

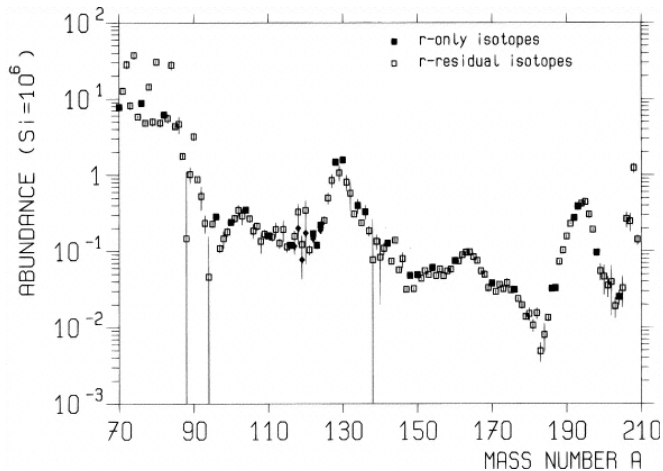
# Developments in DFT for Weak Decay/Capture

J. Engel

Work with **M. Mustonen**, **T. Shafer**, **E. Ney**, **Q. Liu**,  
C. Fröhlich, D. Gambacurta, M. Grasso, G. McLaughlin, M. Mumpower,  
N. Paar, A. Ravlić, N. Schunck, R. Surman, R. Zegers, ...

May 24, 2023

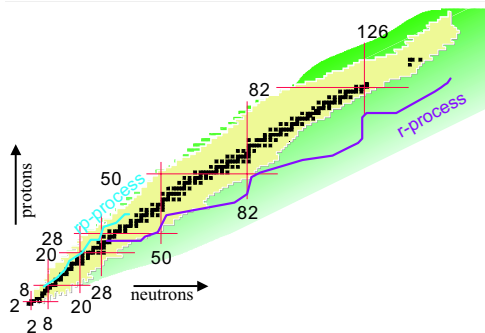
# R-Process Abundances



# Nuclear Landscape

To convincingly locate the site(s) of the  $r$  process, we need to know reaction rates, particularly  $\beta$ -decay rates, in neutron-rich nuclei.

To fully understand supernova evolution, we need to know electron-capture rates for lots of medium-mass nuclei.

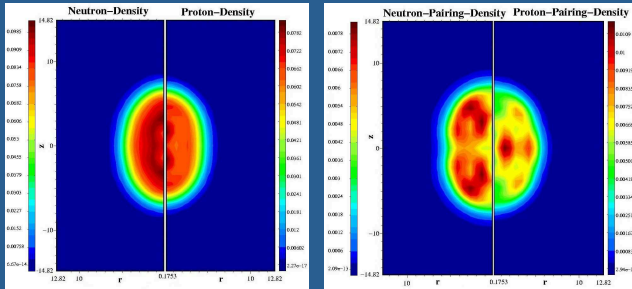


# Starting Point: Mean-Field-Like Calculation (HFB)

Gives you ground state density, etc. This is where Skyrme functionals have made their living.

Zr-102: normal density and pairing density  
HFB, 2-D lattice, SLy4 + volume pairing

Ref: Artur Blazkiewicz, Vanderbilt, Ph.D. thesis (2005)



HFB:  $\beta_2^{(p)}=0.43$

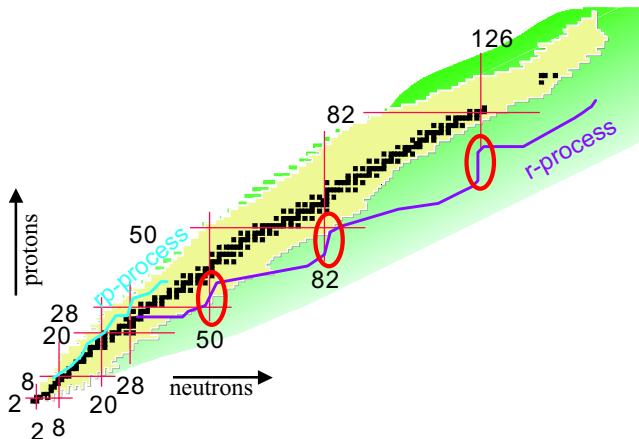
exp:  $\beta_2^{(p)}=0.42(5)$ , J.K. Hwang et al., Phys. Rev. C (2006)

# QRPA

Self-consistent QRPA is time-dependent HFB with small harmonic perturbation. Perturbing operator is  $\beta$ -decay transition operator. Decay matrix elements obtained from response of nucleus to perturbation.

