Physics of level density and thermalization

Vladimir Zelevinsky
FRIB Michigan State University

Sofia Karampagia
Grand Valley State University

FRIB workshop, East Lansing, May 26, 2023

In collaboration with

Alexander Volya
Alex Brown
Mihai Horoi, J. Kaiser, M. Ghita
Roman Sen’kov
Antonio Renzaglia
Alex Berlaga

Work in progress

- **Thermalization, Quantum Chaos,** and Level Density
- Shell model and **Moments method**
- "**Constant temperature**" model

Many-body quantum system with no random elements,
internally developed chaotic behavior

In physical terms, one may say that quantum thermalization occurs in the Hilbert space rather than phase space

/T. Mori et al. J. Phys. B51, 112001 (2018)./

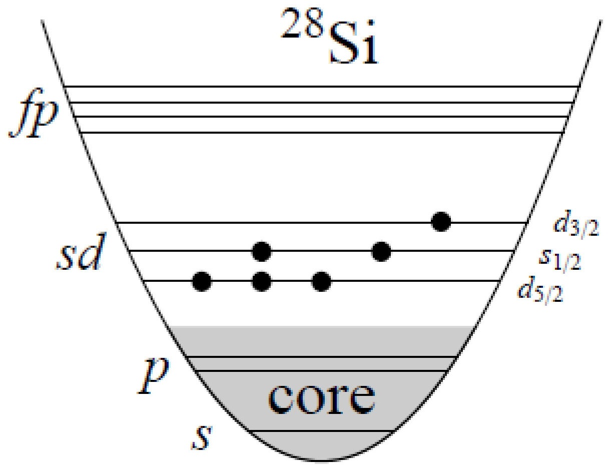
General method – diagonalization of Hamiltonian matrix

"Eigenfunction thermalization hypothesis" – read Landau and Lifshits

Microscopic description of Nuclear Level Density

Shell model (the most successful)

- Restricted model space
  Dim(sd) $\sim 10^6$
  Dim(fp) $\sim 10^{10}$
- Need effective interaction
- Numerical diagonalization
- High accuracy: $\delta E \sim \pm 200 KeV$



How it works:

Many-body states in Shell Model: $|\alpha\rangle = \sum_{k=1}^{Dim} C_k^\alpha |k\rangle$.

Schrödinger equation: $\hat{H}|\alpha\rangle = E_\alpha|\alpha\rangle \quad \Rightarrow \quad \hat{H}\vec{C}_\alpha = E_\alpha \vec{C}_\alpha$.

FAMILY OF ENTROPIES FOR A MESOSCOPIC SYSTEM

- **THERMODYNAMIC** (*Boltzmann*)

$\rho(E) \propto \exp(S_{\rm th})$

- **QUASIPARTICLE** (*Landau Fermi-liquid*)

$S^{\alpha}_{\rm s.p.} = -\sum_i \{ n_i^{\alpha} \ln(n_i^{\alpha})$
$+(1-n_i^{\alpha}) \ln(1-n_i^{\alpha}) \}$

- **INFORMATION** (*Shannon*)

$|\alpha\rangle = \sum_k C_k^{\alpha} |k\rangle, \quad S^{\alpha}_{\rm inf} = -\sum_k \{ |C_k^{\alpha}|^2 \ln |C_k^{\alpha}|^2 \}$

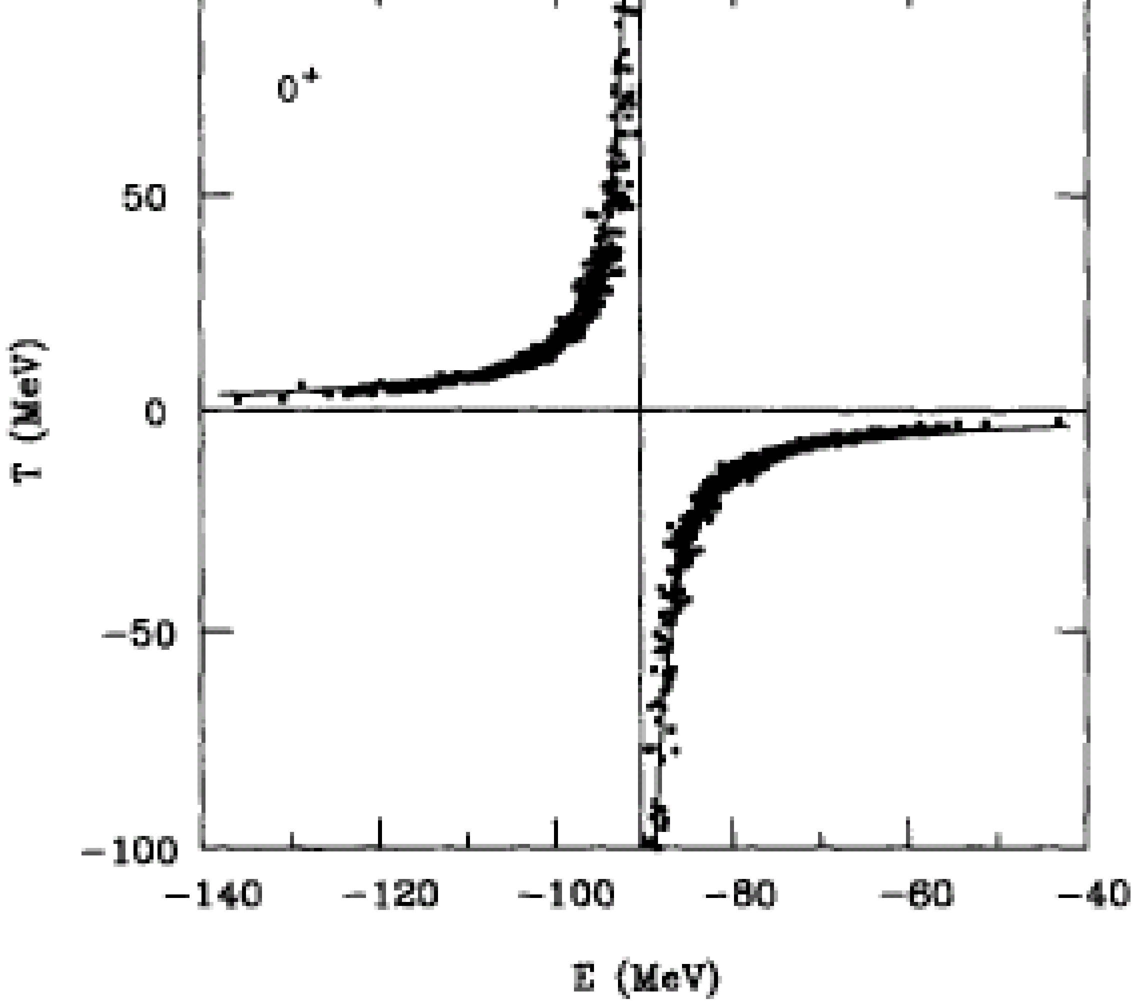
$$\langle n_i \rangle_E = [e^{(\epsilon_i - \mu)/T_{\rm s.p.}} + 1]^{-1}$$

Temperature T(E)

$$T_{\rm th} = \left(\frac{dS_{\rm th}}{dE} \right)^{-1}$$

$$T_{\rm inf} = \left(\frac{d\bar{S}_{\rm inf}}{dE} \right)^{-1}$$

T(s.p.) and T(inf) =
for individual states !



