Shapes, symmetries, and collective behavior in light nuclei

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April. 5, 2023

Ab initio nuclear physics

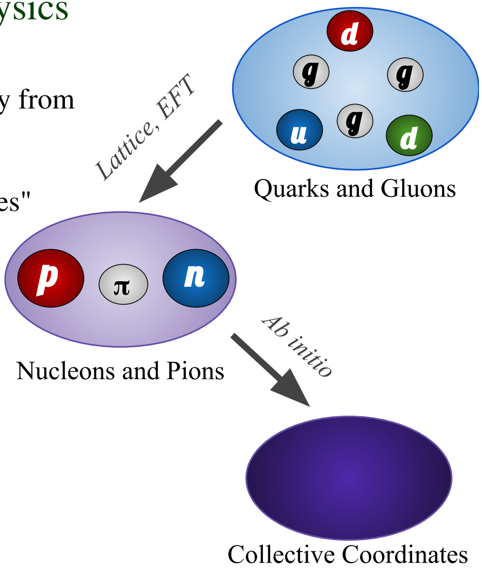
Lofty Goal: Predict nuclear structure and reactions directly from quantum chromodynamics (QCD)

Reality: Treat the nucleus as a "Tower of Effective Theories"

New effective degrees of freedom emerge at energy scales

– Effective field theory and lattice QCD provide a link between the quark scale and the nucleon scale.

– What are the relevant degrees of freedom at the next scale? Collective degrees of freedom?



Ab initio motivated simple pictures

- Simple pictures are useful for interpreting experimental data or calculated observables
  As we move away from stability, intuition and simple pictures based on $N \approx Z$ nuclei may be incomplete

- Ab initio methods can:
  - Provide insight into underlying correlations and symmetries
  - Test applicability of simple models

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Need: measurements of observables to validate ab initio motivated understanding of simple pictures of nuclear structure.

Outline

– There are many *ab initio* methods:

Pick your favorite combination of abbreviations
"NCSM","SM","MC","SA", "SRG", "IM", "RGM", "-C", etc.

– Emergence of collective behavior in beryllium isotopes

Enhanced E2 transitions.

– Simple pictures: rotations, dynamical symmetry, two-state mixing

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No-core shell model

– Emergence of collective behavior in beryllium isotopes

Enhanced E2 transitions.

– Simple pictures: rotations, dynamical symmetry, two-state mixing

No-core shell model

Solve many-body Schrodinger equation

$$\sum_{i}^{A} -\frac{\hbar^2}{2m_i}\nabla_i^2 \Psi + \frac{1}{2}\sum_{i,j=1}^{A} V(|r_i - r_j|)\Psi = E\Psi$$

Expanding wavefunctions in a basis

$$\Psi = \sum_{k=1}^{\infty} \alpha_k \phi_k$$

Reduces to Hamiltonian matrix eigenproblem

$$\begin{pmatrix} H_{11} & H_{12} & \dots \\ H_{21} & H_{22} & \dots \\ \vdots & \vdots & \end{pmatrix}\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \end{pmatrix} = E\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \end{pmatrix}$$

Harmonic oscillator basis

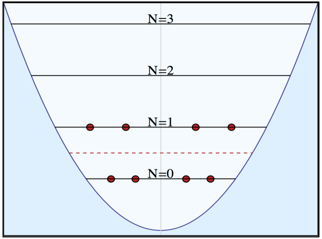
- Basis states are configurations, i.e., distributions of particles over harmonic oscillator shells (*nlj substates*)

- States are organized by total number of oscillator quanta above the lowest Pauli allowed number N_{ex}

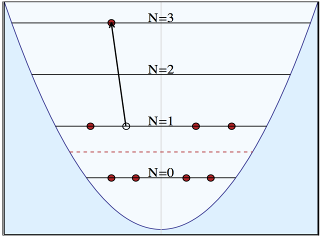
- States with higher N_{ex} contribute less to the wavefunction

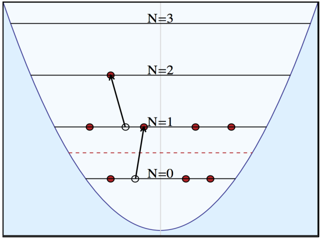
- Basis must be truncated:
  Restrict $N_{\text{ex}} \leq N_{\text{max}}$

$N = 2n + l$



$N_{\text{ex}} = 0$





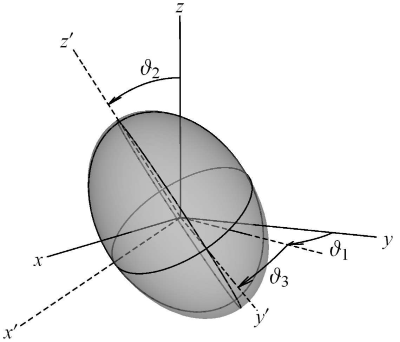
$N_{\text{ex}} = 2$

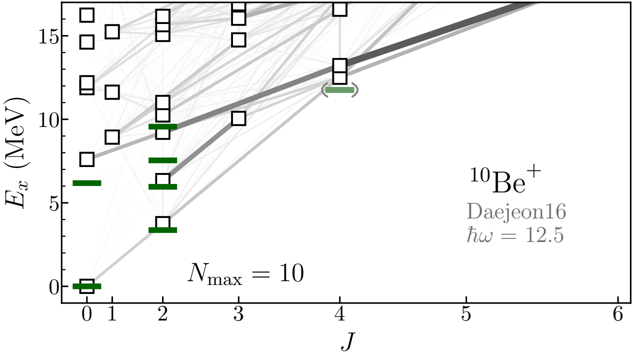
Nuclear rotations

Characterized by rotation of intrinsic state $|\phi_K\rangle$ by Euler angles ϑ $(J = K, K+1, \ldots)$

$$|\psi_{JKM}\rangle \propto \int d\vartheta \left[\mathscr{D}^J_{MK}(\vartheta)|\phi_K;\vartheta\rangle + (-)^{J+K}\mathscr{D}^J_{M-K}(\vartheta)|\phi_{\bar{K}};\vartheta\rangle \right]$$

