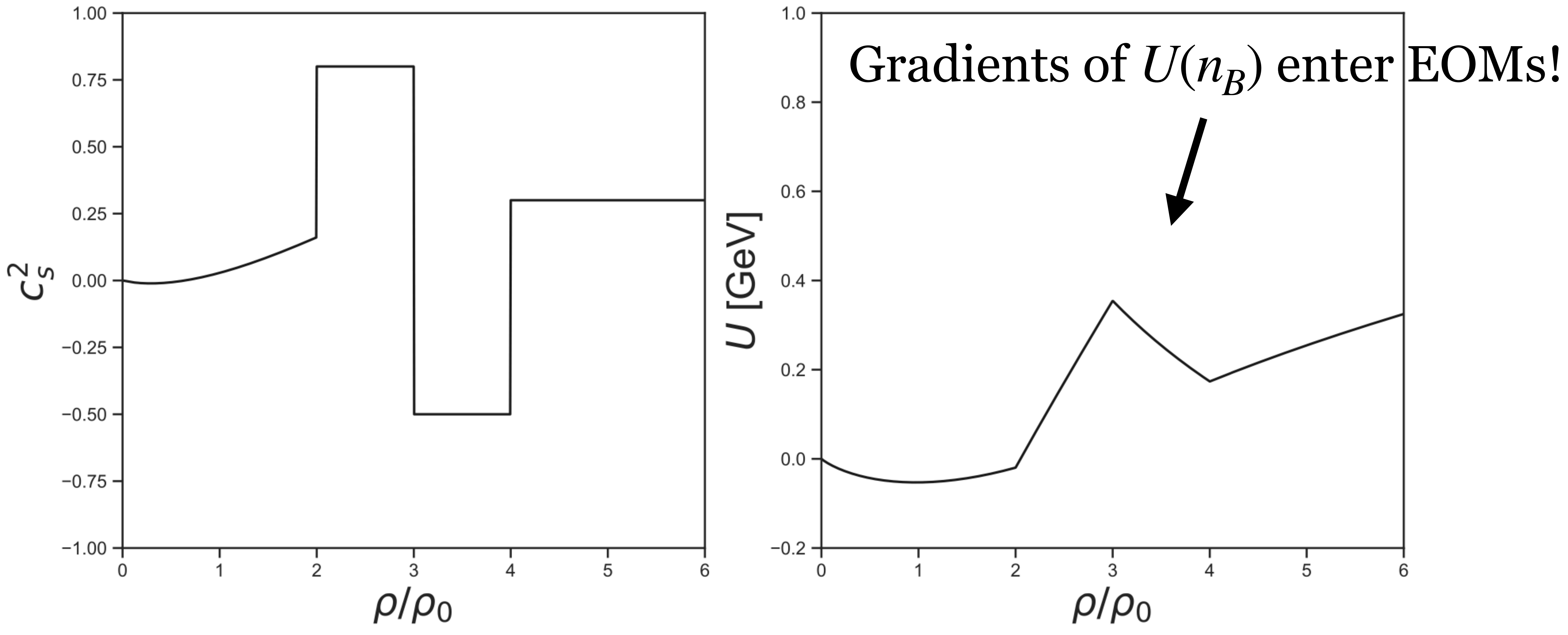


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# Hadronic transport with $c_s^2$-parametrized mean-fields

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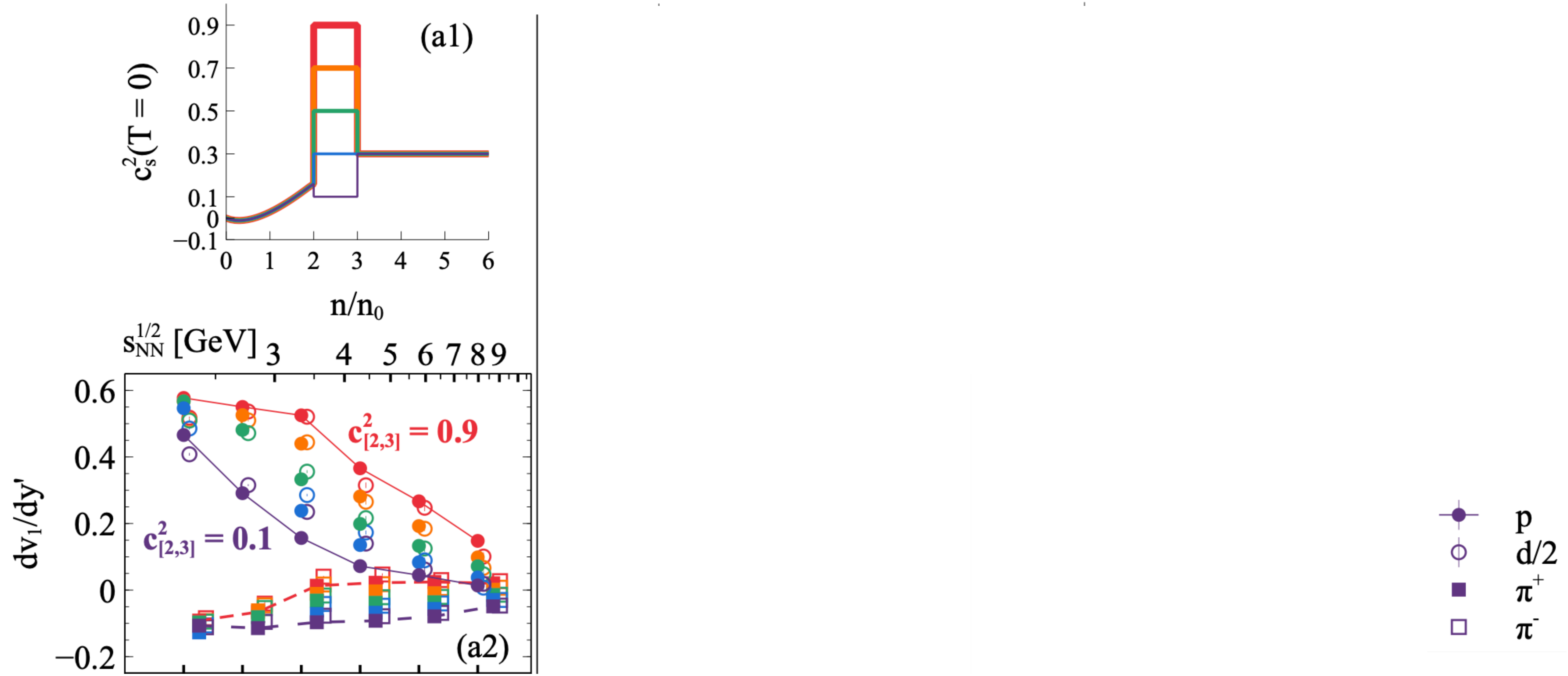
arXiv:2208.11996

Generalized VDF ($n_B$-dependent interaction coefficients):

mean-field potential piecewise parametrized by (constant) values of $c_s^2$ for $n_i < n_B < n_j$



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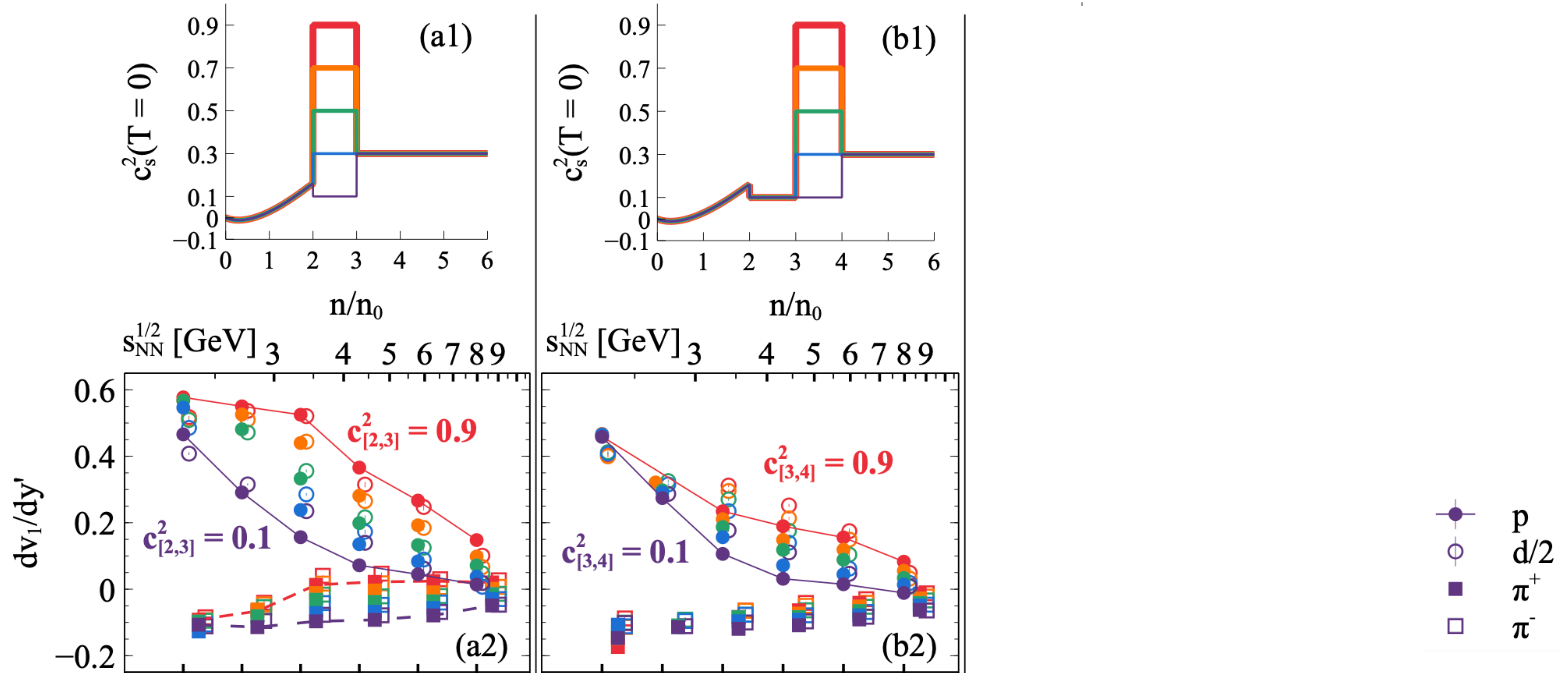

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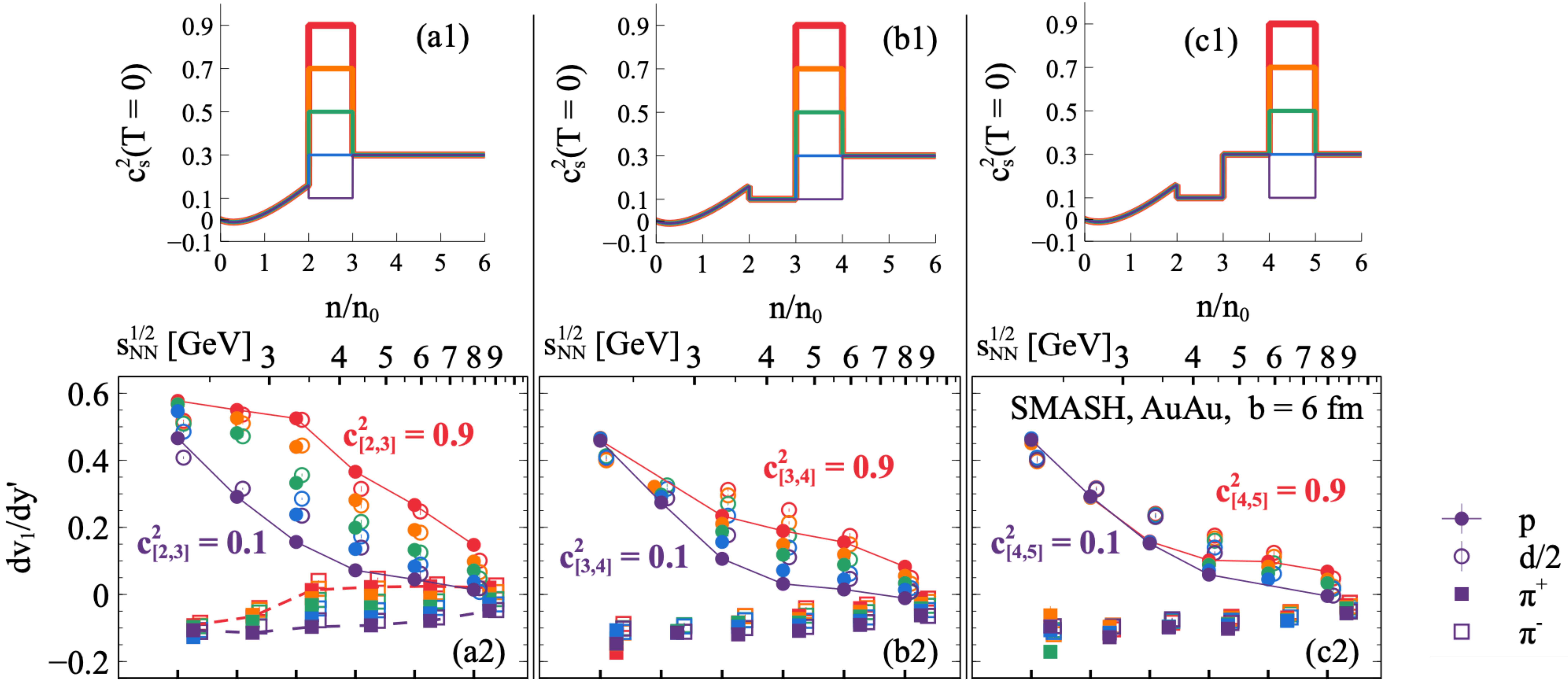

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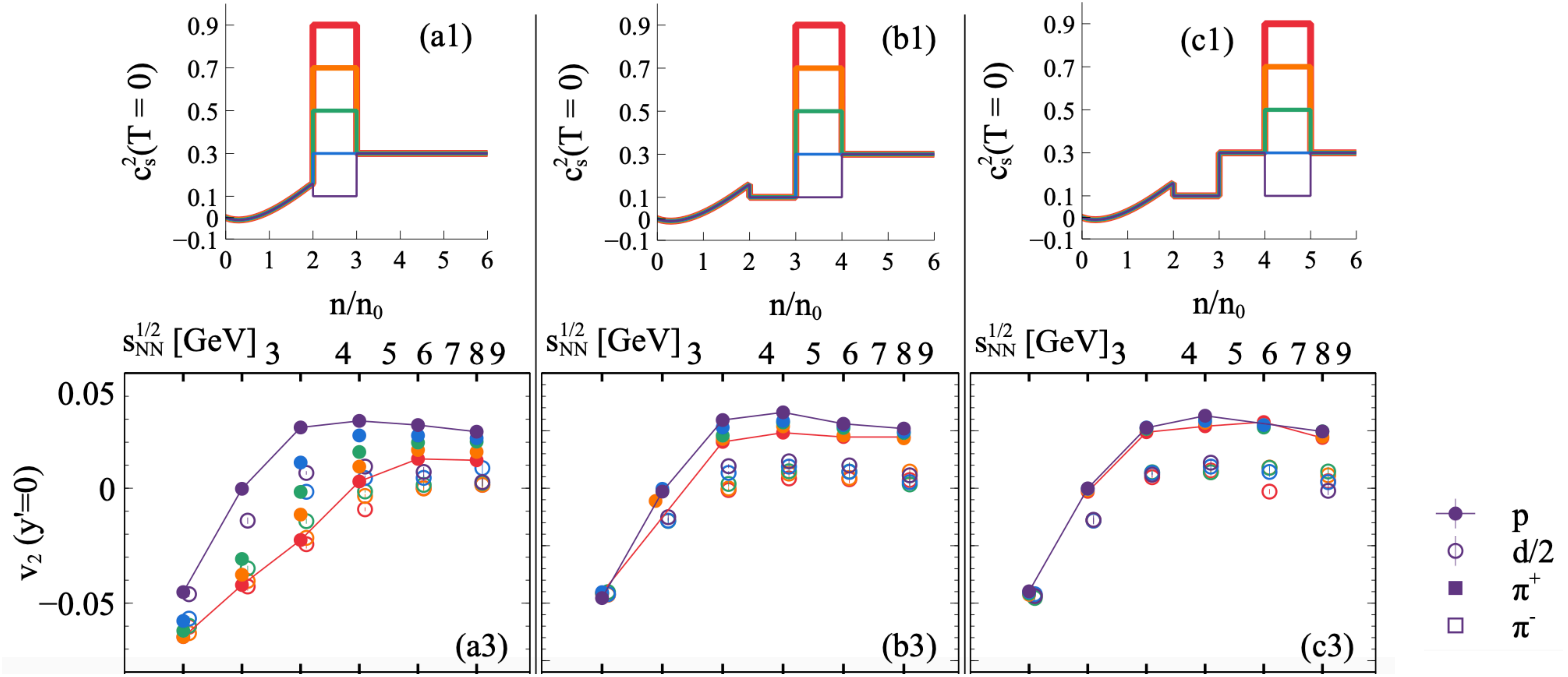
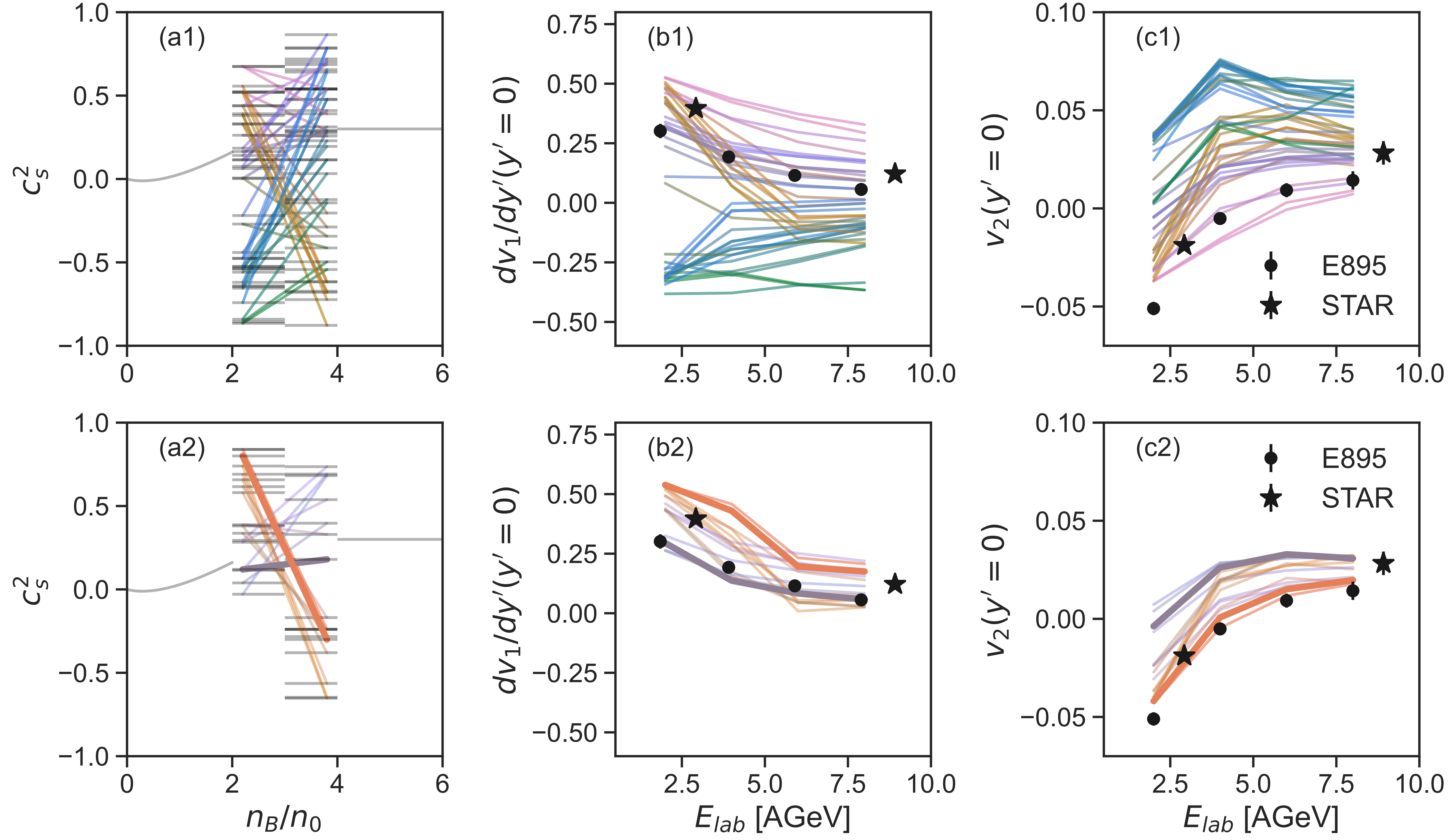


# STAR (new) and E895 (old) data cannot be simultaneously described

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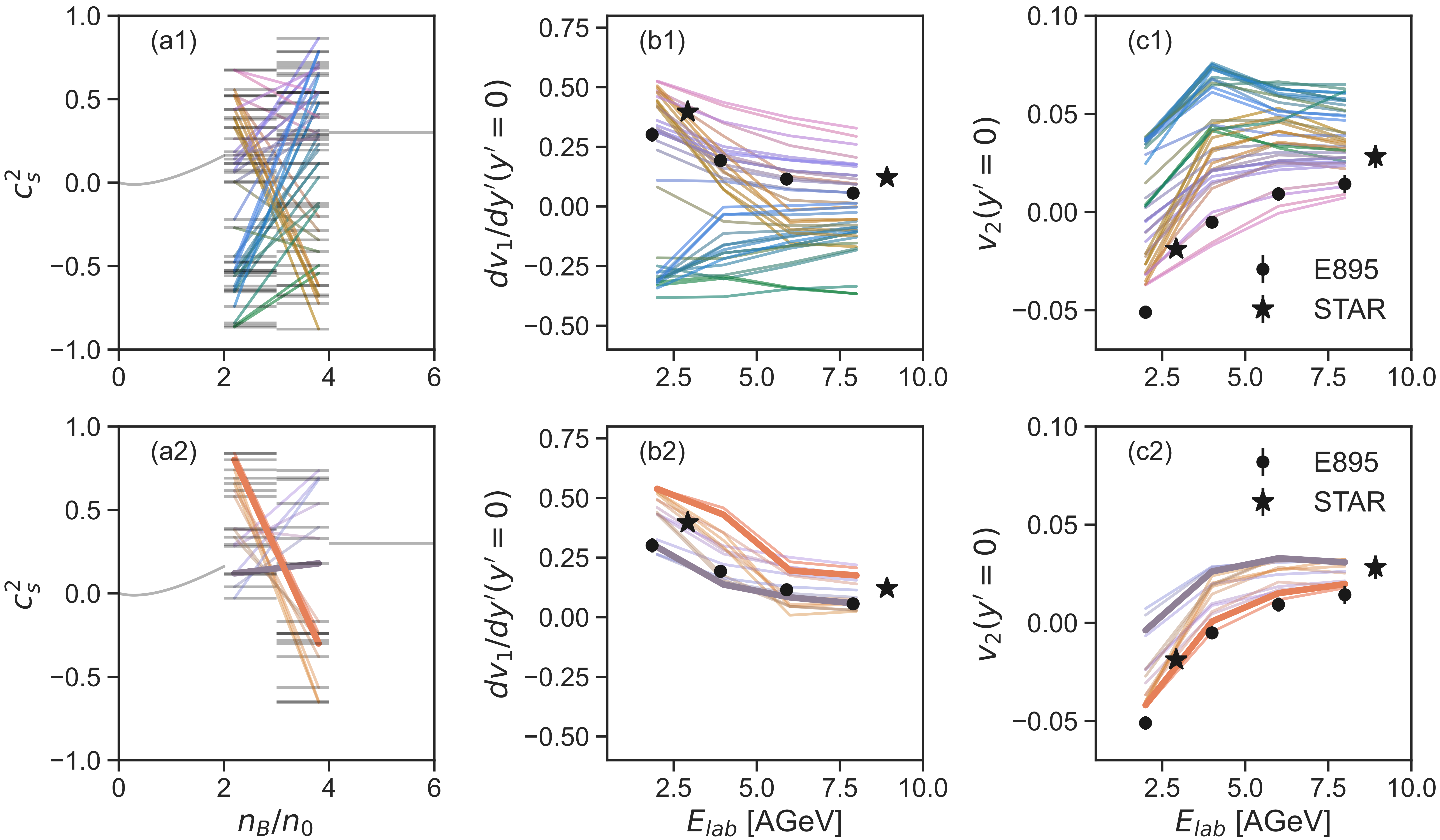


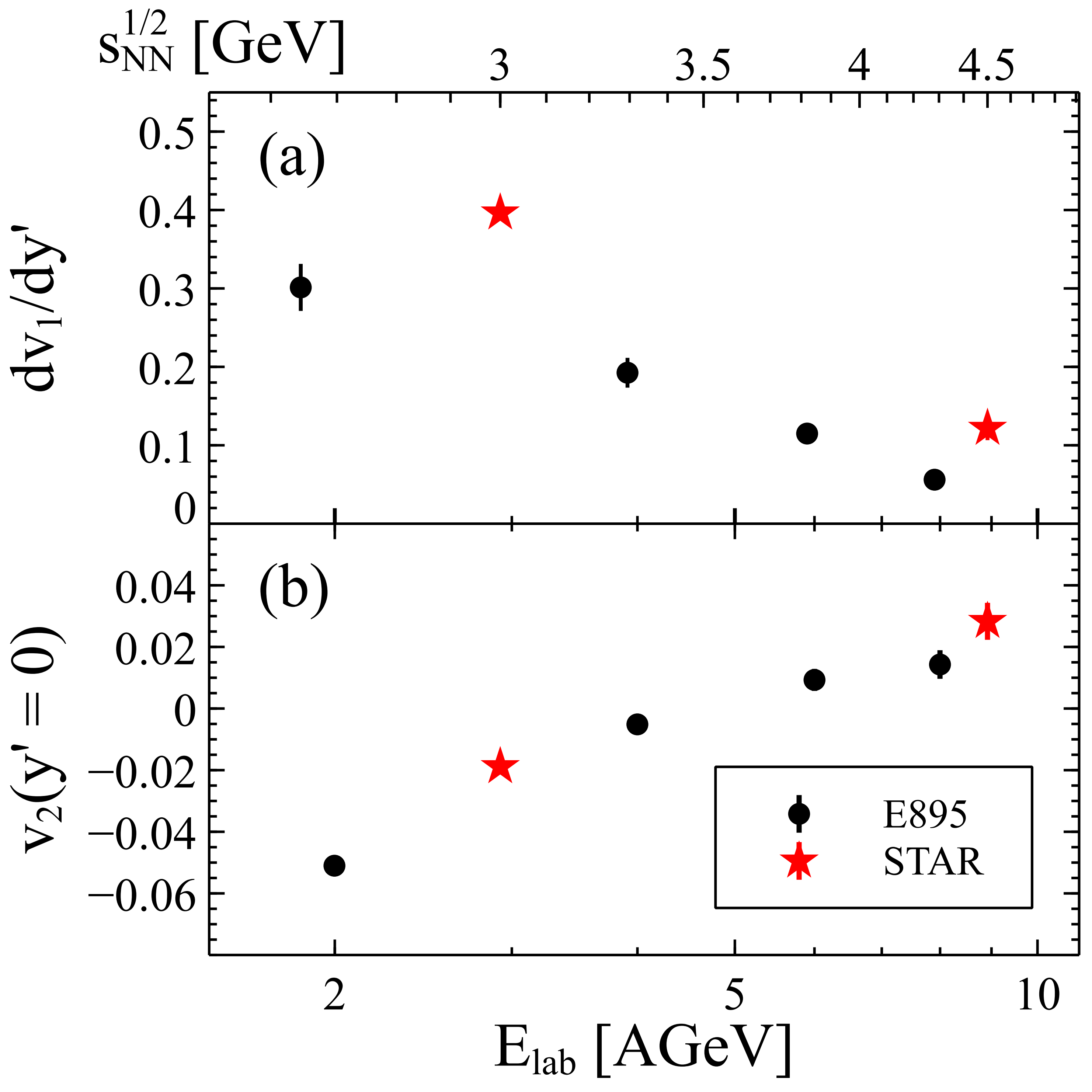


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D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

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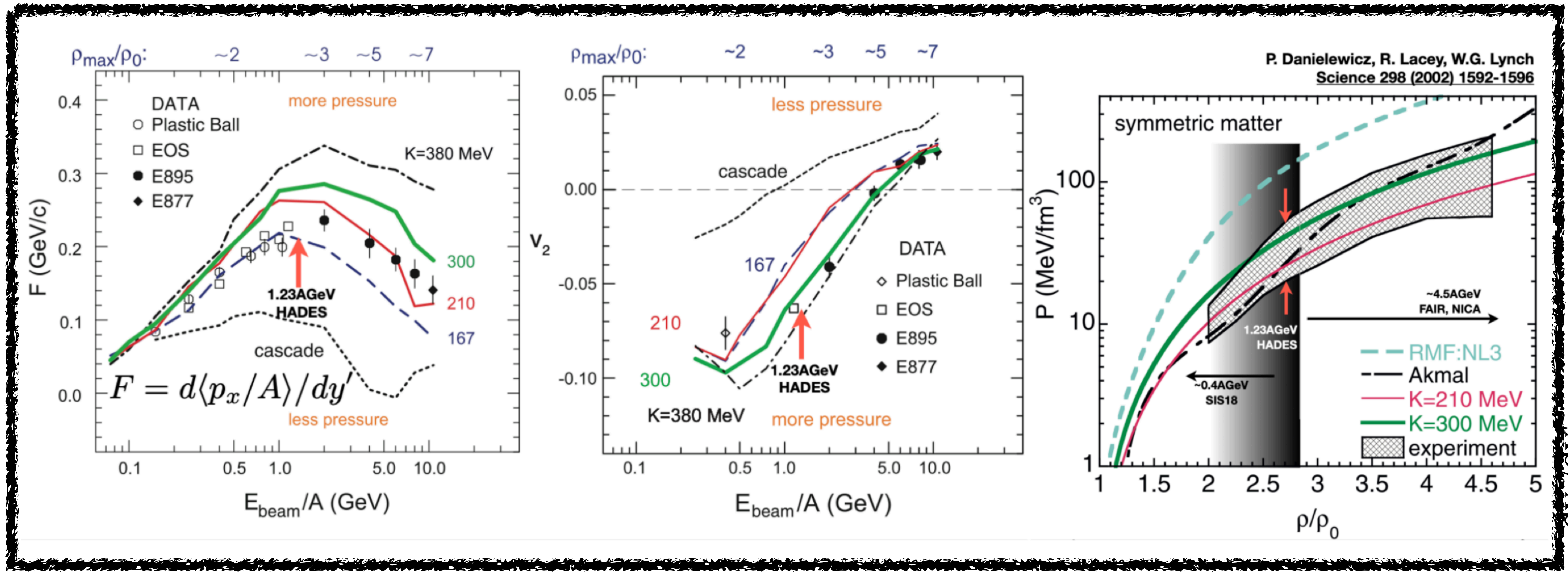
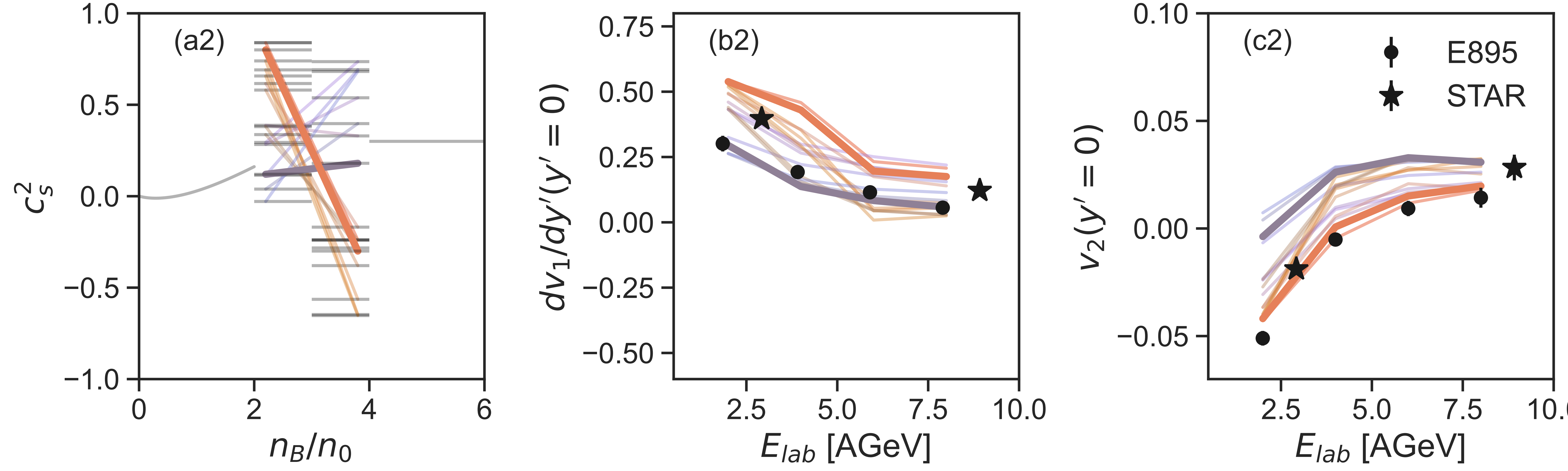
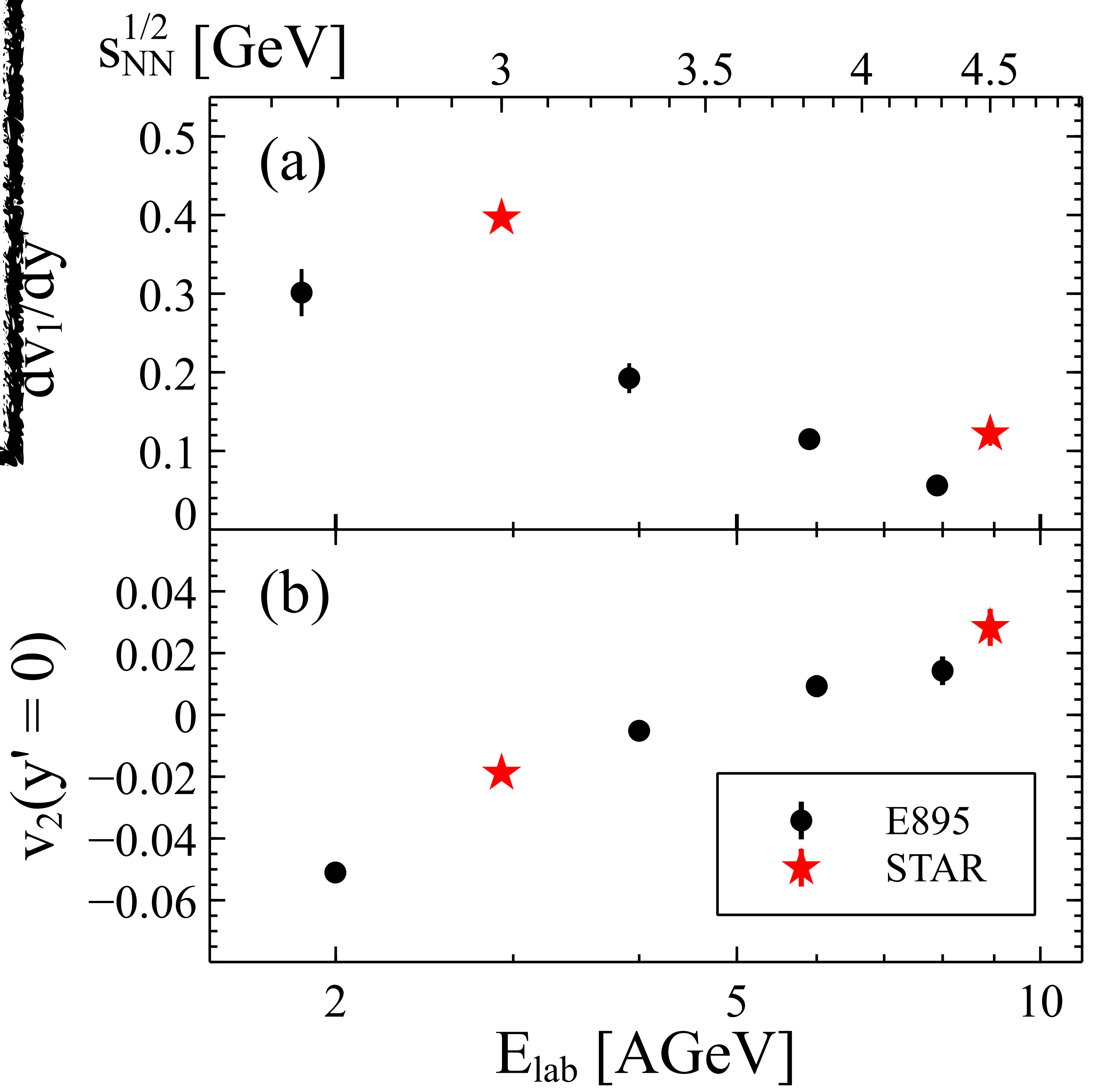



Same problem as in the DLL constraint!