Gaussian level density

CENTROID E_0

WIDTH σ_E

Microcanonical temperature

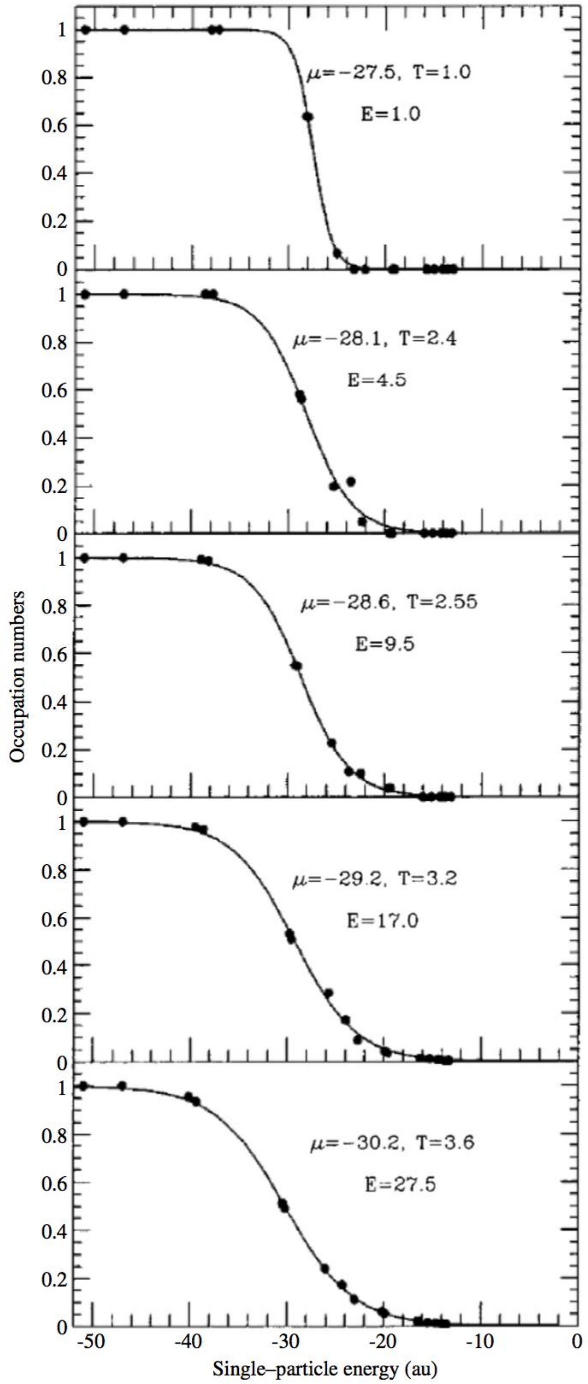
$$T_{\rm th} = \sigma_E^2/(E_0 - E)$$

839 states (28 Si) J=0

EFFECTIVE TEMPERATURE of INDIVIDUAL STATES

From occupation numbers in the shell model solution **(dots)**

From thermodynamic entropy defined by level density **(lines)**



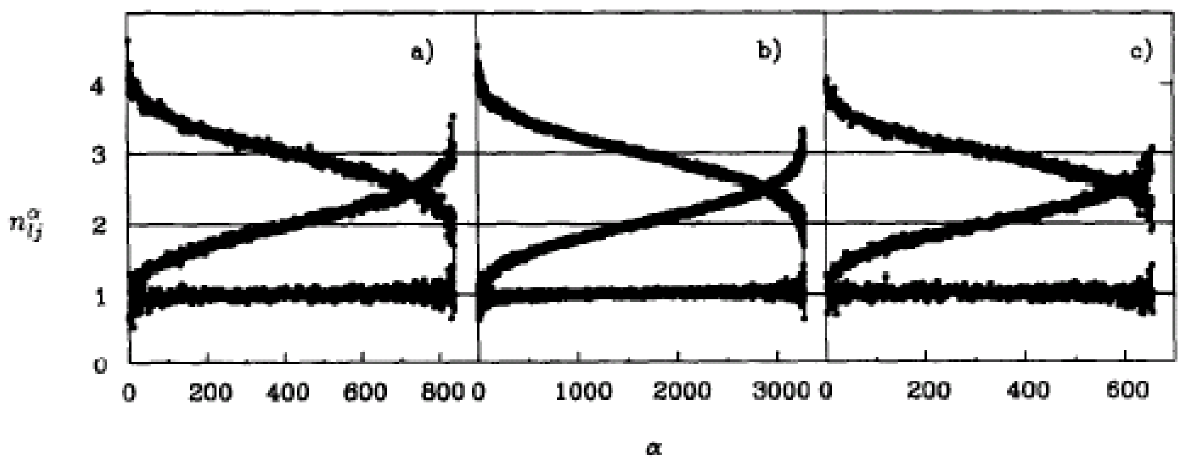
Occupation numbers in multicharged ions Au25+

(recombination as analog of neutron resonances in nuclei)

$$n_s^\alpha = \langle\alpha|\,\hat{n}_s\,|\alpha\rangle = \sum_k \left|C_k^\alpha\right|^2 \langle k|\,\hat{n}_s\,|k\rangle$$

/G. Gribakin, A. Gribakina, V. Flambaum/

Average over individual states is equivalent to a thermal ensemble d5/2, d3/2, s1/2