# Initial Skyrme Application: Spherical QRPA

Even Isotopes Only

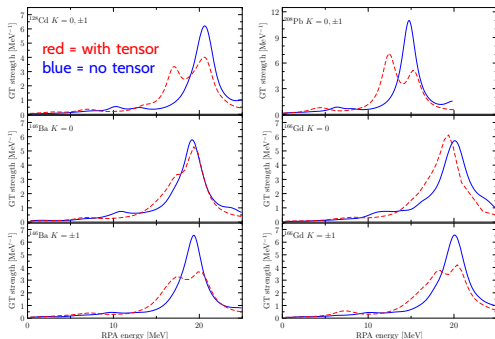


Closed shell nuclei are spherical.

# Later: Fast Skyrme QRPA in Deformed Nuclei

Finite-Amplitude Method (FAM) — Nakatsukasa et al.

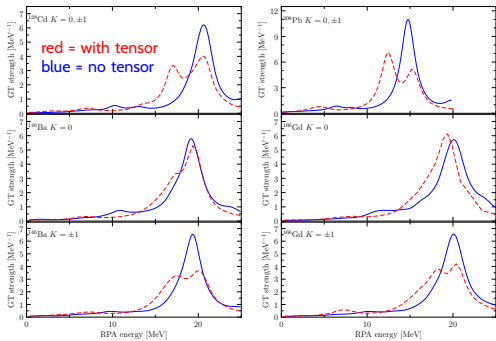
Strength functions  
computed directly from  
linear response, in orders  
of magnitude less time  
than with matrix QRPA.



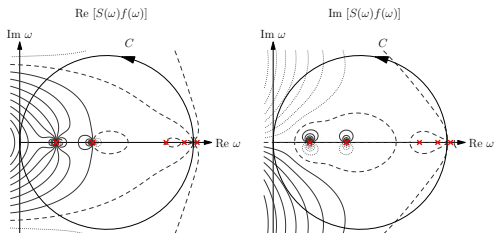
# Later: Fast Skyrme QRPA in Deformed Nuclei

Finite-Amplitude Method (FAM) — Nakatsukasa et al.

Strength functions computed directly from linear response, in orders of magnitude less time than with matrix QRPA.



Beta-decay rates obtained by integrating strength with phase-space weighting function in contour around excited states below threshold.





# Global Skyrme Fit for Even Nuclei

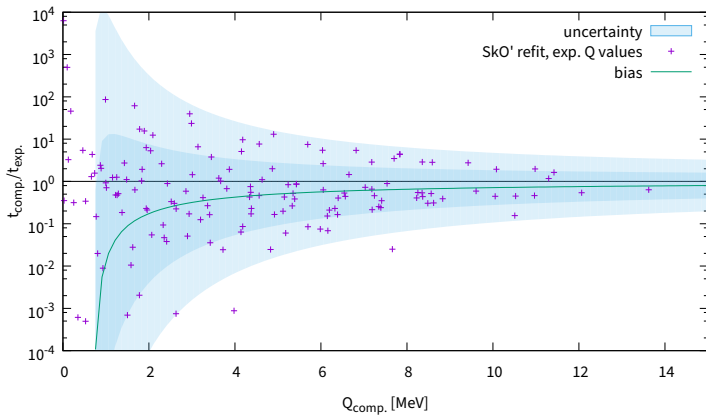
Mika Mustonen

Fit the charge-changing time-odd functional

$$\mathcal{H}_{\text{odd}}^{\text{c.c.}} = C_1^S \mathbf{s}_{11}^2 + C_1^{\Delta S} \mathbf{s}_{11} \cdot \nabla^2 \mathbf{s}_{11} + C_1^T \mathbf{s}_{11} \cdot \mathbf{T}_{11} + C_1^j \mathbf{j}_{11}^2 \\ + C_1^{\nabla j} \mathbf{s}_{11} \cdot \nabla \times \mathbf{j}_{11} + C_1^F \mathbf{s}_{11} \cdot \mathbf{F}_{11} + C_1^{\nabla s} (\nabla \cdot \mathbf{s}_{11})^2 + V_0 \times pn \text{ pair.}$$