Danielewicz, Lacey, Lynch, Science **298**,1592–1596 (2002)

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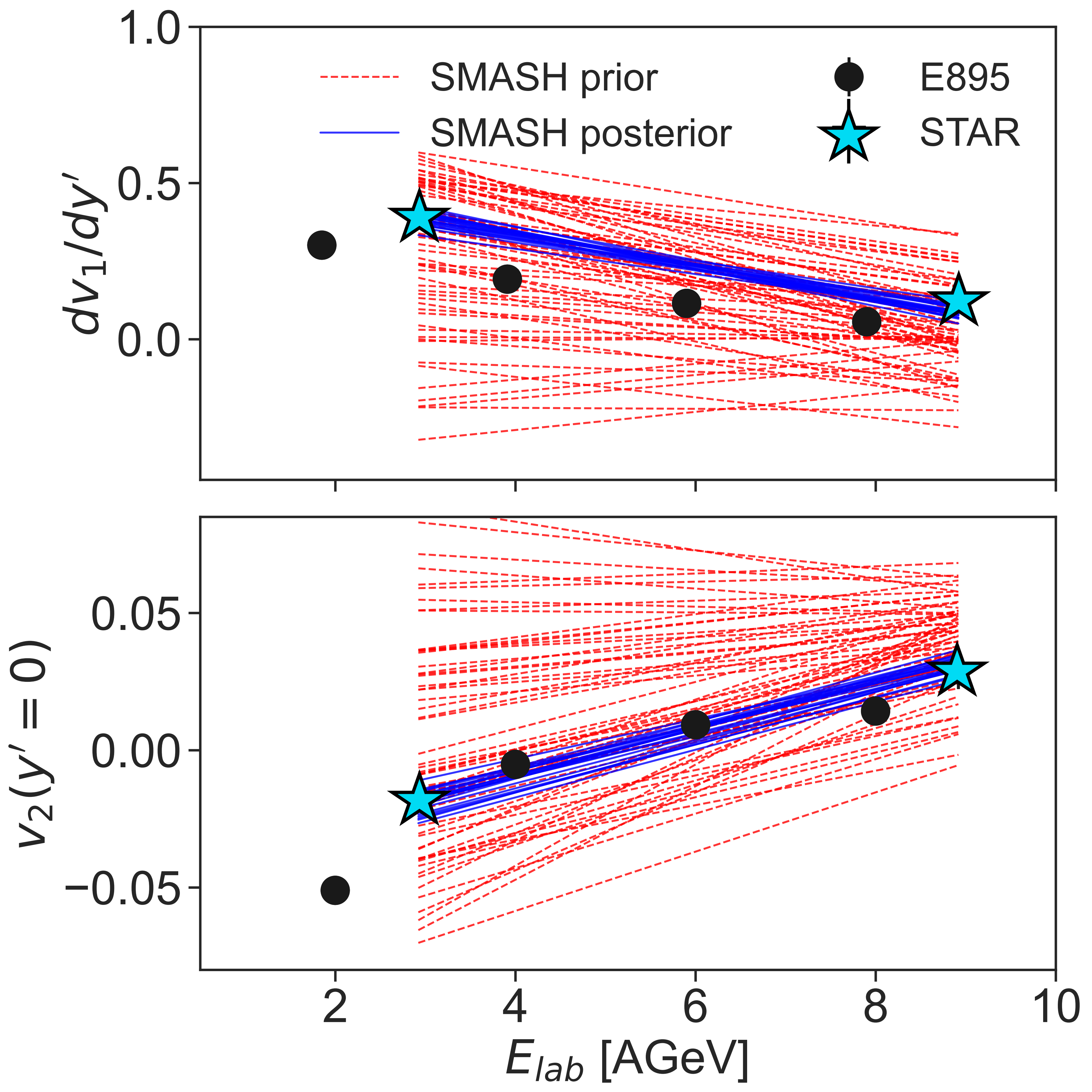
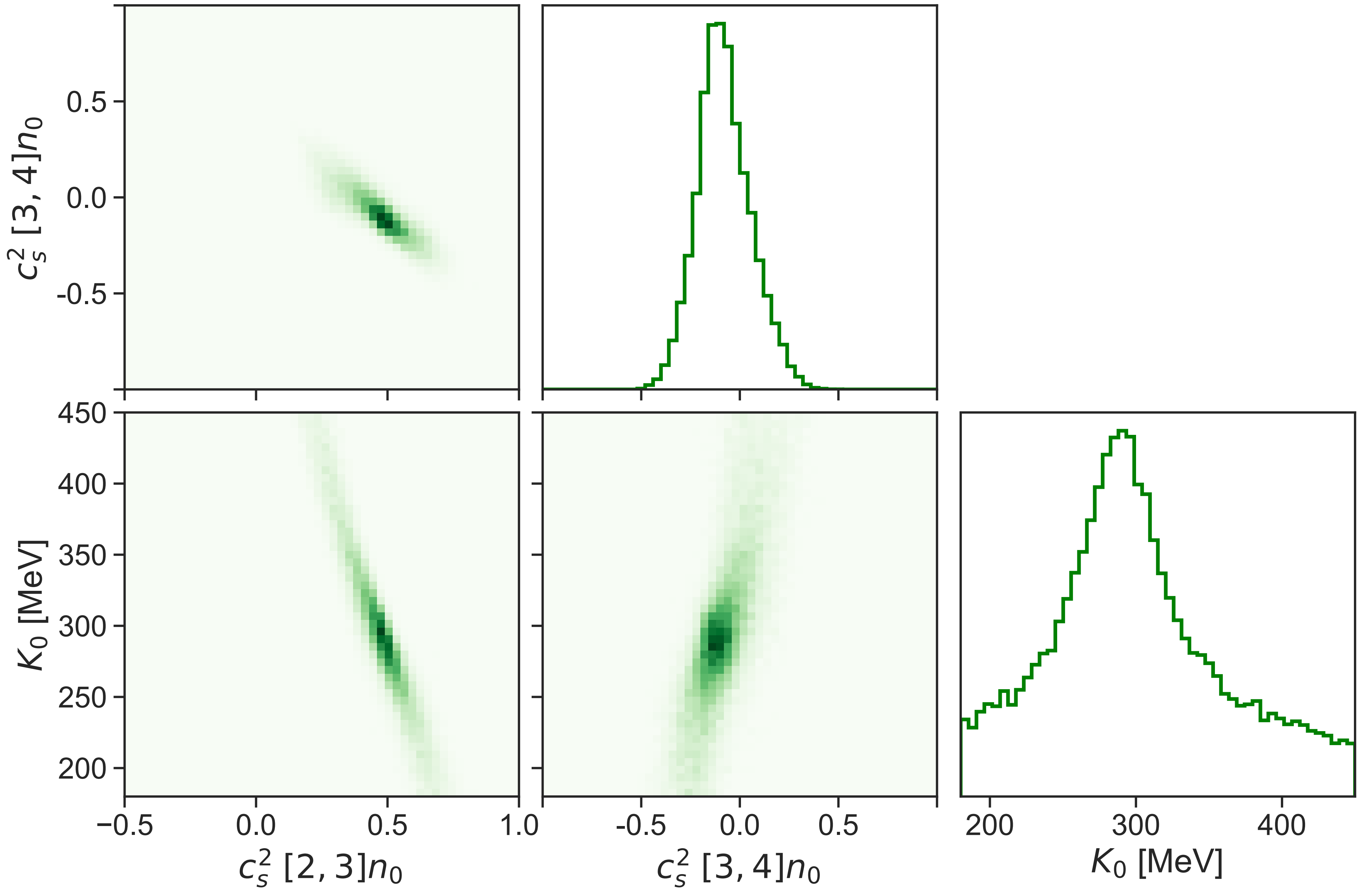
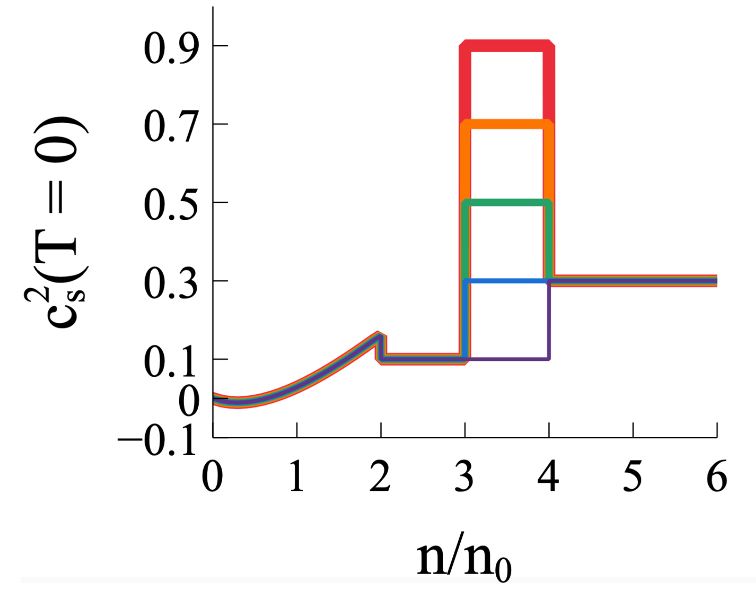
# Bayesian analysis of STAR flow data with varying $K_0$, $c^2_{[2,3]n_0}$, $c^2_{[3,4]n_0}$




The maximum a posteriori probability (MAP) parameters are

$K_0 = 300 \pm 60\text{MeV}, \quad c^2_{[2,3]n_0} = 0.47 \pm 0.12, \quad c^2_{[3,4]n_0} = -0.08 \pm 0.14$

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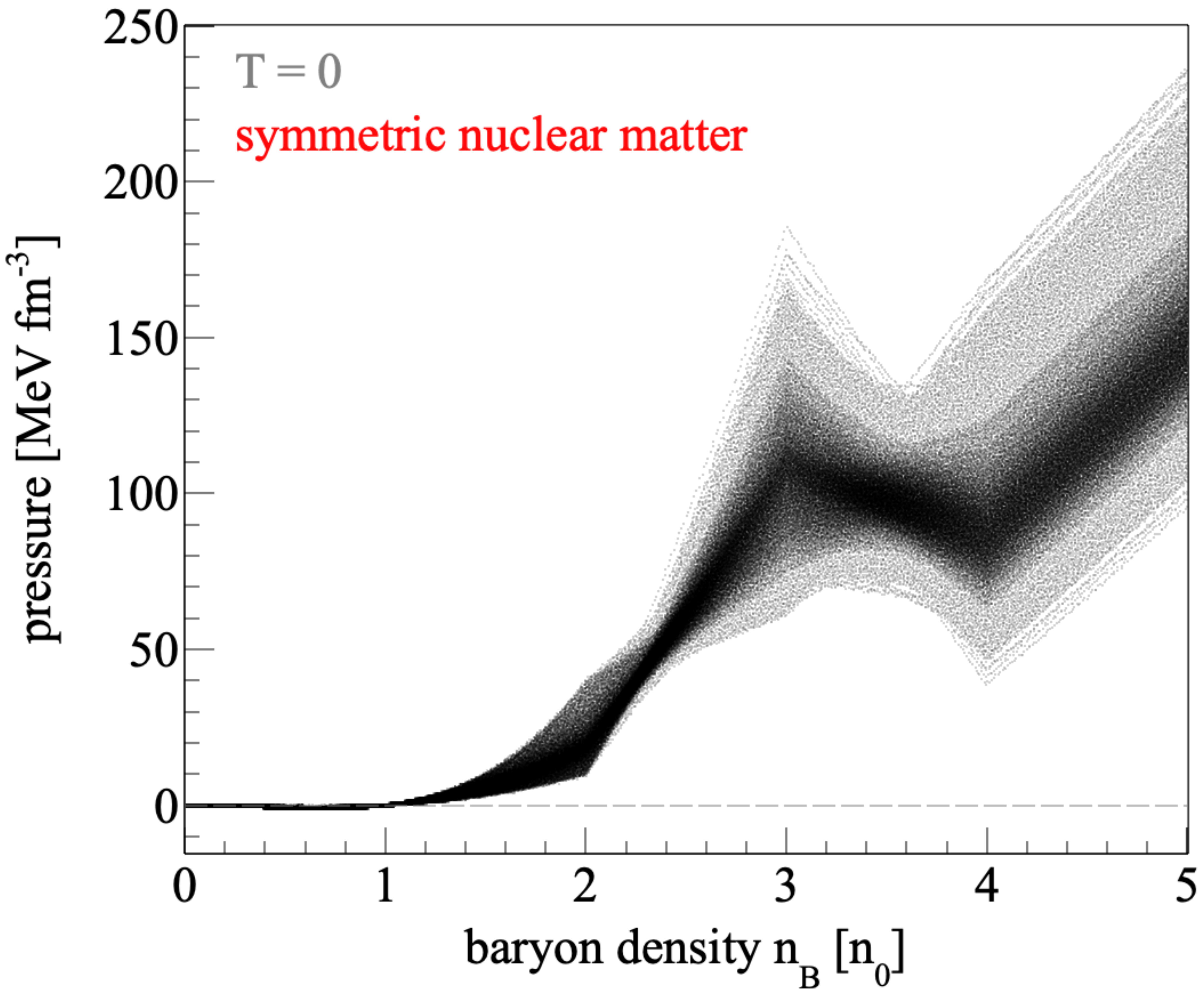


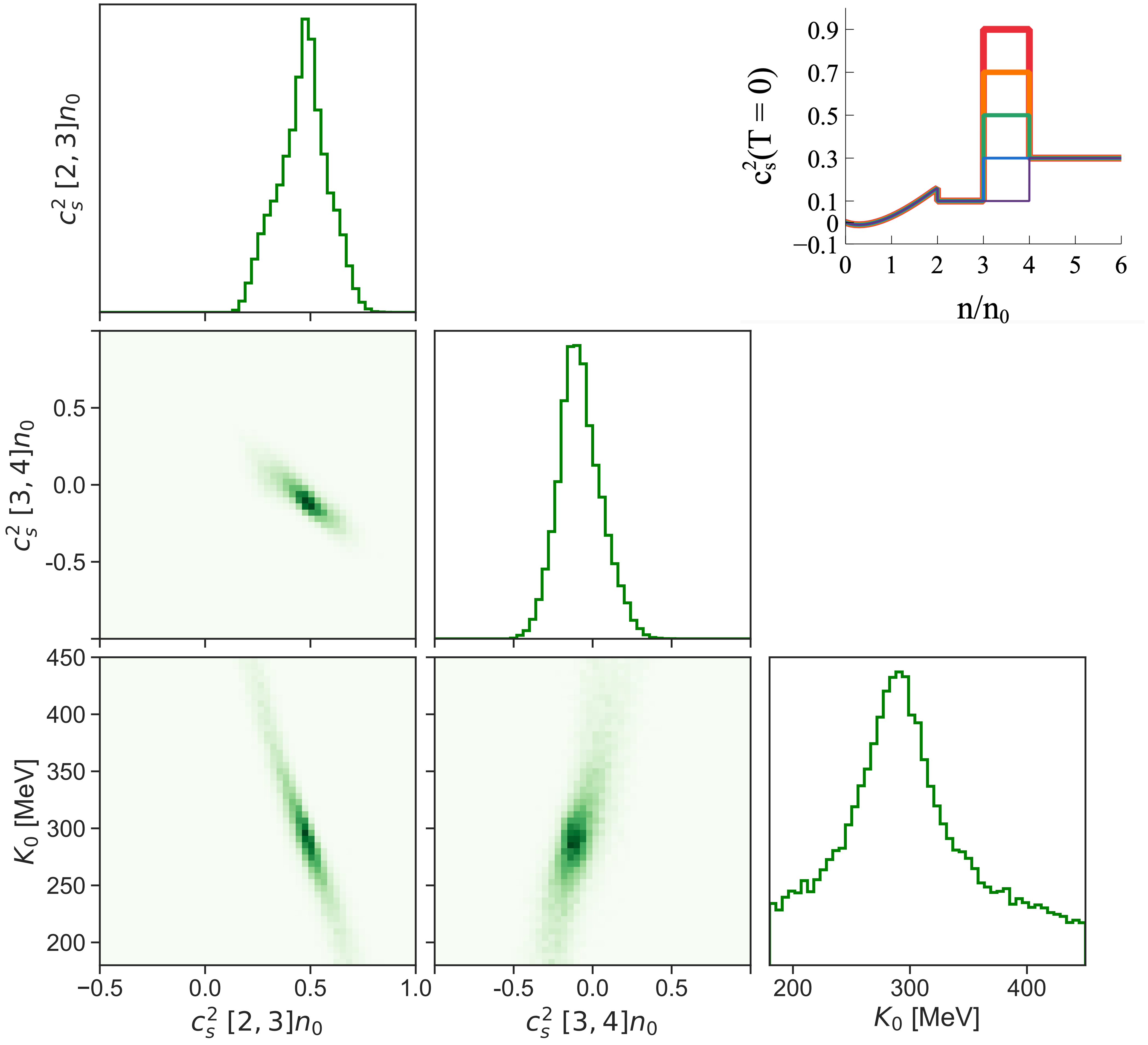


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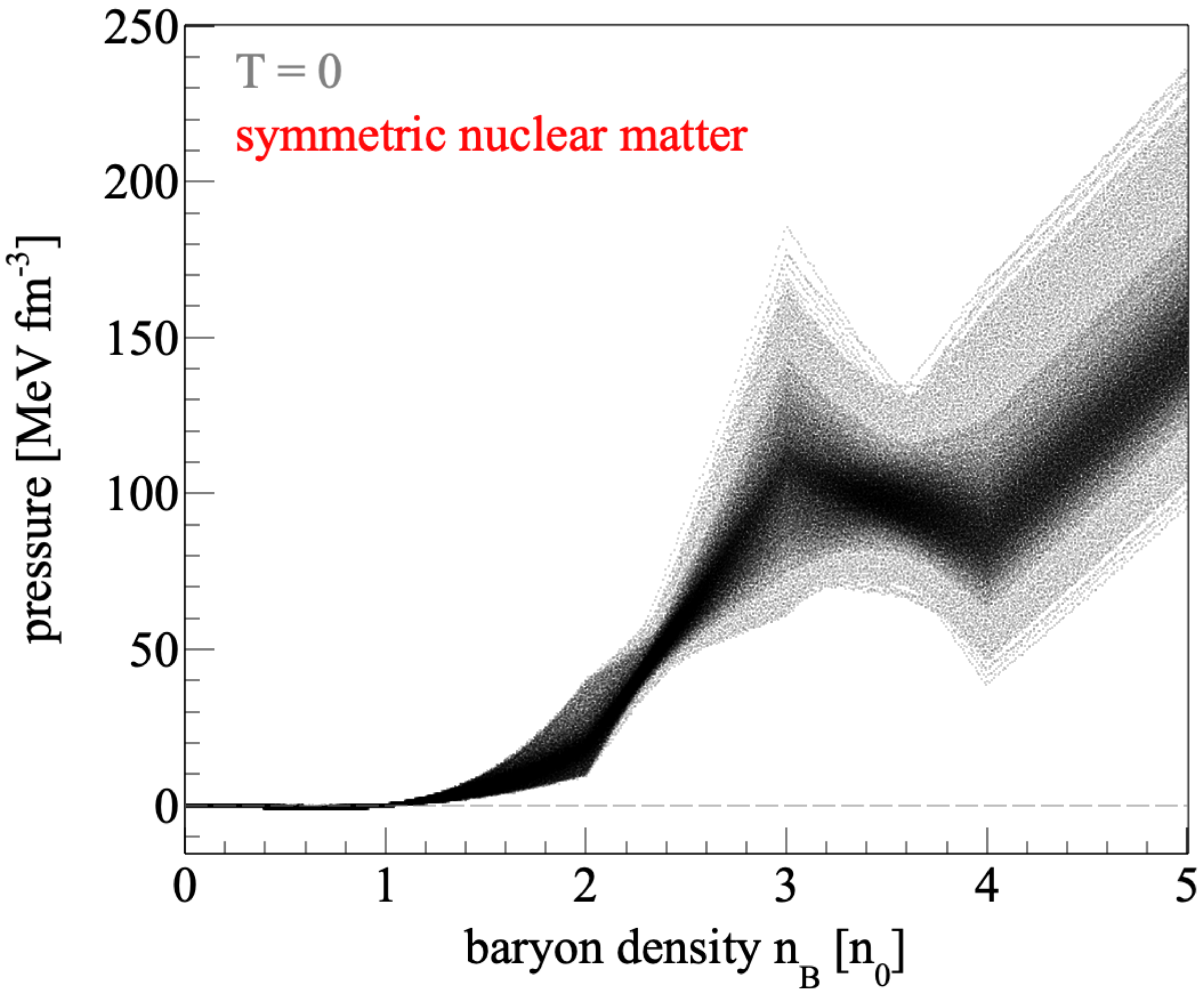
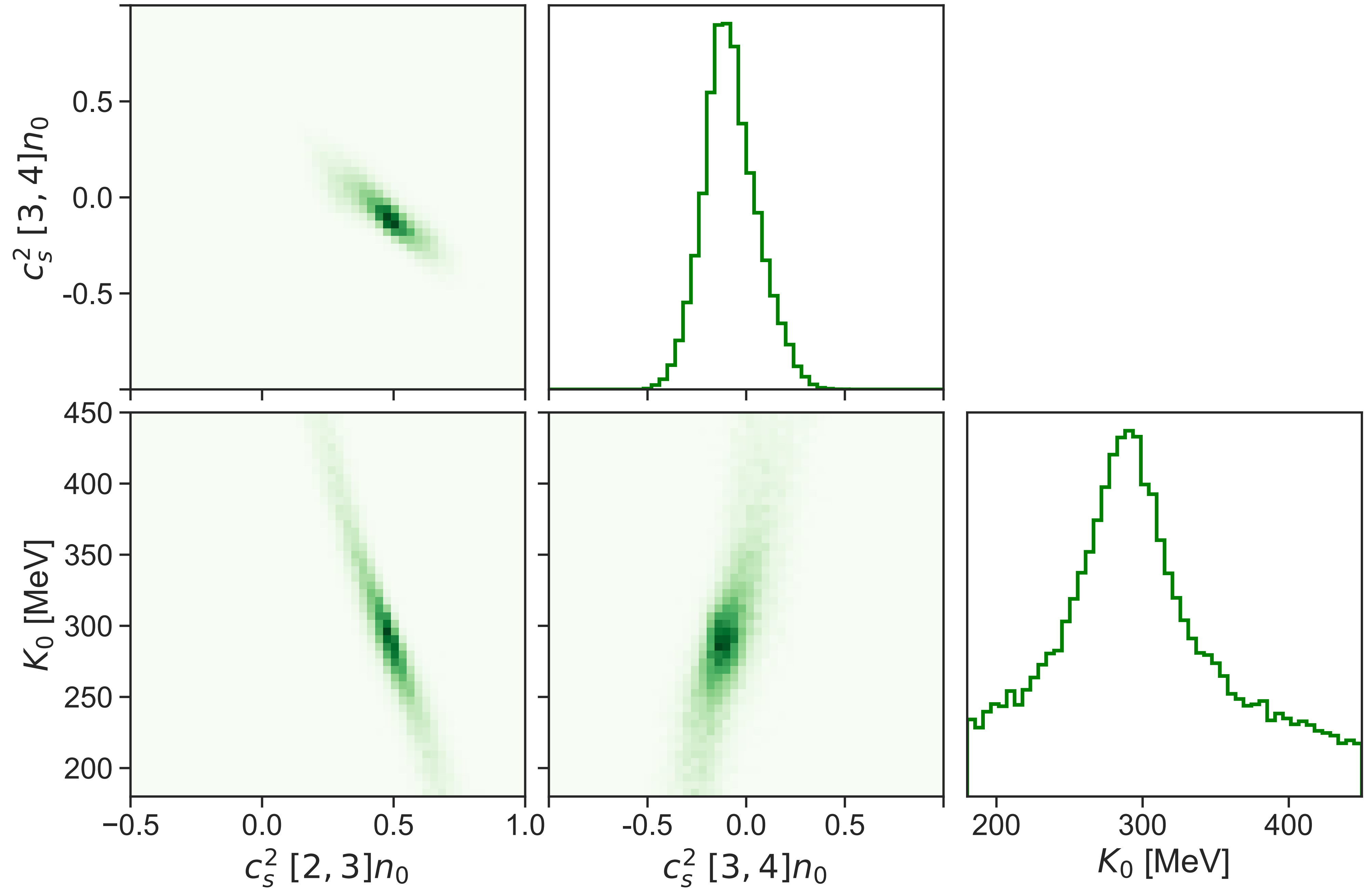



The constrained EOS is very stiff at $n_B \in (2,3)n_0$ and very soft at $n_B \in (3,4)n_0$ !

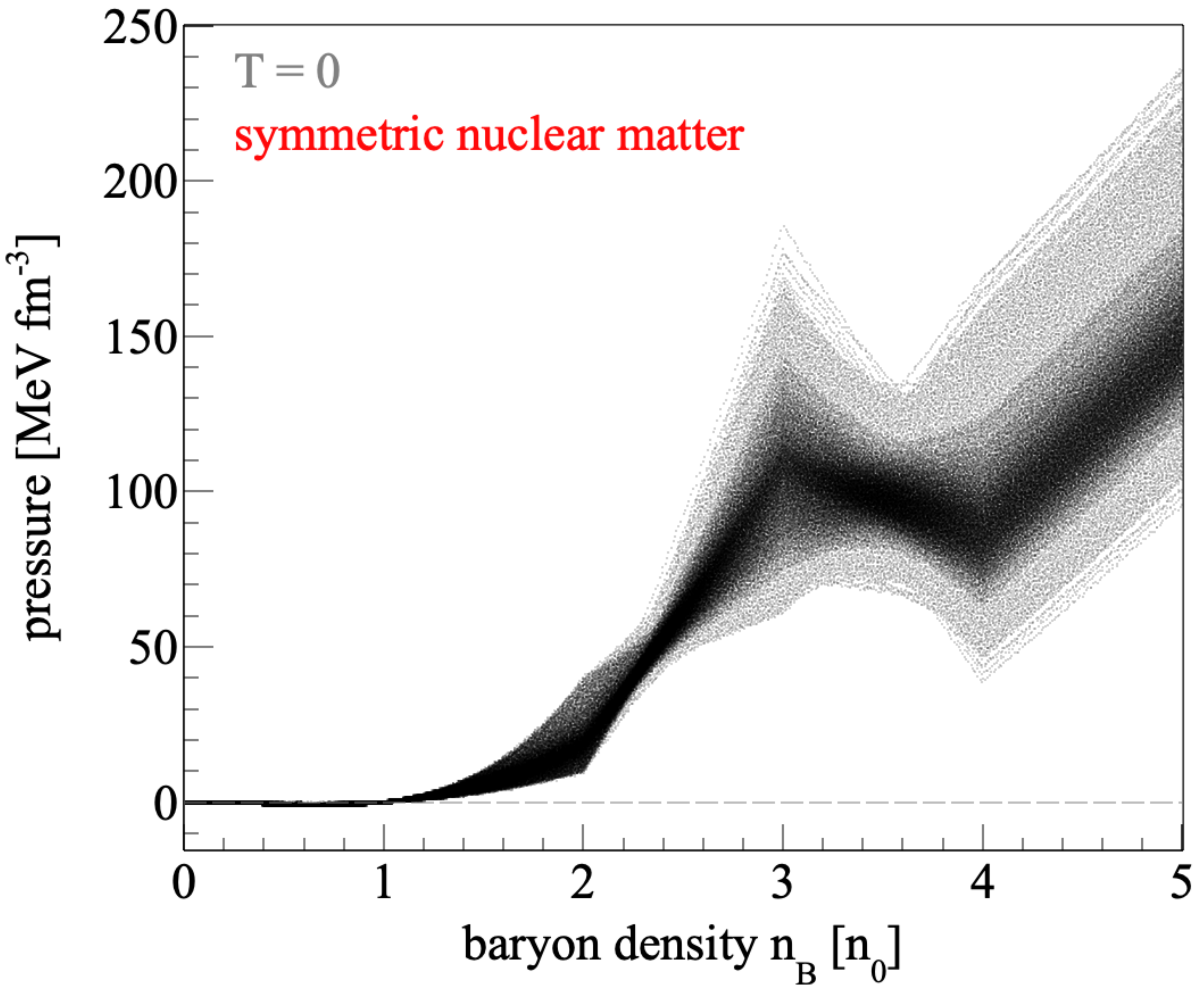


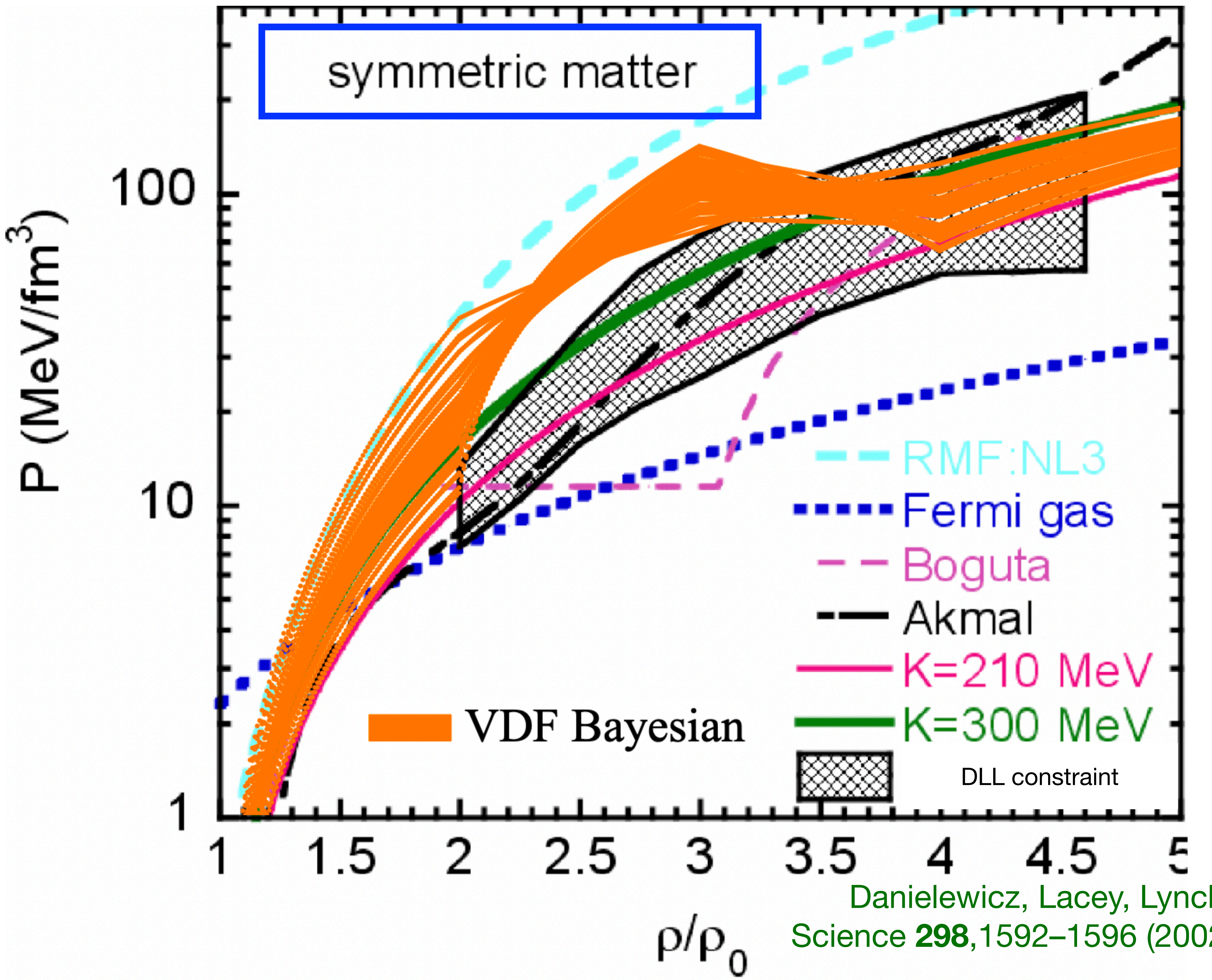

The maximum a posteriori probability (MAP) parameters are

$K_0 = 300 \pm 60\text{MeV}, \quad c^2_{[2,3]n_0} = 0.47 \pm 0.12, \quad c^2_{[3,4]n_0} = -0.08 \pm 0.14$

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

# Bayesian analysis of STAR flow data with varying $K_0$, $c^2_{[2,3]n_0}$, $c^2_{[3,4]n_0}$




Danielewicz, Lacey, Lynch, Science **298**,1592–1596 (2002)

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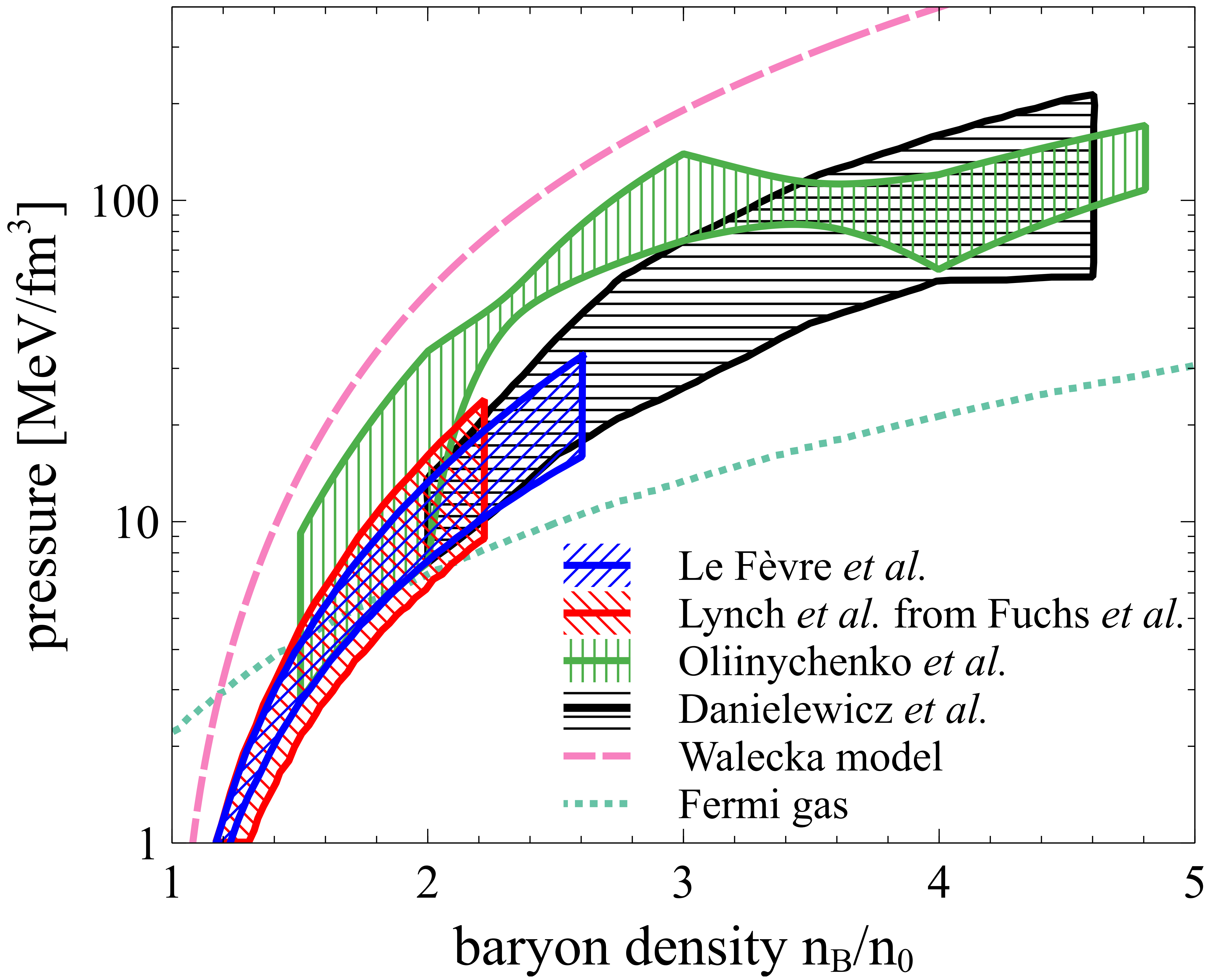
D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