28 Si

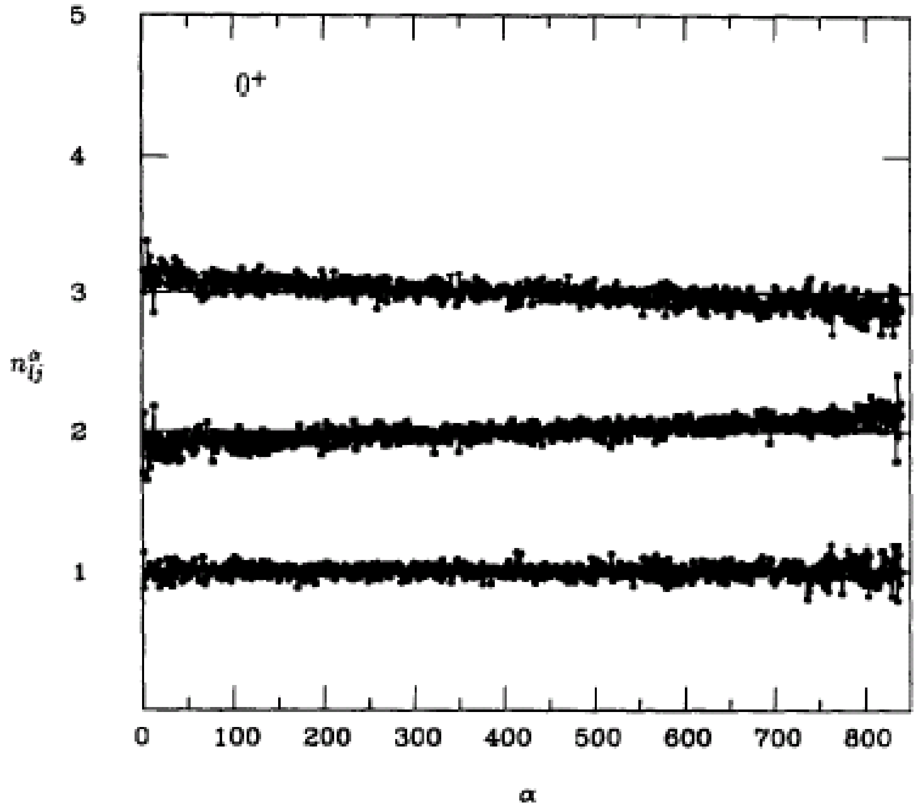
J=0 *J=2* *J=9*

Single – particle occupation numbers

Thermodynamic behavior
identical in all symmetry classes

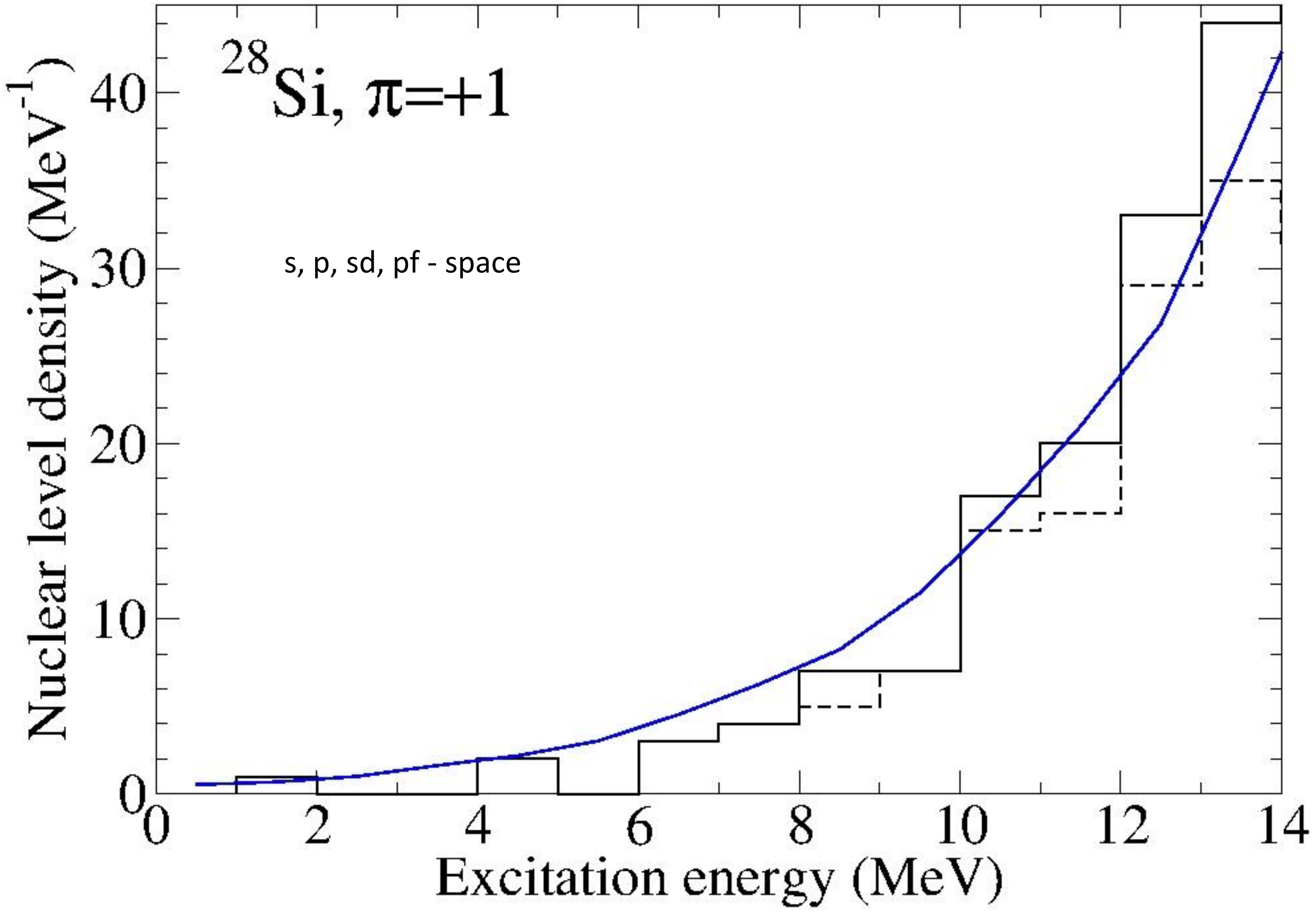
FERMI-LIQUID PICTURE

J=0

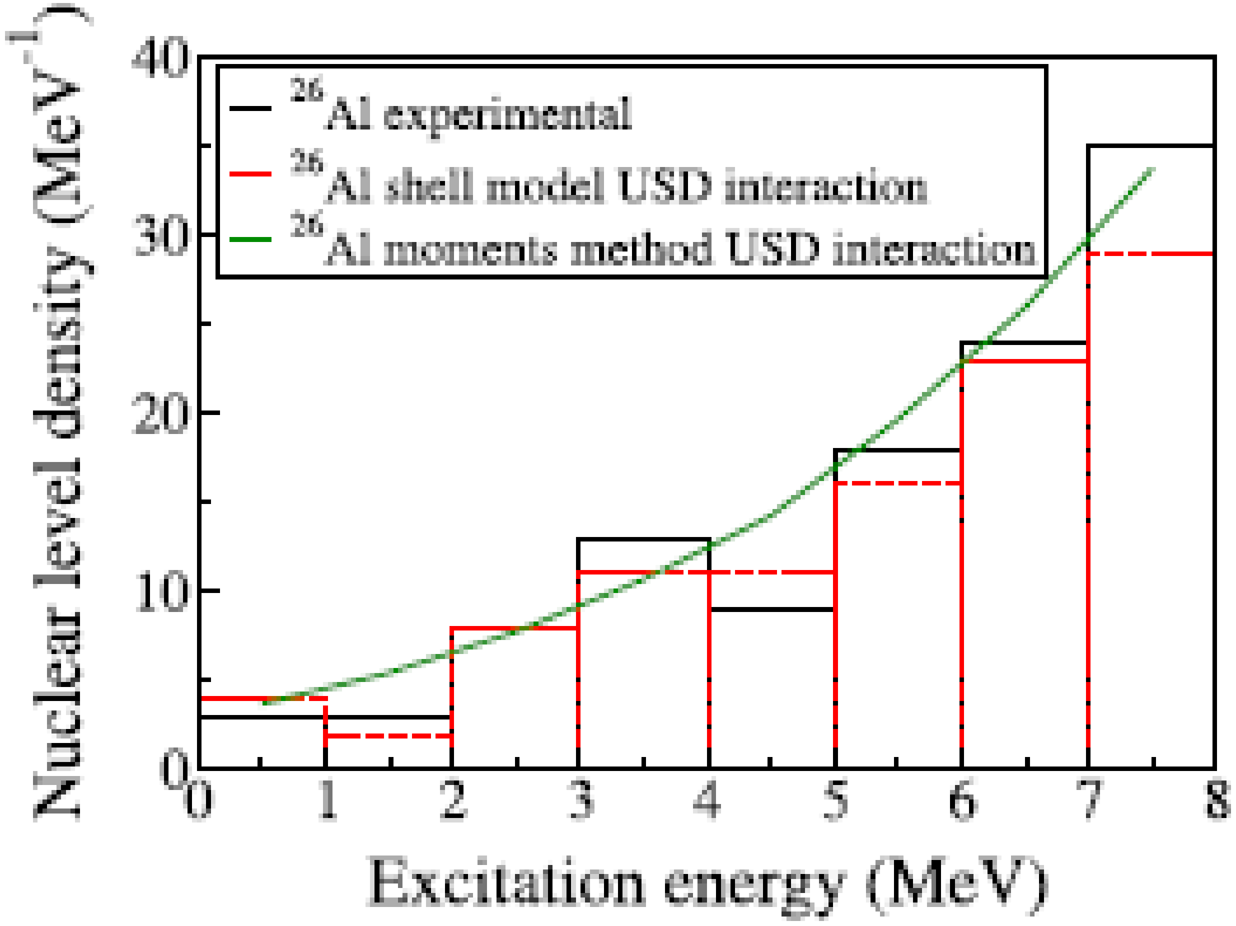


Artificially strong interaction (factor of 10)

Single-particle thermometer cannot resolve spectral evolution



^{28}Si, π=+1
s, p, sd, pf - space
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
40
0
2
4
6
8
10
12
14



S. Karampagia, V.Z.
Nucl. Phys. A962 (2017)

J = 0 – 7, positive parity level density

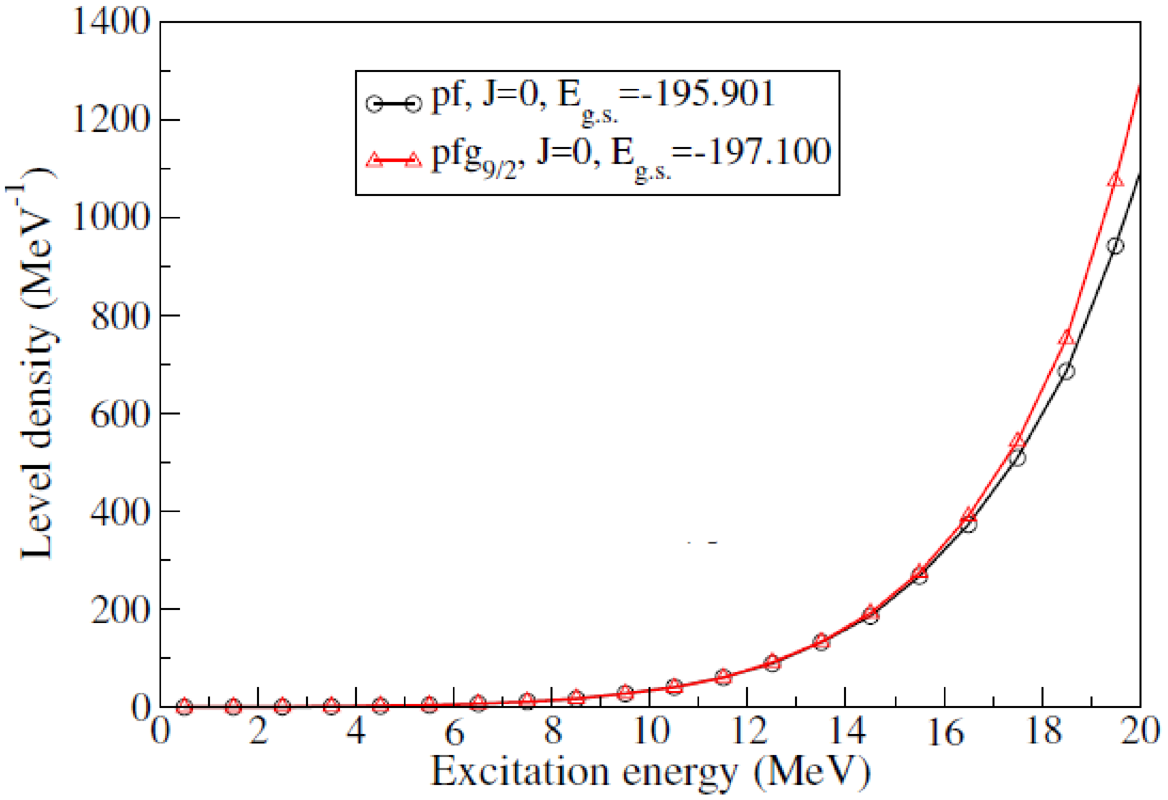


FIG. 3: Comparison of the level density of ^{56}Fe calculated in the pf model space (black line with circles) versus the one calculated in the $pf + g_{9/2}$ model space (red line with triangles), using the $E_{g.s.}$=-197.100, which minimizes the difference of the low-lying level densities between the two model spaces.

Moments method

No diagonalization required

$$\rho(E,\alpha)=\sum_{\kappa} D_{\alpha\kappa}\cdot G_{\alpha\kappa}(E)$$

[Wong]

$$\alpha=\{n,J,T_z,\pi\}$$

Quantum numbers

Exact quantum numbers

$$\kappa=\{n_1,n_2,\dots,n_q\}$$

Partitions

$$G_{\alpha\kappa}(E)=G(E+E_{g.s.}-E_{\alpha\kappa},\sigma_{\alpha\kappa})$$

$$G(x,\sigma)=C\cdot\begin{cases}\exp\left(-x^2/2\sigma^2\right) & , \ |x|\le\eta\cdot\sigma\\ 0 & , \ |x|>\eta\cdot\sigma\end{cases}$$

Finite range Gaussian

$$D_{\alpha\kappa}$$

Many-body dimension

$$E_{\alpha\kappa}=\langle H\rangle_{\alpha\kappa};$$

$$\sigma_{\alpha\kappa}=\sqrt{\langle H^2\rangle_{\alpha\kappa}-\langle H\rangle_{\alpha\kappa}^2}$$

$$\mathrm{Tr}^{(J)}[\cdots]=\mathrm{Tr}^{(J_z)}[\cdots]_{J_z=J}-\mathrm{Tr}^{(J_z)}[\cdots]_{J_z=J+1}$$

$$\langle H\rangle_{\alpha\kappa}=\mathrm{Tr}^{(\alpha\kappa)}[H]/D_{\alpha\kappa},$$

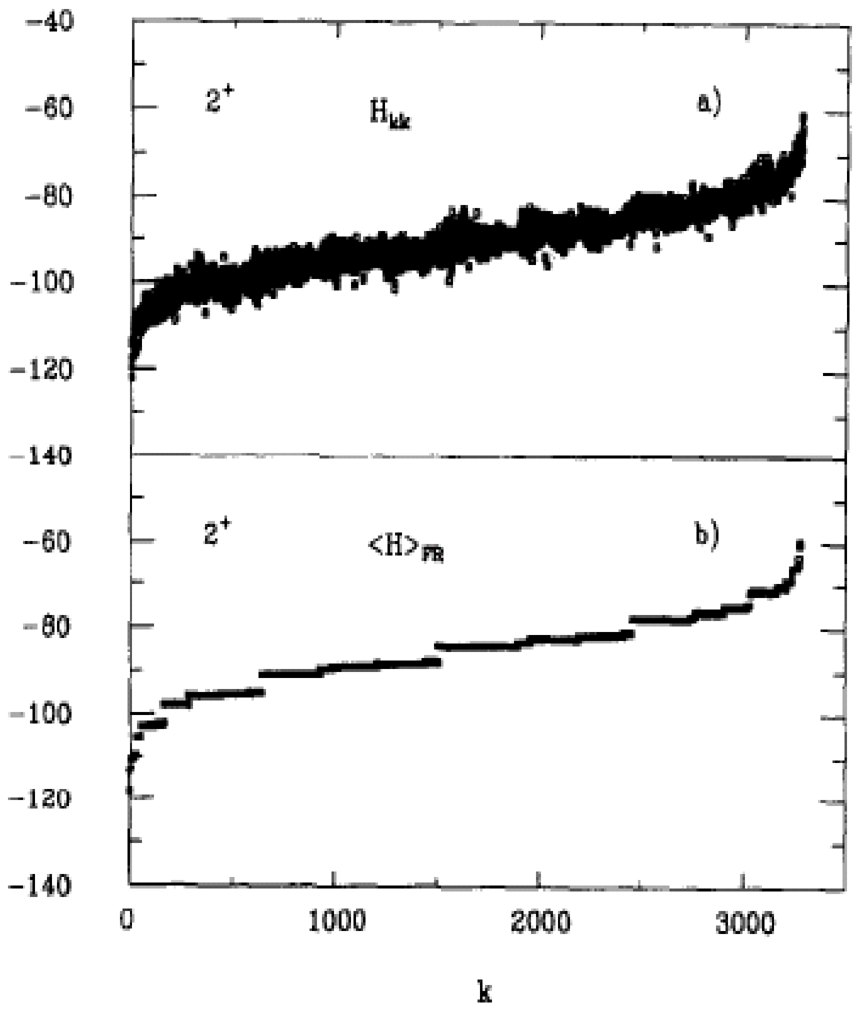
Centroids – first moment

$$\langle H^2\rangle_{\alpha\kappa}=\mathrm{Tr}^{(\alpha\kappa)}[H^2]/D_{\alpha\kappa}$$