Included 7 GT resonance energies, 2 spin-dipole resonance energies, 7  $\beta$ -decay rates in selected spherical and well-deformed nuclei from light to heavy.

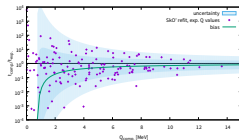
# Results



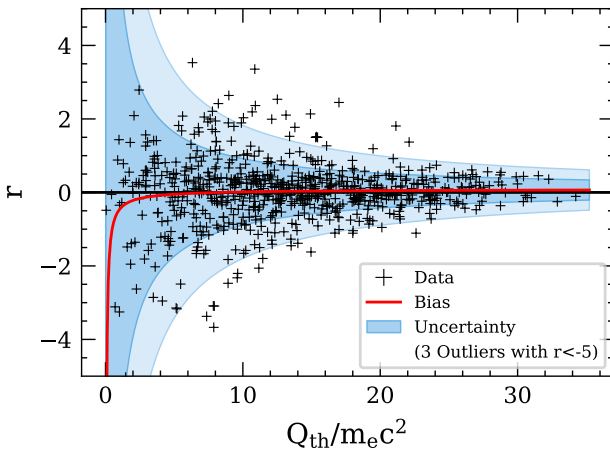
# Results with All Nuclei

Evan Ney

Figured out how to adapt  
FAM to treat odd-A and  
odd-odd nuclei.

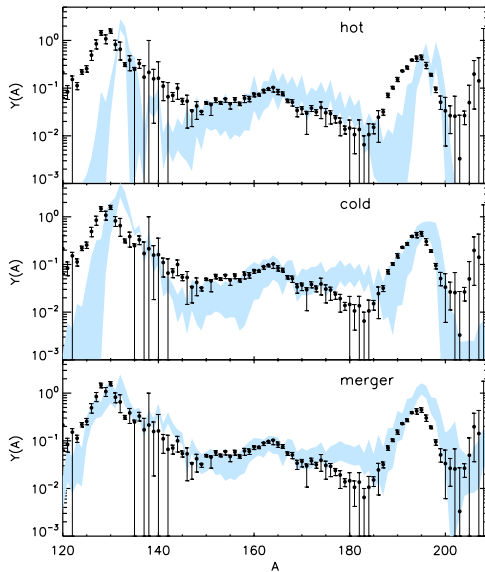


Even-even results



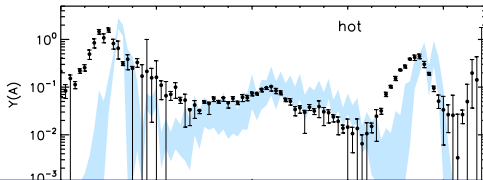
# What's at Stake Here?