# EOS of symmetric nuclear matter: selected results

Symmetric nuclear matter



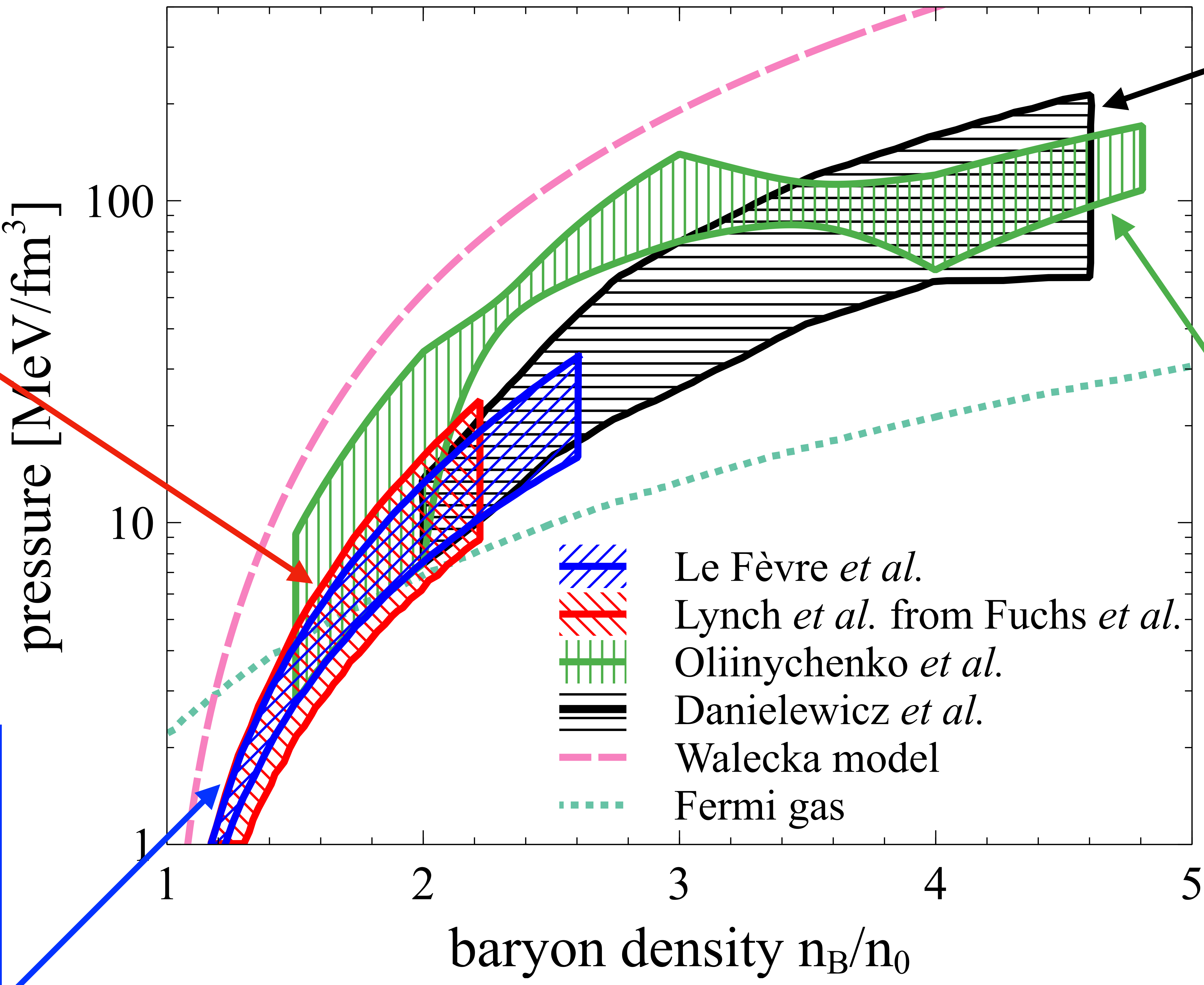

**A. Sorensen** *et al.*, arXiv:2301.13253

# EOS of symmetric nuclear matter: selected results

**Symmetric nuclear matter**

197Au+197Au & 12C+12C @ < 1.5 GeV/u
($\sqrt{s_{NN}}$ < 2.5 GeV)
observables: subthreshold kaon production (KaoS)
model used: QMD w/ nucleons, Δ, N*(1440), pions, kaons;
EOS parametrized by $K_0$;
kaon potentials, momentum dependence
C. Fuchs *et al.*, Prog. Part. Nucl. Phys. **53**, 113–124 (2004) arXiv:nucl-th/0312052

197Au+197Au @ 0.4—1.5 GeV/u
($\sqrt{s_{NN}}$ = 2.07 — 2.52 GeV)
observables: proton flow (FOPI)
model used: isospin QMD (IQMD) w/ nucleons, Δ, N*(1440), deuterons, tritons;
EOS parametrized by $K_0$;
momentum dependence
A. Le Fèvre, Y. Leifels, W. Reisdorf, J. Aichelin, C. Hartnack, Nucl. Phys. A 945, 112 (2016), arXiv:1501.05246



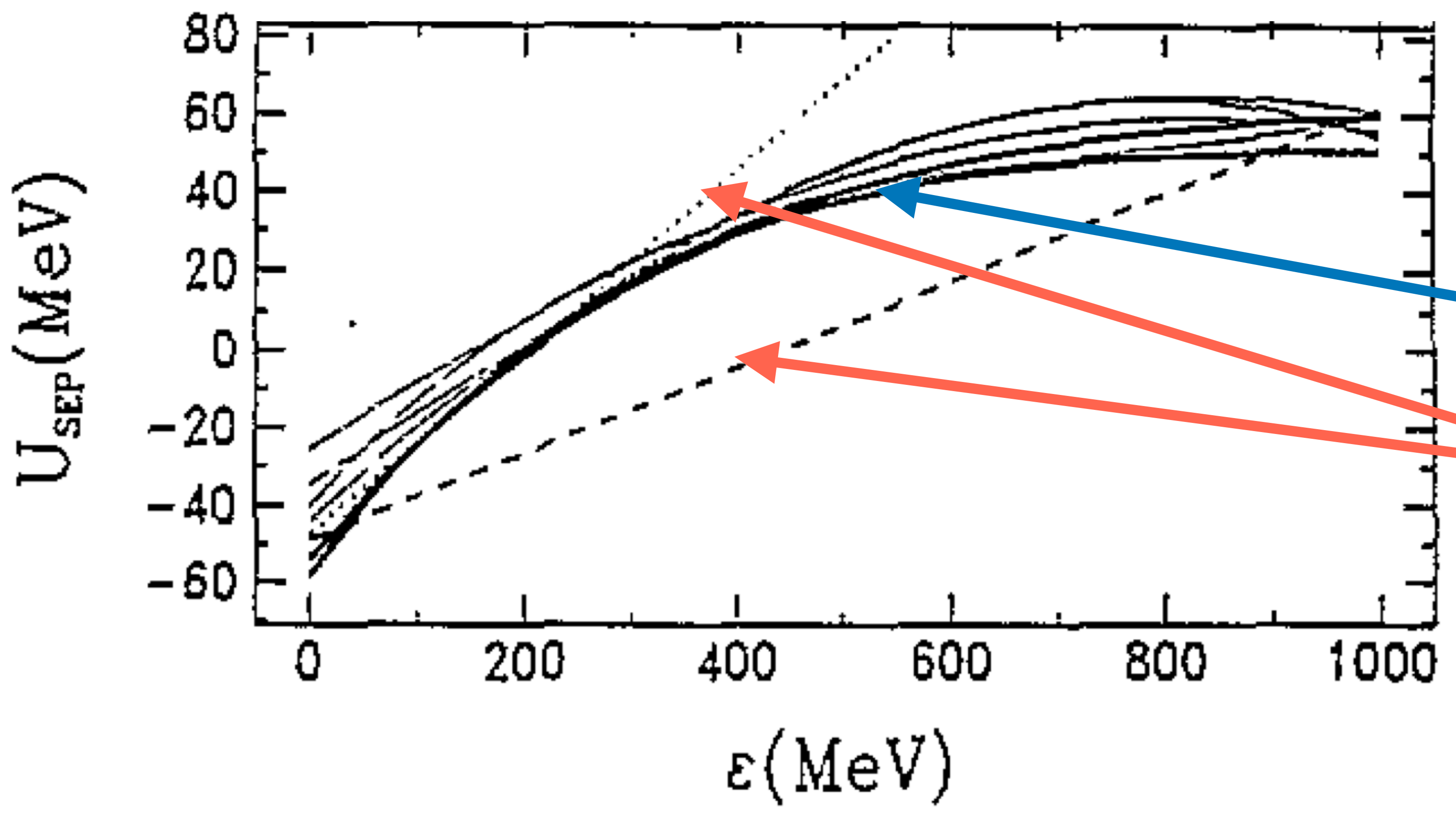

**A. Sorensen** *et al.*, arXiv:2301.13253

197Au+197Au @ 0.15—10 GeV/u
($\sqrt{s_{NN}}$ = 1.95 — 4.72 GeV)
observables: proton flow (Plastic Ball, EOS, E877, E895)
model used: pBUU w/ nucleons, Δ, N*(1440), pions;
EOS parametrized by $K_0$;
momentum dependence
Danielewicz, Lacey, Lynch, Science **298**,1592–1596 (2002)

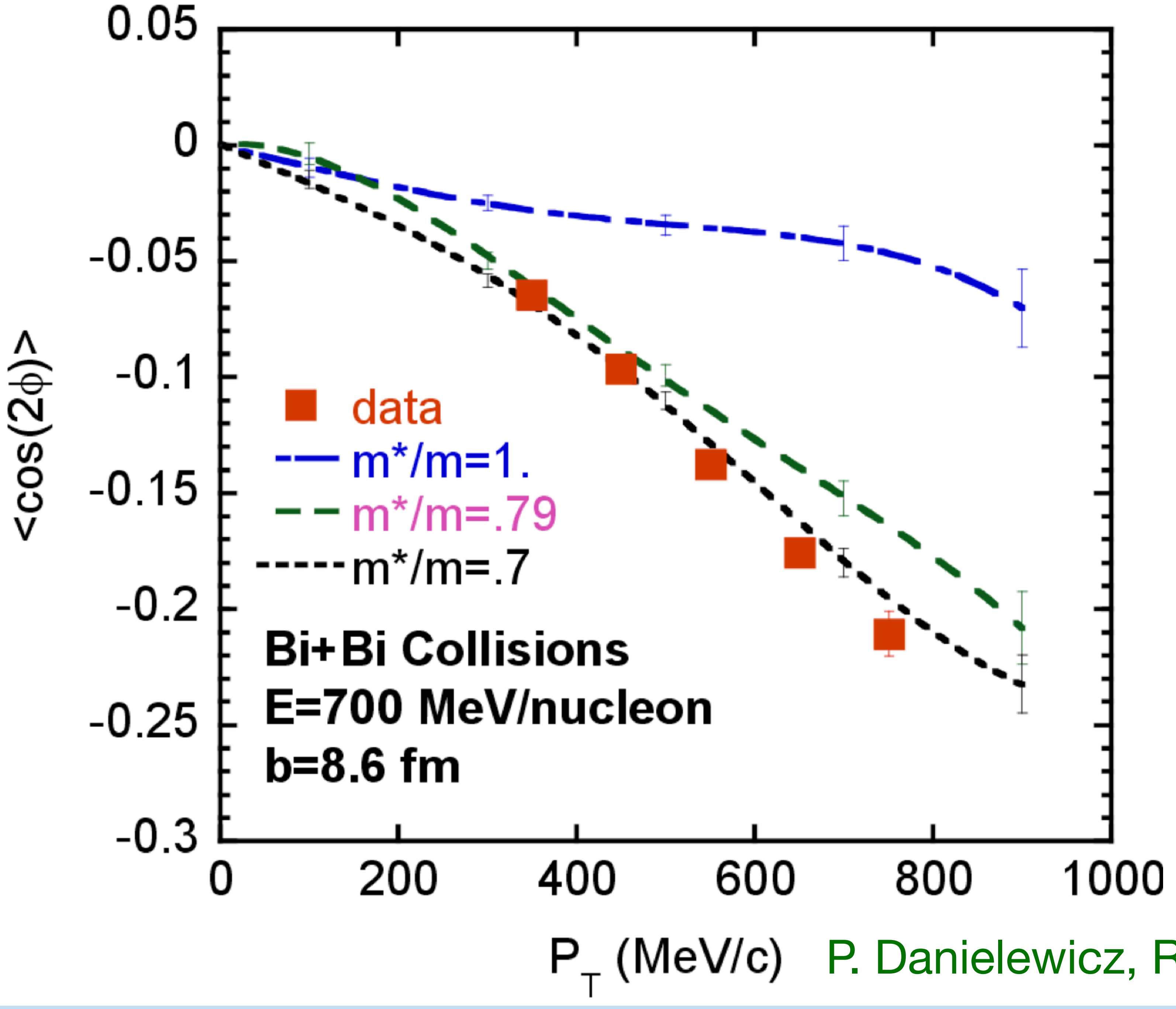
197Au+197Au @ 2.9—9 GeV/u
($\sqrt{s_{NN}}$ = 3 — 4.5 GeV)
observables: proton flow (STAR)
model used: SMASH w/ over 120 hadronic species, including deuterons;
relativistic EOS parametrized independently in different density regions;
**NO momentum dependence**
D. Oliinychenko, AS, V. Koch, L. McLerran, arXiv:2208.11996

# Momentum-dependent mean-fields are a necessary component

Measured in scattering experiments:




B. Blaettel, V. Koch, U. Mosel, Rept. Prog. Phys. **56**,1–62 (1993)




Affects the $p_T$-dependence of the elliptic flow

P. Danielewicz, R. Lacey, and W. G. Lynch, Science **298**, 1592 (2002), arXiv:nucl-th/0208016

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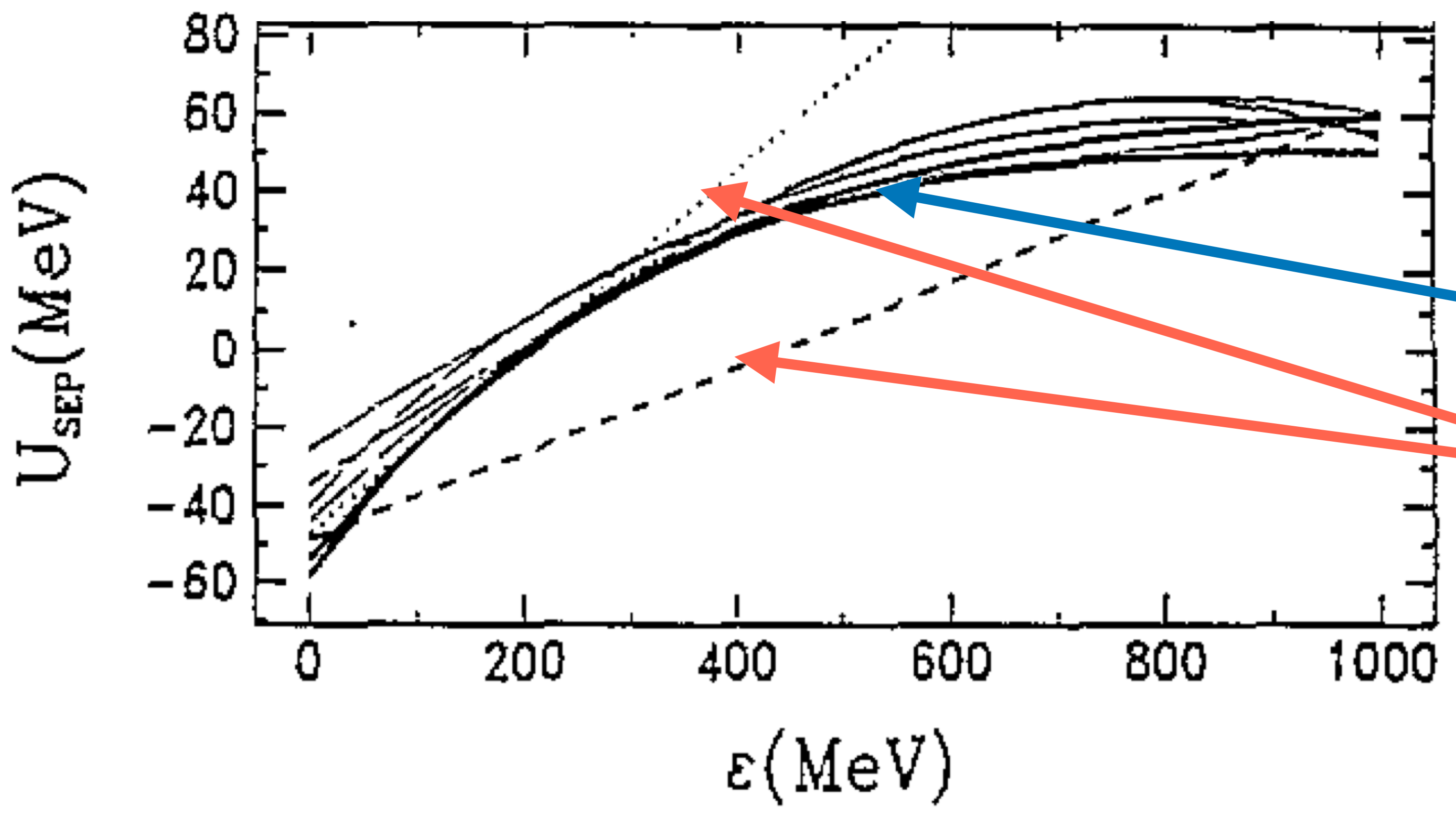
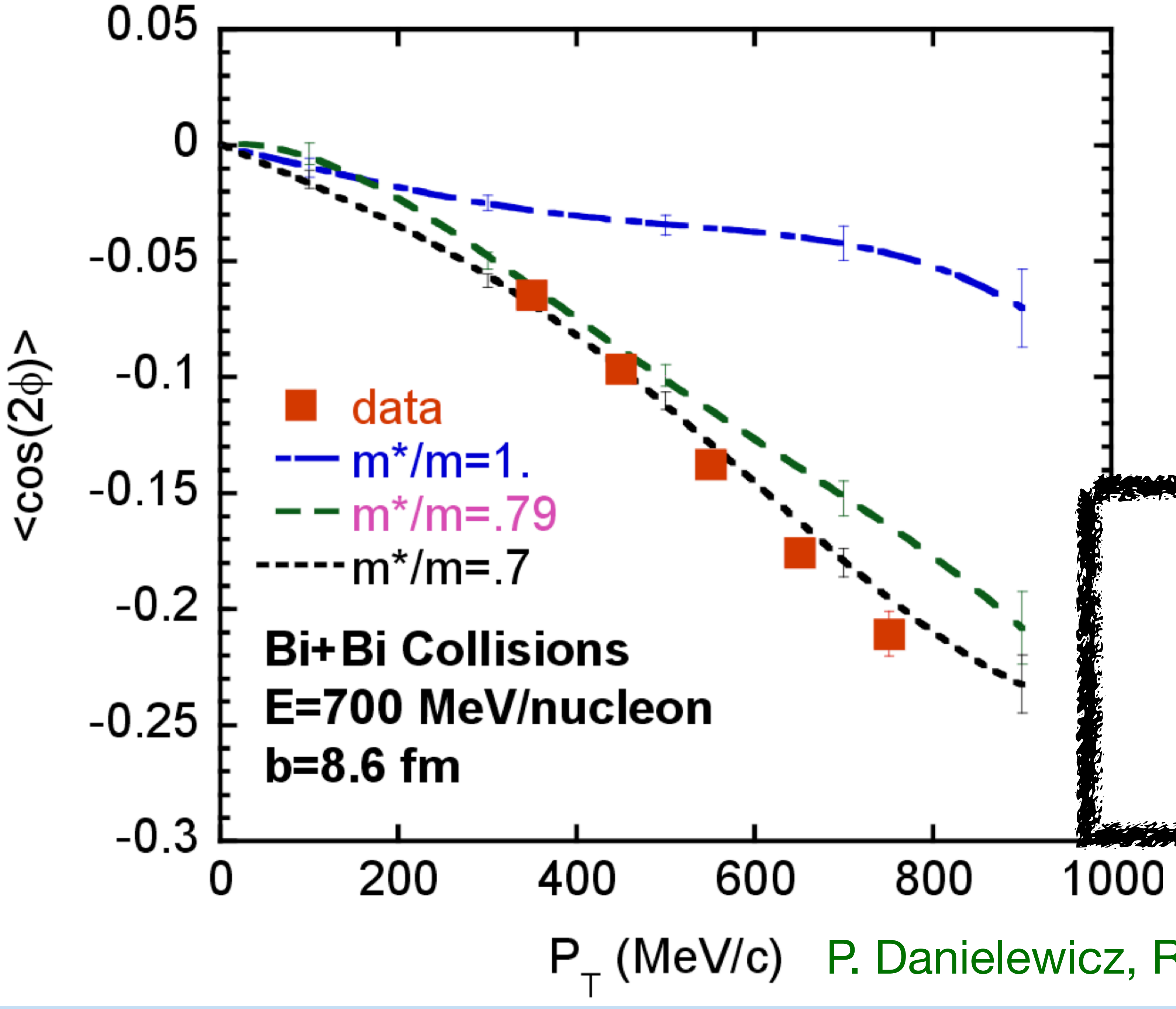



B. Blaettel, V. Koch, U. Mosel, Rept. Prog. Phys. **56**,1–62 (1993)

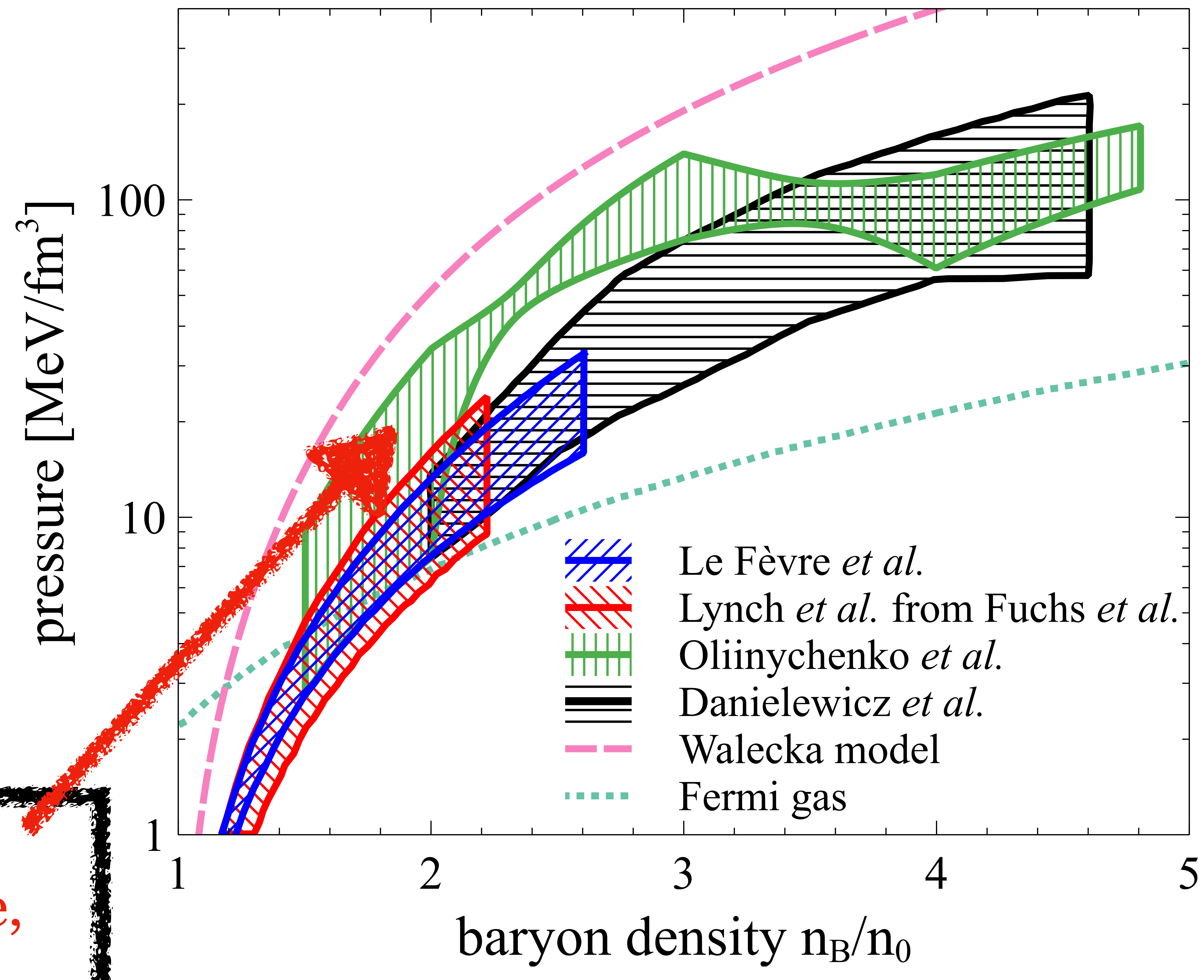
fits to data

parametrizations of the Walecka model




Affects the $p_T$-dependence of the elliptic flow

Without momentum dependence, the extracted EOS is too stiff!

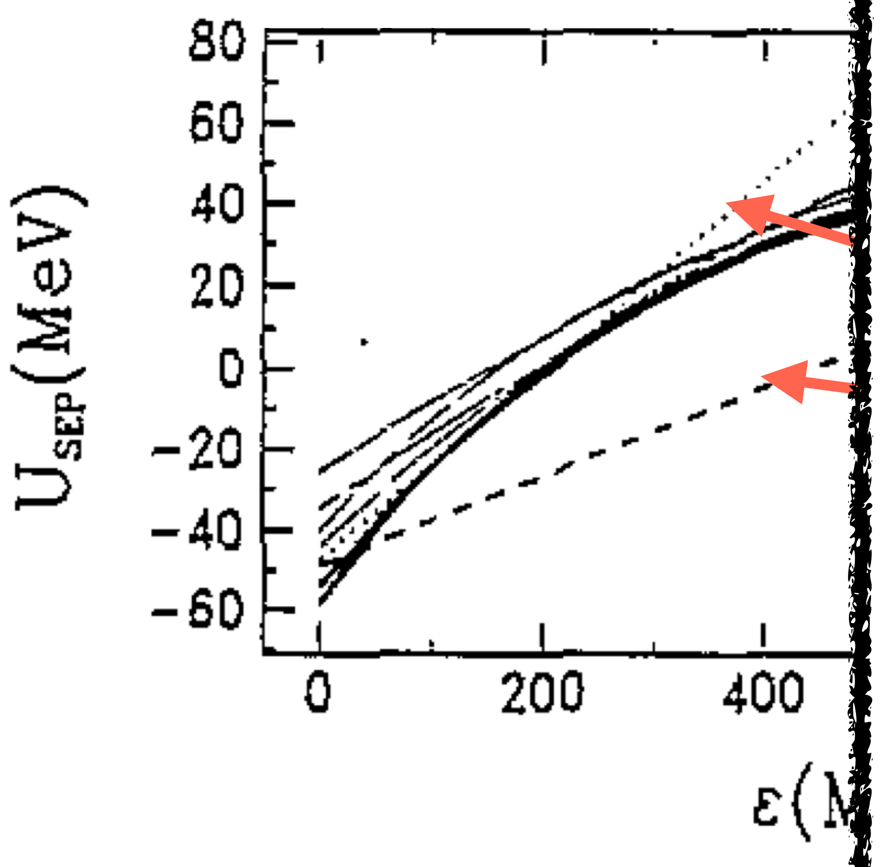


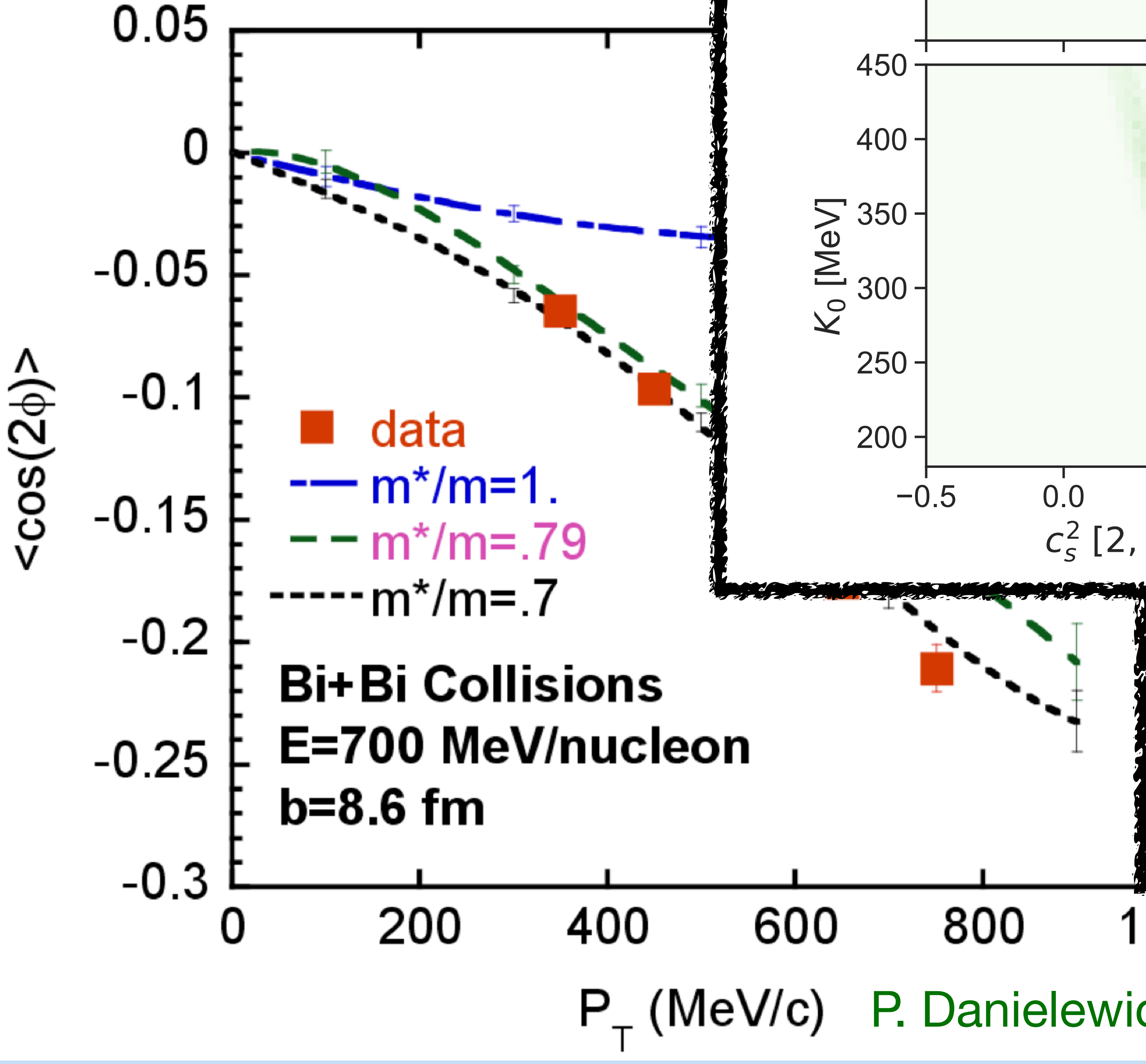

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

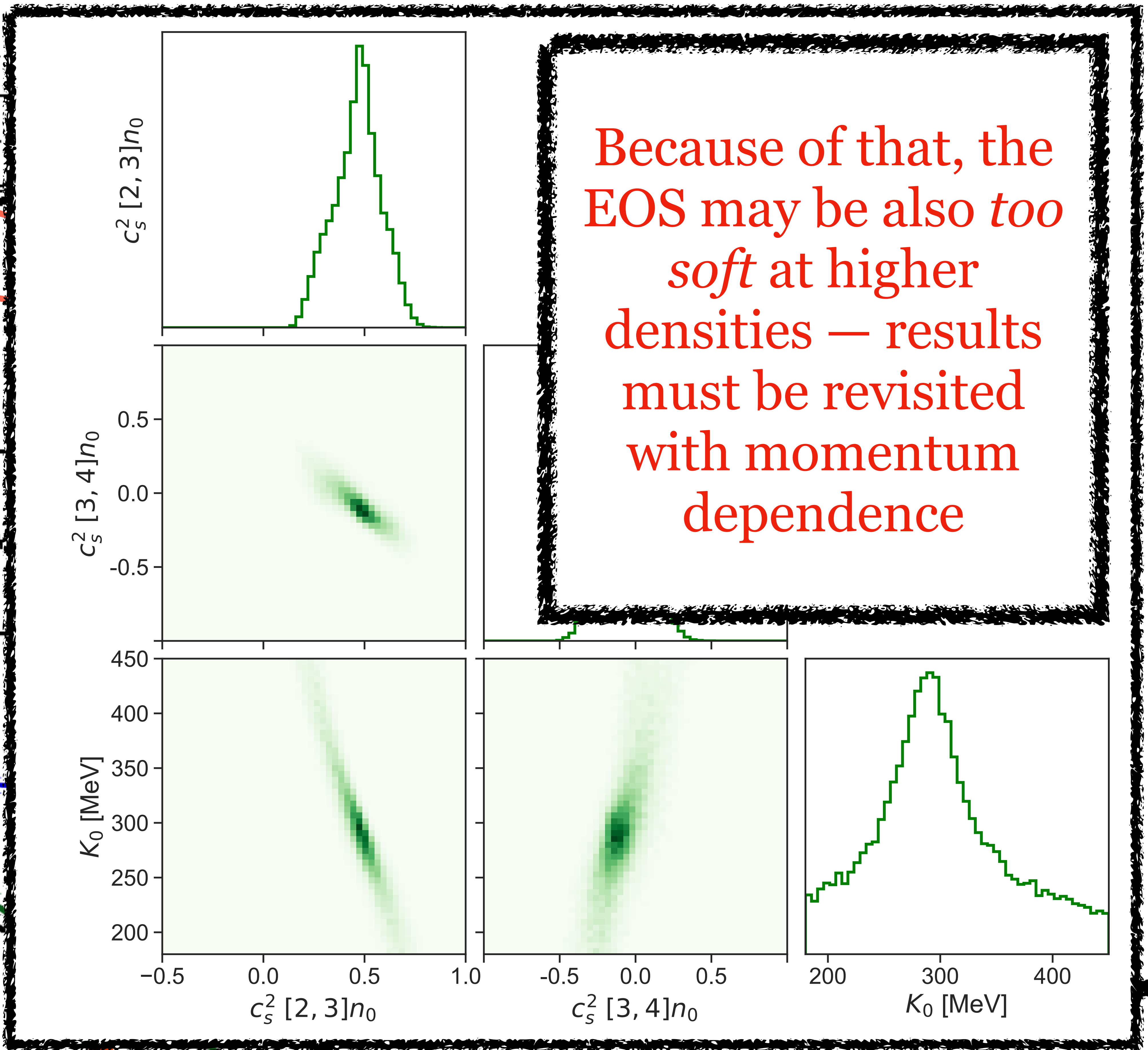
P. Danielewicz, R. Lacey, and W. G. Lynch, Science **298**, 1592 (2002), arXiv:nucl-th/0208016

# Momentum-dependent mean-fields are a necessary component

Measured in scatteri

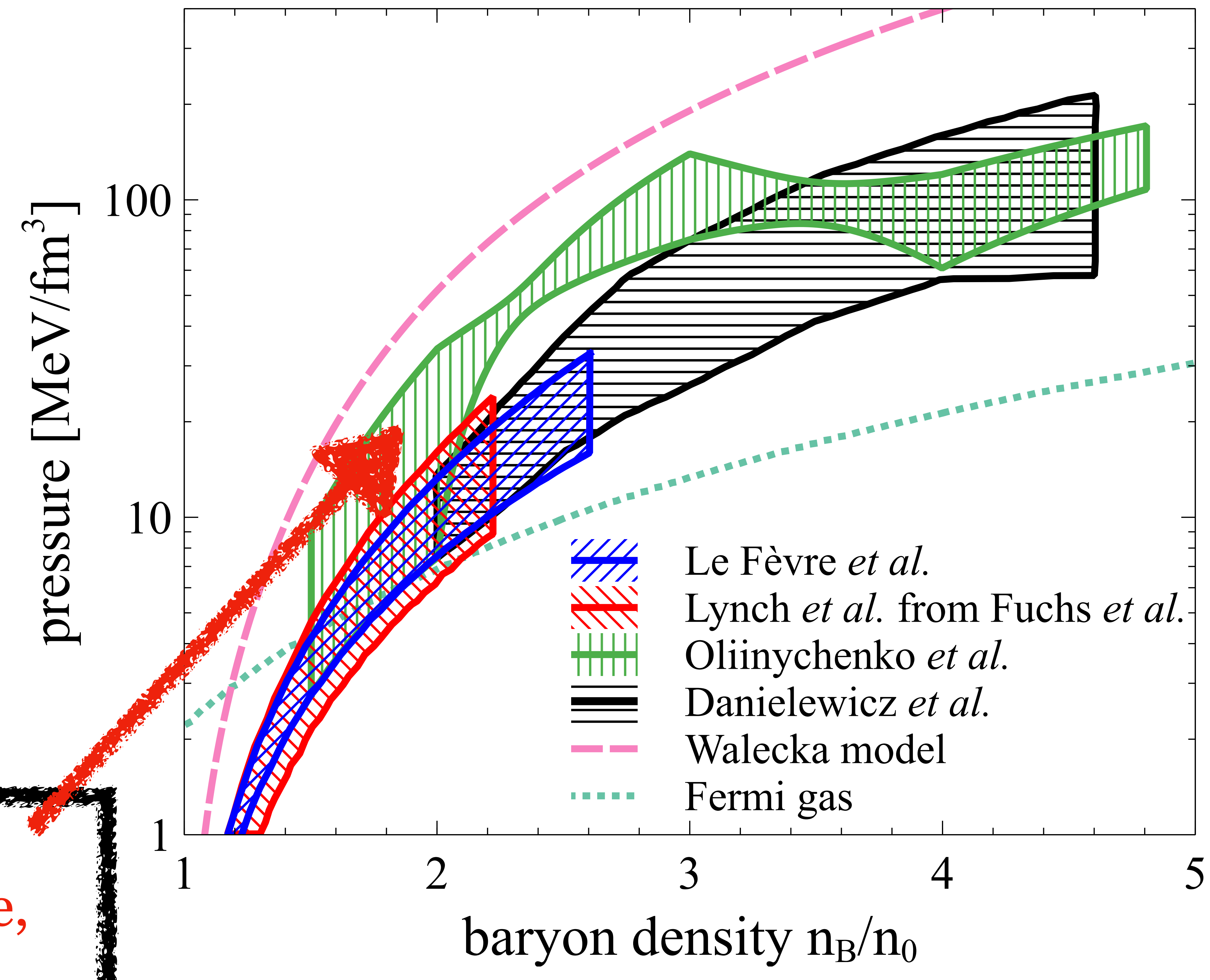



Because of that, the EOS may be also *too soft* at higher densities — results must be revisited with momentum dependence

Without momentum dependence, the extracted EOS is too stiff!