Widths - second moment

^{28}Si

Diagonal matrix elements of the Hamiltonian in the mean-field representation

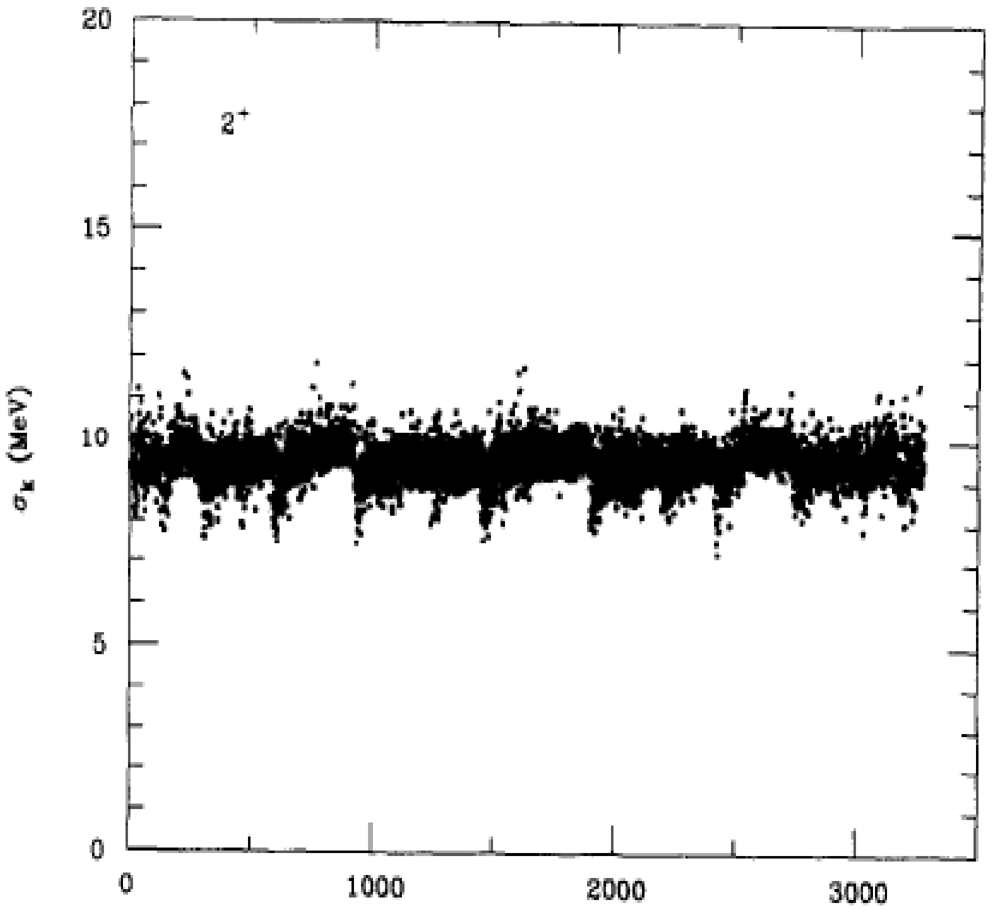


Partition structure in the shell model

(a) *All 3276 states* ; (b) *energy centroids*

28

Si



Energy dispersion for individual states is nearly constant

(result of **geometric chaoticity**!)

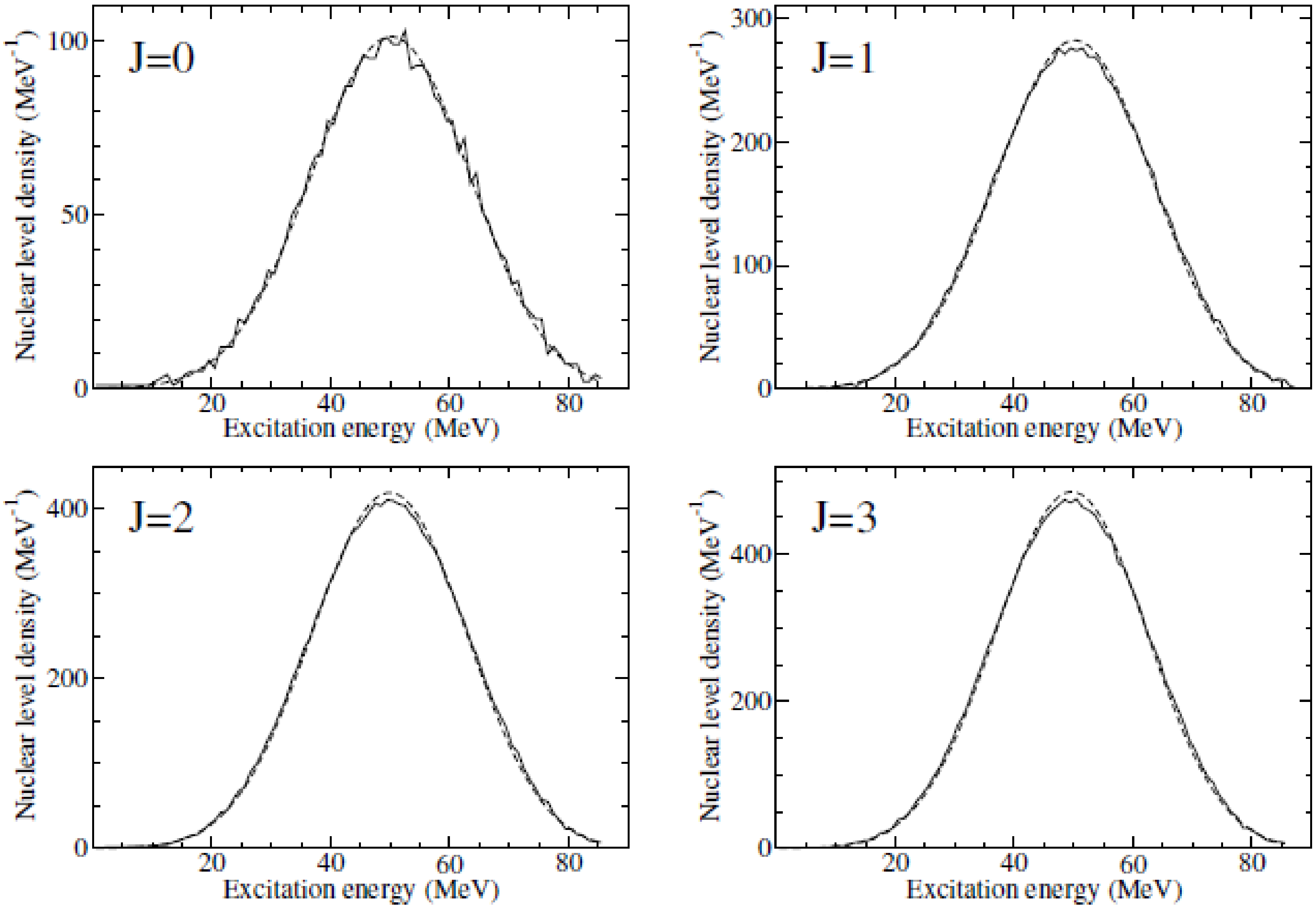
Also in multiconfigurational method (hybrid of shell model and density functional)

$$\sigma_k^2 = \langle k|(H - H_{kk})^2|k\rangle = \sum_{l \neq k} H_{kl}^2 ,$$

Widths add in quadratures

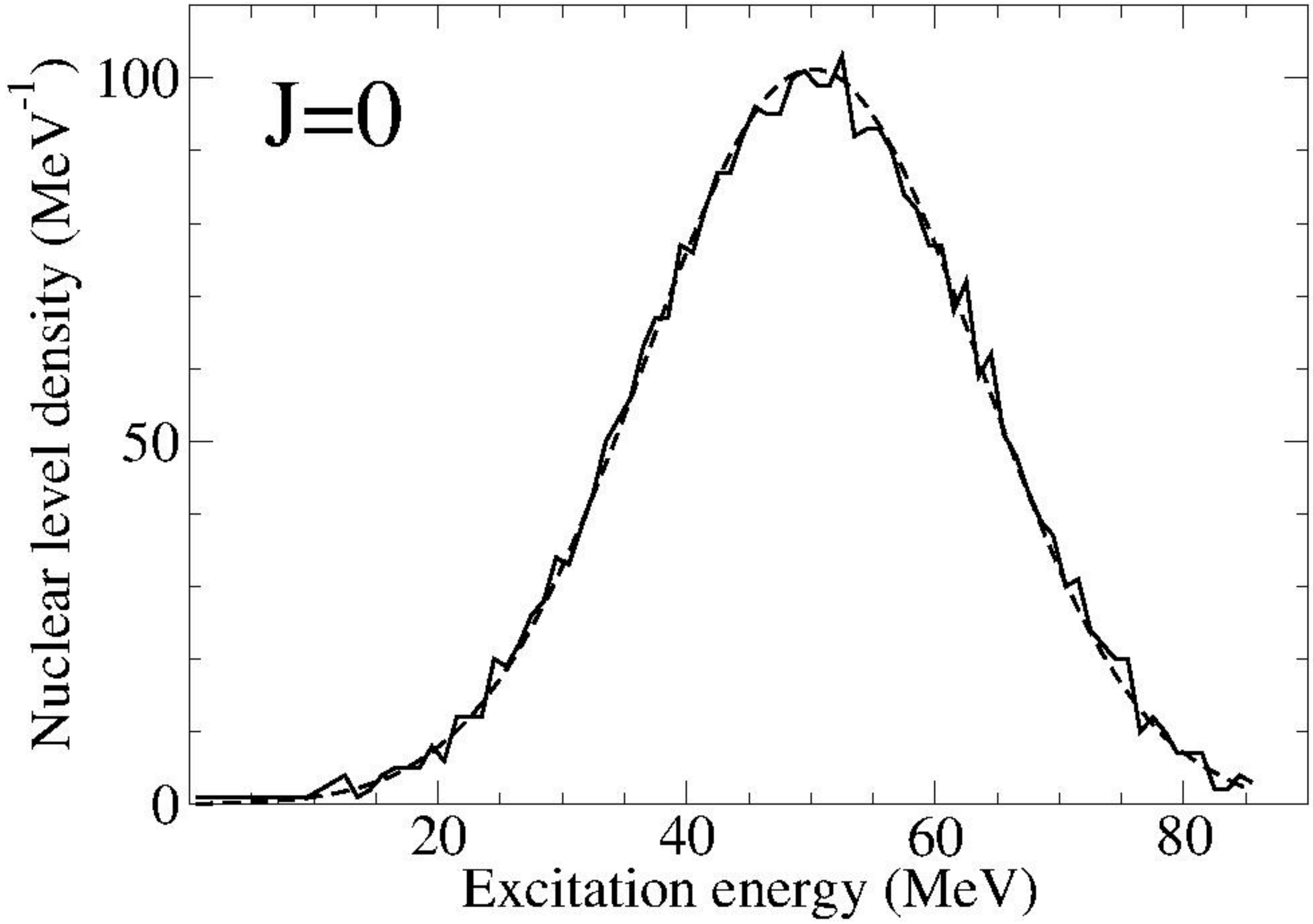
^{28}Si, parity=+1, some *J*, *sd*-shell

Shell Model (solid line) vs. Moments Method (dashed line).