Rotational energy: $E(J) = E_0 + A[J(J+1)]$





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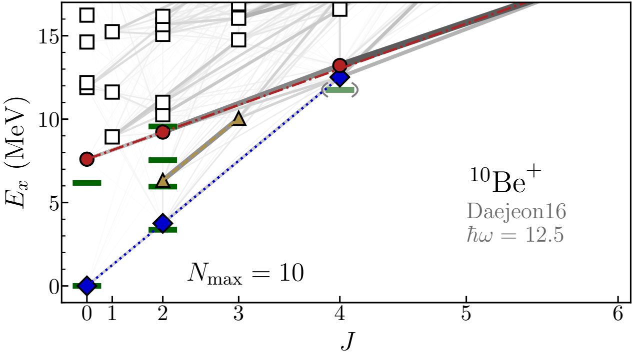
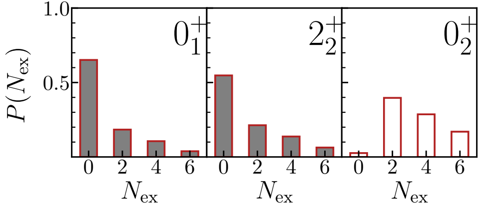
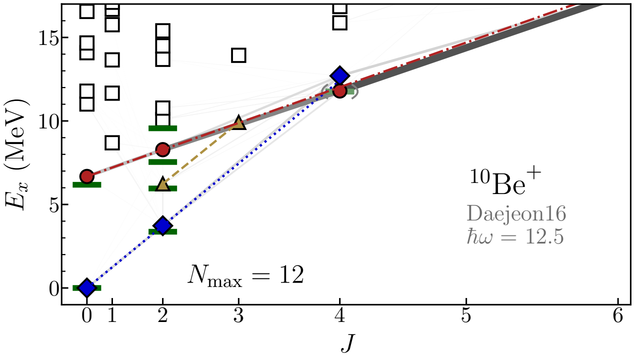


Probing underlying symmetries

- *Ab initio* calculations provides access to underlying wave functions of the collective states

- Using the “Lanczos trick” we can decompose the wave functions according to different symmetries
  C. W. Johnson. Phys. Rev. C **91** *(2015) 034313.*





Probing underlying symmetries

- Elliott's SU(3): In limit of large quantum numbers, labels (λ,μ) are associated with deformation parameters

  O. Castanos, J. P. Draayer, Y. Leschber, Z. Phys. A **329** *(1988) 3.*

$$\beta^2 \propto r^{-4}(\lambda^2 + \lambda\mu + \mu^2 + 3\lambda + 3\mu + 3)$$

$$\gamma = \tan^{-1}[\sqrt{3}(\mu+1)/(2\lambda+\mu+3)]$$

- Elliott rotation model: Bands arise from projecting out states with good L and K_L from intrinsic state with definite $(\lambda\mu)$

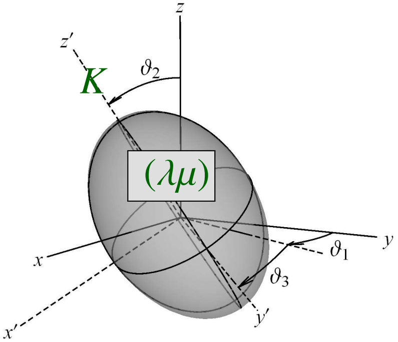
$$|(\lambda\mu)K_L L M_L\rangle$$

- Couple to spin to get good J states

$$L \times S \rightarrow J, \qquad K = K_L + K_S$$

SU(3) generators

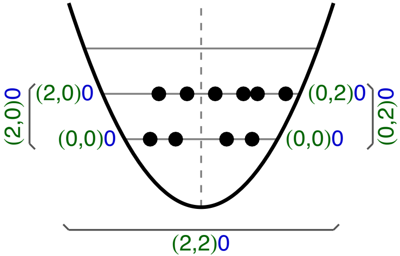
Q_{2M}	*Algebraic quadrupole*
L_{1M}	*Orbital angular momentum*



Elliott SU(3)

SU(3) symmetry of a configuration

- Each particle has SU(3) symmetry $(N, 0)$, $N = 2n + \ell$
- SU(3) couple particles to get total SU(3)
- Allowed spins dictated by antisymmetry constraints
- Final quantum numbers are $N_{\mathrm{ex}}(\lambda\mu)S$.



Lowest energies correspond to most deformed state

$\langle Q \cdot Q \rangle / r^4 \propto \beta^2$

$$H \propto -Q \cdot Q$$

$$= -6C_{\mathbf{SU(3)}}(\lambda, \mu) + 3\mathbf{L}^2$$

Elliott SU(3) → U(3)

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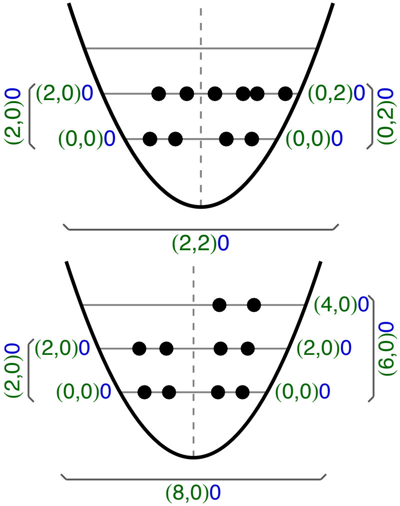


Lowest energies correspond to most deformed state

$\langle Q \cdot Q \rangle / r^4 \propto \beta^2$

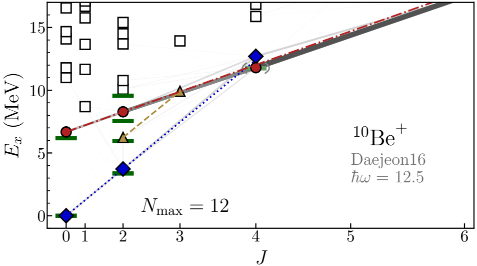
$$H \propto -Q \cdot Q + E(N_{\mathrm{ex}})$$