## Significance of Factor-of-Two Uncertainty

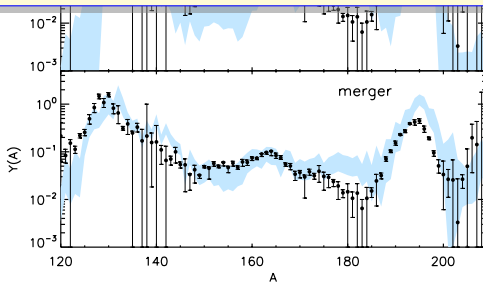


# What's at Stake Here?

## Significance of Factor-of-Two Uncertainty

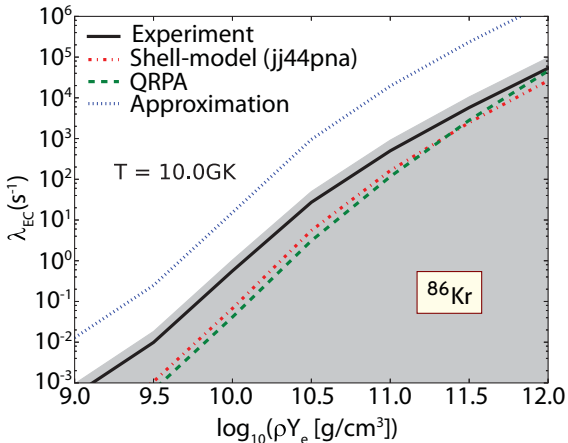


Real uncertainty is larger, though.



# Electron Capture

Evan Ney



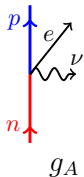
For terrestrial or astrophysical environments.

Developed a non-zero-temperature FAM.

# Most Recently: Two-Body Current

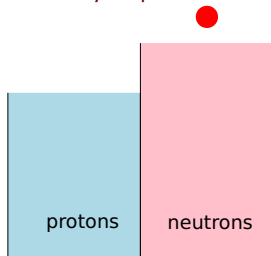
Evan Ney

Leading order:



Usual  $\beta$ -decay current

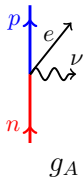
Consider very simple wave function



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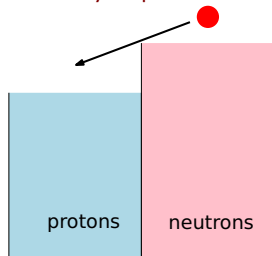
Evan Ney

Leading order:



Usual  $\beta$ -decay current

Consider very simple wave function

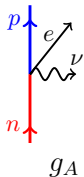




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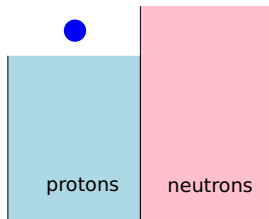
Evan Ney

Leading order:



Usual  $\beta$ -decay current

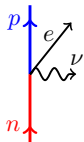
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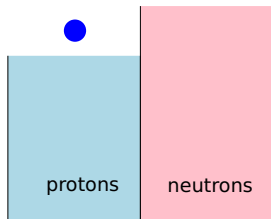
Leading order:



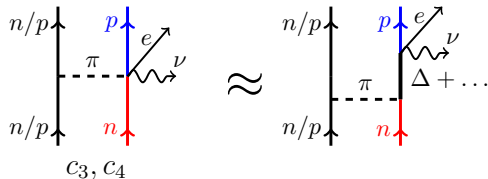
$g_A$

Usual  $\beta$ -decay current

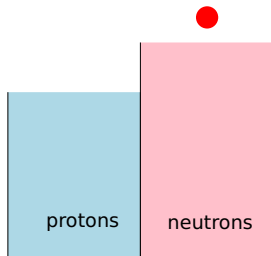
Consider very simple wave function



Higher order:



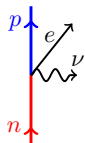
There are also contact terms...



# Most Recently: Two-Body Current

Evan Ney

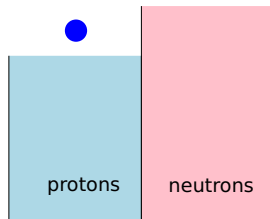
Leading order:



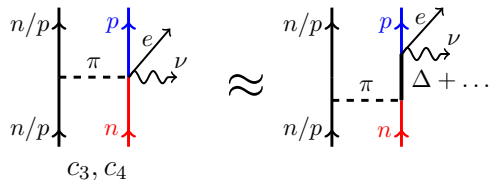
$g_A$

Usual  $\beta$ -decay current

Consider very simple wave function

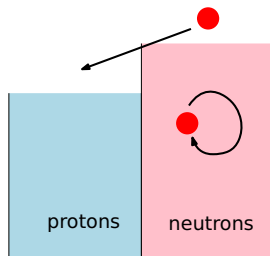


Higher order:



$C_3, C_4$

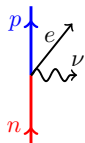
There are also contact terms...