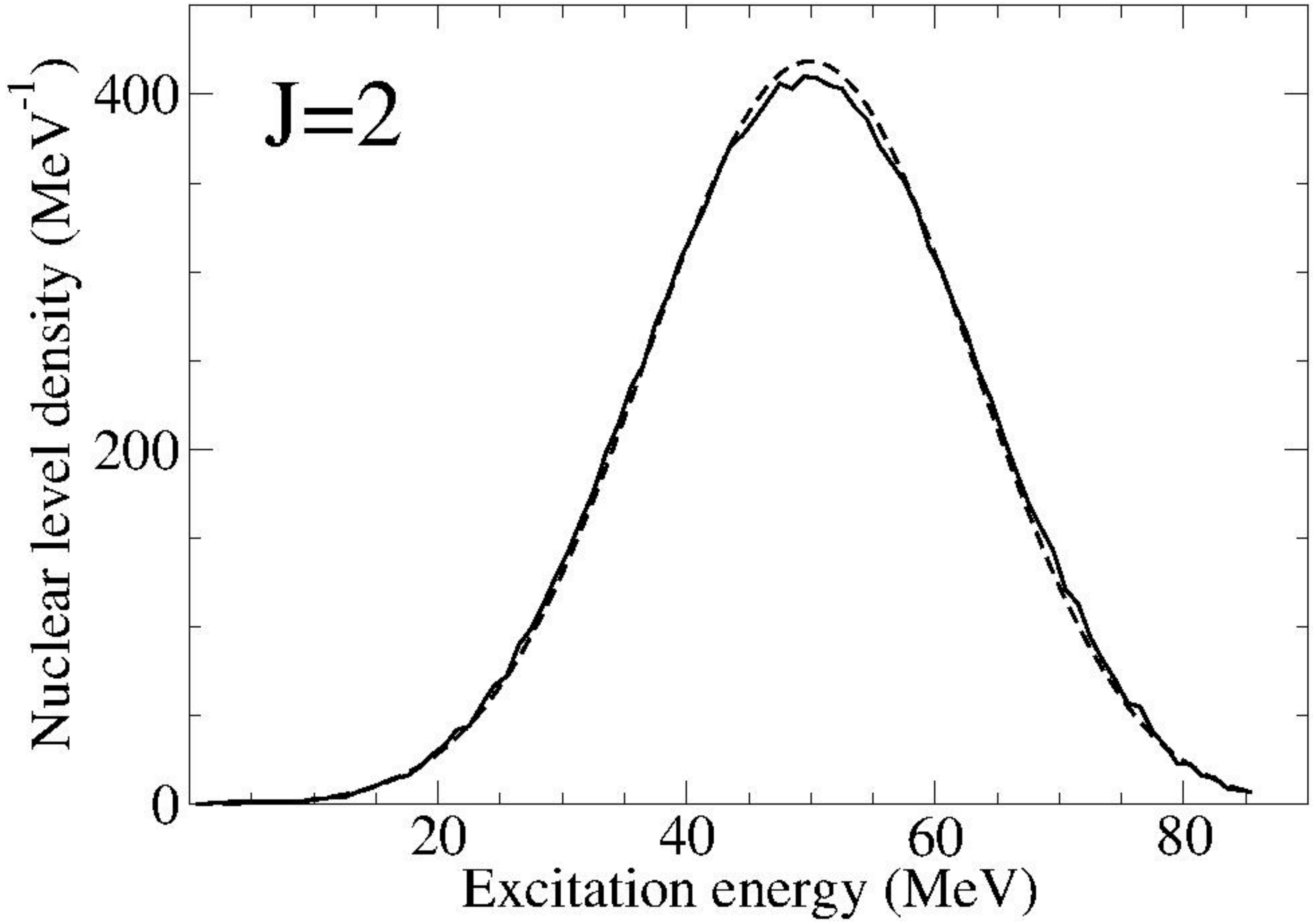


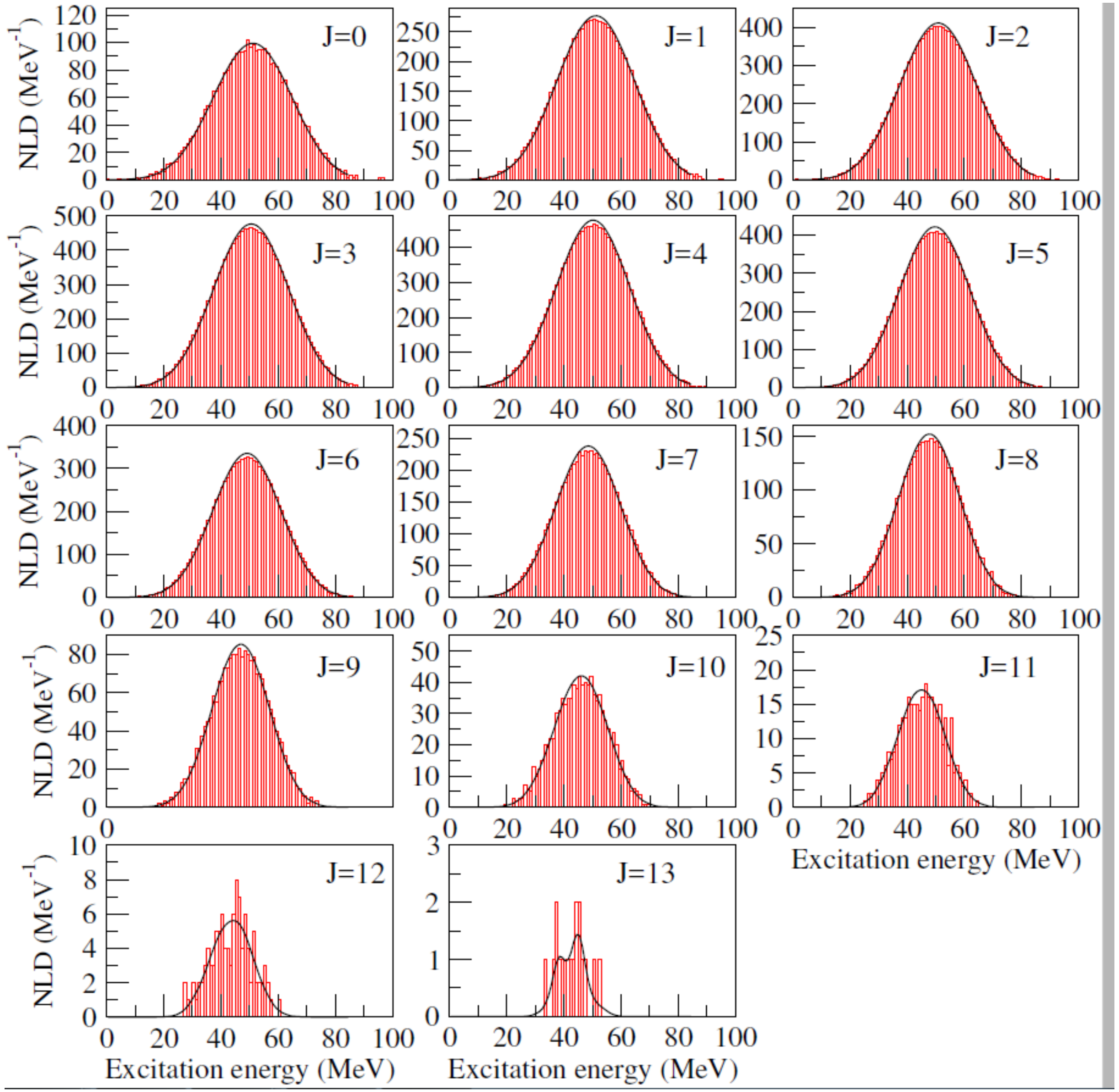
Shell-model level density.

Moments method (no diagonalization)



J=0
Nuclear level density (MeV^{-1})
100
50
0
20
40
60
80
Excitation energy (MeV)


J=2
Nuclear level density (MeV^{-1})
0
200
400
20
40
60
80
Excitation energy (MeV)