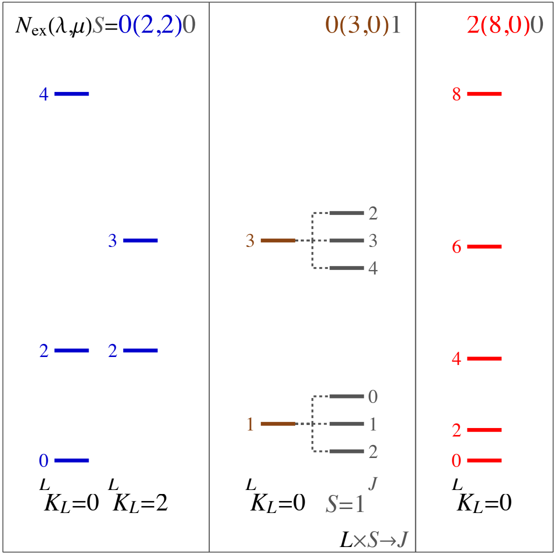
$$= -6C_{\mathrm{SU(3)}}(\lambda,\mu) + 3\mathbf{L}^2 + E(N_{\mathrm{ex}})$$

Elliott rotational bands: ^{10}Be

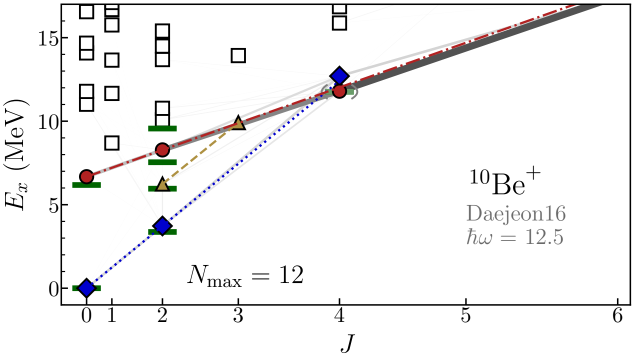
$$H \propto -Q \cdot Q = -6C_{\mathrm{SU}(3)} + 3\mathbf{L}^2 + E(N_{\mathrm{ex}})$$

$N_{\mathrm{ex}}(\lambda,\mu)$	S	$\Rightarrow$	$\langle C_{\mathrm{SU}(3)} \rangle$
0(0,0)	0,1,2	$\Rightarrow$	0
0(1,1)	0,1,2	$\Rightarrow$	6
0(0,3)	1	$\Rightarrow$	12
0(3,0)	1	$\Rightarrow$	12
0(2,2)	0	$\Rightarrow$	16
$\cdots$			
2(8,0)	0	$\Rightarrow$	58.67

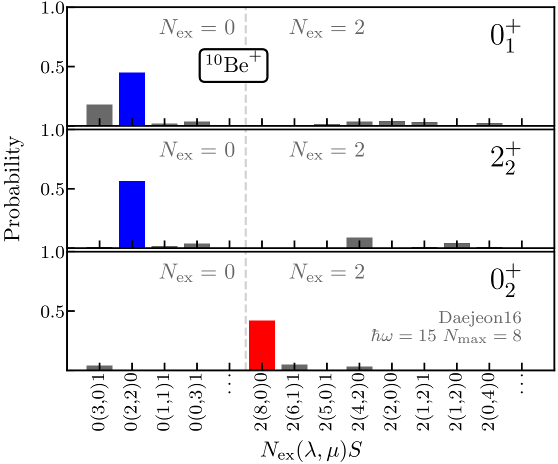




Elliott rotational bands: ^{10}Be



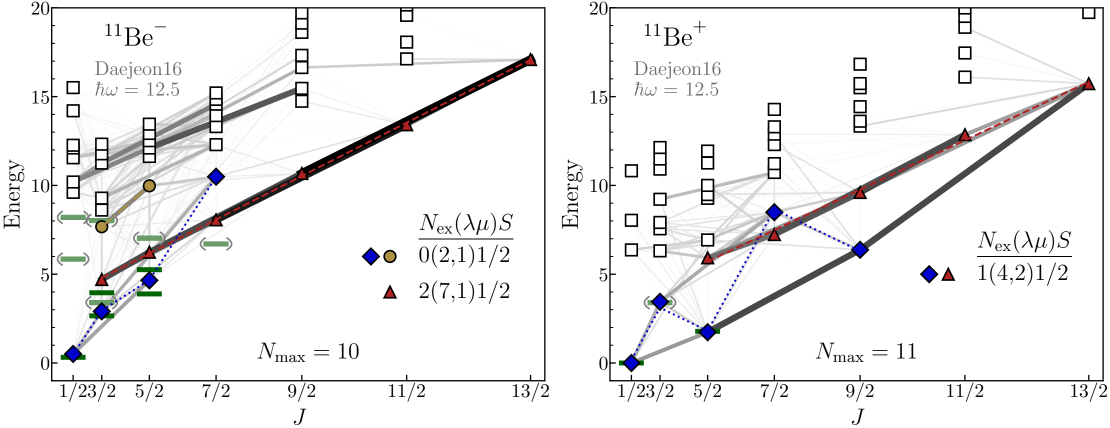
Approximate dynamical symmetry



- Ground state band: $N_{ex}(\lambda\mu)S = 0(2,2)0$
  $\beta = 0.16$, $\gamma = 30°$
- Intruder band: $N_{ex}(\lambda\mu)S = 2(8,0)0$
  $\beta = 0.27$, $\gamma = 5°$

^{11}Be

Parity inversion



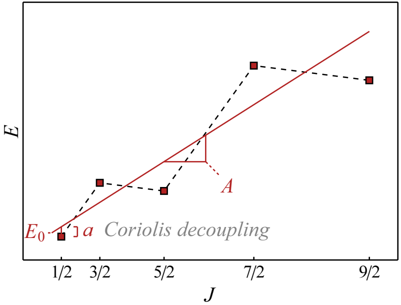
$0(2,1)\frac{1}{2} \rightarrow \beta = 0.15, \quad 1(4,2)\frac{1}{2} \rightarrow \beta = 0.22, \quad 2(7,1)\frac{1}{2} \rightarrow \beta = 0.28$

Nuclear rotations

Characterized by rotation of intrinsic state $|\phi_K\rangle$ by Euler angles ϑ $\quad (J = K, K+1, \ldots)$