# Most Recently: Two-Body Current

Evan Ney

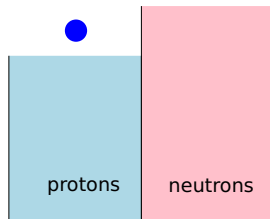
Leading order:



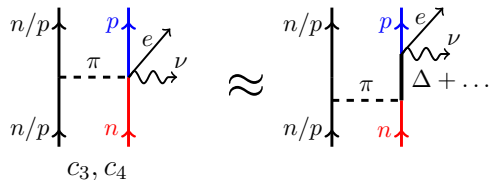
$g_A$

Usual  $\beta$ -decay current

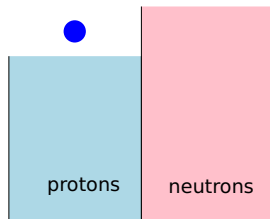
Consider very simple wave function



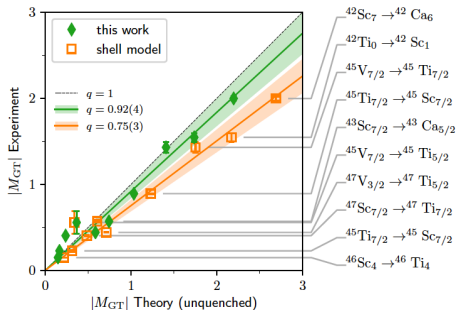
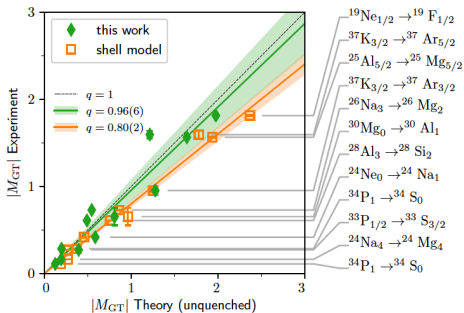
Higher order:



There are also contact terms...



# Quenching in the *sd* and *pf* Shells



IMSRG calculation, Gysbers et al

Some quenching from correlations omitted by the shell model.

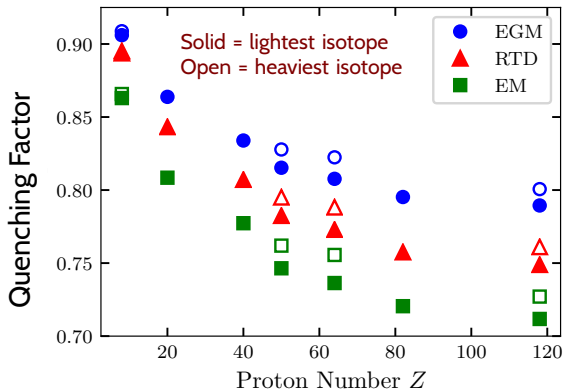
But a lot comes from the two-body current.

In these  $A < 50$  nuclei,  $\beta$ -decay quenching doesn't much depend on  $Z$  and  $N$ . But what about in heavier nuclei?

# Z- and N-Dependence of Quenching from Currents

Integrated GT Strength

Three sets of chiral parameters, no contact



EGM - E. Epelbaum, W. Glöckle, and U.-G. Meißner, Nucl. Phys. A 747, 362 (2005).