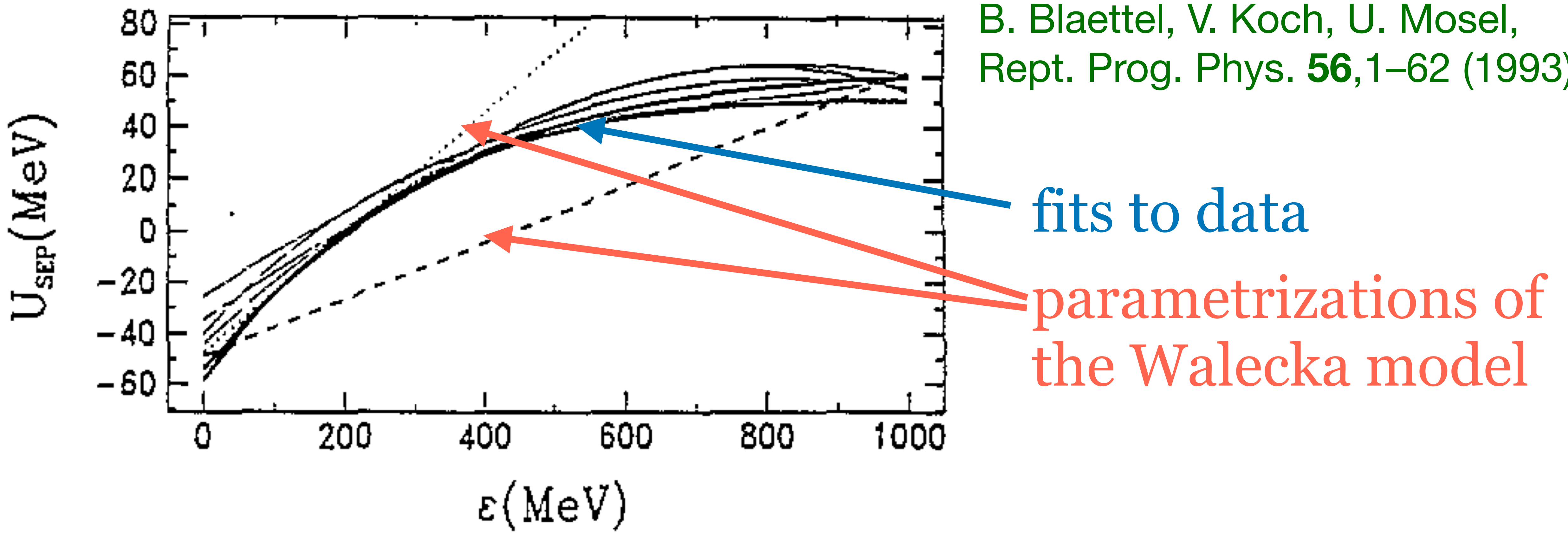



D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

P. Danielewicz, R. Lacey, and W. G. Lynch, Science **298**, 1592 (2002), arXiv:nucl-th/0208016

# Work in progress: Flexible momentum-dependent mean-fields

Measured in scattering experiments:




B. Blaettel, V. Koch, U. Mosel, Rept. Prog. Phys. **56**,1–62 (1993)

Solution:
vector+scalar density functional model (VSDF)

Challenge: scalar fields are costly to compute

**VDF model:**

$$\mathcal{E}_N = g\int \frac{d^3p}{(2\pi)^3}\,\epsilon_{\text{kin}}\, f_{\mathbf{p}} + \sum_{i=1}^{N} A_k^0 j_0 - g^{00}\sum_{i=1}^{N}\left(\frac{b_i-1}{b_i}\right)A_k^\lambda j_\lambda$$

$$A_k^\mu = C_k\left(j_\lambda j^\lambda\right)^{\frac{b_k}{2}-1} j^\mu\,, \qquad j_\mu j^\mu = n_B^2\,, \qquad j^\mu = g\int \frac{d^3p}{(2\pi)^3}\,\frac{p^\mu - A^\mu}{\epsilon^*_{kin}}\, f_{\mathbf{p}}$$

**VSDF model:**

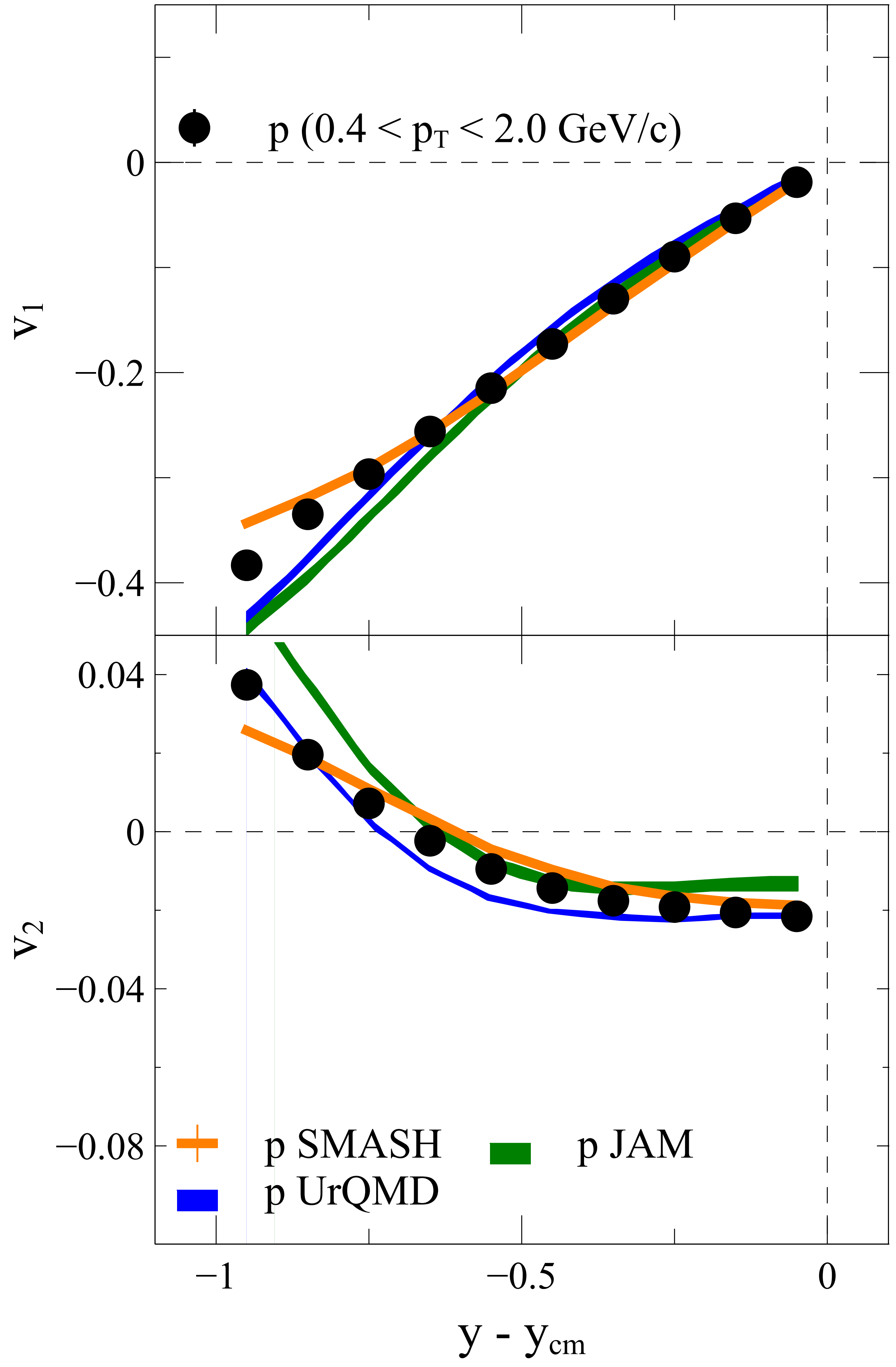
$$\mathcal{E}_{N,\,M} = g\int \frac{d^3p}{(2\pi)^3}\,\epsilon^*_{\text{kin}}\, f_{\mathbf{p}} + \sum_{i=1}^{N} A_k^0 j_0 - g^{00}\sum_{i=1}^{N}\left(\frac{b_i-1}{b_i}\right)A_k^\lambda j_\lambda + g^{00}\sum_{m=1}^{M} G_m\left(\frac{d_m-1}{d_m}\right)n_s^{d_m}$$

$$m^* = m_0 - \sum_{m=1}^{M} G_m n_s^{d_m-1} \qquad n_s = g\int \frac{d^3p}{(2\pi)^3}\,\frac{m^*}{\epsilon^*_{\text{kin}}}\, f_{\mathbf{p}}$$

**A. Sorensen**, "Density Functional Equation of State and Its Application to the Phenomenology of Heavy-Ion Collisions," arXiv:2109.08105, Sorensen:2021zxd

# Describing proton flow is not enough

$\sqrt{s_{NN}} = 3$ GeV




STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2108.00908
D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996
**A. Sorensen** *et al.*, arXiv:2301.13253

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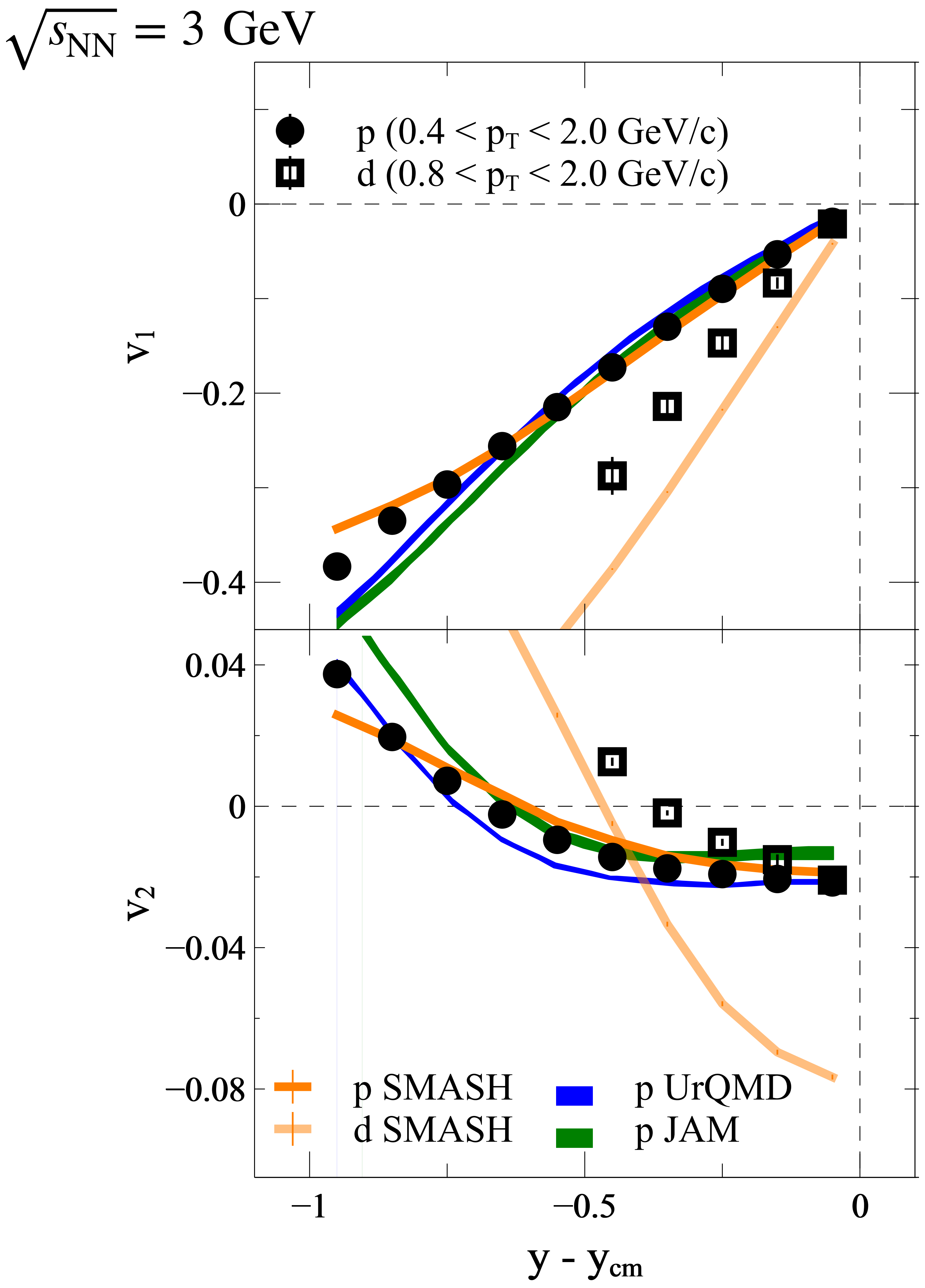


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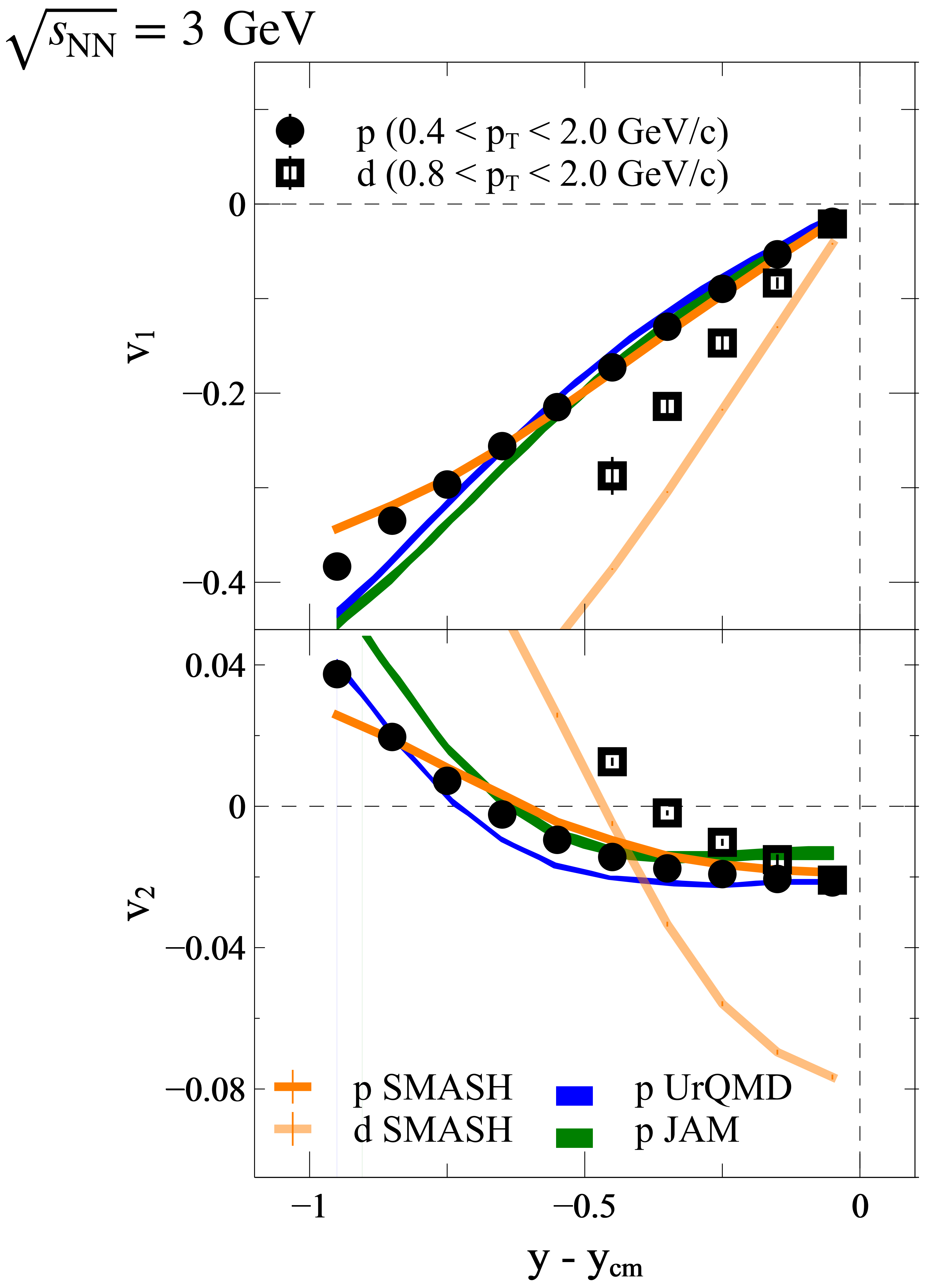


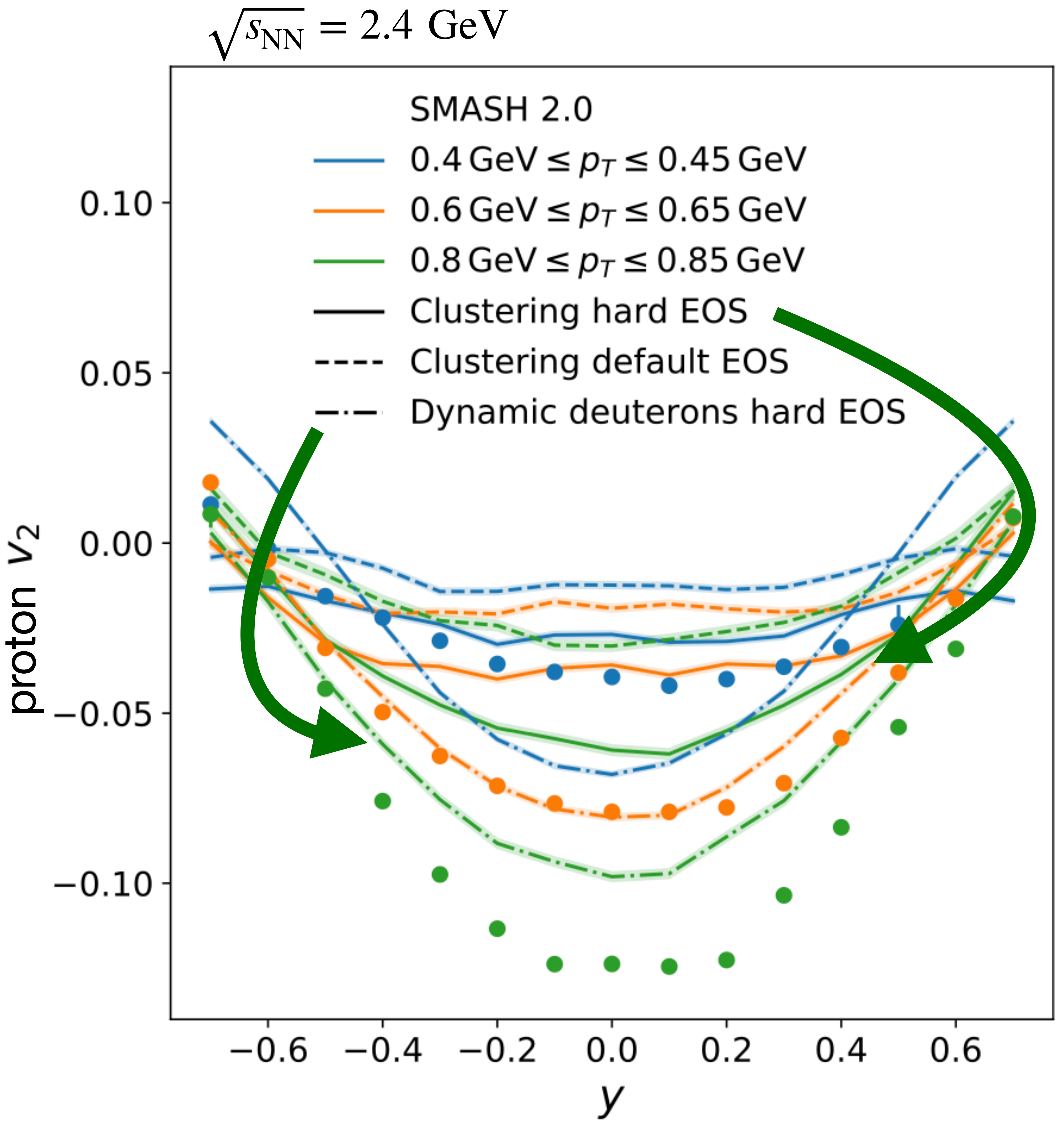


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D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996
**A. Sorensen** *et al.*, arXiv:2301.13253

J. Mohs, M. Ege, H. Elfner, M. Mayer, Phys. Rev. C **105** 3, 034906 (2022), arXiv:2012.11454

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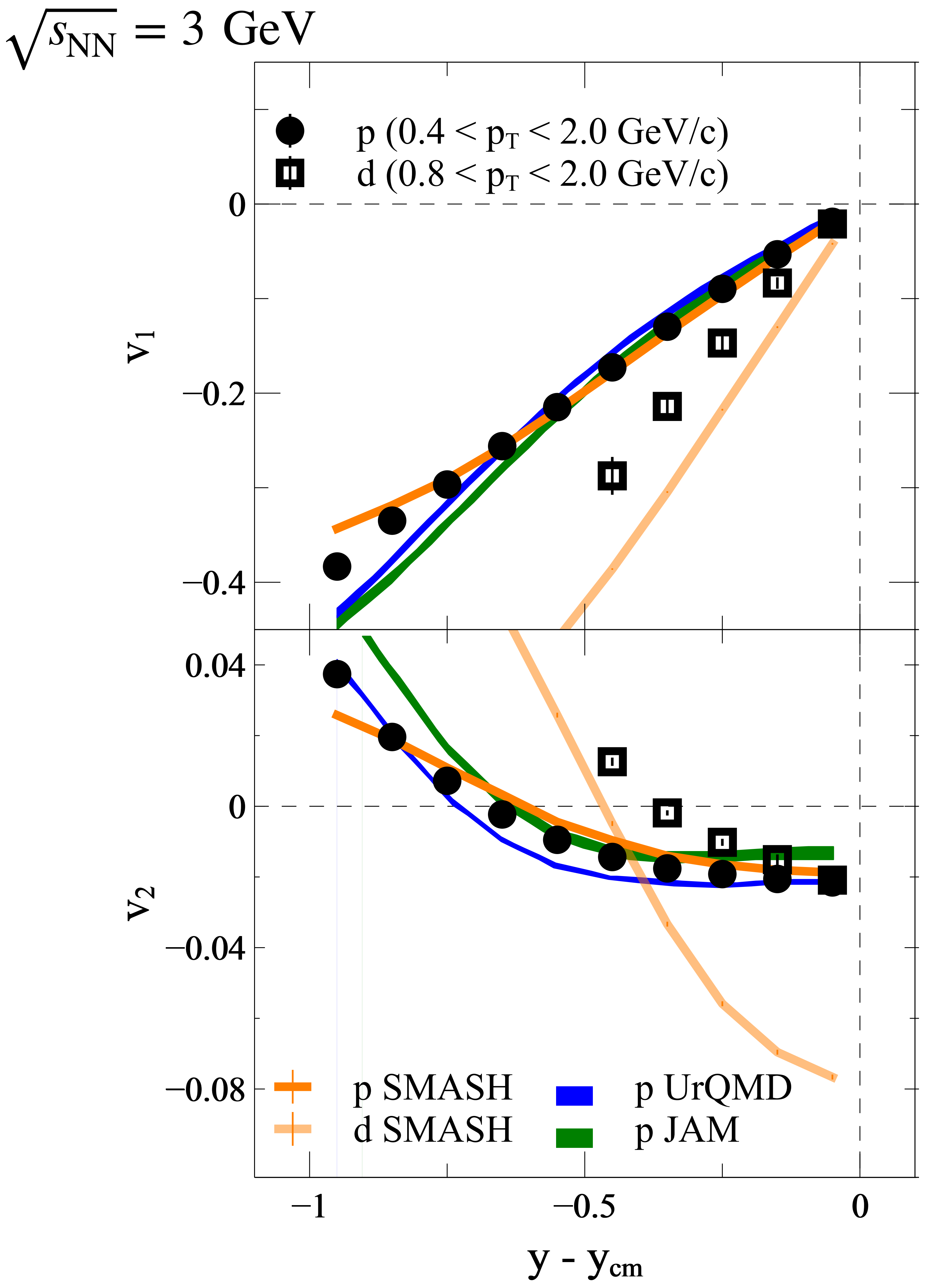
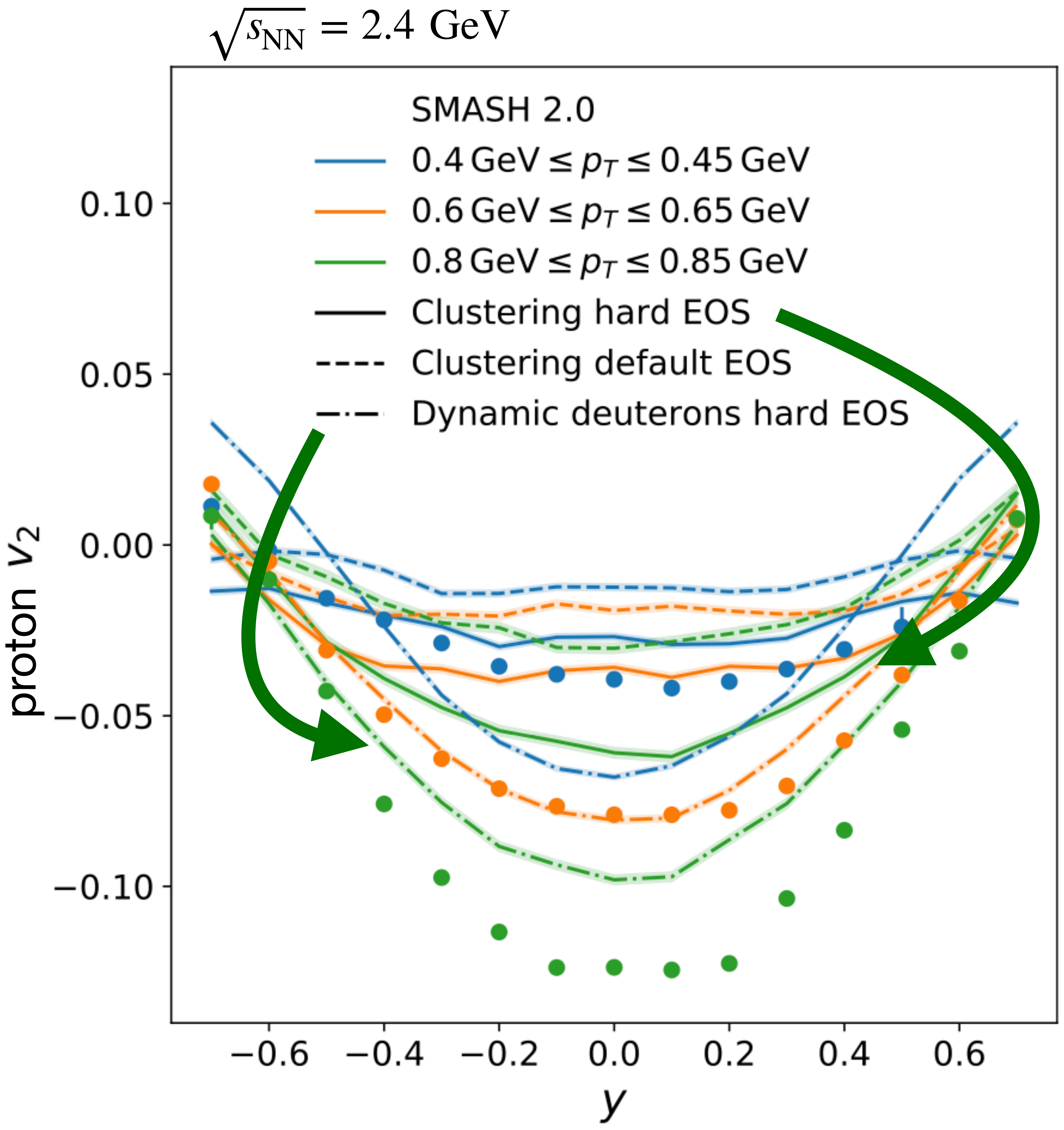



Realistic description of light cluster production needed:

- coalescence: doesn't take into account the dynamic role of light clusters throughout the evolution
- nucleon/pion catalysis: consider as separate degrees of freedom (pBUU, SMASH), produced through $N$ or $\pi$ collisions
- the Holy Grail: dynamical production through potentials

STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2108.00908
D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996
**A. Sorensen** *et al.*, arXiv:2301.13253

J. Mohs, M. Ege, H. Elfner, M. Mayer, Phys. Rev. C **105** 3, 034906 (2022), arXiv:2012.11454

# Connection between HICs and NSs: the symmetry energy

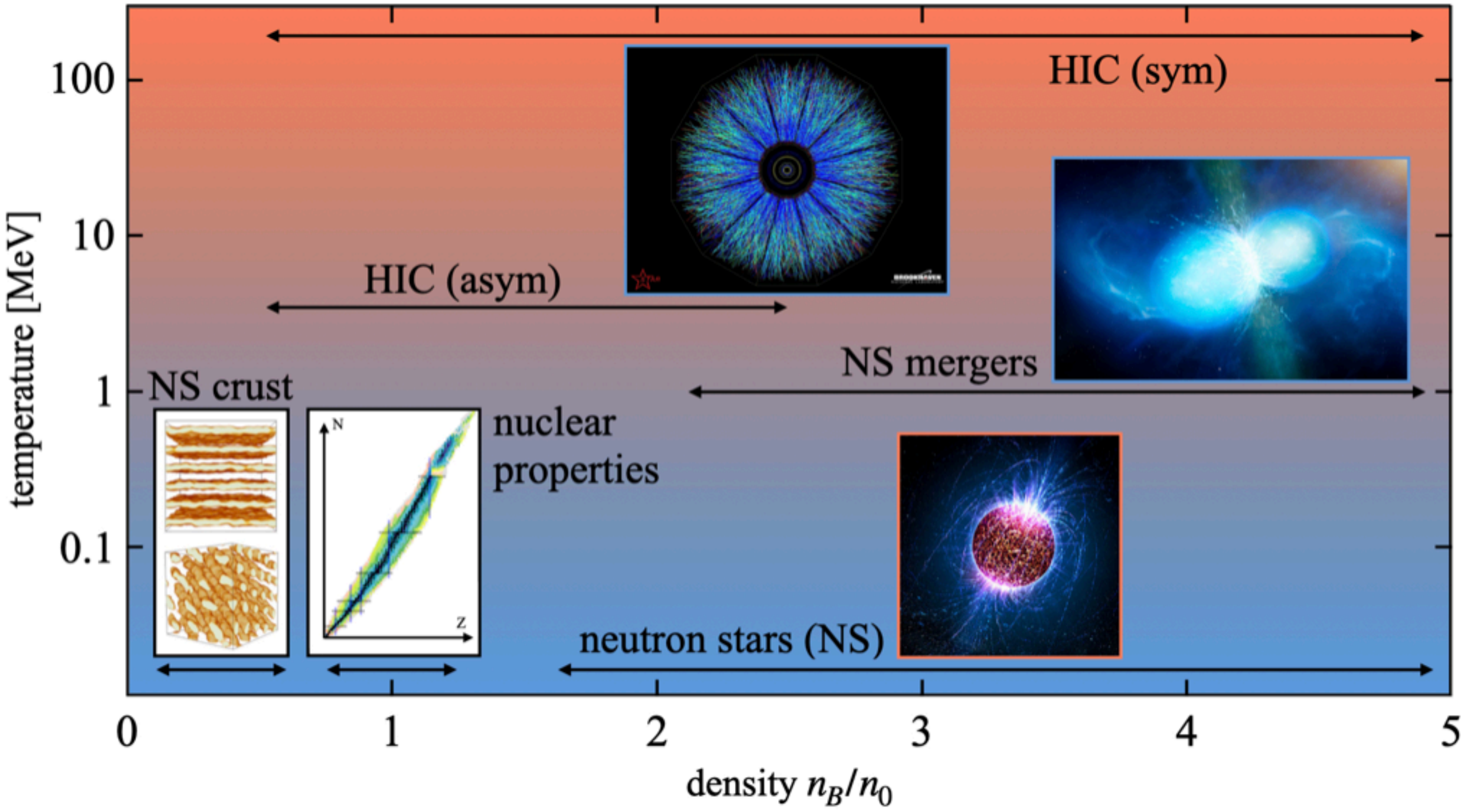
Energy per baryon:

$$\frac{E}{A}(n_B) \equiv \epsilon(n_B) = \epsilon_{\mathrm{SNM}}(n_B) + S(n_B)\delta^2$$

symmetric nuclear matter

symmetry energy

isospin asymmetry: $\delta \equiv \dfrac{N_n - N_p}{N_n + N_p}$




A. Sorensen *et al.*, arXiv:2301.13253

for $^{197}$Au: $\delta_{^{197}\mathrm{Au}} \equiv \dfrac{118-79}{118+79} \approx 0.198 \quad \Rightarrow \quad \delta^2_{^{197}\mathrm{Au}} \approx 0.039$

for $^{108}$Sn: $\delta_{^{108}\mathrm{Sn}} \equiv \dfrac{58-50}{58+50} \approx 0.074 \quad \Rightarrow \quad \delta^2_{^{108}\mathrm{Sn}} \approx 0.006$

for $^{132}$Sn: $\delta_{^{132}\mathrm{Sn}} \equiv \dfrac{82-50}{82+50} \approx 0.24 \quad \Rightarrow \quad \delta^2_{^{132}\mathrm{Sn}} \approx 0.059$

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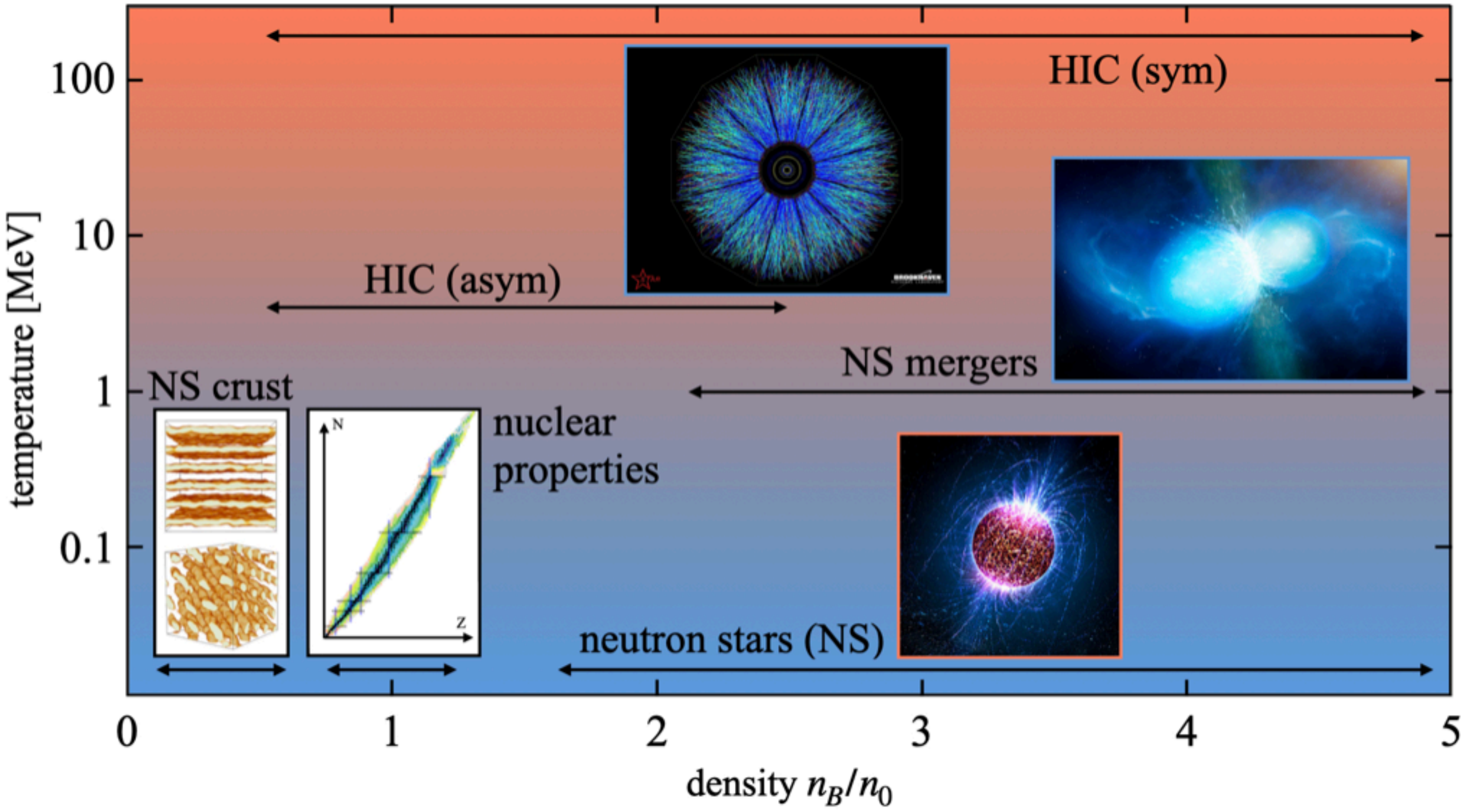


**A. Sorensen** *et al.*, arXiv:2301.13253

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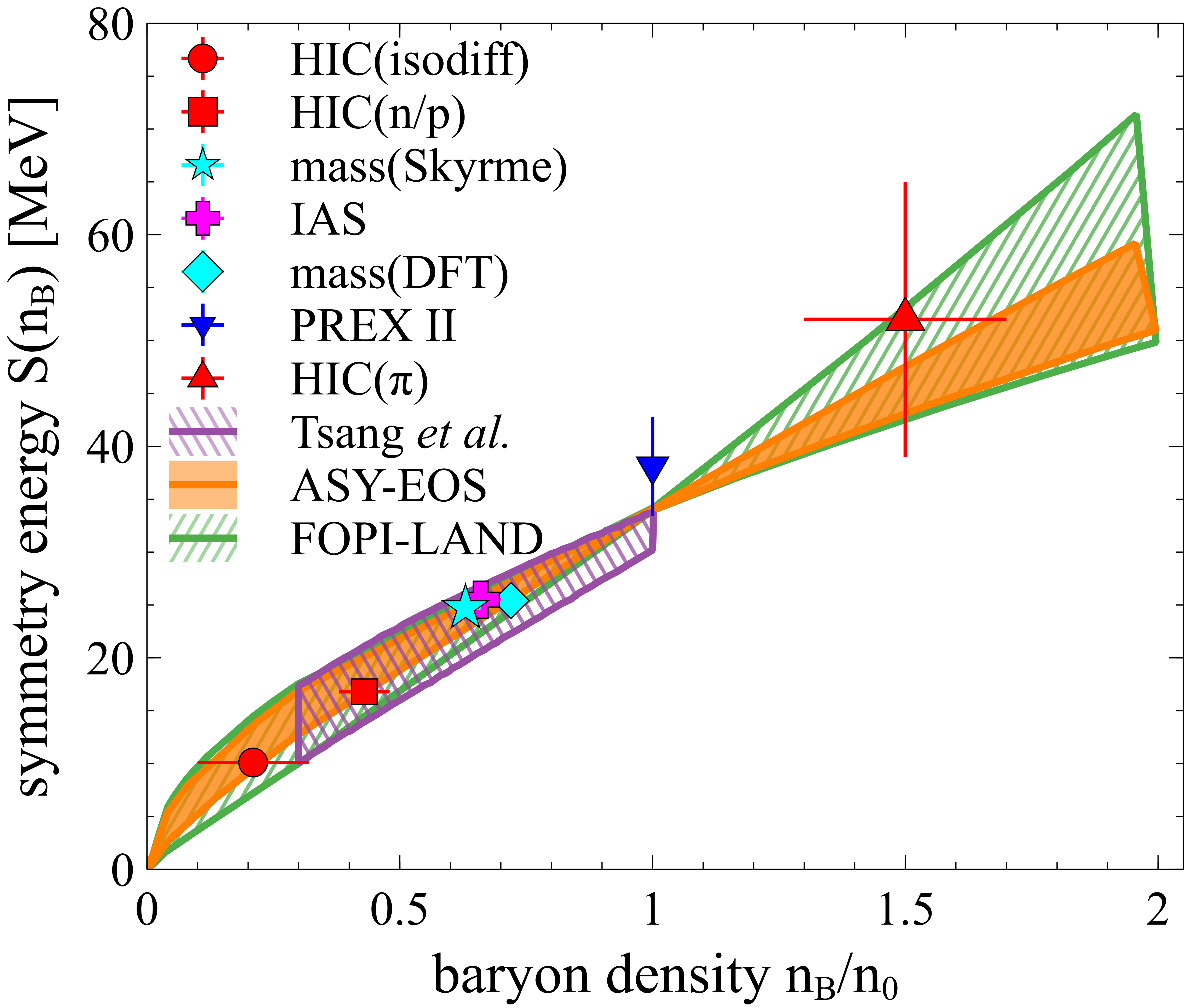
Contributions from the symmetry energy in HICs are generally small!