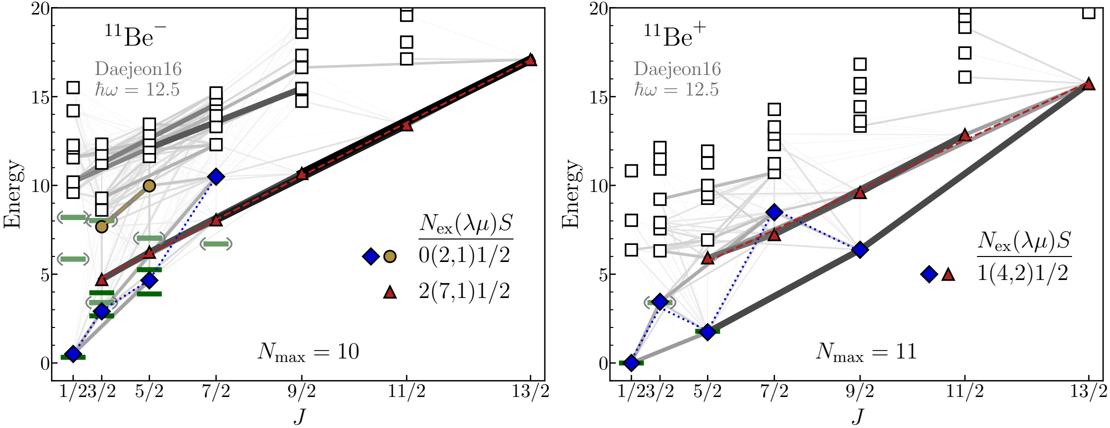
$$|\psi_{JKM}\rangle \propto \int d\vartheta \left[\mathscr{D}^J_{MK}(\vartheta)|\phi_K;\vartheta\rangle + (-)^{J+K}\mathscr{D}^J_{M-K}(\vartheta)|\phi_{\bar{K}};\vartheta\rangle \right]$$

Rotational energy: $E(J) = E_0 + A[J(J+1)] + \underbrace{a(-)^{J+1/2}(J+\frac{1}{2})}_{\text{Coriolis}\,(K=1/2)}$



^{11}Be

Parity inversion



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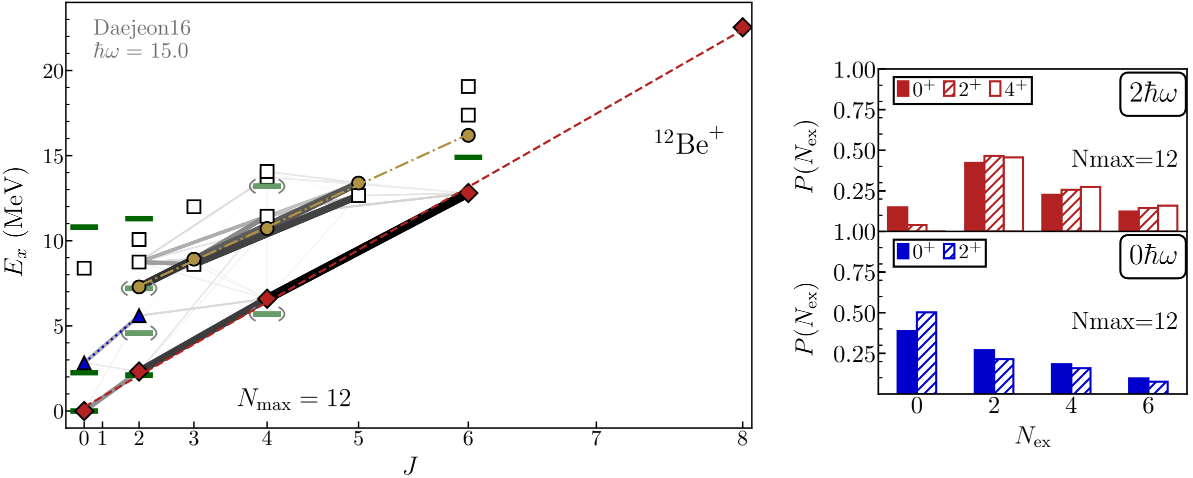
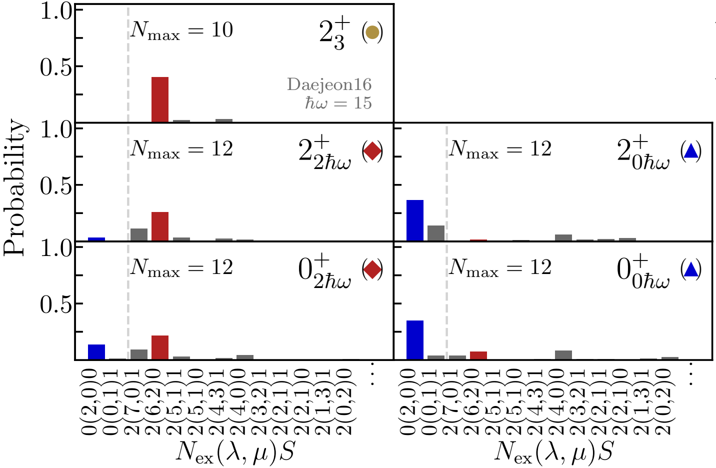
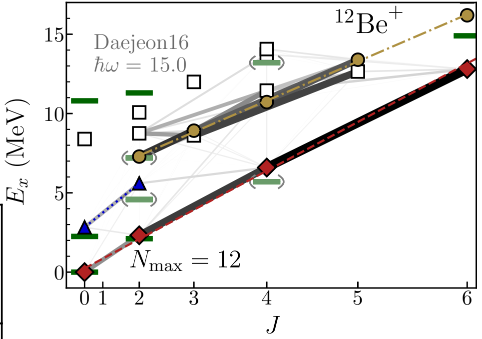
^{12}Be Bands


Daejeon16
$\hbar\omega = 15.0$
^{12}Be$^+$
$N_{max} = 12$
E_x (MeV)
J
0
5
10
15
20
0 1 2 3 4 5 6 7 8
$2\hbar\omega$
0^+ 2^+ 4^+
Nmax=12
$0\hbar\omega$
0^+ 2^+
Nmax=12
$P(N_{ex})$
N_{ex}
0.25
0.50
0.75
1.00
0 2 4 6