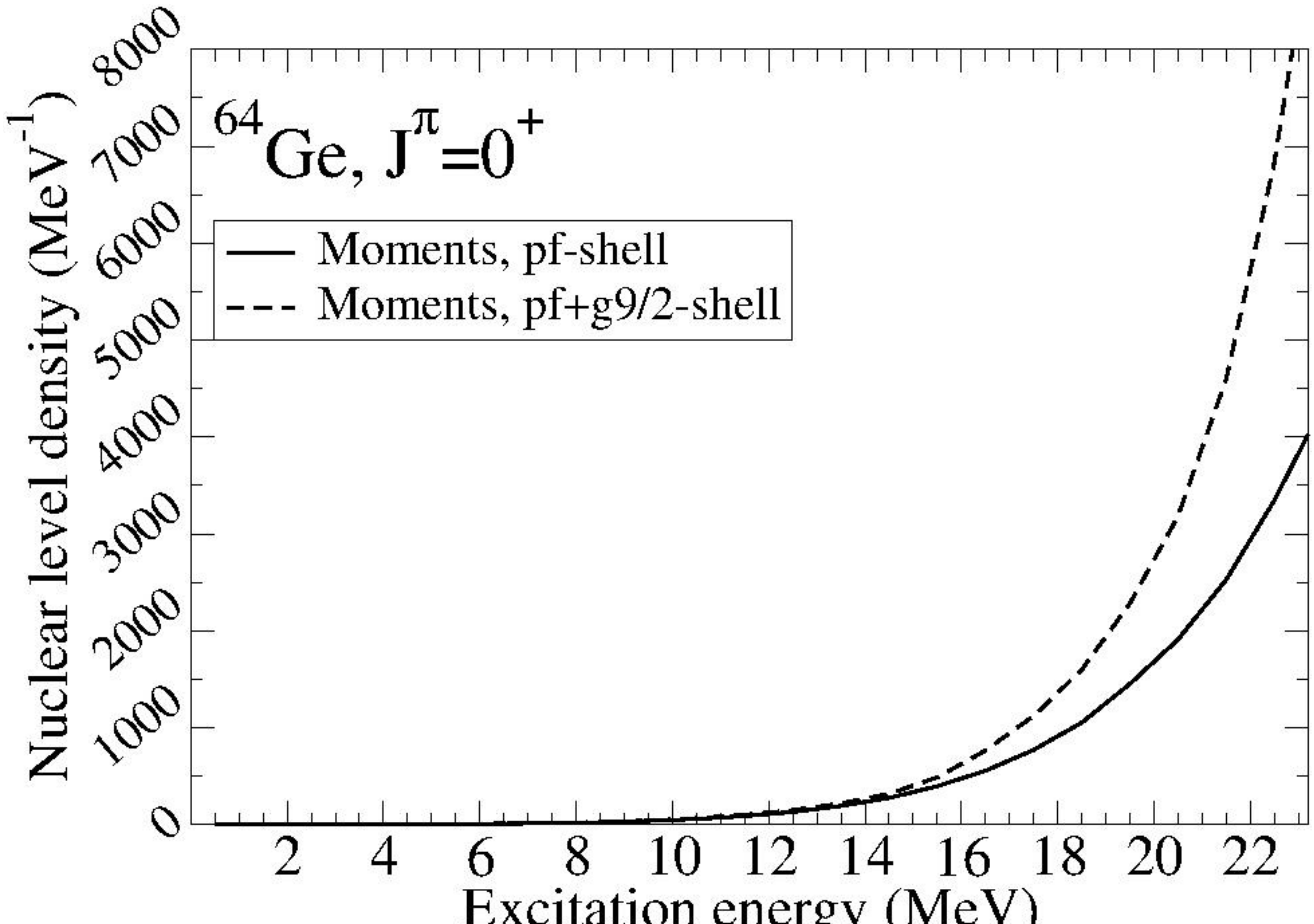


Generic shape (Gaussian)

Level density for different classes of states in 28Si

Full agreement between <u>exact shell model</u> and <u>moments method</u>

<u>Problems</u>: truncated orbital space, only positive parity in sd-model, …



R.Sen’kov, V.Z.
PRC 93 (2016)

MEAN FIELD COMBINATORICS

S. Goriely et al. Phys. Rev. C 78, 064307 (2008)
C 79, 024612 (2009)

http://www.astro.ulb.ac.be/pmwiki/Brusslin/Level

Hartree – Fock – Bogoliubov plus
Collective enhancement with certain phonons

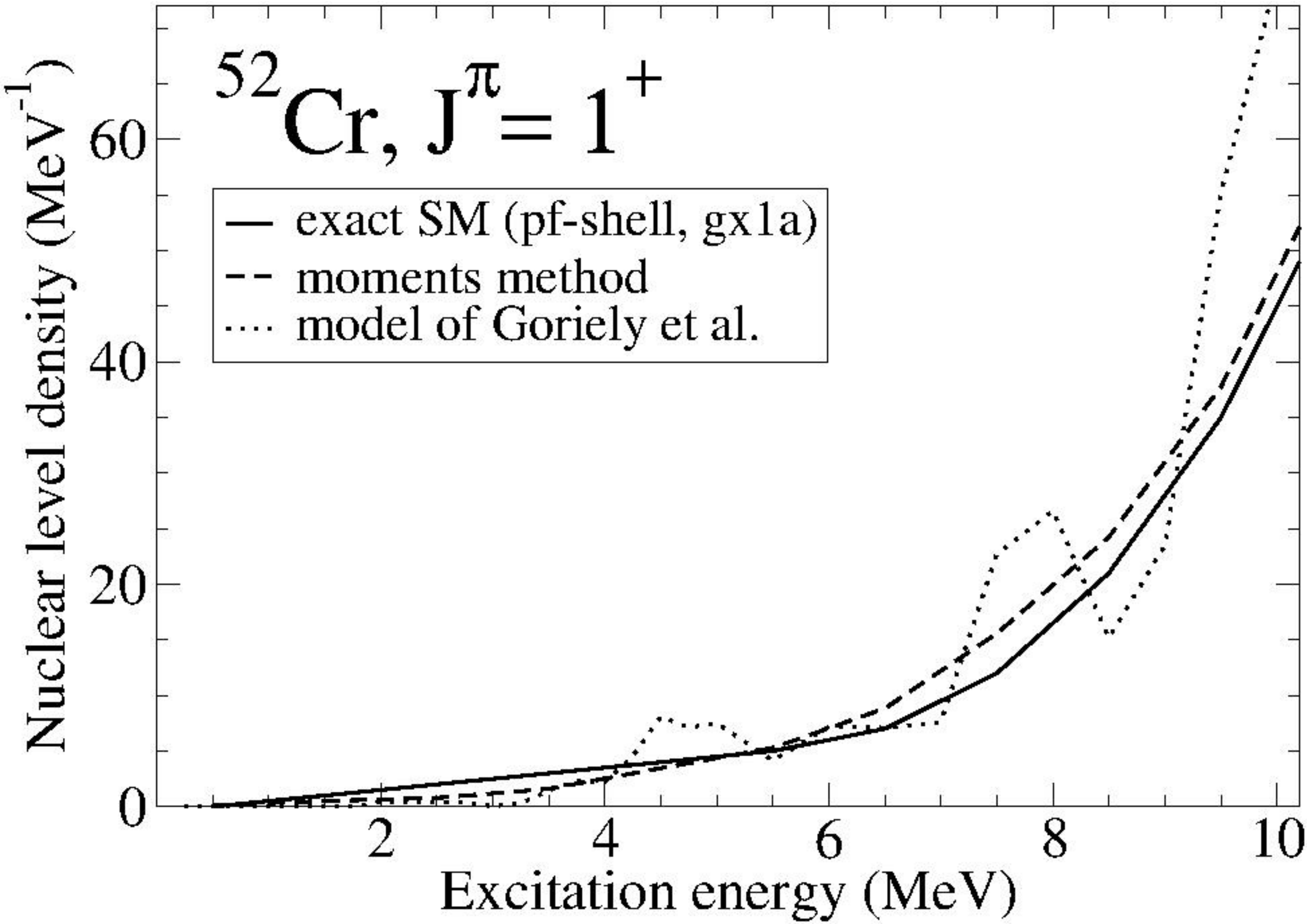
Monte Carlo Shell model – Y. Alhassid +…

S. Goriely, A.-C. Larsen, D. Muecher

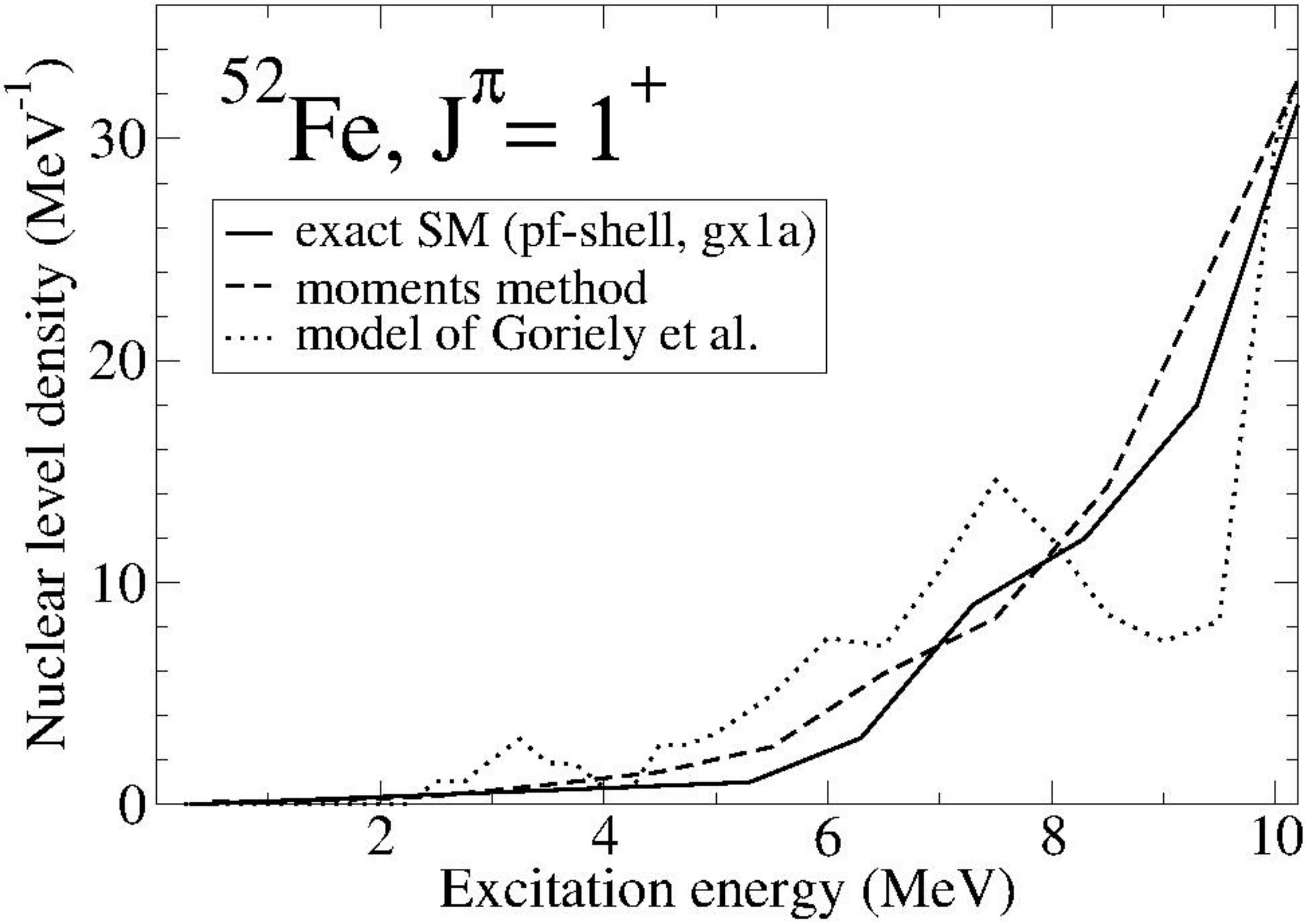
Comprehensive test of nuclear level density models

Phys. Rev. C **106**, 044315 (2022)