*but* FRIB will allow for an unprecedented spread in values of $\delta$

# EOS of asymmetric nuclear matter: selected results

Symmetry energy



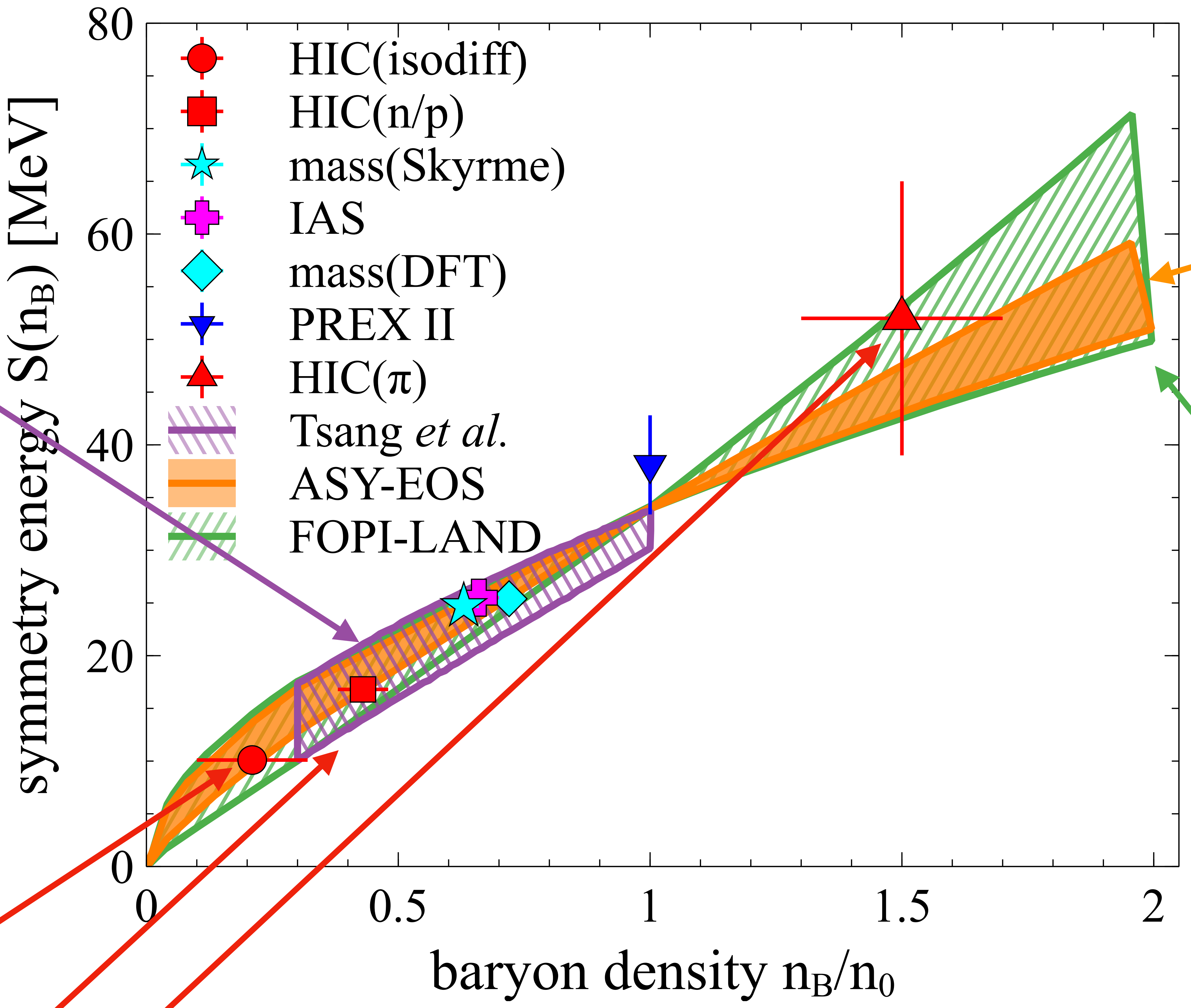

**A. Sorensen** *et al.*, arXiv:2301.13253

# EOS of asymmetric nuclear matter: selected results

**Symmetry energy**

112Sn+124Sn @ = 0.05 GeV/u
($\sqrt{s_{NN}}$ = 1.97 GeV)
observables: isospin diffusion, ratio of neutron to proton spectra
model used: ImQMD
M. B. Tsang, Y. Zhang, P. Danielewicz, M. Famiano, Z. Li, W. G. Lynch, A. W. Steiner, Phys. Rev. Lett. **102**, 122701 (2009), arXiv:0811.3107

Sn systems @ < 0.27 GeV/u
($\sqrt{s_{NN}}$ < 2.01 GeV)
observables: isospin diffusion, neutron to proton energy spectra, pion ratios (S$\pi$RIT)
model used: ImQMD, dcQMD, momentum dependence
W. G. Lynch and M. B. Tsang, Phys. Lett. B 830, 137098 (2022), arXiv:2106.10119




**A. Sorensen** *et al.*, arXiv:2301.13253

197Au+197Au @ 0.4 GeV/u
($\sqrt{s_{NN}}$ = 2.07 GeV)
observables: ratio of neutron to charged fragments (ASY-EOS)
model used: UrQMD, momentum dependence
P. Russotto *et al.*, Phys. Rev. C **94**, 034608 (2016), arXiv:1608.04332

197Au+197Au @ 0.4 GeV/u
($\sqrt{s_{NN}}$ = 2.07 GeV)
observables: ratio of elliptic flow of neutrons and hydrogen nuclei (FOPI-LAND)
model used: UrQMD, momentum dependence
P. Russotto *et al.*, Phys. Lett. B **697**, 471 (2011), arXiv:1101.2361

# Better modeling is necessary for obtaining $S(n_B)$

Ideas to explore:

- threshold effects,
- light cluster production,
- neutron-proton effective mass splitting, …

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Strong efforts by the Transport Model Evaluation Project (TMEP) collaboration to identify code-dependencies and best model practices!

Transport model comparison studies of intermediate-energy heavy-ion collisions #1
TMEP Collaboration • Hermann Wolter (Munich U.) et al. (Feb 14, 2022)
Published in: *Prog.Part.Nucl.Phys.* 125 (2022) 103962 • e-Print: 2202.06672 [nucl-th]
pdf DOI cite claim reference search 31 citations

Comparison of heavy-ion transport simulations: Mean-field dynamics in a box #2
TMEP Collaboration • Maria Colonna (INFN, LNS) et al. (Jun 23, 2021)
Published in: *Phys.Rev.C* 104 (2021) 2, 024603 • e-Print: 2106.12287 [nucl-th]
pdf DOI cite claim reference search 29 citations

Symmetry energy investigation with pion production from Sn+Sn systems #3
SpiRIT and TMEP Collaborations • G. Jhang et al. (Dec 13, 2020)
Published in: *Phys.Lett.B* 813 (2021) 136016 • e-Print: 2012.06976 [nucl-ex]
pdf DOI cite claim reference search 34 citations

Comparison of heavy-ion transport simulations: Collision integral with pions and Δ resonances in a box #4
TMEP Collaboration • Akira Ono (Tohoku U.) et al. (Apr 5, 2019)
Published in: *Phys.Rev.C* 100 (2019) 4, 044617 • e-Print: 1904.02888 [nucl-th]
pdf DOI cite claim reference search 59 citations

Comparison of heavy-ion transport simulations: Collision integral in a box #5
TMEP Collaboration • Ying-Xun Zhang (Beijing, Inst. Atomic Energy and Guangxi Normal U.) et al. (Nov 16, 2017)
Published in: *Phys.Rev.C* 97 (2018) 3, 034625 • e-Print: 1711.05950 [nucl-th]
pdf DOI cite claim reference search 103 citations

Understanding transport simulations of heavy-ion collisions at 100A and 400A MeV: Comparison of heavy-ion transport codes under controlled conditions #6
TMEP Collaboration • Jun Xu (SINAP, Shanghai) et al. (Mar 26, 2016)
Published in: *Phys.Rev.C* 93 (2016) 4, 044609 • e-Print: 1603.08149 [nucl-th]

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pdf DOI cite claim reference search 31 citations

#2 Comparison of heavy-ion transport simulations: Mean-field dynamics in a box
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Published in: *Phys.Rev.C* 104 (2021) 2, 024603 • e-Print: 2106.12287 [nucl-th]
pdf DOI cite claim reference search 29 citations

#3 Symmetry energy investigation with pion production from Sn+Sn systems
SpiRIT and TMEP Collaborations • G. Jhang et al. (Dec 13, 2020)
Published in: *Phys.Lett.B* 813 (2021) 136016 • e-Print: 2012.06976 [nucl-ex]
pdf DOI cite claim reference search 34 citations

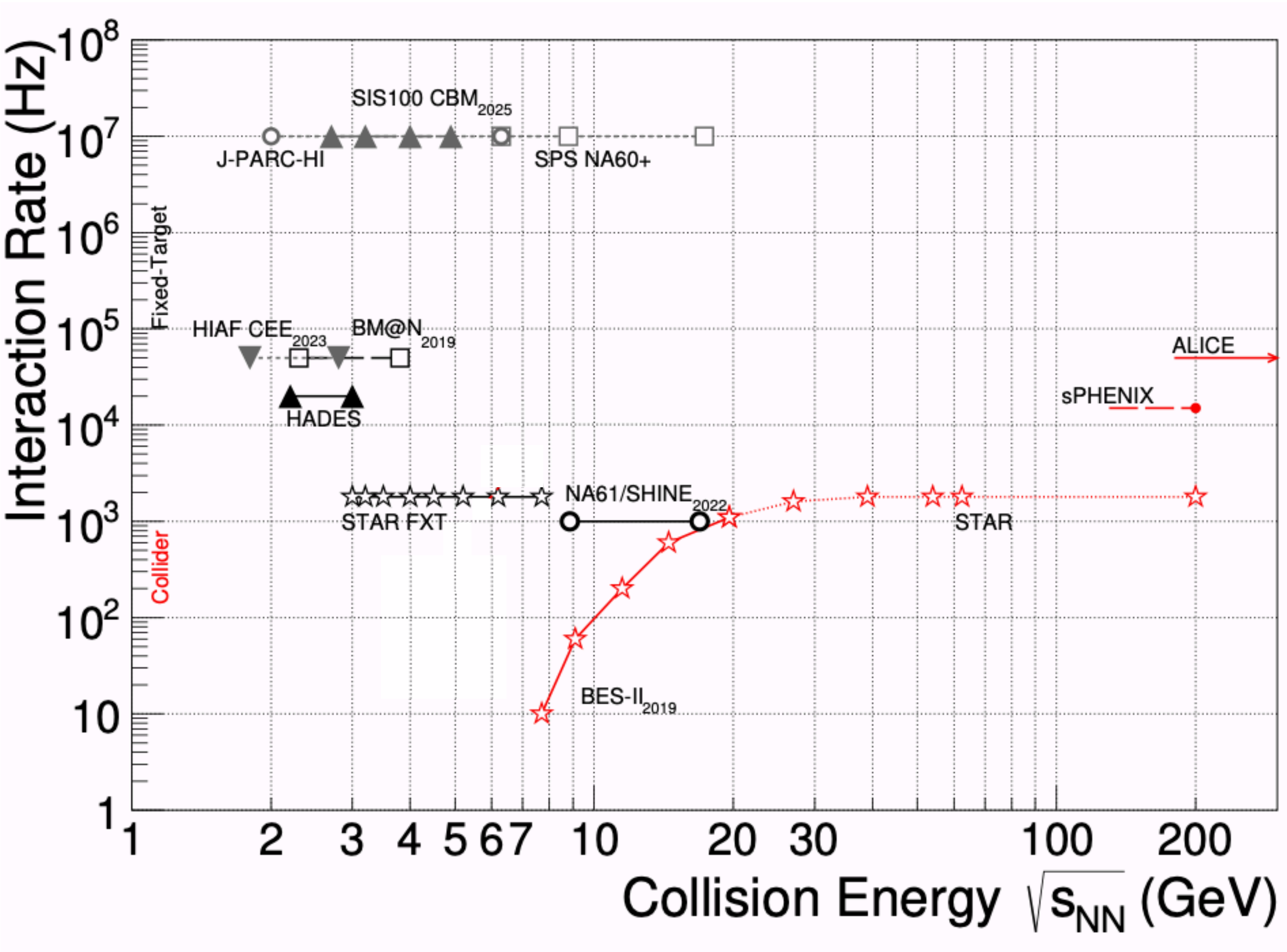
#4 Comparison of heavy-ion transport simulations: Collision integral with pions and Δ resonances in a box
TMEP Collaboration • Akira Ono (Tohoku U.) et al. (Apr 5, 2019)
Published in: *Phys.Rev.C* 100 (2019) 4, 044617 • e-Print: 1904.02888 [nucl-th]
pdf DOI cite claim reference search 59 citations

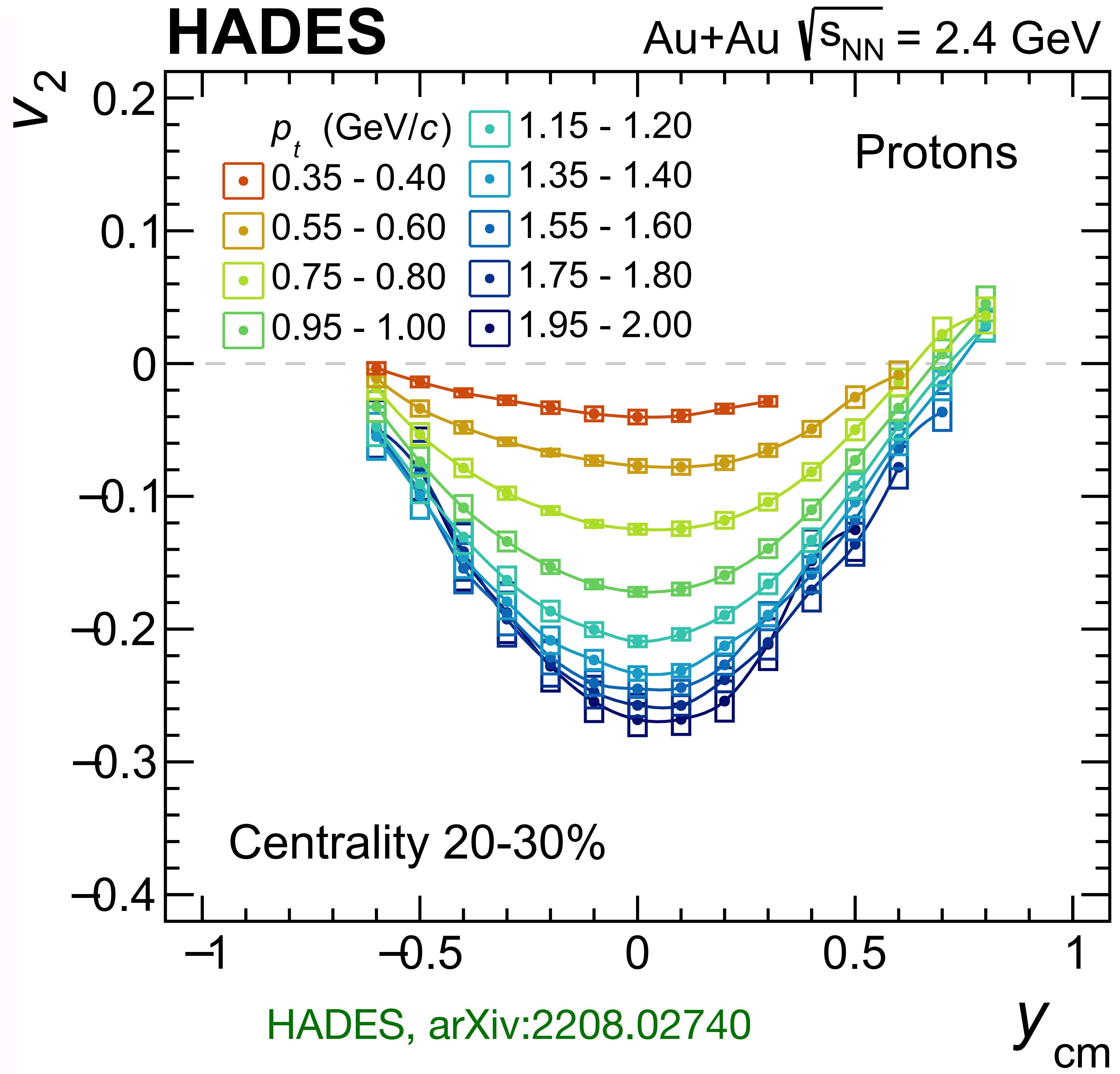
#5 Comparison of heavy-ion transport simulations: Collision integral in a box
TMEP Collaboration • Ying-Xun Zhang (Beijing, Inst. Atomic Energy and Guangxi Normal U.) et al. (Nov 16, 2017)
Published in: *Phys.Rev.C* 97 (2018) 3, 034625 • e-Print: 1711.05950 [nucl-th]
pdf DOI cite claim reference search 103 citations

#6 Understanding transport simulations of heavy-ion collisions at 100A and 400A MeV: Comparison of heavy-ion transport codes under controlled conditions
TMEP Collaboration • Jun Xu (SINAP, Shanghai) et al. (Mar 26, 2016)
Published in: *Phys.Rev.C* 93 (2016) 4, 044609 • e-Print: 1603.08149 [nucl-th]

- *very* high-quality, high-statistics data are imminent from BES FXT & HADES: perhaps observables are now available which were previously inaccessible?

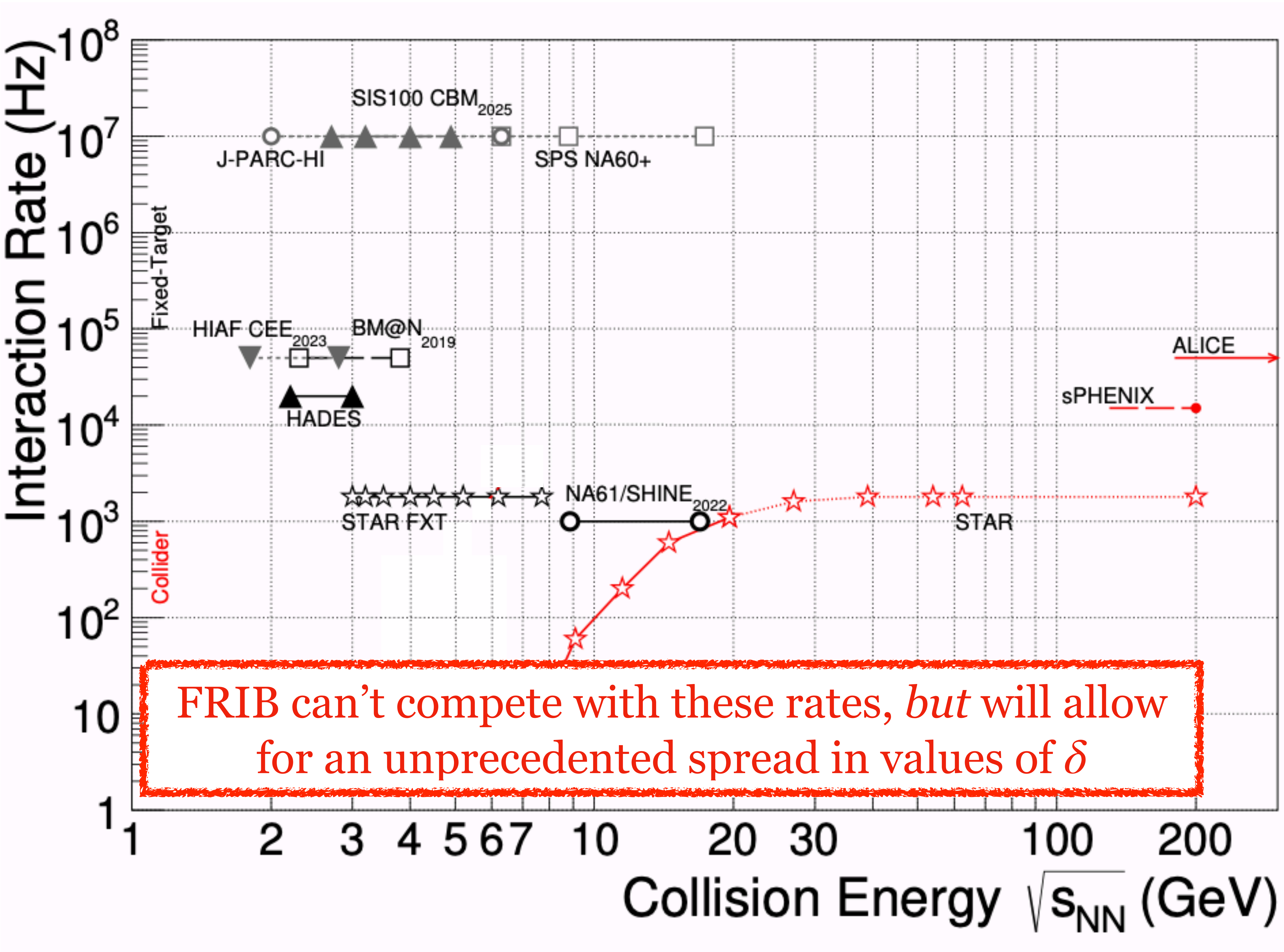
# Precision era of heavy-ion collisions



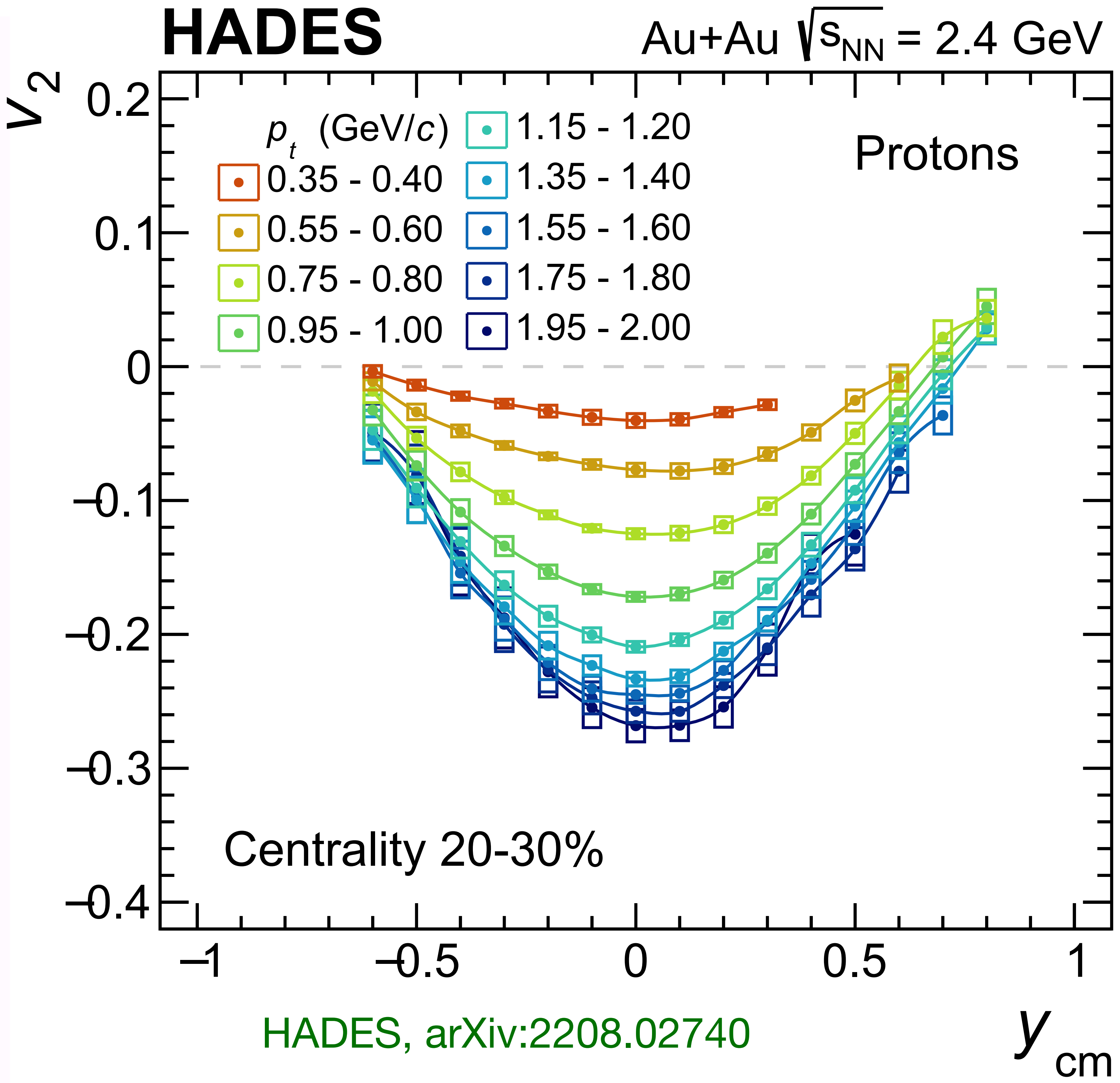




Precision experiments
NEED precision simulations

# Precision era of heavy-ion collisions



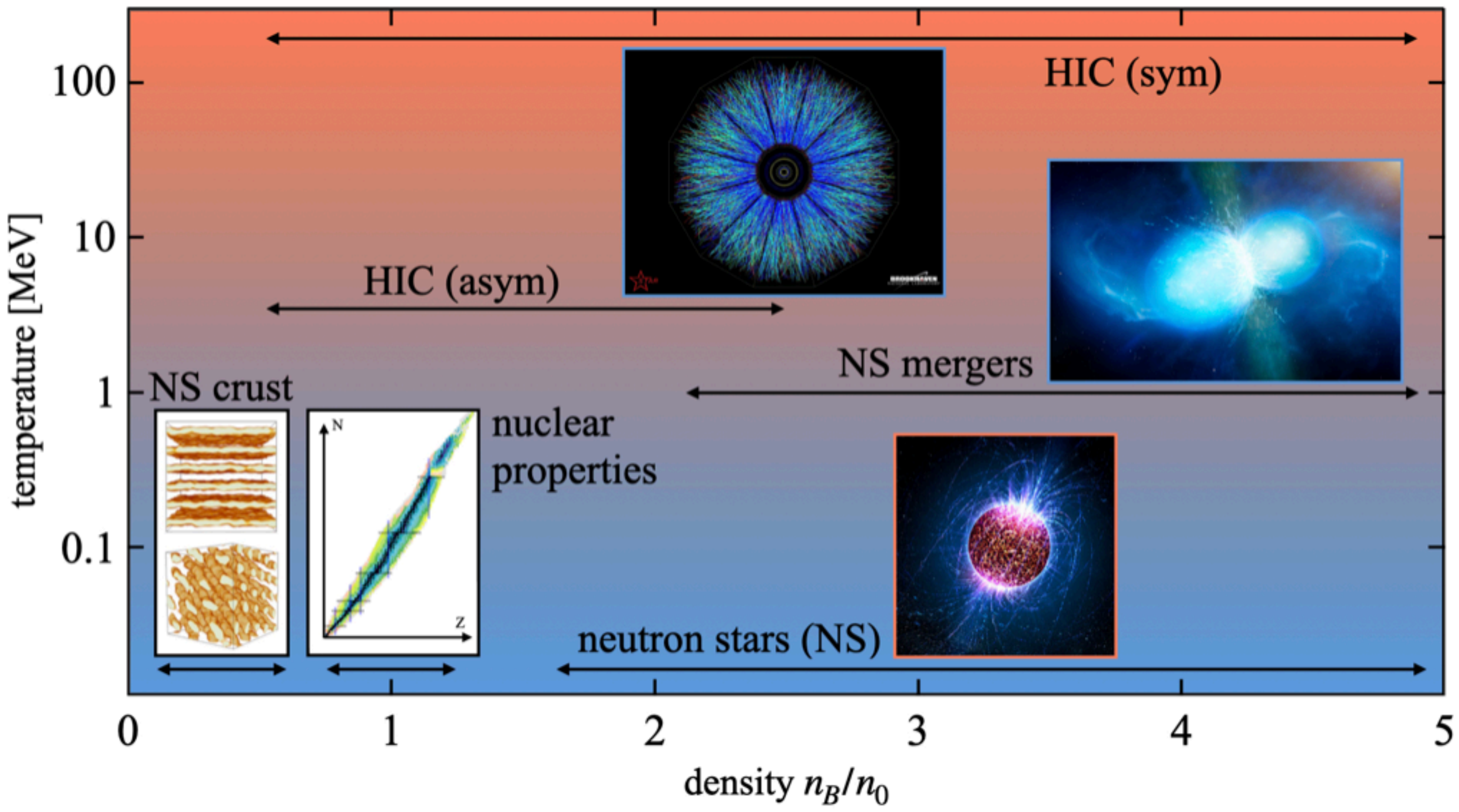

FRIB can't compete with these rates, *but* will allow for an unprecedented spread in values of $\delta$




HADES, arXiv:2208.02740

Precision experiments NEED precision simulations

# Precision era of heavy-ion collisions needs precision simulations




**A. Sorensen** *et al.*, arXiv:2301.13253

**Dense Nuclear Matter Equation of State from Heavy-Ion Collisions** *


Agnieszka Sorensen[1], Kshitij Agarwal[2], Kyle W. Brown[3,4], Zbigniew Chajecki[5], Paweł Danielewicz[3,6], Christian Drischler[7], Stefano Gandolfi[8], Jeremy W. Holt[9,10], Matthias Kaminski[11], Che-Ming Ko[9,10], Rohit Kumar[3], Bao-An Li[12], William G. Lynch[3,6], Alan B. McIntosh[10], William G. Newton[12], Scott Pratt[3,6], Oleh Savchuk[3,13], Maria Stefaniak[14], Ingo Tews[8], ManYee Betty Tsang[3,6], Ramona Vogt[15,16], Hermann Wolter[17], Hanna Zbroszczyk[18]

**Endorsing authors:**

Navid Abbasi[19], Jörg Aichelin[20,21], Anton Andronic[22], Steffen A. Bass[23], Francesco Becattini[24,25], David Blaschke[26,27,28], Marcus Bleicher[29,30], Christoph Blume[31], Elena Bratkovskaya[14,29,30], B. Alex Brown[3,6], David A. Brown[32], Alberto Camaiani[33], Giovanni Casini[25], Katerina Chatziioannou[34,35], Abdelouahad Chbihi[36], Maria Colonna[37], Mircea Dan Cozma[38],


2023

**II. THE EQUATION OF STATE FROM 0 TO $5n_0$**

**A. Transport model simulations of heavy-ion collisions**

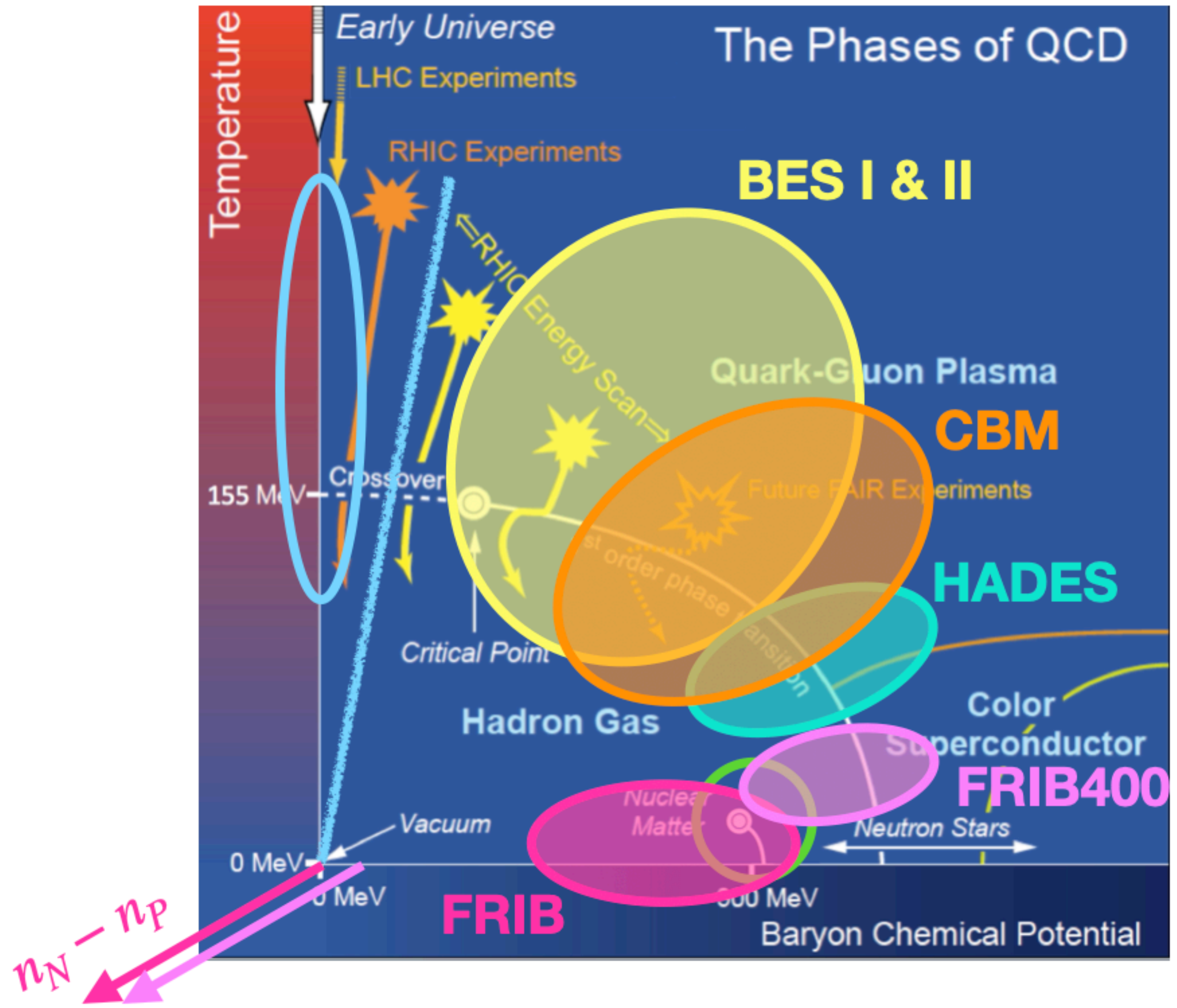
*3. Challenges and opportunities*

Selected results presented in Fig. 9 showcase significant achievements in determining the EOS and, simultaneously, the need to develop improved transport models to obtain tighter and more reliable constraints. Answering this need will require support for a sustained collaborative effort within the community to address remaining challenges in modeling collisions, in particular in the intermediate energy range ($E_{\rm lab} \approx 0.05$–$25$ $A$GeV, or $\sqrt{s_{NN}} \approx 1.9$–$7.1$ GeV). In the following, we will address selected areas where we see the need for such developments: (1) comprehensive treatment of both mean-field potentials and the collision term in transport codes, (2) use of microscopic information on mean fields and in-medium cross sections, such as discussed in Section II B, in transport, (3) better description of the initial state of heavy-ion collisions in hadronic transport codes, (4) deeper understanding of fluctuations in transport approaches, which affect many aspects of simulations, (5) inclusion of correlations beyond the mean field into transport, which is crucial for a realistic description of, e.g., light-cluster production, (6) treatment of short-range-correlations in transport, which are tightly connected to multi-particle collisions as well as off-shell transport, (7) sub-threshold particle production, (8) connections between quantum many-body theory and semiclassical transport theory, (9) investigations focused on extending the limits of applicability of hadronic transport approaches, (10) studies of new observables, e.g., azimuthally resolved spectra, to obtain tighter constraints on the EOS, (11) the question of quantifying the uncertainty of results obtained in transport simulations, and (12) the use of emulators and flexible parametrizations for wide-ranging explorations of all possible EOSs. Fortunately, advances in transport theory as well as the greater availability of high-performance computing make many of these improvements possible. Support for these developments will lead to a firm control and greater understanding of multiple complex aspects of the collision dynamics, allowing comparisons of transport model calculations and heavy-ion experiment measurements to provide an important contribution to the determination of the EOS of dense nuclear matter, which, in particular, cannot be determined by any other method at intermediate densities $(1$–$5)n_0$.