Decompose wave functions by Elliott U(3)





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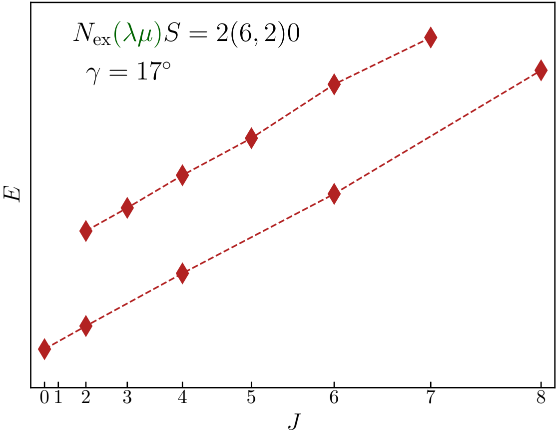
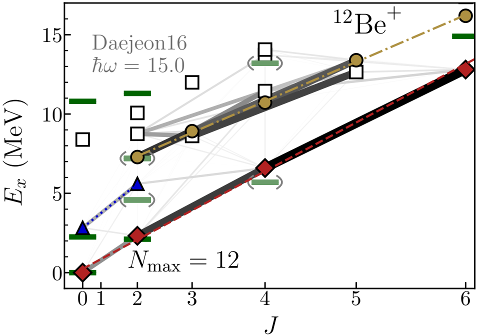




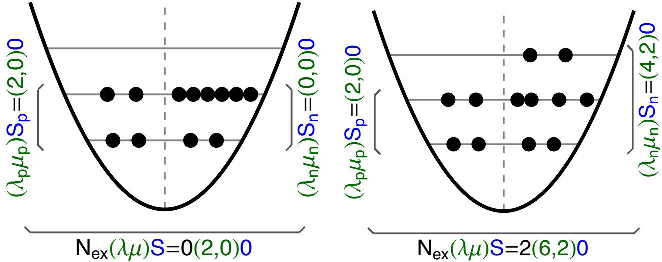
$$0(2,0)0 \rightarrow \beta = 0.11$$

$$2(6,2)0 \rightarrow \beta = 0.25$$

Rotor Hamiltonian: A. S. Davydov and G. F. Filippov. Nucl. Phys. **8** (1958) 237.

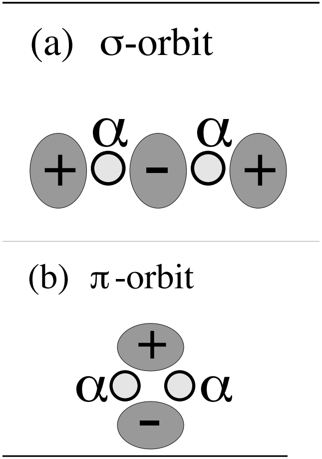
SU(3) configurations

In SU(3) picture ground state and isomeric 0_2^+ state have very different neutron shape. $\beta_n(0^+_{2\hbar\omega})/\beta_n(0^+_{0\hbar\omega}) \approx 4$.



$$Q_2 \sim C^{(1,1)} + A^{+2(2,0)} + B^{-2(0,2)}$$

$$r^2 \sim H^{(0,0)} - \sqrt{\tfrac{3}{2}}A^{+2(2,0)} - \sqrt{\tfrac{3}{2}}B^{-2(0,2)}$$



Antisymmetrized molecular dynamics (AMD)

Washington University in St. Louis

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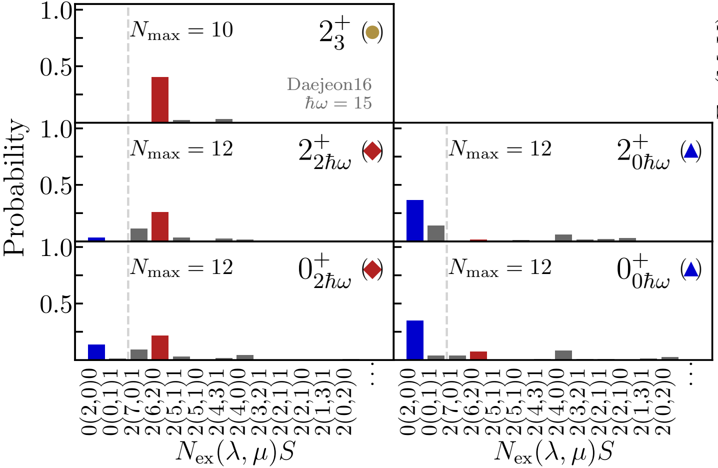
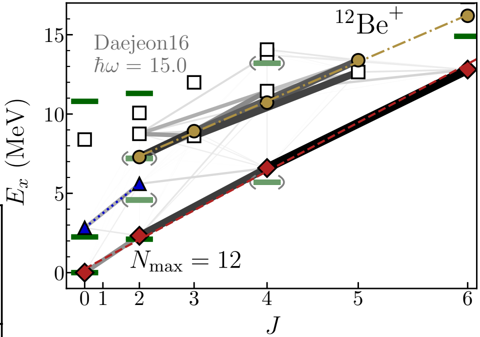
Decompose wave functions by Elliott U(3)