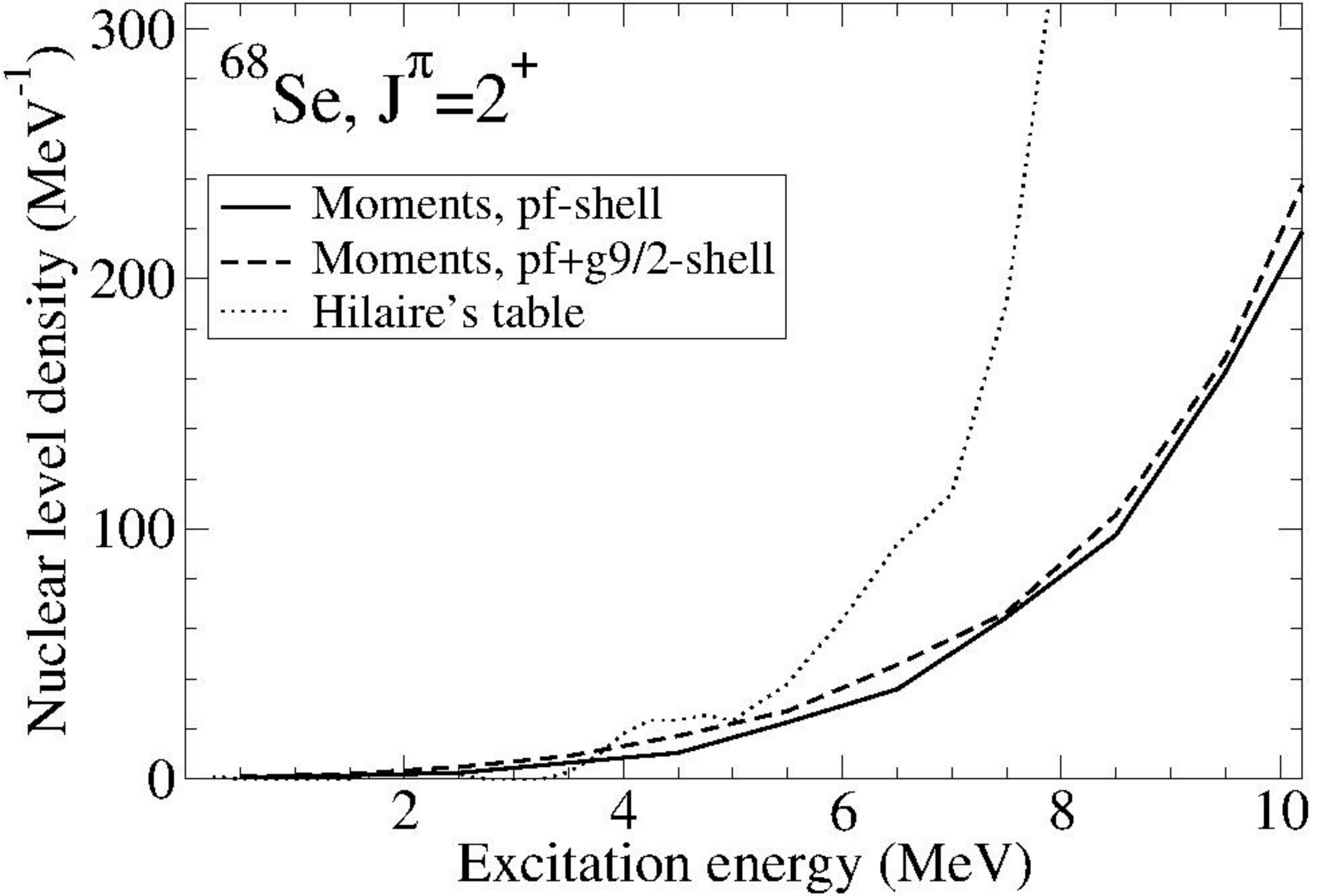
Constant temperature model



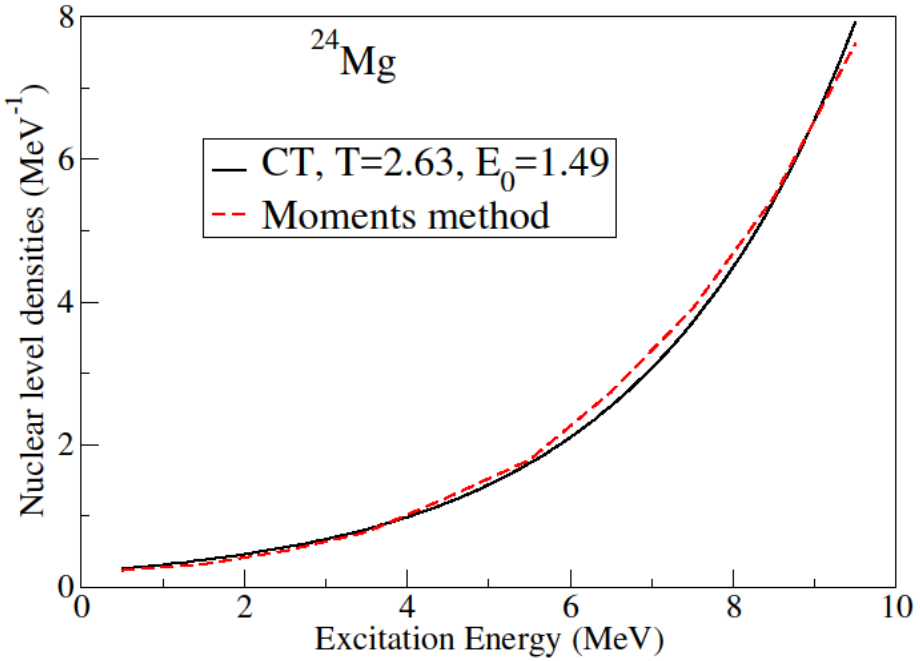
^{52}Cr, $J^{\pi} = 1^{+}$
exact SM (pf-shell, gx1a)
moments method
model of Goriely et al.
Nuclear level density (MeV^{-1})
0
20
40
60
2
4
6
8
10
Excitation energy (MeV)

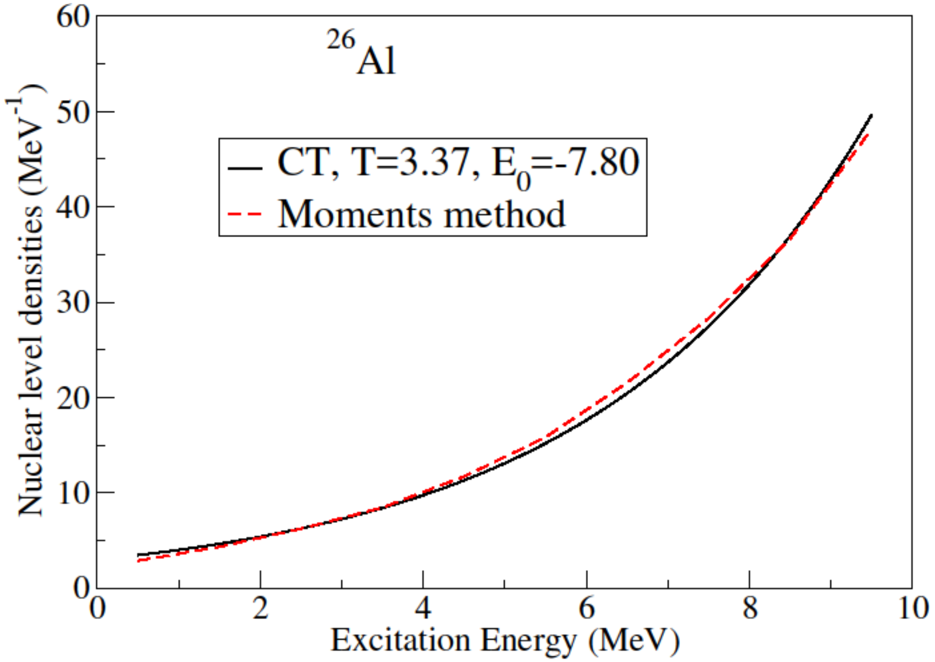


$^{52}Fe, J^{\pi} = 1^{+}$
exact SM (pf-shell, gx1a)
moments method
model of Goriely et al.
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
2
4
6
8
10



300
200
100
0
2
4
6
8
10
^{68}Se, $J^{\pi}=2^{+}$
Moments, pf-shell
Moments, pf+g9/2-shell
Hilaire's table
Nuclear level density (MeV^{-1})
Excitation energy (MeV)





CONSTANT TEMPERATURE PHENOMENOLOGY

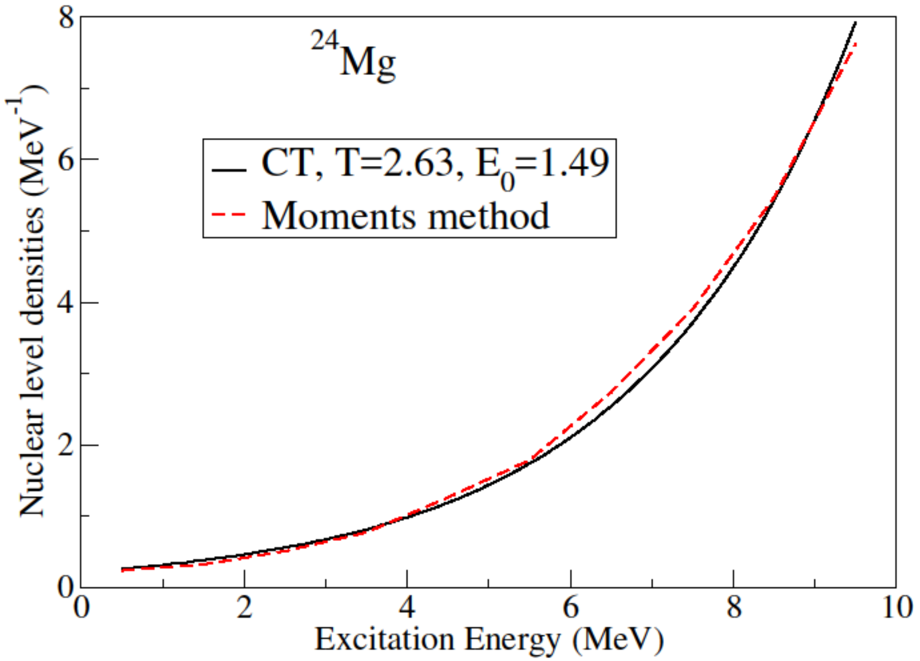
LEVEL DENSITY (E) = (const) exp (E/T)