# Summary: A new beginning of the Dense QCD era

What's different, new, exciting about *now*?

- New analyses, new understanding: e.g., triangular flow, initial state fluctuations, cumulants
- New detectors, new data: unprecedented measurements, from ultra-precise triple-differential flow observables to hyperon-hyperon interactions
- New computing capabilities: large-scale simulations possible with state-of-the-art, benchmarked hadronic transport codes
- New approach to constraining the EOS: Bayesian analyses using flexible parametrizations of the EOS



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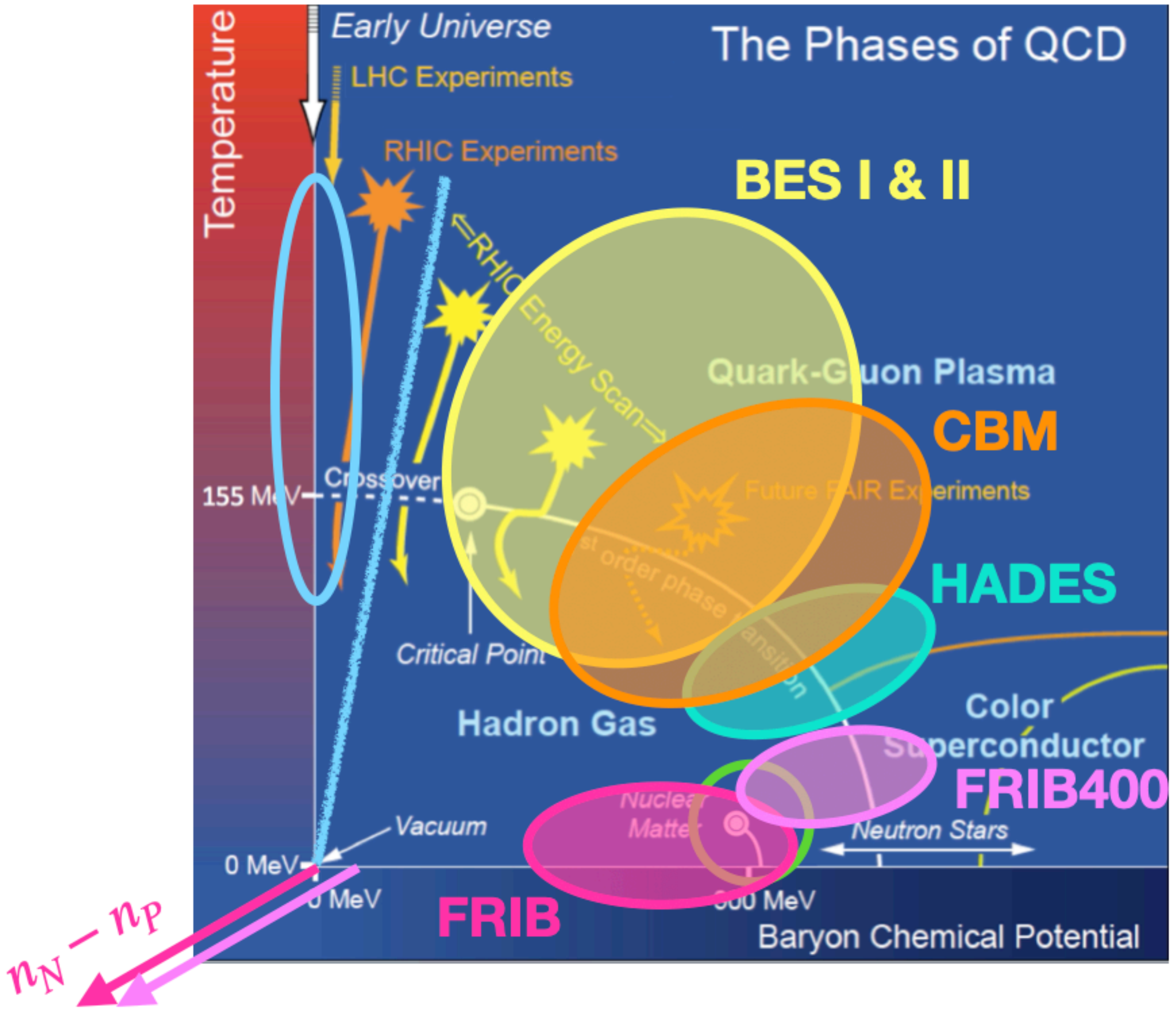
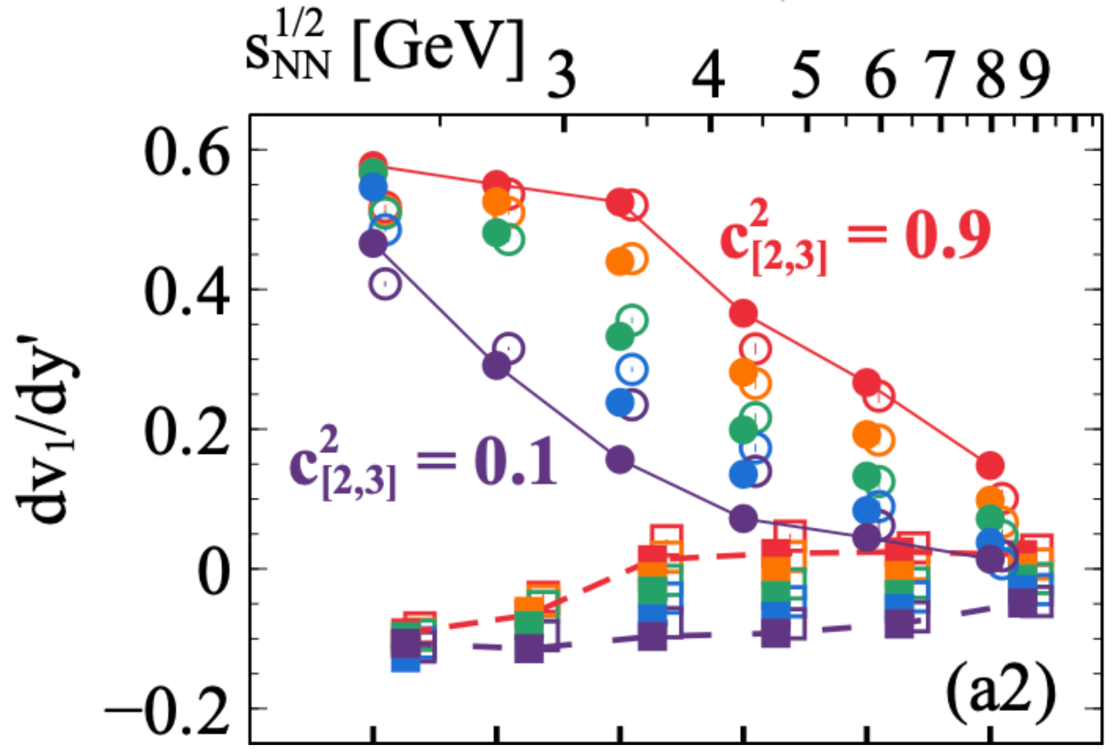
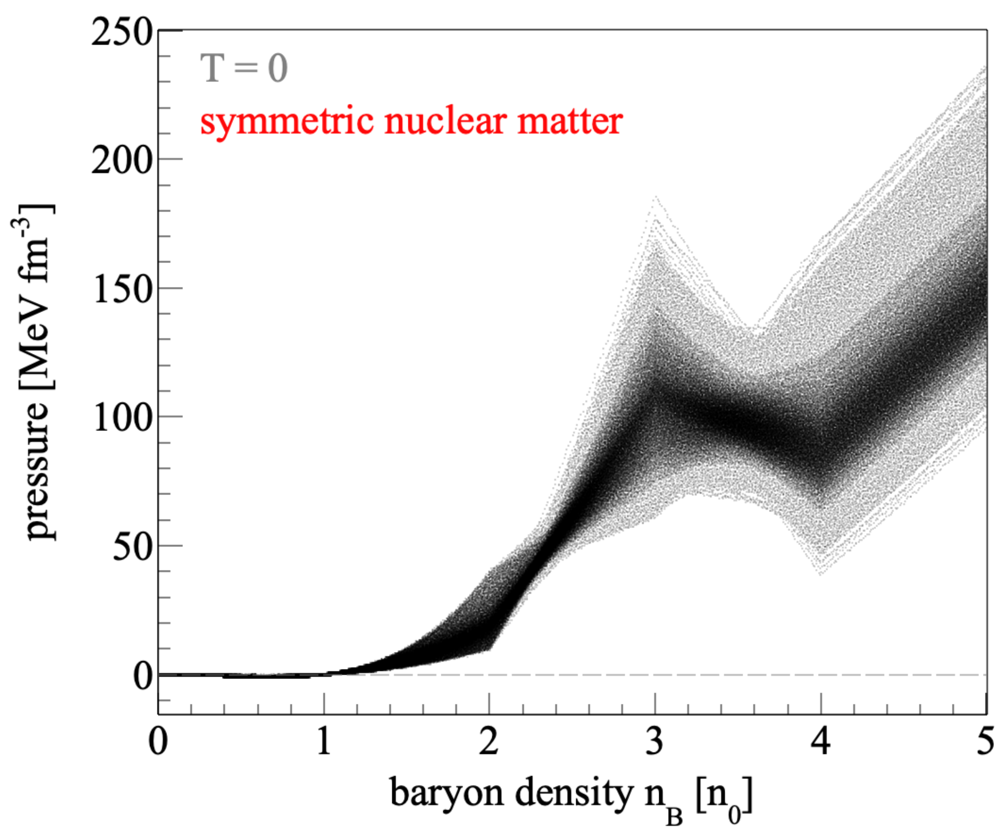
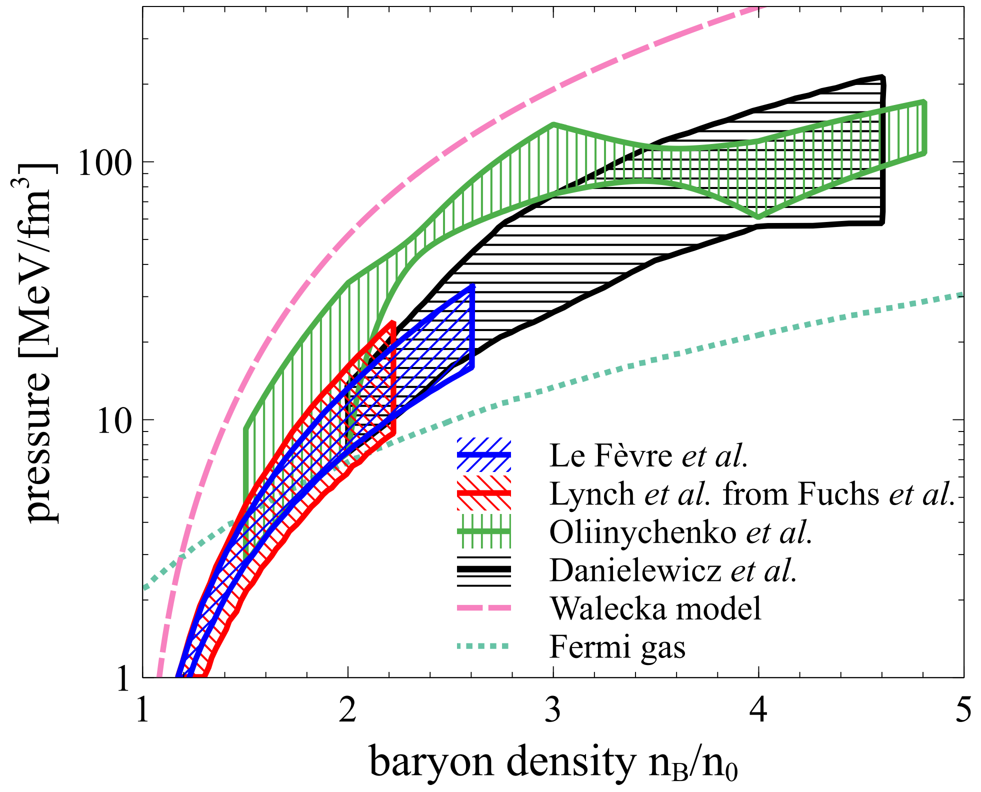


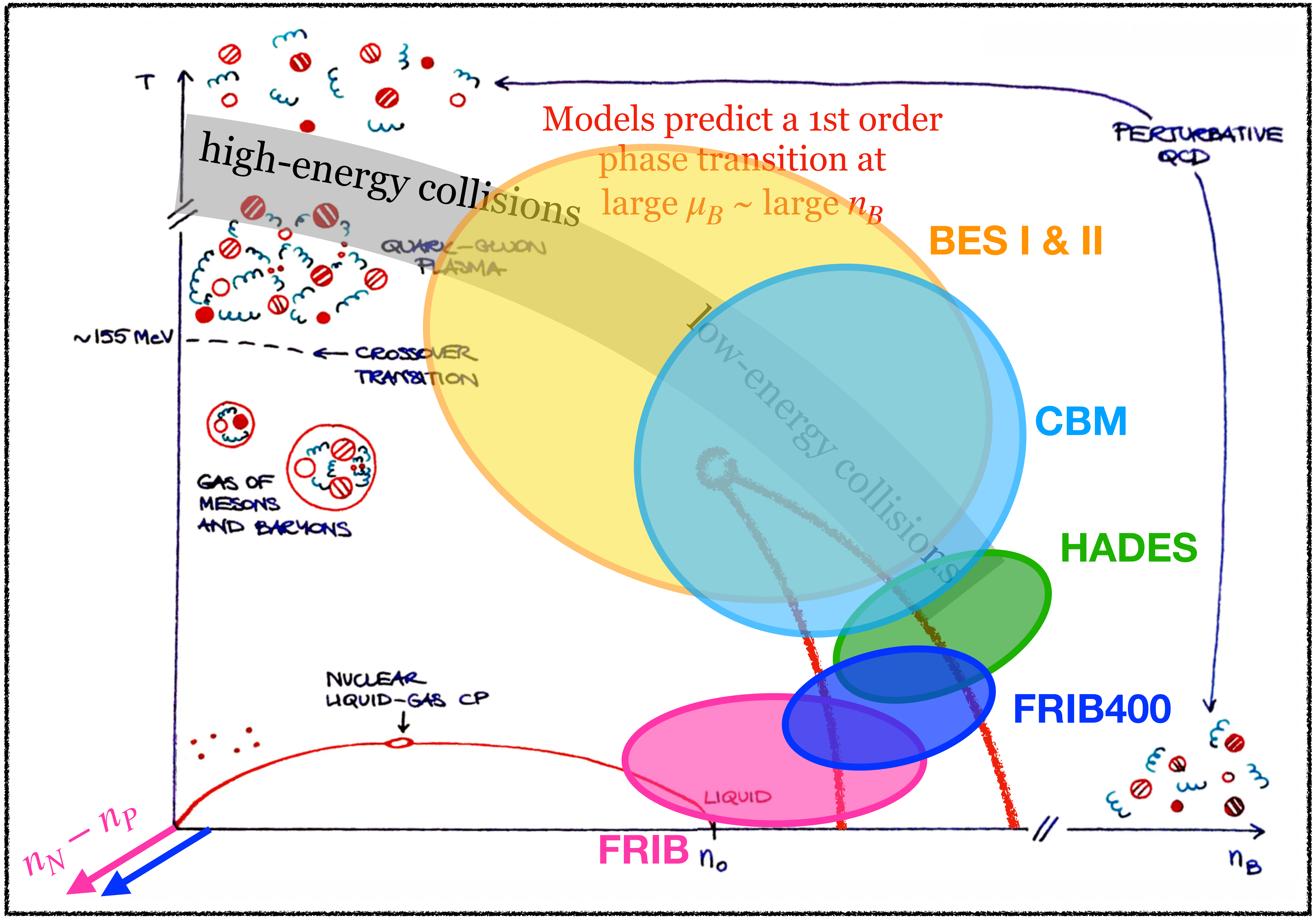



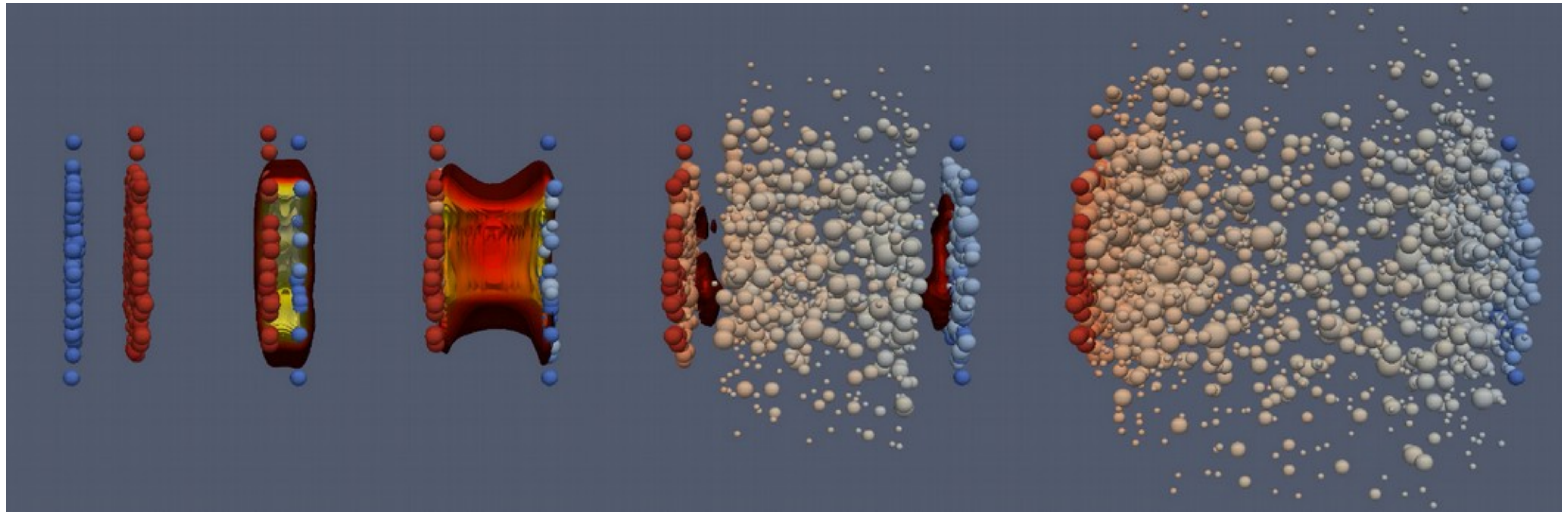

Thank you for your attention

# The QCD phase diagram: great interest in behavior at high $n_B$



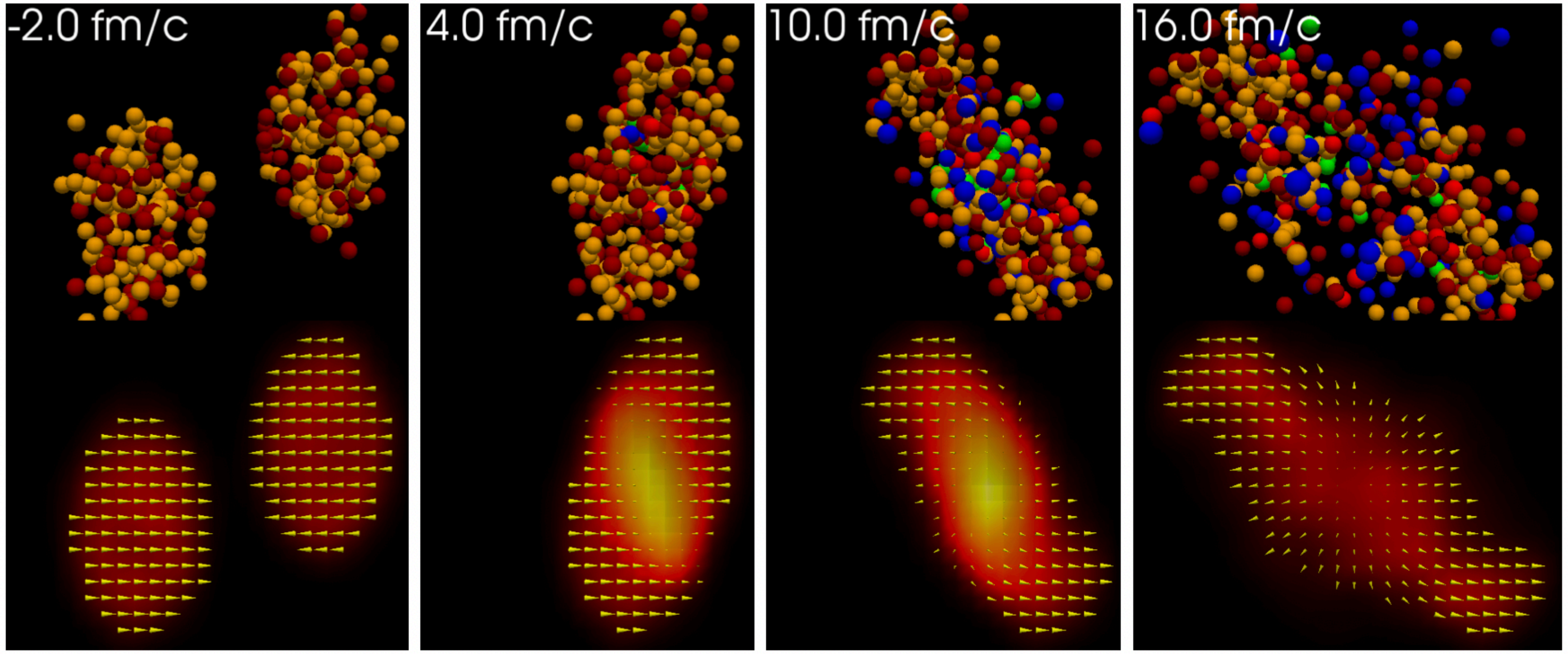
$\sqrt{s_{NN}} = 200$ GeV:

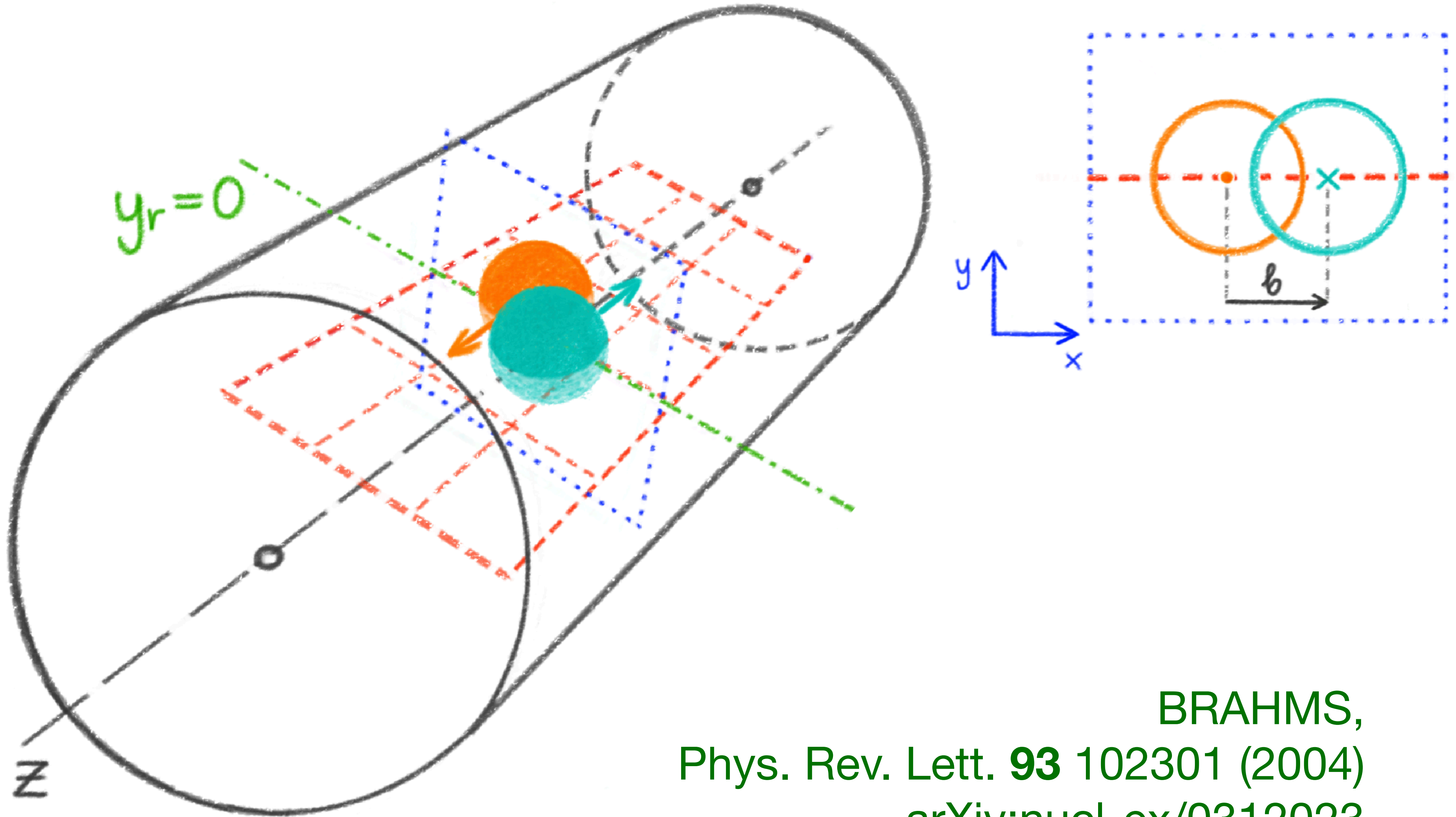
H. Elfner (Petersen), J. Bernhard, MADAI collaboration



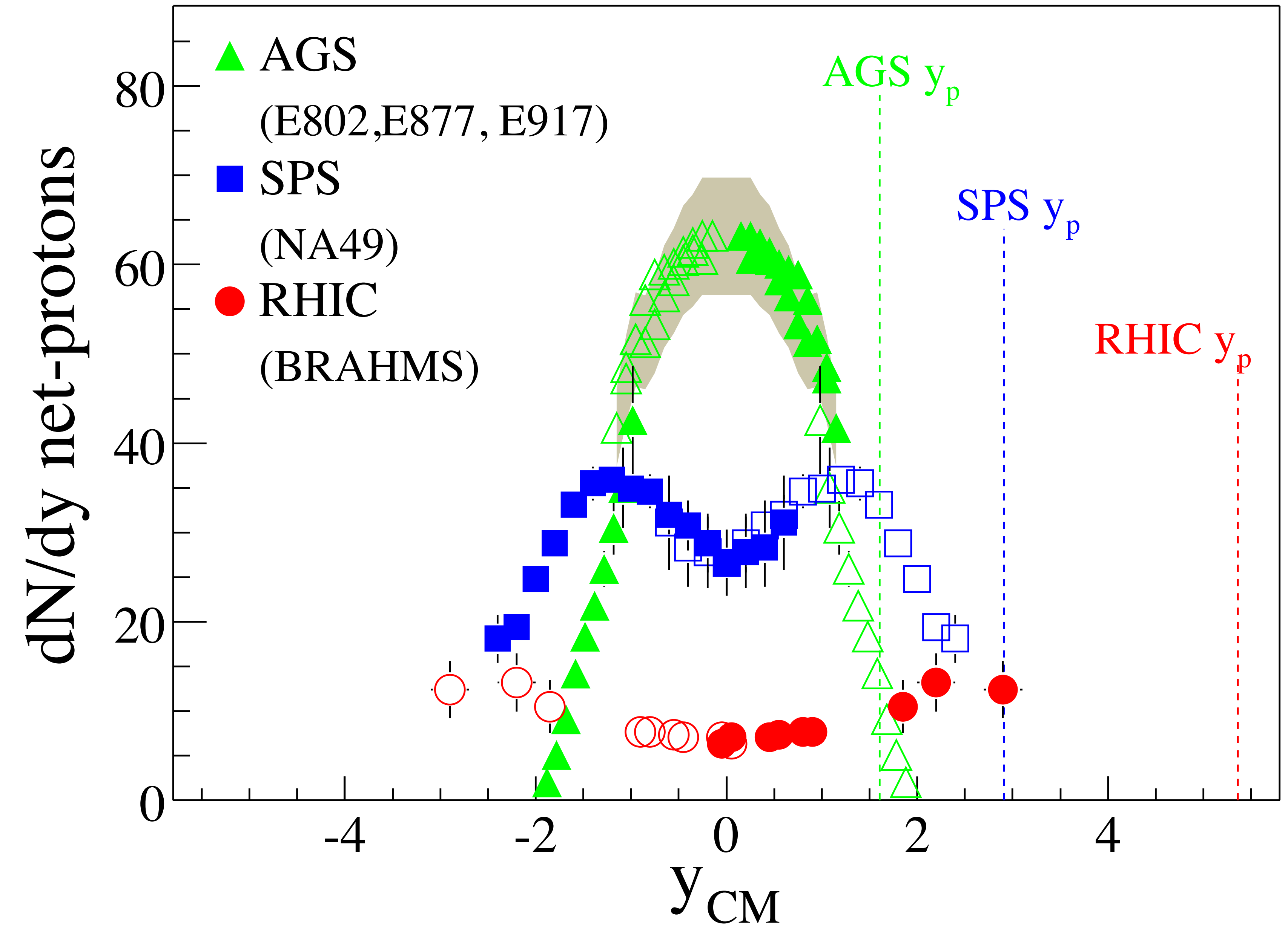
$\sqrt{s_{NN}} = 3$ GeV:

from D. Oliinychenko's slides



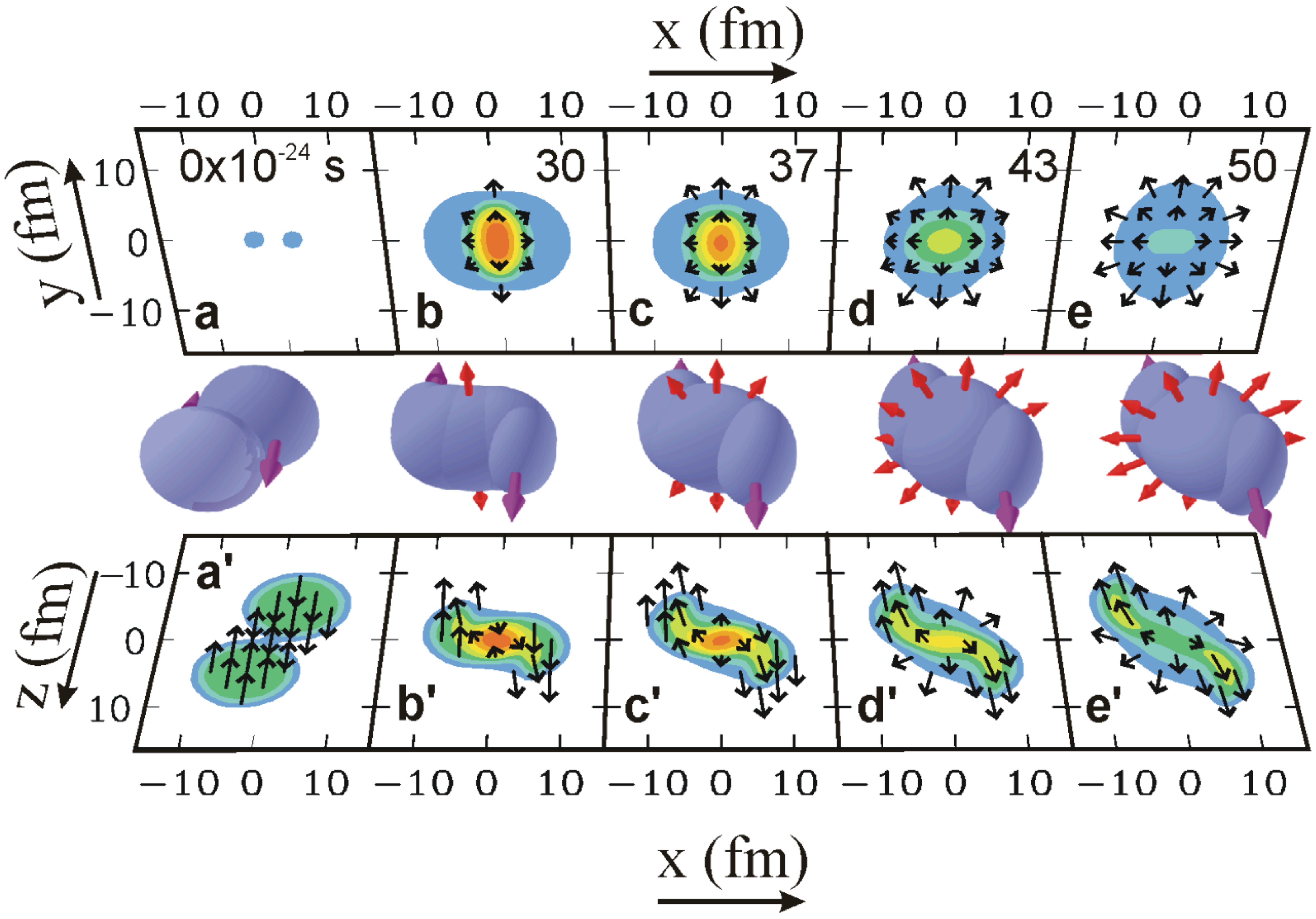




BRAHMS,
Phys. Rev. Lett. **93** 102301 (2004)
arXiv:nucl-ex/0312023



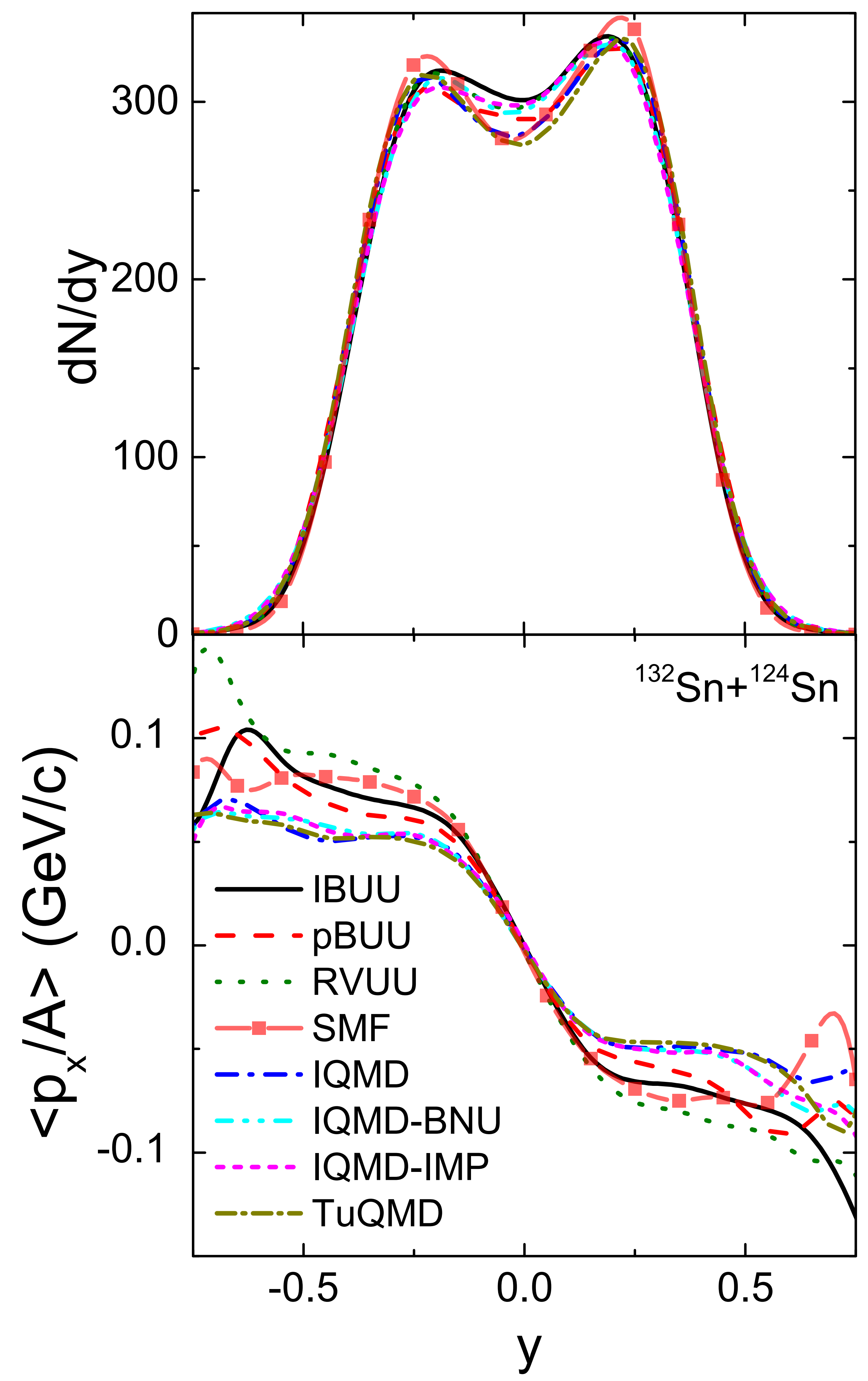
# Flow observables in heavy-ion collisions