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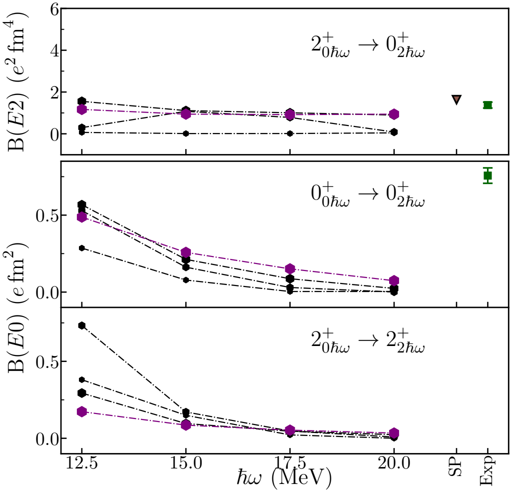
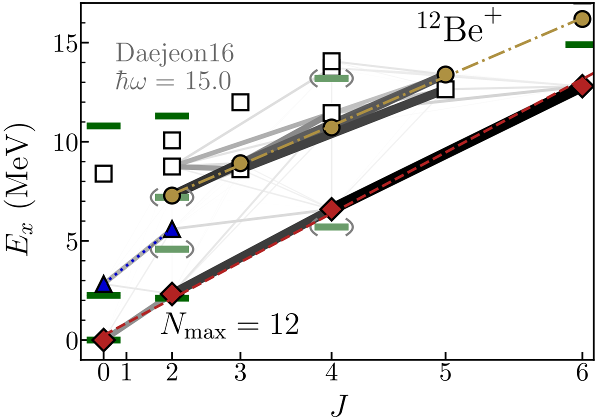
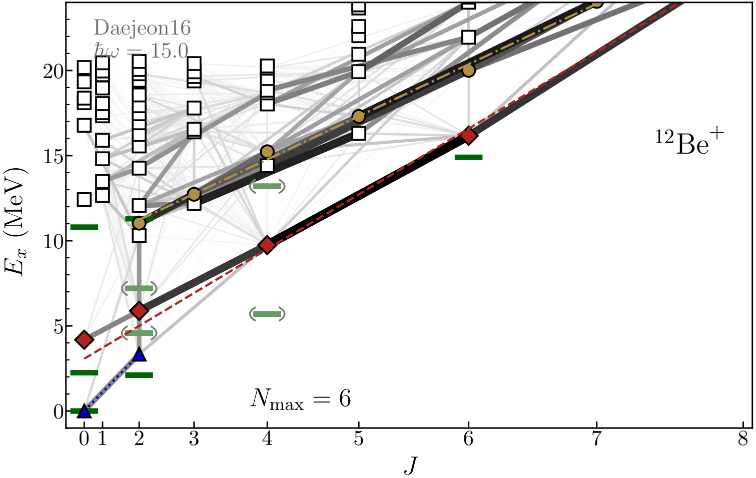
^{12}Be transitions

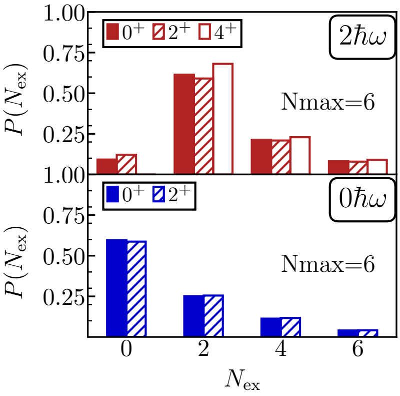




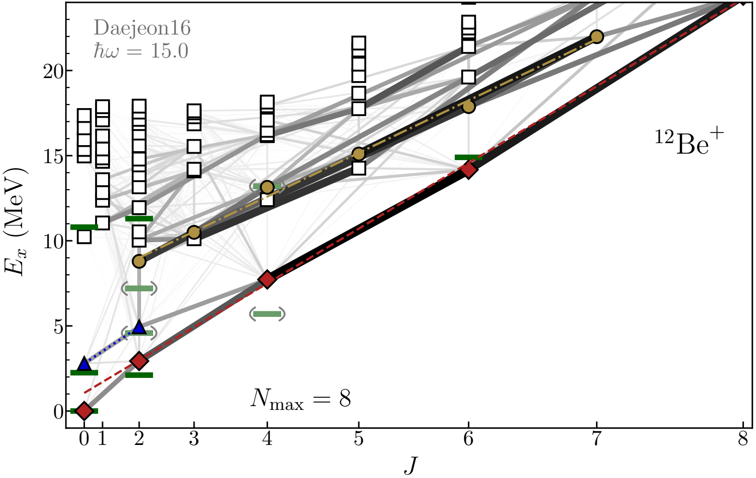
^{12}Be Bands

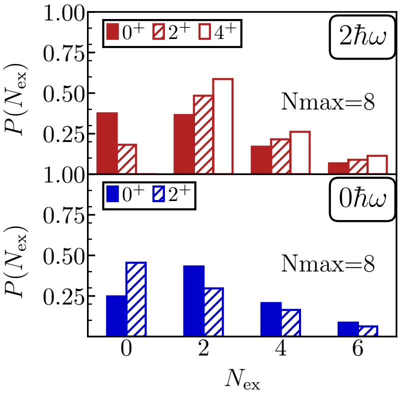


In previous NCSM calculations, bands were insufficiently converged to cross.

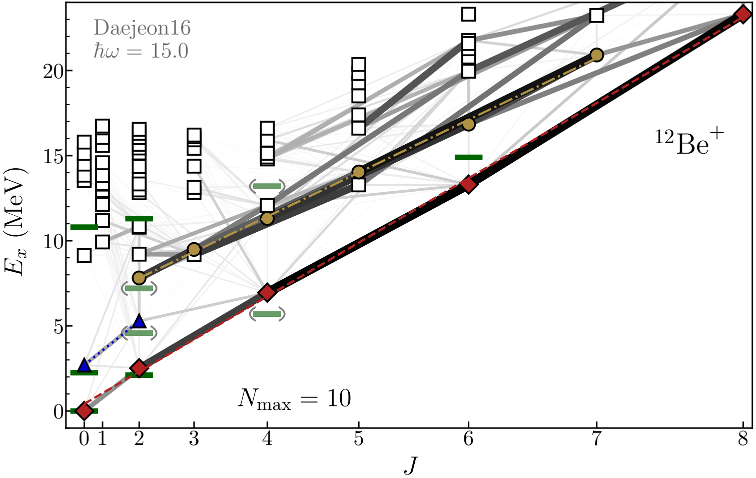


^{12}Be Bands

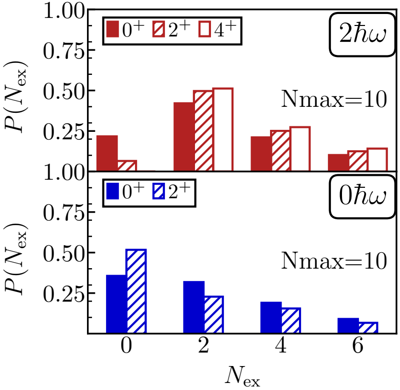

Daejeon16
$\hbar\omega = 15.0$
^{12}Be^{+}
$N_{\rm max} = 8$
E_x (MeV)
J
0
5
10
15
20
0 1 2 3 4 5 6 7 8