RTS - M. C. M. Rentmeester, R. G. E. Timmermans, and J. J. de Swart, Phys. Rev. C 67, 044001 (2003)

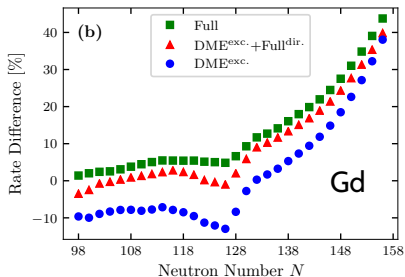
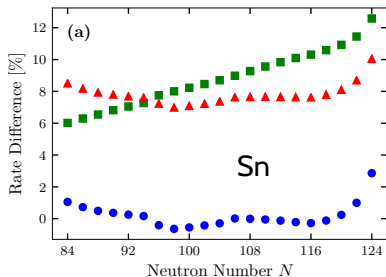
EM - D. R. Entem and R. Machleidt, Phys. Rev. C 68, 041001(R) (2003).

$$g_A = 1 \quad \longrightarrow \quad q = .79$$

# Effect on $\beta$ -Decay Rates

Difference from rate with one-body operator, with  $g_A = 1.0$

Focus on green squares



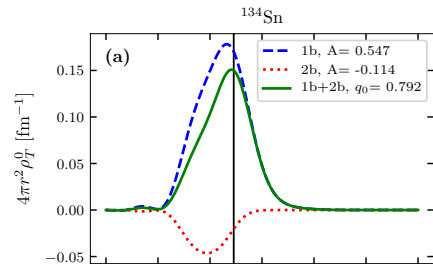
Two-body current has larger effect in neutron-rich nuclei.  
Quenching of rates decreases and can even become enhancement near the drip line.

Why?

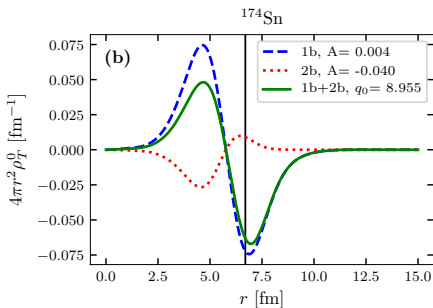
# Enhancement of Low-Lying Strength

Can occur in neutron-rich isotopes

Density-dependence of current means it does very little beyond the nuclear surface.



← Typical transition in <sup>134</sup>Sn



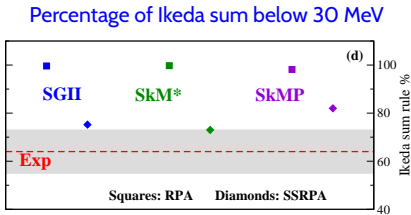
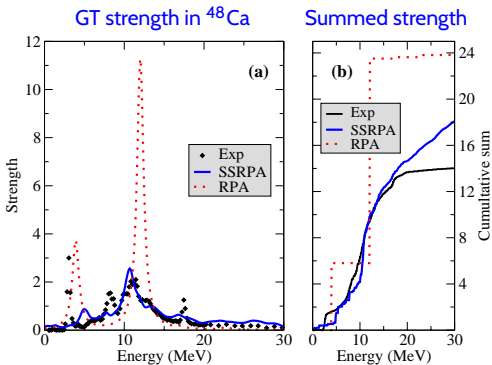
← Unusual (and lowest-lying) transition in <sup>174</sup>Sn



# Beyond QRPA

Second RPA with D. Gambacurta and M. Grasso

**Second RPA:** Add 4p-4h basic excitations to RPA 2p-2h excitations.  
Should better describe spreading widths and low-lying strength.



# Simpler Version: QRPA + Quasiparticle-Vibration Coupling

RPA response function

$$\Pi = \begin{array}{c} \text{---} \times \\ \updownarrow \\ \text{---} \times \end{array} + \begin{array}{c} \text{---} \times \\ \updownarrow \\ \updownarrow \\ \text{---} \times \end{array} + \begin{array}{c} \text{---} \times \\ \updownarrow \\ \updownarrow \\ \updownarrow \\ \text{---} \times \end{array} + \begin{array}{c} \text{---} \times \\ \updownarrow \\ \updownarrow \\ \updownarrow \\ \updownarrow \\ \text{---} \times \end{array} + \dots$$