Flow observables are the canonical observables for extracting the EOS




P. Danielewicz, R. Lacey, W. G. Lynch,
Science **298**, 1592–1596 (2002), arXiv:nucl-th/0208016

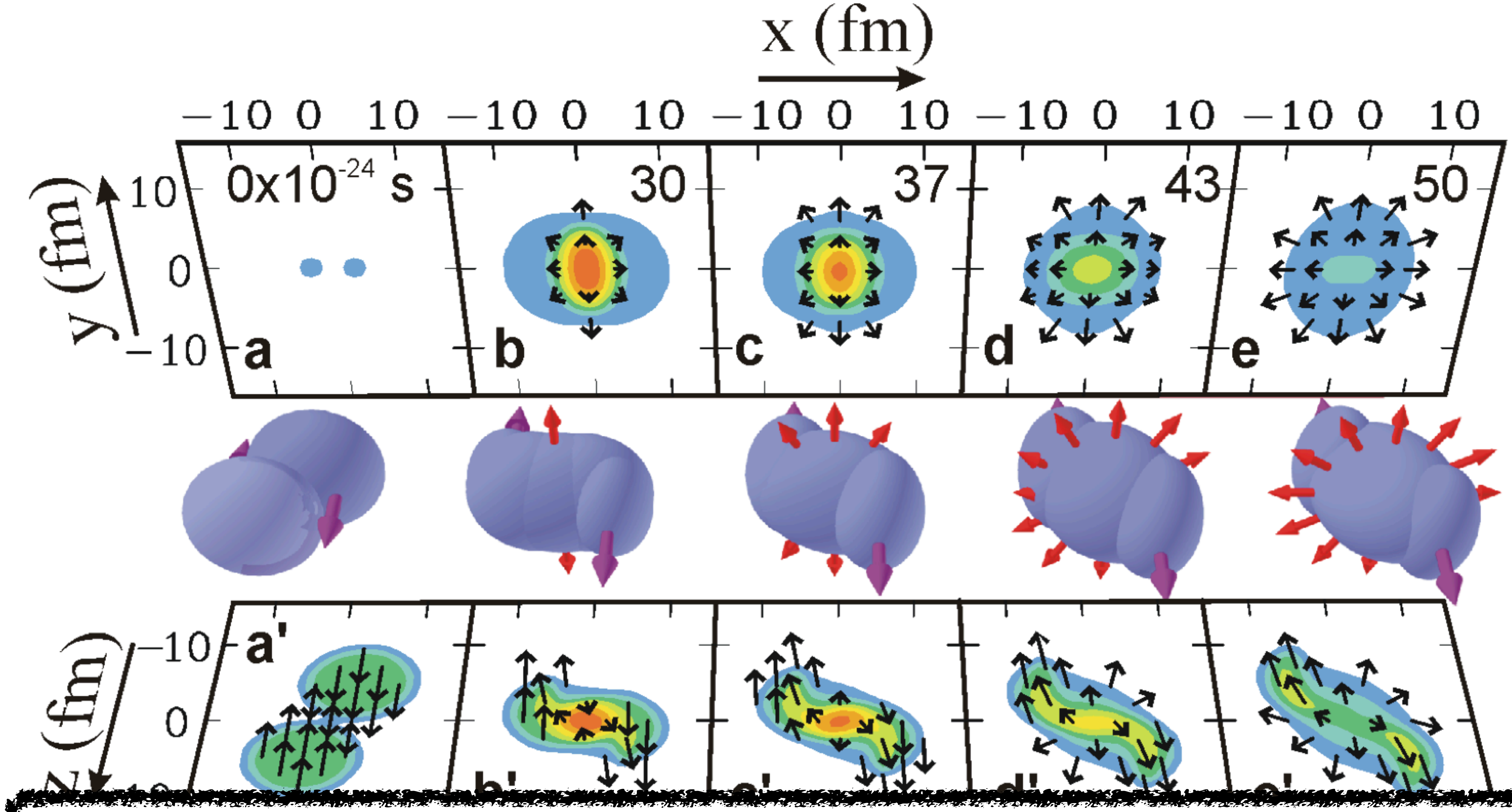
There is code-dependence:




J. Xu *et al.* (TMEP Collaboration), *in preparation*

# Flow observables in heavy-ion collisions

Flow observables are the canonical observables for extracting the EOS

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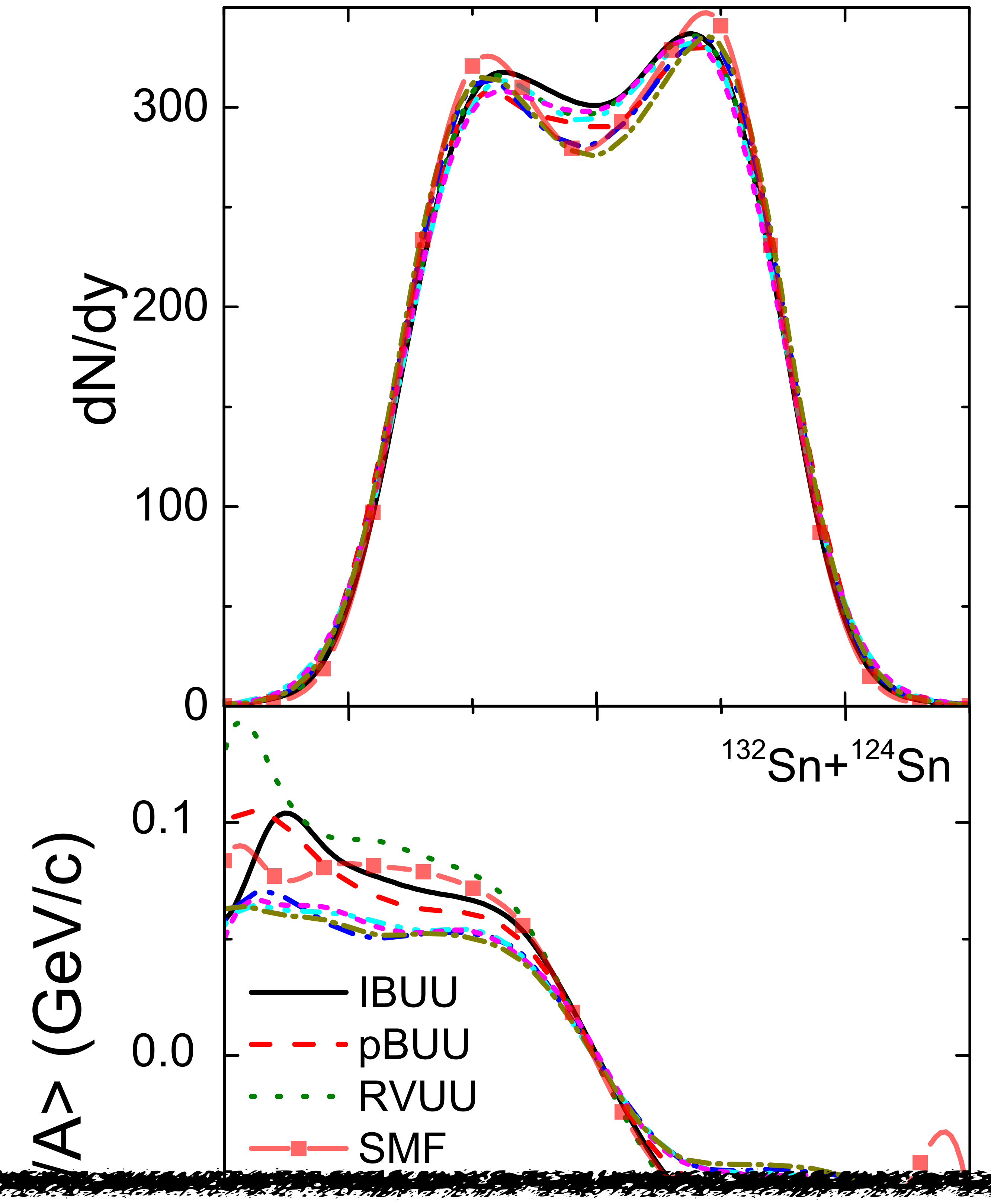
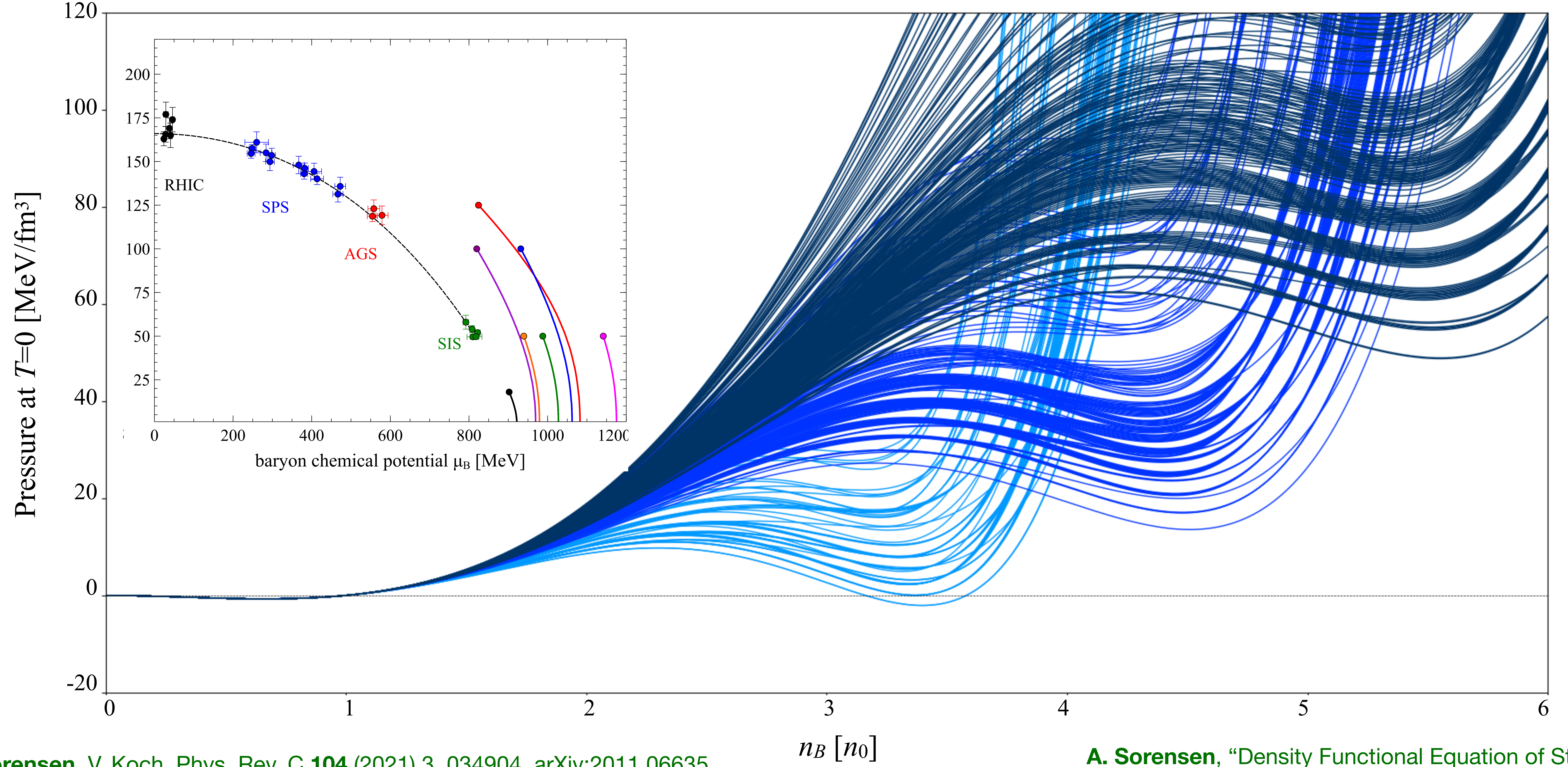



Comparisons between different codes are needed to understand the dependence on:
1) different physics assumptions 2) different implementation solutions
See efforts by, e.g., TMEP collaboration

# VDF model: two 1st order phase transitions (EOSs)

Properties of ordinary nuclear matter are well known, but few constraints for $n_B \geq 1.5 n_0$




**A. Sorensen**, V. Koch, Phys. Rev. C **104** (2021) 3, 034904, arXiv:2011.06635

**A. Sorensen**, “Density Functional Equation of State and Its Application to the Phenomenology of Heavy-Ion Collisions,” arXiv:2109.08105

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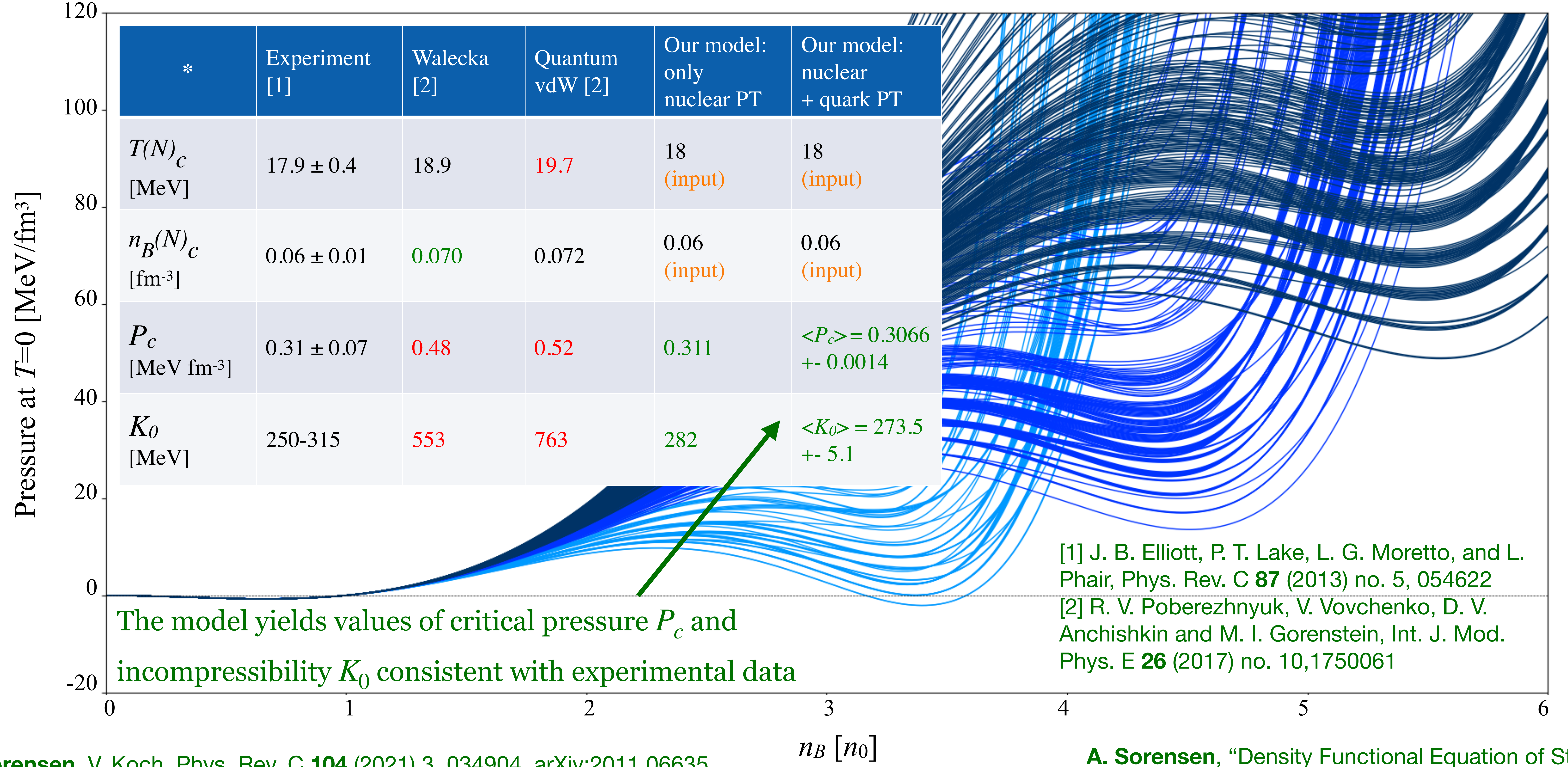



| * | Experiment [1] | Walecka [2] | Quantum vdW [2] | Our model: only nuclear PT | Our model: nuclear + quark PT |
|---|---|---|---|---|---|
| $T(N)_c$ [MeV] | 17.9 ± 0.4 | 18.9 | 19.7 | 18 (input) | 18 (input) |
| $n_B(N)_c$ [fm$^{-3}$] | 0.06 ± 0.01 | 0.070 | 0.072 | 0.06 (input) | 0.06 (input) |
| $P_c$ [MeV fm$^{-3}$] | 0.31 ± 0.07 | 0.48 | 0.52 | 0.311 | $\langle P_c \rangle$ = 0.3066 +- 0.0014 |
| $K_0$ [MeV] | 250-315 | 553 | 763 | 282 | $\langle K_0 \rangle$ = 273.5 +- 5.1 |

The model yields values of critical pressure $P_c$ and incompressibility $K_0$ consistent with experimental data


[1] J. B. Elliott, P. T. Lake, L. G. Moretto, and L. Phair, Phys. Rev. C **87** (2013) no. 5, 054622

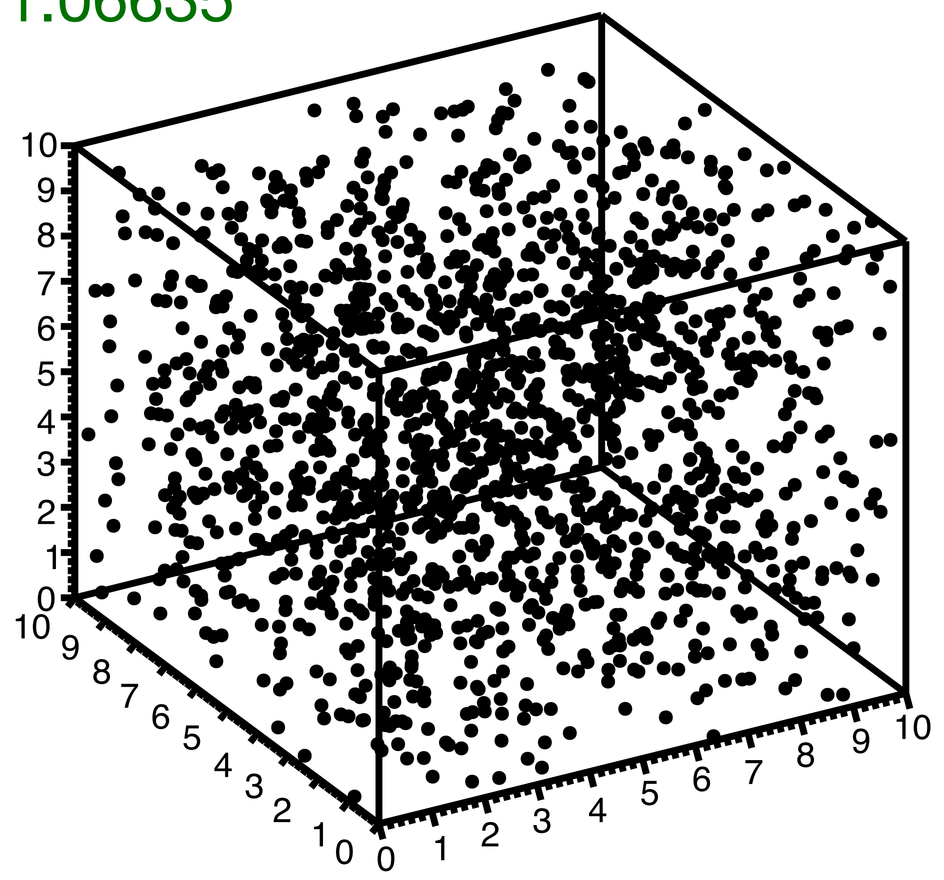
[2] R. V. Poberezhnyuk, V. Vovchenko, D. V. Anchishkin and M. I. Gorenstein, Int. J. Mod. Phys. E **26** (2017) no. 10,1750061

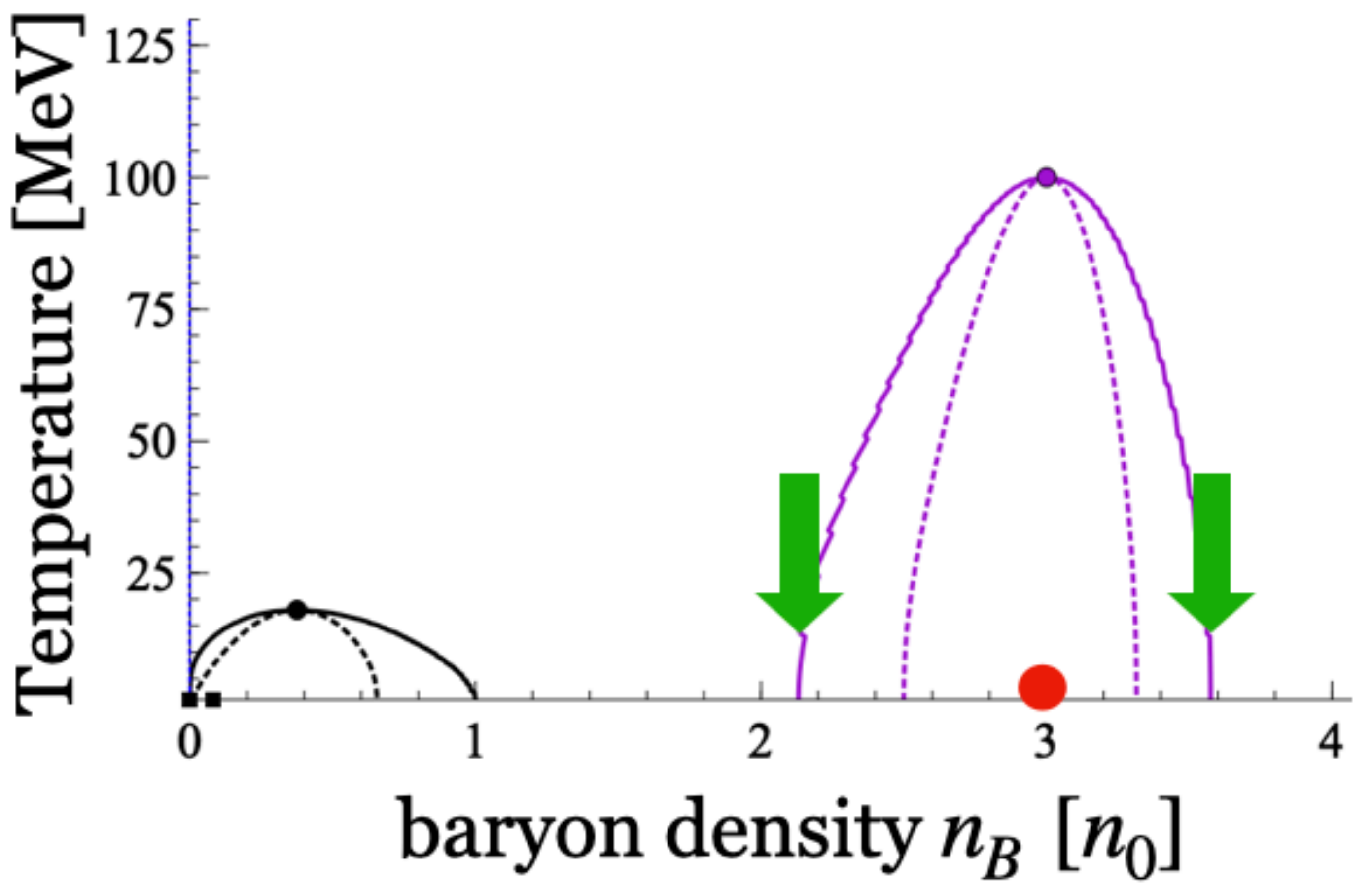
**A. Sorensen**, V. Koch, Phys. Rev. C **104** (2021) 3, 034904, arXiv:2011.06635

**A. Sorensen**, “Density Functional Equation of State and Its Application to the Phenomenology of Heavy-Ion Collisions,” arXiv:2109.08105

# VDF in SMASH: tests in the spinodal region

**A. Sorensen**, V. Koch, Phys. Rev. C **104**, 3, 034904 (2021)
arXiv:2011.06635

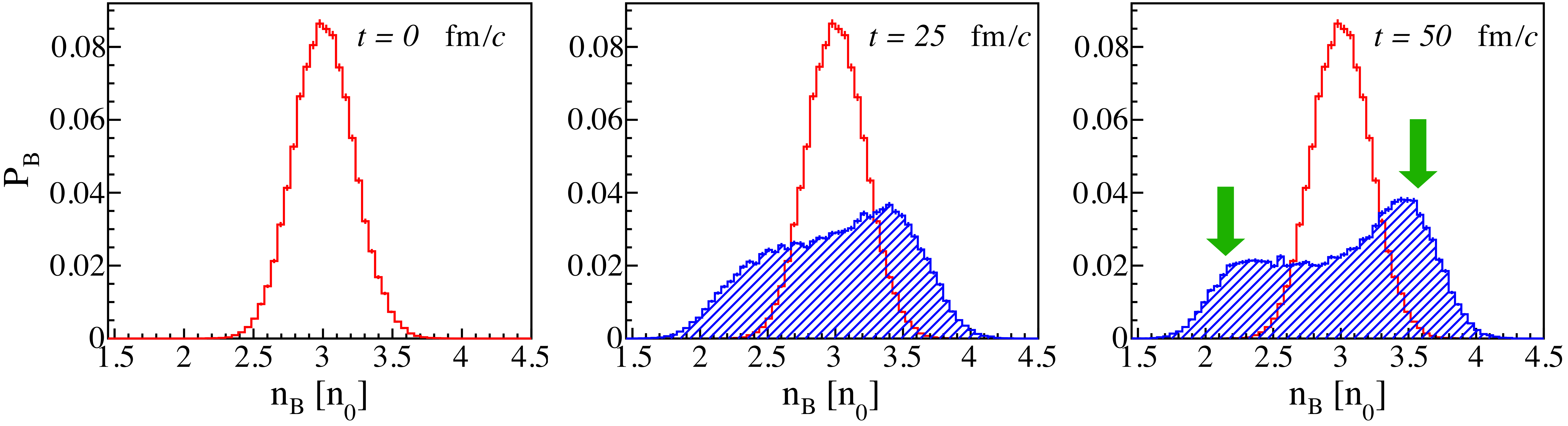



Simulation info for practitioners:

time step: 0.1 fm/c
smearing: triangular with range 2 fm
lattice: cubic cells with 1 fm on a side
collisions: off

500 events
bin width = 2 fm




**The distribution becomes bimodal as the system separates!**

# Transport model simulations of heavy-ion collisions

- Boltzmann-Uehling-Uhlenbeck (BUU)-type codes:

  - solve coupled Boltzmann equations $\forall i:\quad \frac{\partial f_i}{\partial t} + \frac{d\mathbf{x}_i}{dt}\frac{\partial f_i}{\partial \mathbf{x}_i} + \frac{d\mathbf{p}_i}{dt}\frac{\partial f_i}{\partial \mathbf{p}_i} = I_{\text{coll}}^{(i)}$

  with the method of test particles: the distribution is *over*sampled with a *large* number of discrete test-particles, which are evolved according to the single-particle EOMs
  (test particles probe the evolution in the phase space)

  - forces from gradients of single-particle energies (mean-fields: needs a robust density calculation!)

  - collision term based on measured cross-sections for scatterings and decays

- Quantum Molecular Dynamics (QMD)-type codes
  - solve molecular dynamics problem (evolve nucleons according to their EOMs)

  - forces: in principle distance-dependent particle-particle interactions, in practice: often mean-fields!

  - collisions based on measured cross-sections for scatterings and decays

# Transport model simulations of heavy-ion collisions

- Boltzmann-Uehling-Uhlenbeck (BUU)-type codes:

  - solve coupled Boltzmann equations $\forall i: \quad \frac{\partial f_i}{\partial t} + \frac{d\mathbf{x}_i}{dt}\frac{\partial f_i}{\partial \mathbf{x}_i} + \frac{d\mathbf{p}_i}{dt}\frac{\partial f_i}{\partial \mathbf{p}_i} = I^{(i)}_{\text{coll}}$

  with the meth ber of discrete
  test-particles,
  (test particles

  - forces from gr y calculation!)

  - collision term

Transport ***automatically*** includes:

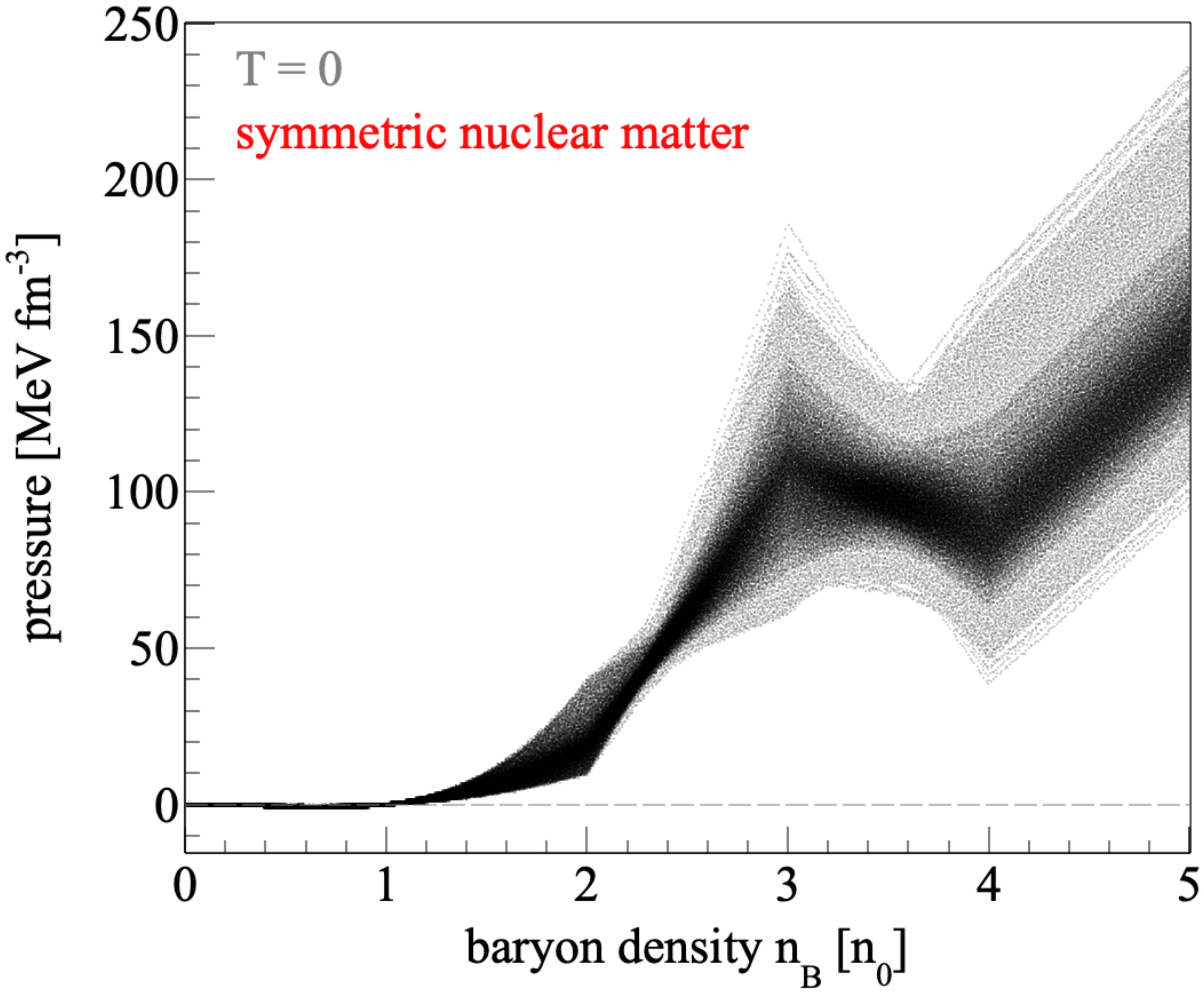
- non-equilibrium evolution, including triggered by probing unstable regions of the phase diagram
- effects due to the interplay between participants and spectators
- baryon, strangeness, charge transport/diffusion

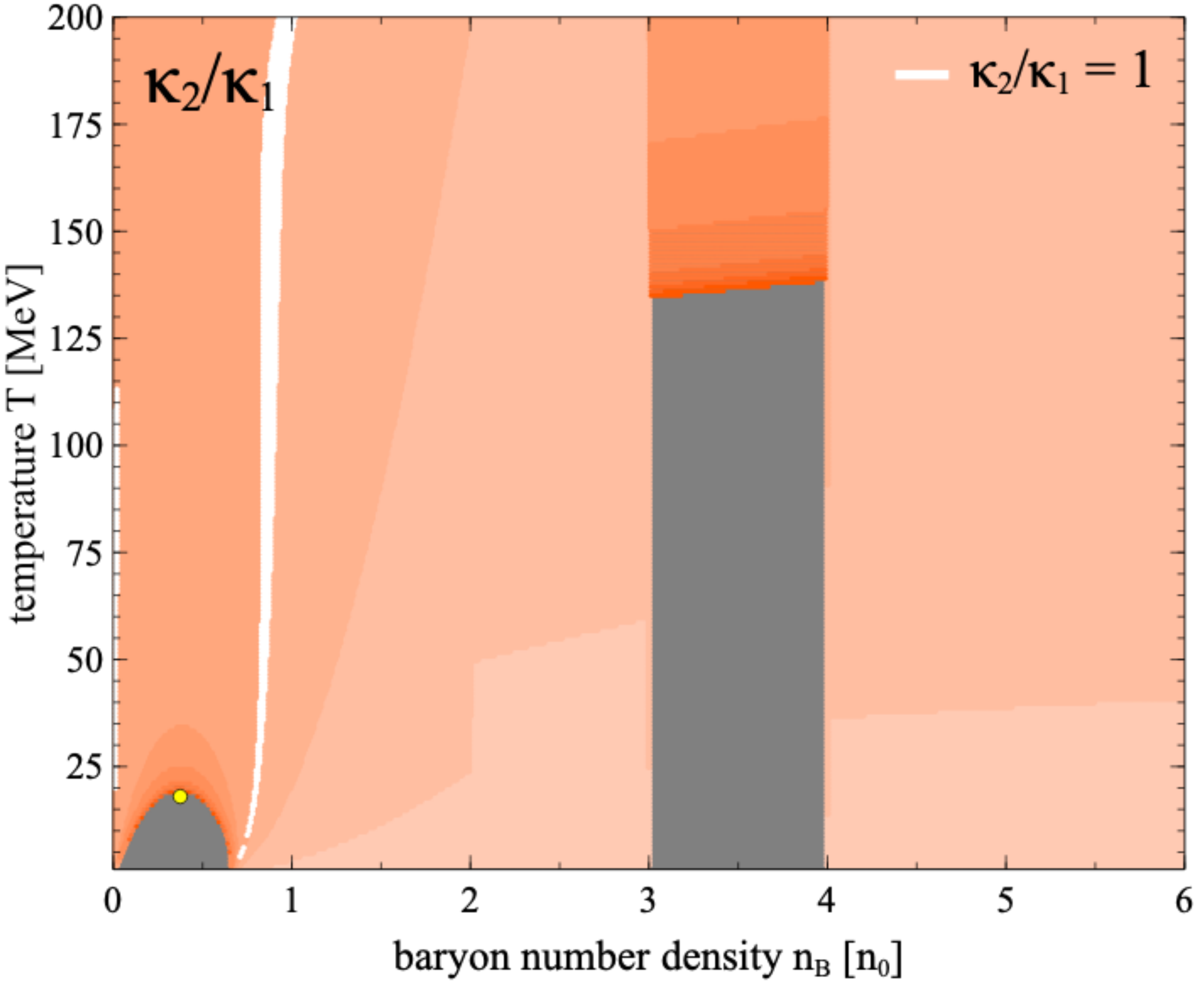
- Quantum Mole
  - solve molecul

  - forces: in principle distance-dependent particle-particle interactions, in practice: often mean-fields!