0^+ 2^+ 4^+
$2\hbar\omega$
Nmax=8
0^+ 2^+
$0\hbar\omega$
Nmax=8
$P(N_{\rm ex})$
$P(N_{\rm ex})$
1.00
0.75
0.50
0.25
1.00
0.75
0.50
0.25
0 2 4 6
$N_{\rm ex}$

^{12}Be Bands

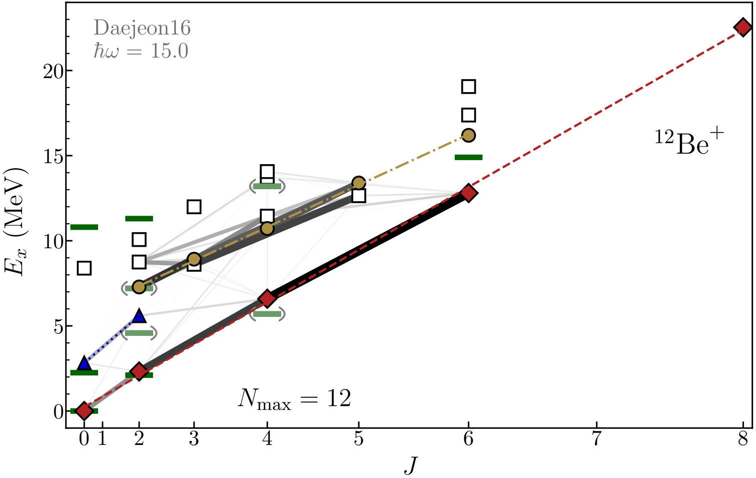


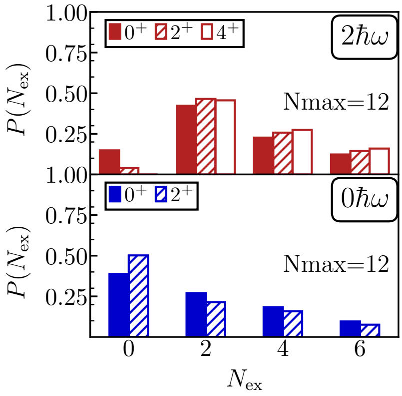
Daejeon16
$\hbar\omega = 15.0$
^{12}Be^{+}
$N_{\max} = 10$
E_x (MeV)
J
0
5
10
15
20
0 1 2 3 4 5 6 7 8



0^+ 2^+ 4^+
$2\hbar\omega$
Nmax=10
0^+ 2^+
$0\hbar\omega$
Nmax=10
$P(N_{\rm ex})$
$P(N_{\rm ex})$
1.00
0.75
0.50
0.25
1.00
0.75
0.50
0.25
0 2 4 6
$N_{\rm ex}$

^{12}Be Bands

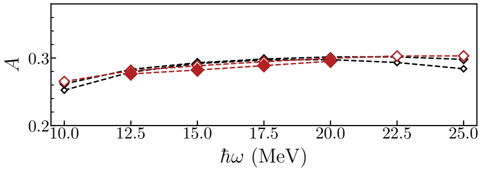

Daejeon16
ħω = 15.0
^{12}Be^{+}
$N_{\text{max}} = 12$
E_x (MeV)
J
0
5
10
15
20
0 1 2 3 4 5 6 7 8

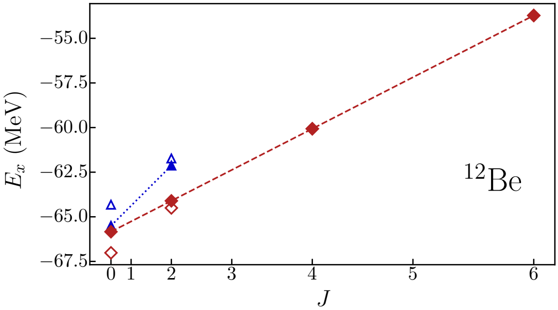

0+ 2+ 4+
2ħω
Nmax=12
0+ 2+
0ħω
Nmax=12
P(N_ex)
N_ex
1.00
0.75
0.50
0.25
0 2 4 6

Two state mixing

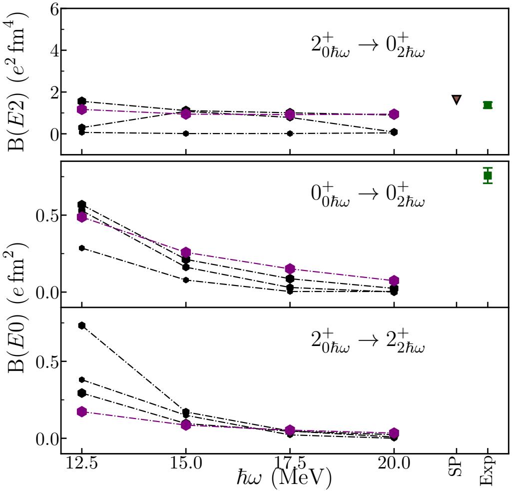
$$\underbrace{H_{\text{mix}} = \begin{pmatrix} E_1 & V \\ V & E_2 \end{pmatrix}}_{\text{mixing Hamiltonian}} \quad \rightarrow \quad \underbrace{\begin{pmatrix} \Psi'_1 \\ \Psi'_2 \end{pmatrix}}_{\text{“mixed"}} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\text{mixing matrix}} \underbrace{\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}}_{\text{“unmixed"}}$$

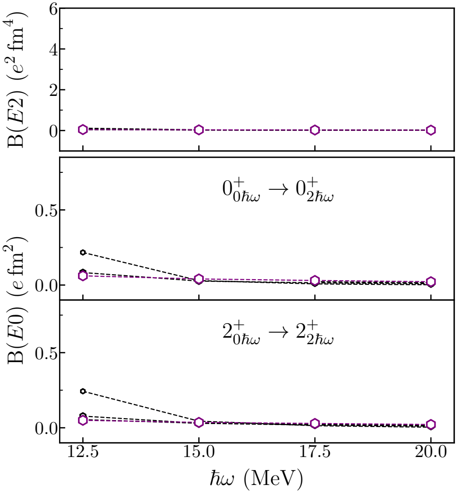
- Mixing angle θ depends on mixing matrix element V and $\Delta E = E_1 - E_2$
- Get “unmixed" energy from $E(J) = E_0 + A[J(J+1)]$