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RPA response function

$$\Pi = \begin{array}{c} \text{---} \times \\ \circlearrowleft \\ \text{---} \times \end{array} + \begin{array}{c} \text{---} \times \\ \circlearrowleft \\ \text{---} \times \\ \circlearrowleft \\ \text{---} \times \end{array} + \begin{array}{c} \text{---} \times \\ \circlearrowleft \\ \text{---} \times \\ \circlearrowleft \\ \text{---} \times \\ \circlearrowleft \\ \text{---} \times \end{array} + \begin{array}{c} \text{---} \times \\ \circlearrowleft \\ \text{---} \times \\ \circlearrowleft \\ \text{---} \times \\ \circlearrowleft \\ \text{---} \times \\ \circlearrowleft \\ \text{---} \times \end{array} + \dots$$

Phonon-exchange potential

$$\text{~~~~~} = \begin{array}{c} \text{---} \\ \circlearrowleft \\ \text{---} \\ V_{Sk} \end{array} + \begin{array}{c} \text{---} \\ \circlearrowleft \\ \text{---} \\ \circlearrowleft \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \circlearrowleft \\ \text{---} \\ \circlearrowleft \\ \text{---} \\ \circlearrowleft \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \circlearrowleft \\ \text{---} \\ \circlearrowleft \\ \text{---} \\ \circlearrowleft \\ \text{---} \\ \circlearrowleft \\ \text{---} \end{array} + \dots$$

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Phonon-exchange potential

$$\text{---} \text{---} = \begin{array}{c} \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \text{---} \\ \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \text{---} \\ \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \text{---} \end{array} + \dots$$

Modification of particle-hole bubble by QVC

$$\begin{array}{c} \text{---} \\ \circlearrowleft \\ \text{---} \end{array} \Rightarrow \begin{array}{c} \text{---} \\ \circlearrowleft \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \circlearrowleft \\ \text{---} \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \circlearrowleft \\ \text{---} \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \circlearrowleft \\ \text{---} \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \circlearrowleft \\ \text{---} \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \\ \text{---} \text{---} \\ \circlearrowleft \\ \text{---} \end{array} + \dots$$

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RPA response function

$$\Pi = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \dots$$

Has been applied in spherical nuclei, but never deformed ones. We figured out how to build it into the FAM.



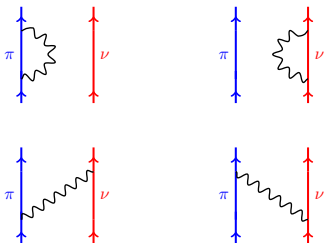
Modification of particle-hole bubble by QVC

$$\text{Diagram 1} \Rightarrow \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} + \dots$$

# QVC Results

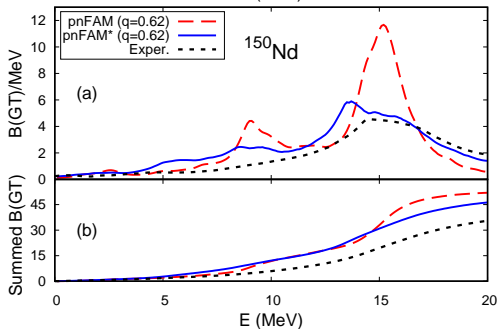
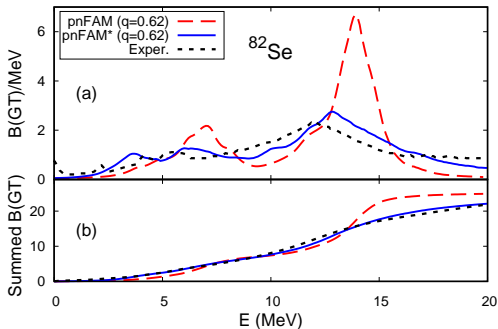
Called pnFAM\* Here

Phonon-exchange diagrams:  
phonons are like-particle  
excitations



With Q. Liu

### GT Distributions ( $g_A = 1$ )



## Next

All these developments will require refitting of the time-odd functional (and constants in the current) and UQ. There's still a lot to do on the road to more realistic DFT-based  $\beta$ -decay rates!

Whatever one can measure far from stability related to  $\beta$  decay,

- ▶ GT distributions
- ▶ charge-changing dipole and spin-dipole distributions
- ▶  $\beta$ -decay half lives
- ▶  $\vdots$

will be helpful.

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*Thanks for Listening*