  - collisions based on measured cross-sections for scatterings and decays

# Bayesian analysis of STAR flow data with varying $K_0$, $c^2_{[2,3]n_0}$, $c^2_{[3,4]n_0}$



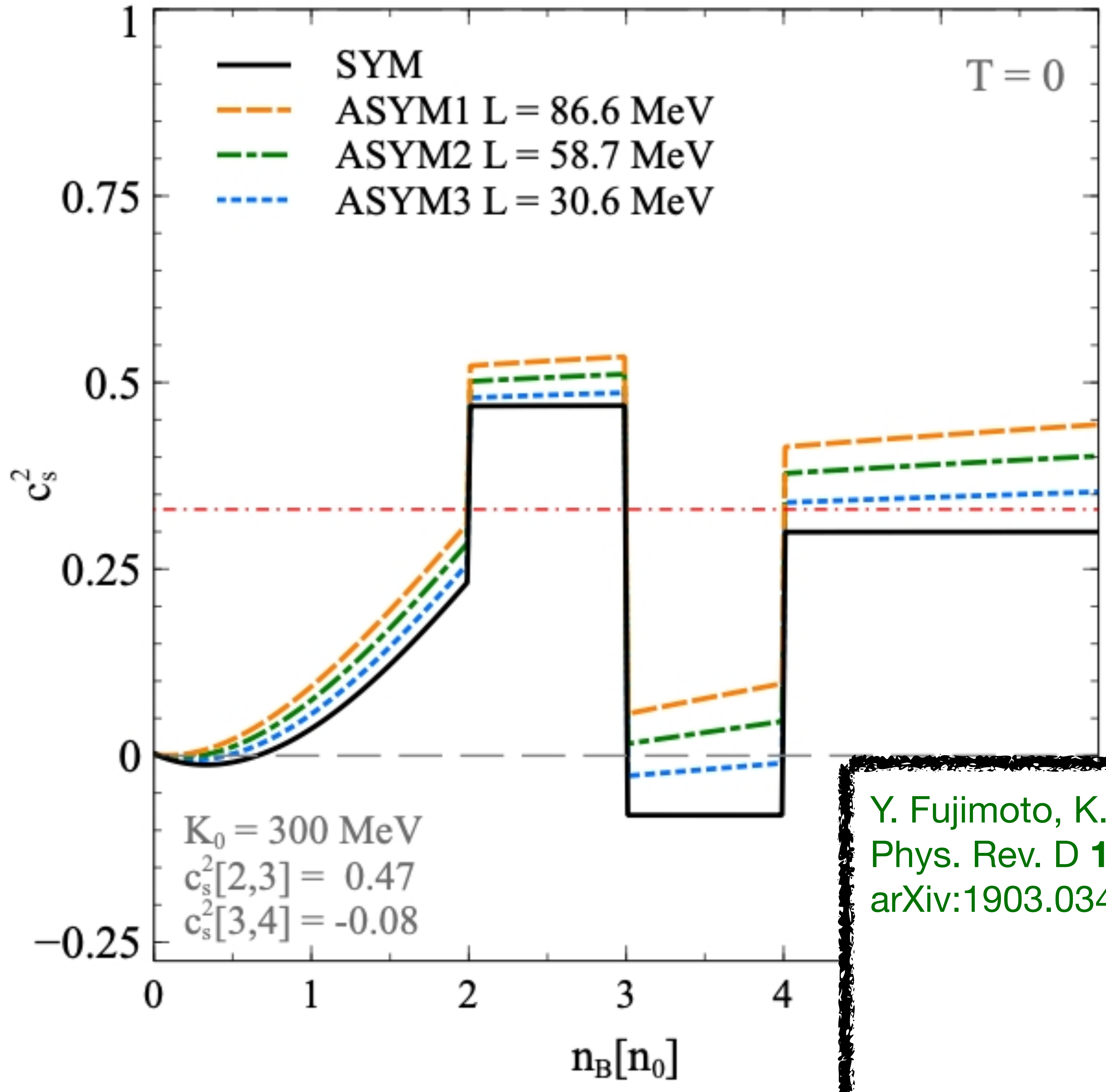



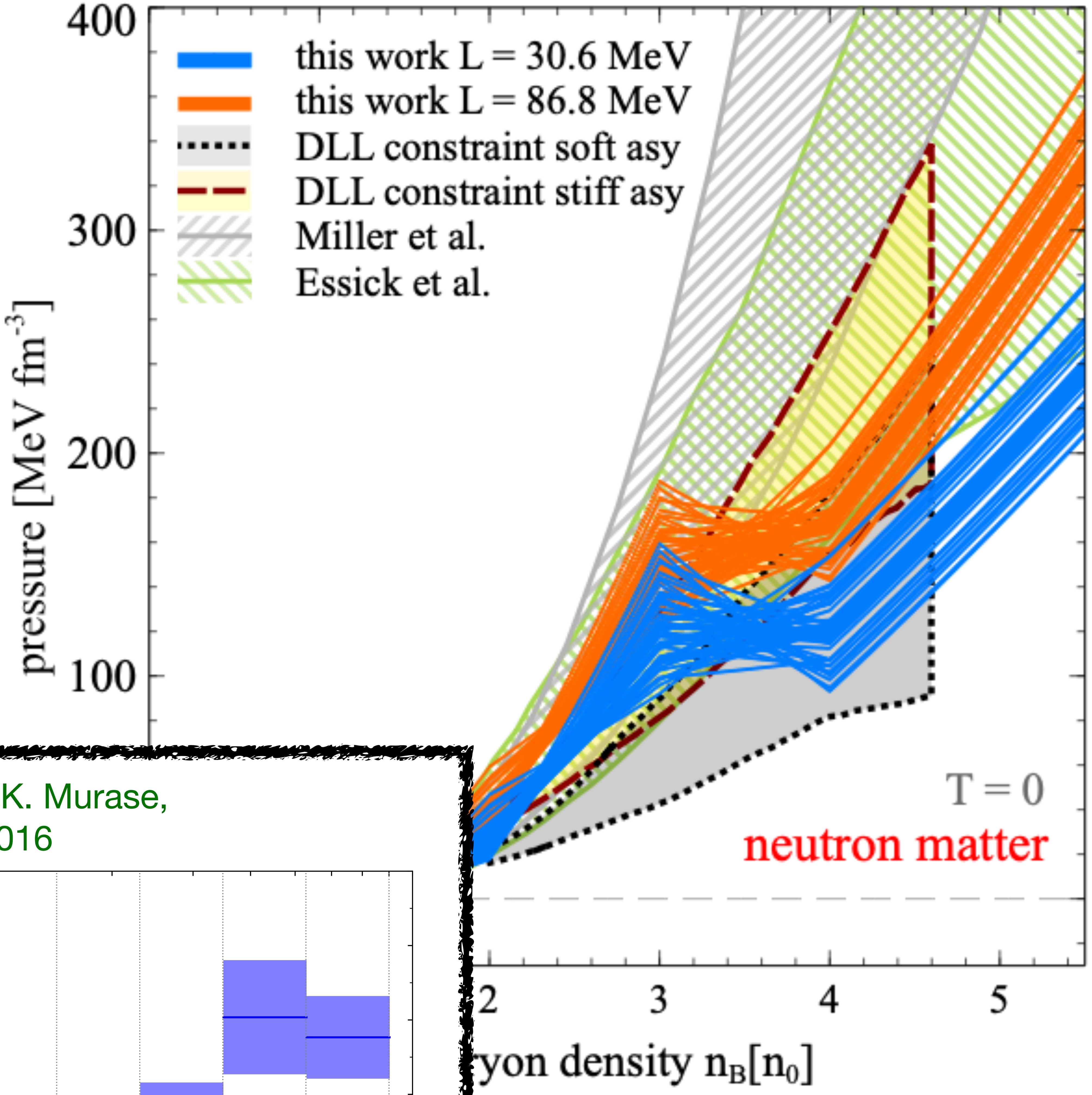

The maximum a posteriori probability (MAP) parameters are

$K_0 = 300 \pm 60\text{MeV}, \quad c^2_{[2,3]n_0} = 0.47 \pm 0.12, \quad c^2_{[3,4]n_0} = -0.08 \pm 0.14$

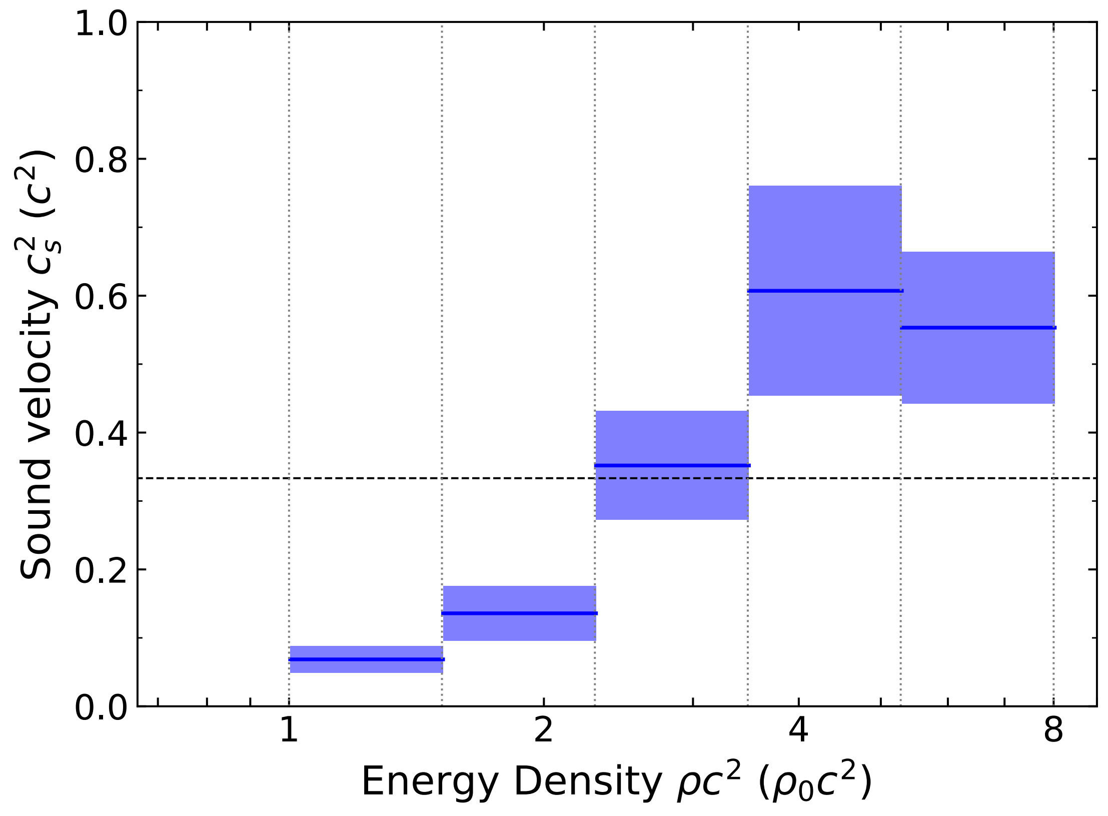
D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

# Tension with neutron star data?




Y. Fujimoto, K. Fukushima and K. Murase, Phys. Rev. D **101** (2020) 5, 054016 arXiv:1903.03400



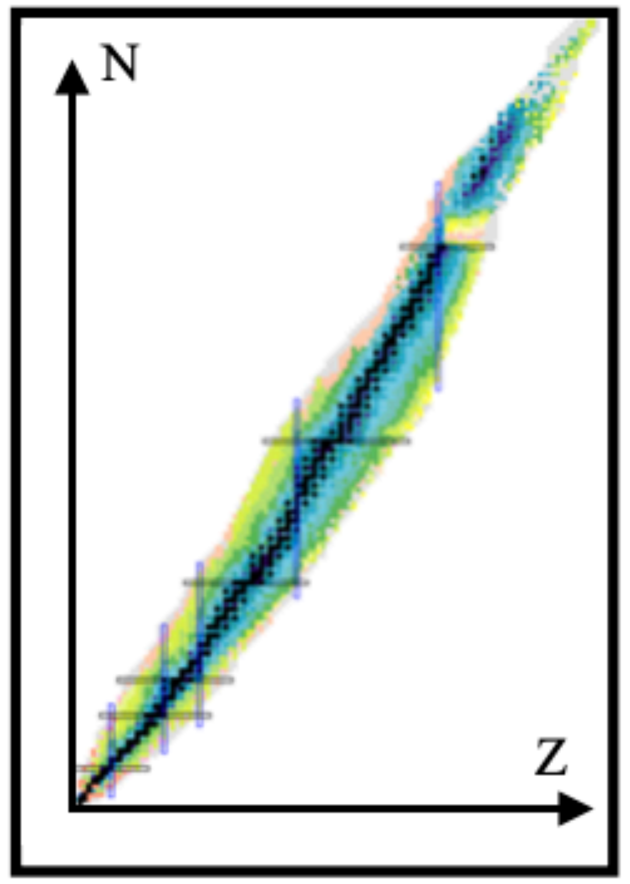

Based on a simple symmetry energ

phase transition in SNM != a phas 28.1 MeV

B.-A. Li, P. G. Krastev, D.-H. Wen, N.-B. Zhang, Eur. Phys. J. A **55** (2019) 7, 117, arXiv:1905.13175

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

# Significant potential in exploring global analyses

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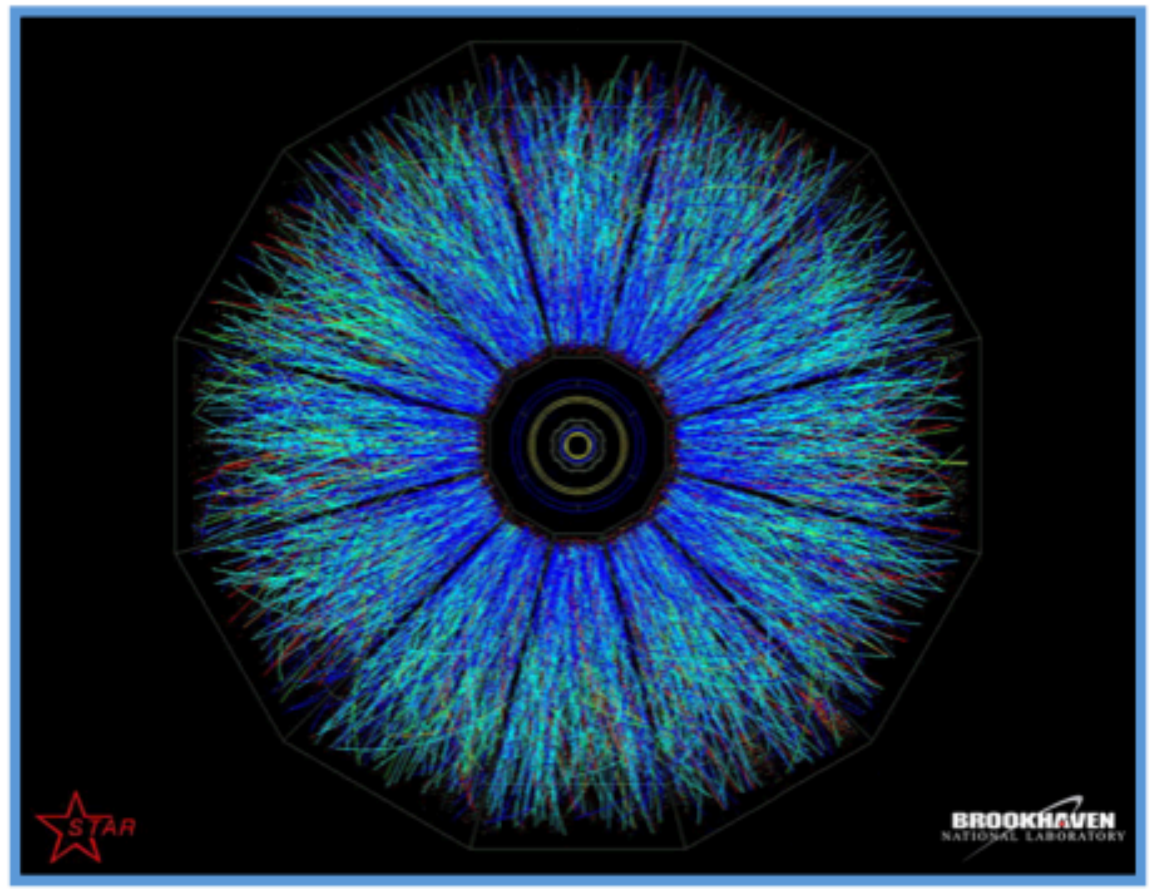


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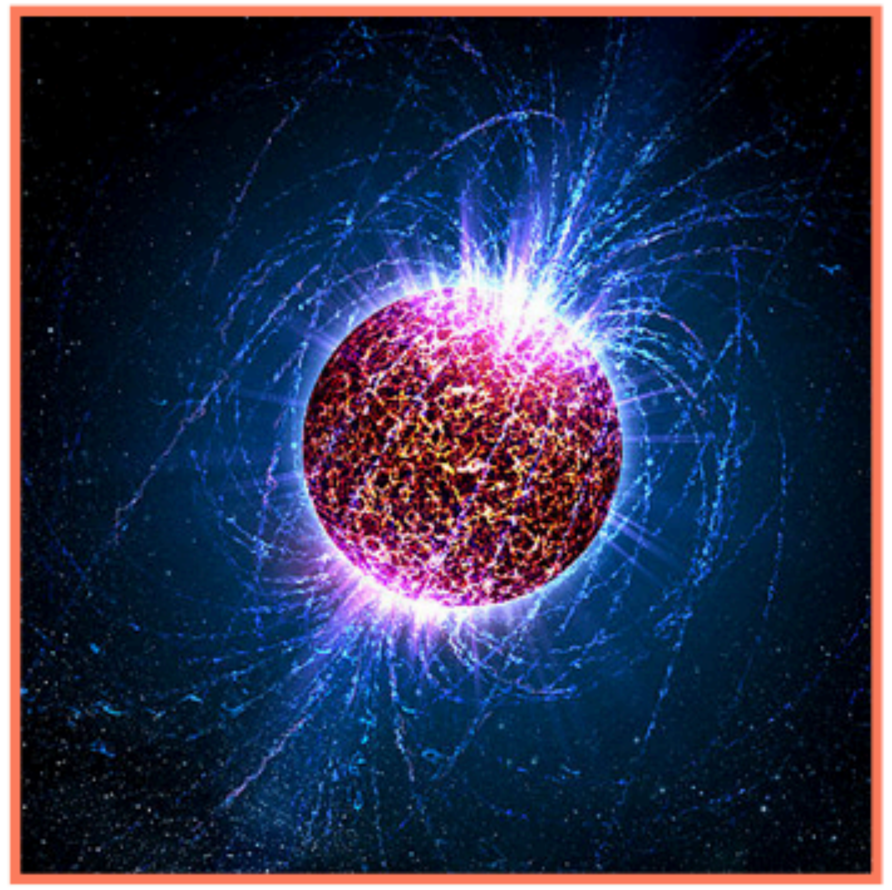

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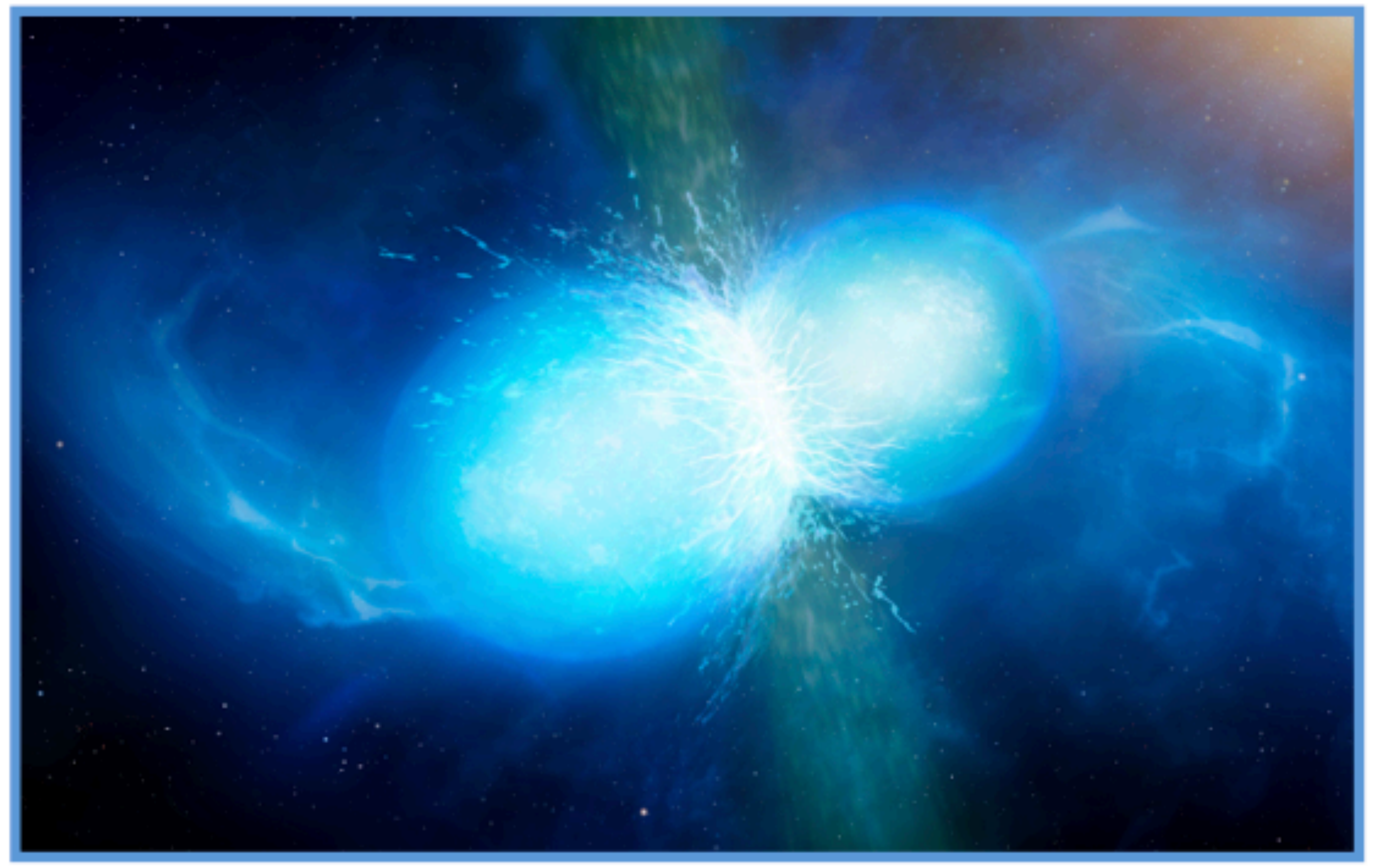
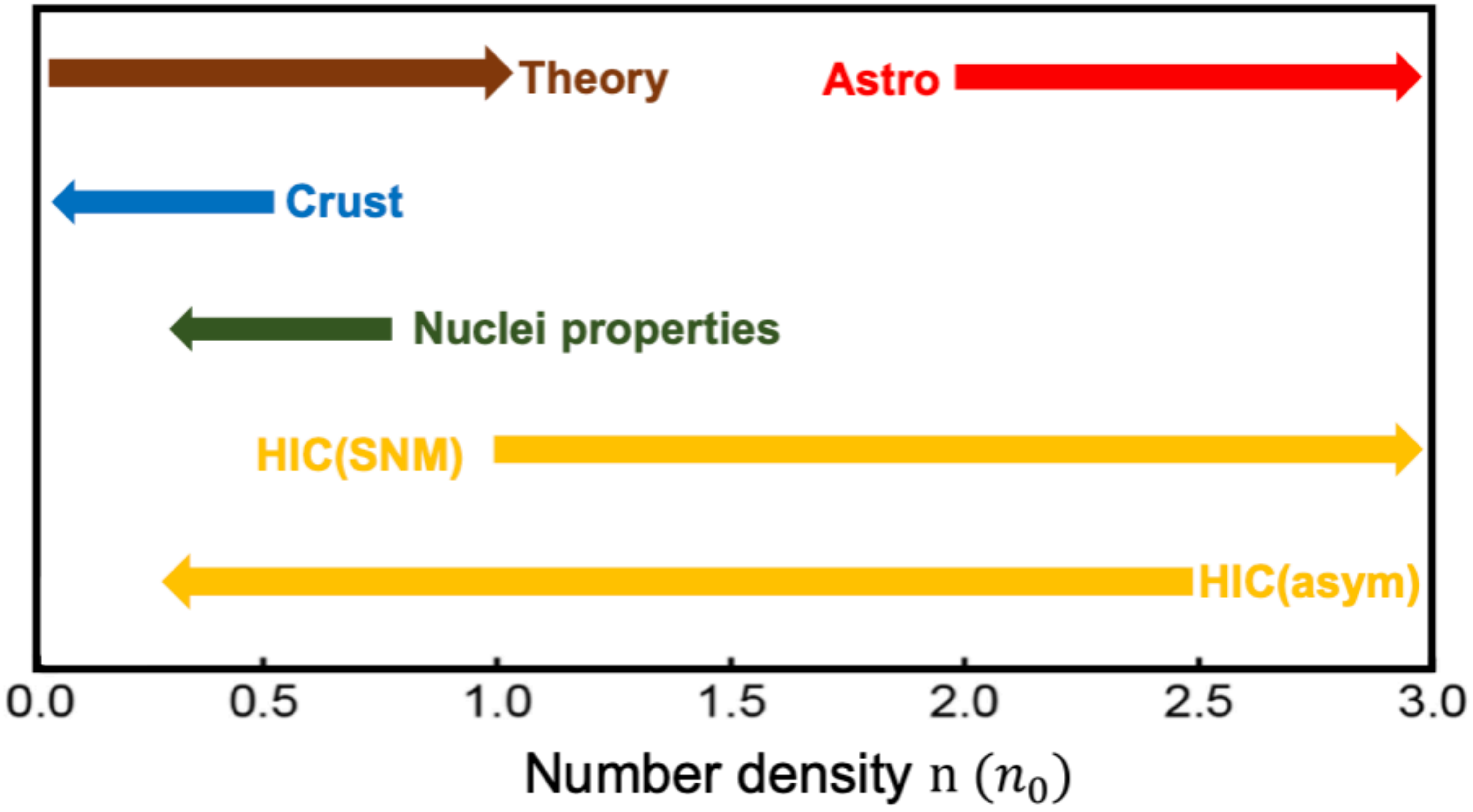
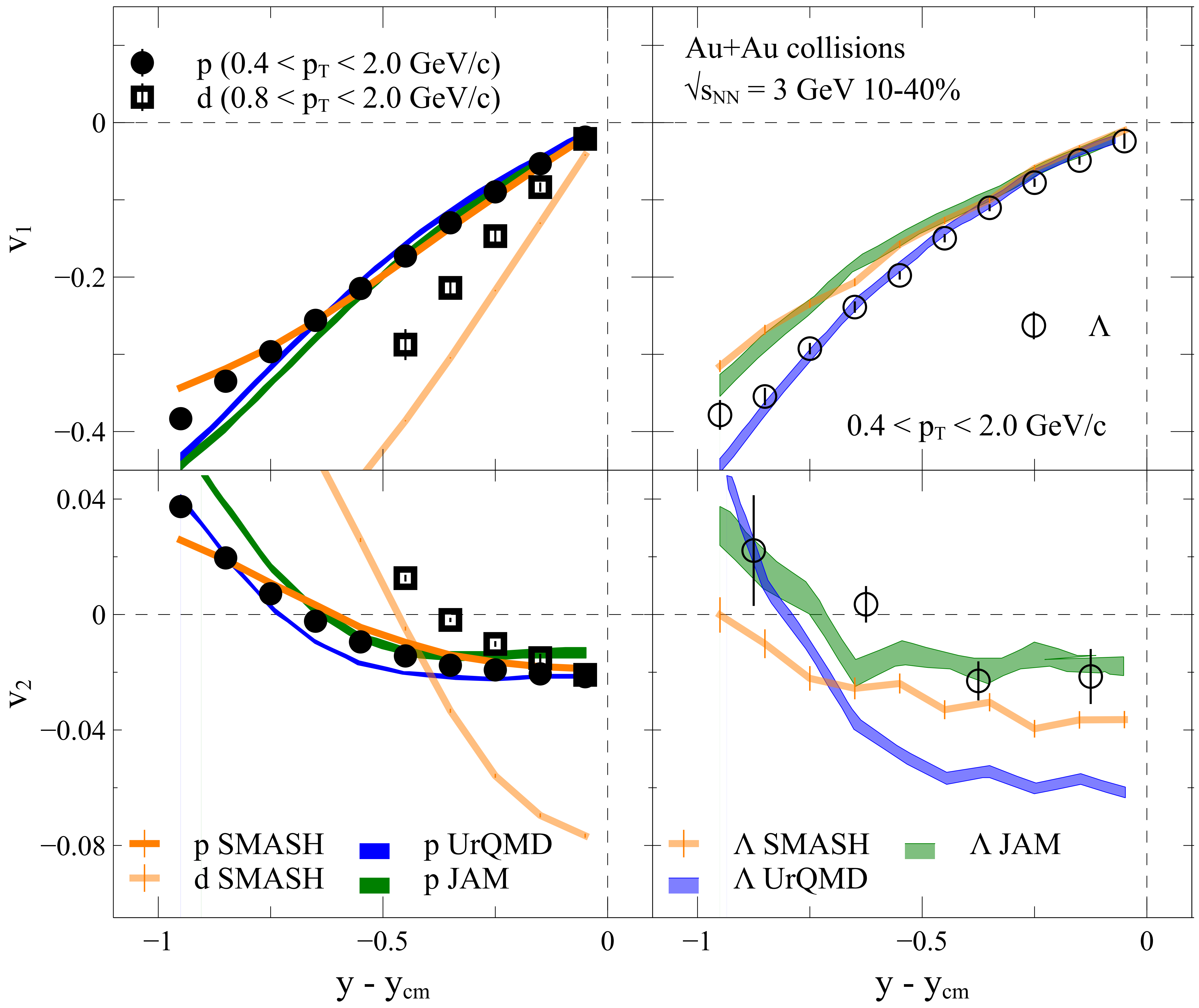




# Describing proton flow is not enough




Strange baryons are not well described
- the results may depend on:

- nucleon-hyperon and hyperon-hyperon interactions
- in-medium modifications of interactions

Models of interactions exists and could be tested; interactions could be based on those obtained within first-principle calculations (e.g., HALQCD collaboration
HAL QCD, Nucl. Phys. A **998** 121737 (2020), arXiv:1912.08630)

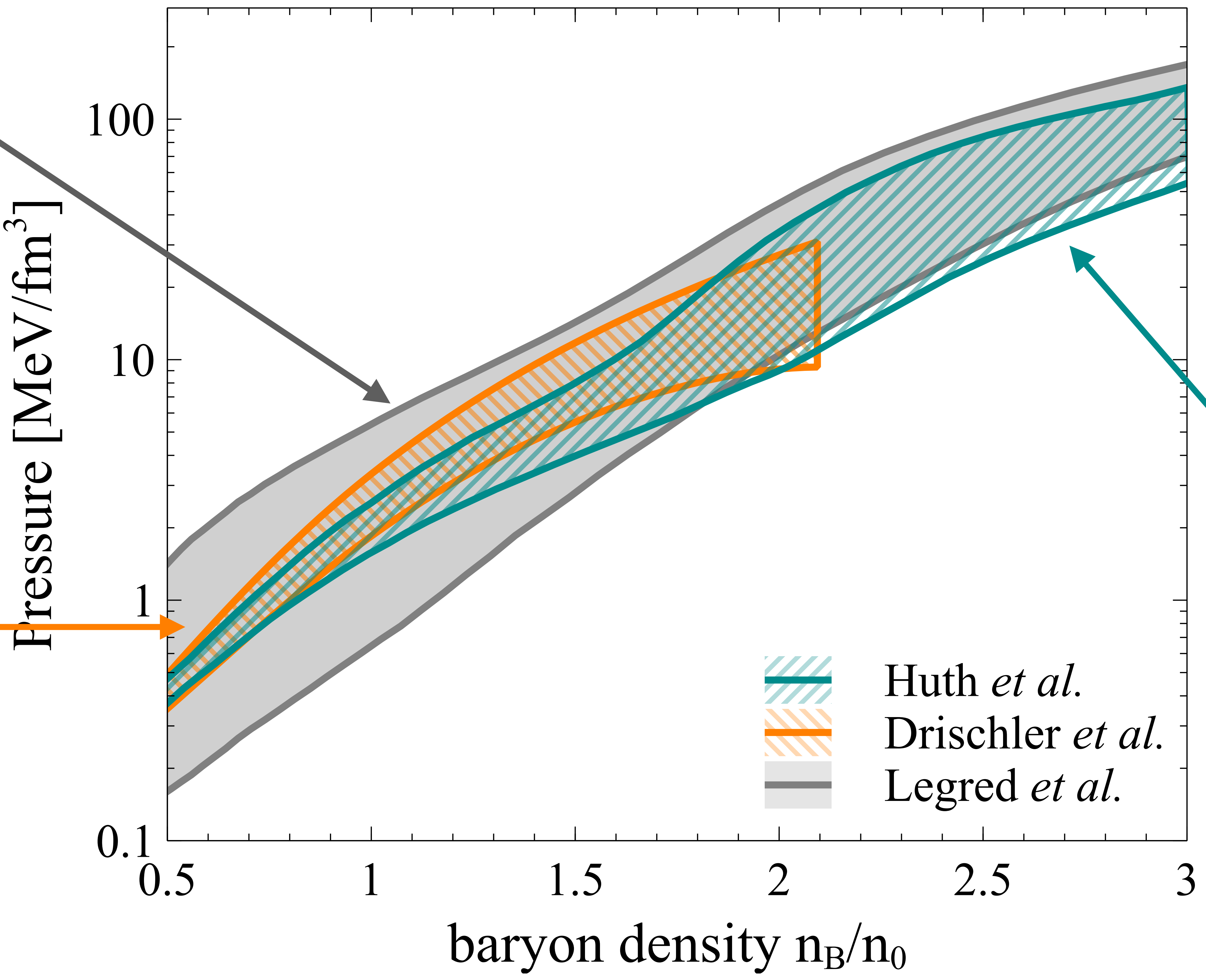
STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2108.00908
D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996
**A. Sorensen** *et al.*, arXiv:2301.13253

# Significant potential in exploring global analyses

Constraints using multiple inputs (nuclear structure, heavy-ion collisions, neutron stars) are tight

nonparametric EOS inference based on Gaussian processes
combines information from X-ray, radio, and gravitational wave observations of neutron stars.

I. Legred, K. Chatziioannou, R. Essick, S. Han, and P. Landry, Phys. Rev. D **104**, 063003 (2021), arXiv:2106.05313




Bayesian analysis of correlated effective field theory truncation errors based on order-by-order calculations up to next-to-next-to-next-to-leading order in the χEFT expansion

C. Drischler, S. Han, J. M. Lattimer, M. Prakash, S. Reddy, and T. Zhao, Phys. Rev. C **103**, 045808 (2021), arXiv:2009.06441

combined nuclear theory via $\chi$EFT calculations (constraining the EOS below 1.5n0), EOS inferences from heavy-ion collisions from FOPI and ASY-EOS experiments, and astrophysical data on bulk neutron star properties

S. Huth et al., Nature **606**, 276 (2022), arXiv:2107.06229

**A. Sorensen** *et al.*, arXiv:2301.13253

# Relativistic vector density functional (VDF) model

"Resonance matter": SMASH (and UrQMD) should be able to handle that well!