^{12}Be transitions





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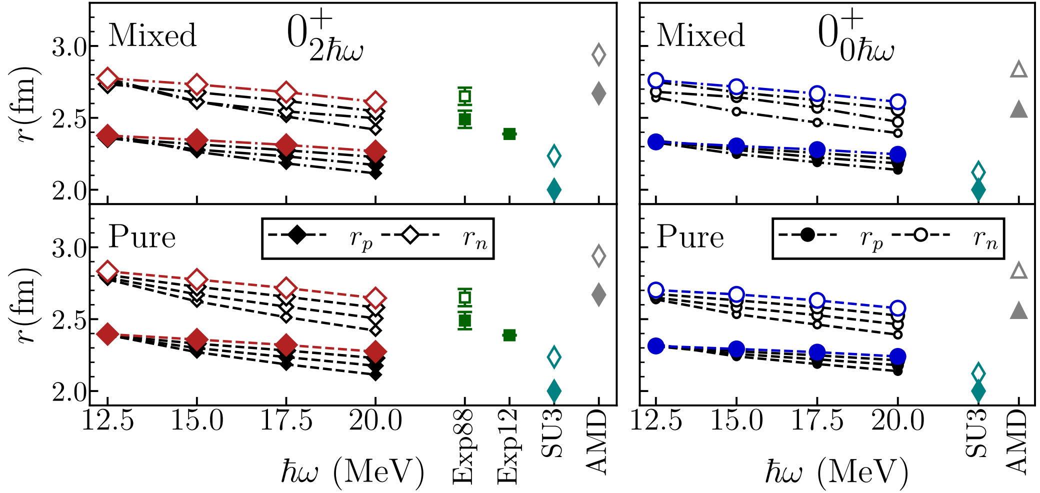
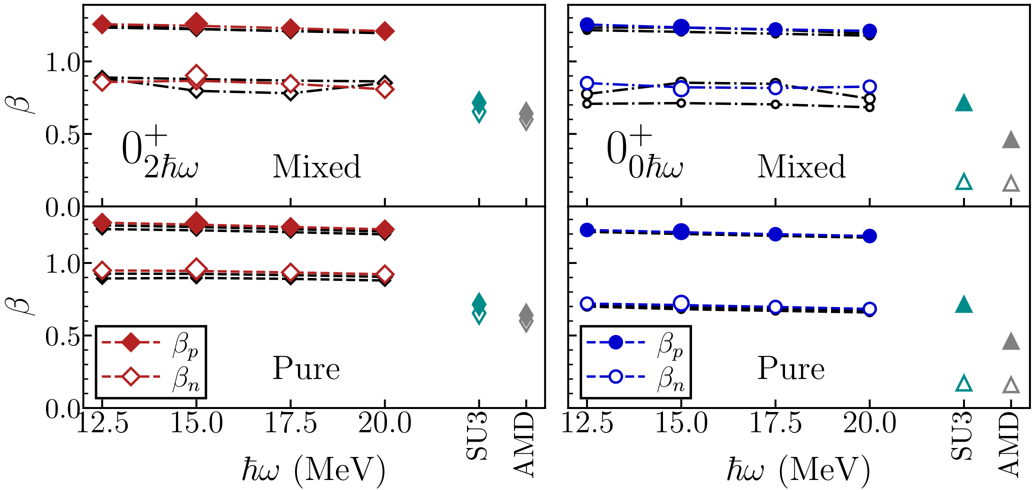
FRIB Theory Alliance

^{12}Be radii



$$\beta^2 \sim \langle Q \cdot Q \rangle / \langle r^2 \rangle^2$$

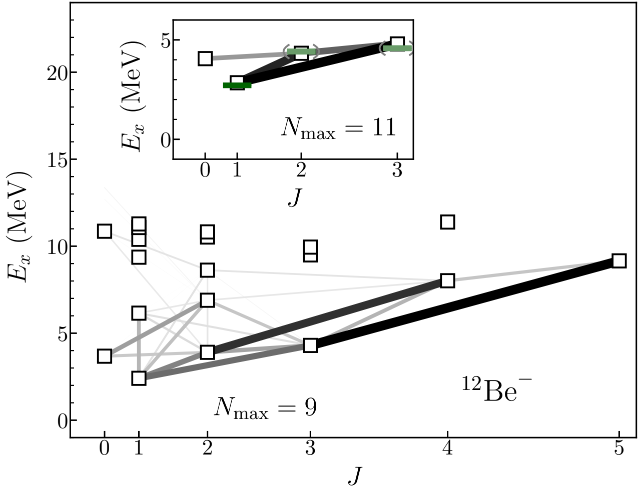


Summary

- Rotational bands emerge in calculated spectrum ^{10,11,12}Be
- Bands exhibit approximate SU(3) dynamical symmetry
- Intruder bands come increasingly lower in the spectrum with additional neutrons
  - Shape coexistence
  - Parity inversion in ^{11}Be
  - Intruder ground state in ^{12}Be
- Mixing of 0^+ states in ^{12}Be can be described in terms of a two-state mixing model.

Would like measured values for radii, E2 and E0 transitions, lifetimes, etc. of nuclei near shell closures, e.g., oxygen isotopes

^{12}Be negative parity spectrum



0^- predicted in:

Phys. Rev. C **68** *(2003), 014319.*

Phys. Lett. B **660** *(2008) 32.*

No 0^- bound state found in ^{11}Be(d,p)-transfer exp.

Phys. Rev. C **88** *(2013) 044619.*

SU(3):

$N_{ex}(\lambda,\mu)S = 1,(4,1)1$

$N_{ex}(\lambda,\mu)S = 1,(4,1)0$

$\beta = 0.18$

$\gamma_{SU(3)} = 16°$

^{12}Be in-band transitions