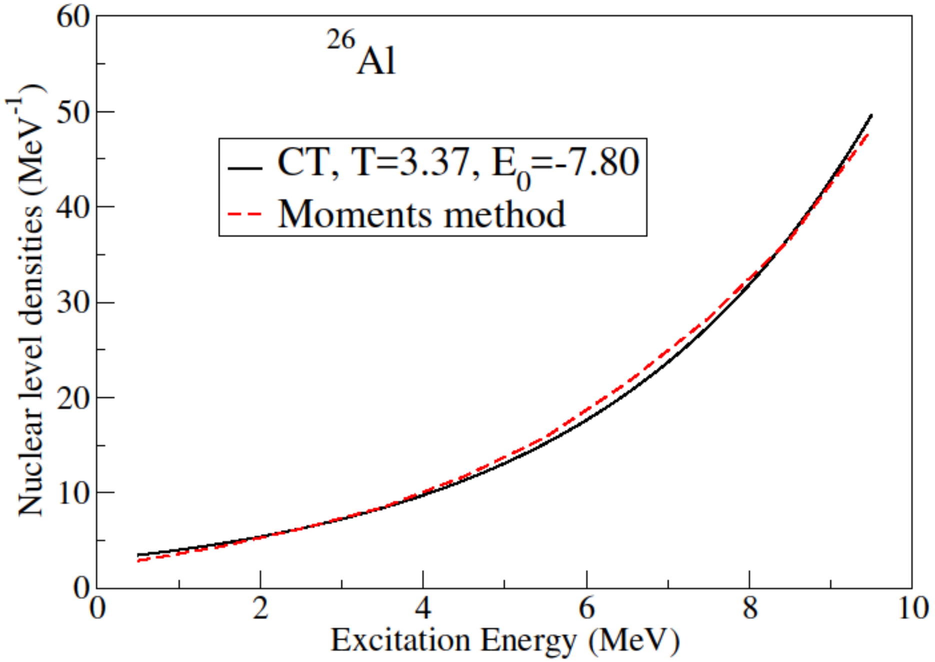
Ericson (1962), Gilbert and Cameron (1965)

Moretto (1975) – pairing phase transition

T – "effective constant temperature"

1/T – rate of increase of level density





CONSTANT TEMPERATURE PHENOMENOLOGY

Level density (const) exp(E/T)

$$T_{\text{t-d}} = \left(\frac{\partial S}{\partial E}\right)^{-1} = T\left(1 - e^{-E/T}\right)$$

Partition function = **Trace{exp[-H/T(t-d)]}** **diverges** at **T > T(t-d)**

Cumulative level number

$N(E) = \exp(S)$,

Entropy **$S(E) = \ln(N)$**

Thermodynamic temperature

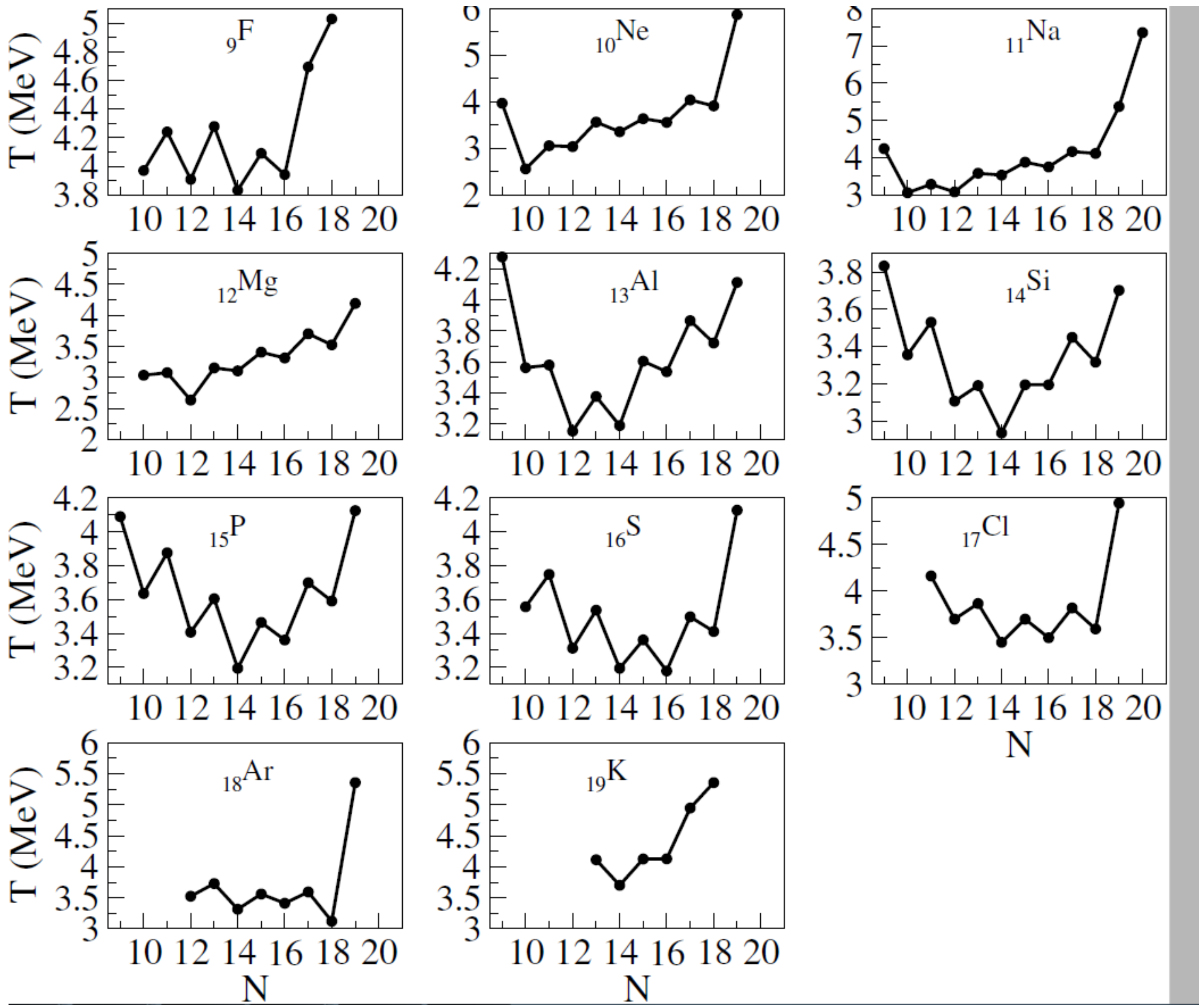
$T(t\text{-}d) = dS/dE = T[1 - \exp(-E/T)]$

Parameter T is ***limiting temperature***

(***Hagedorn temperature*** in particle physics)

Pairing phase transition? (Moretto) - Chaotization

1/T – rate of increase of the level density



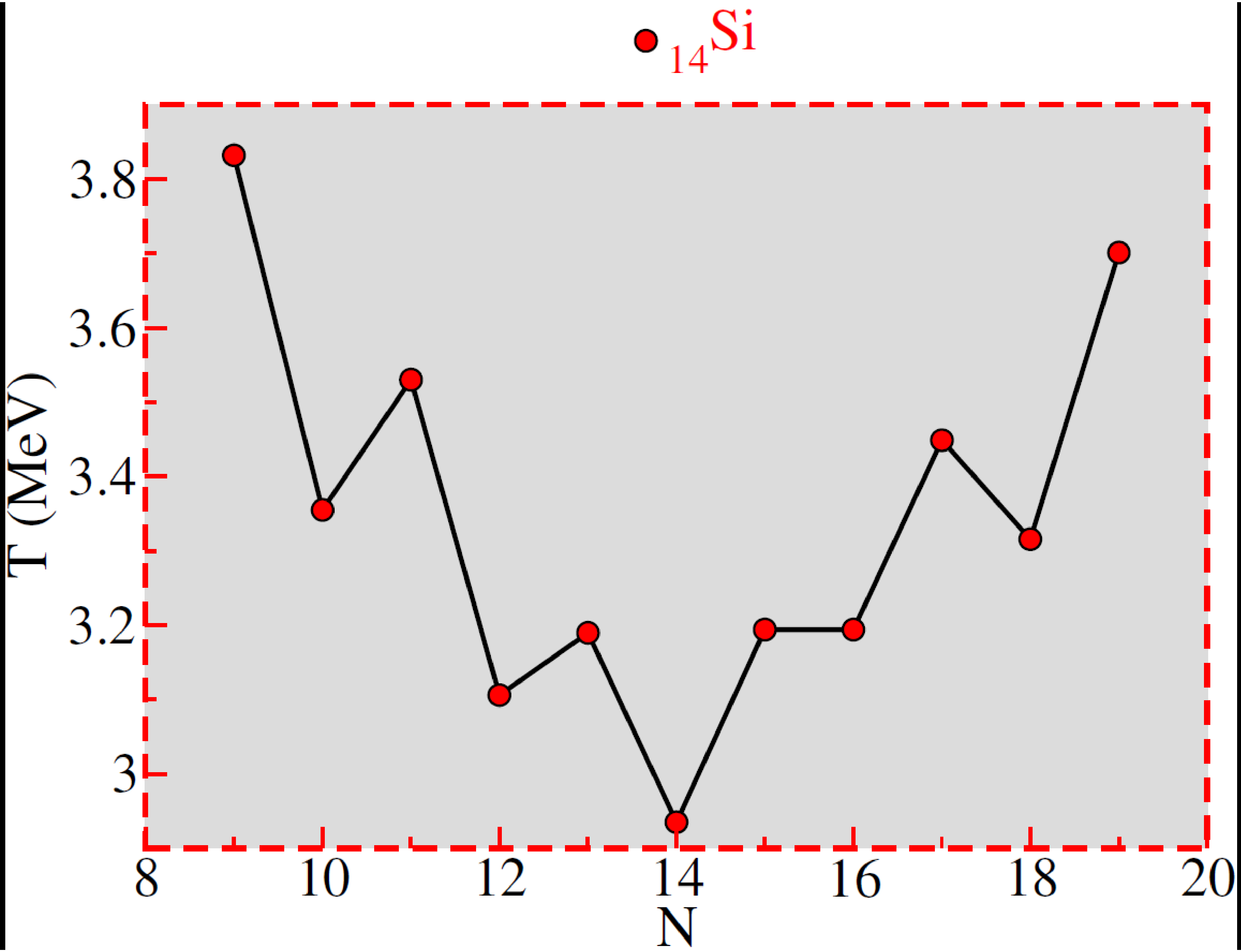
Effective temperature **T**

for **(sd)** – nuclei,

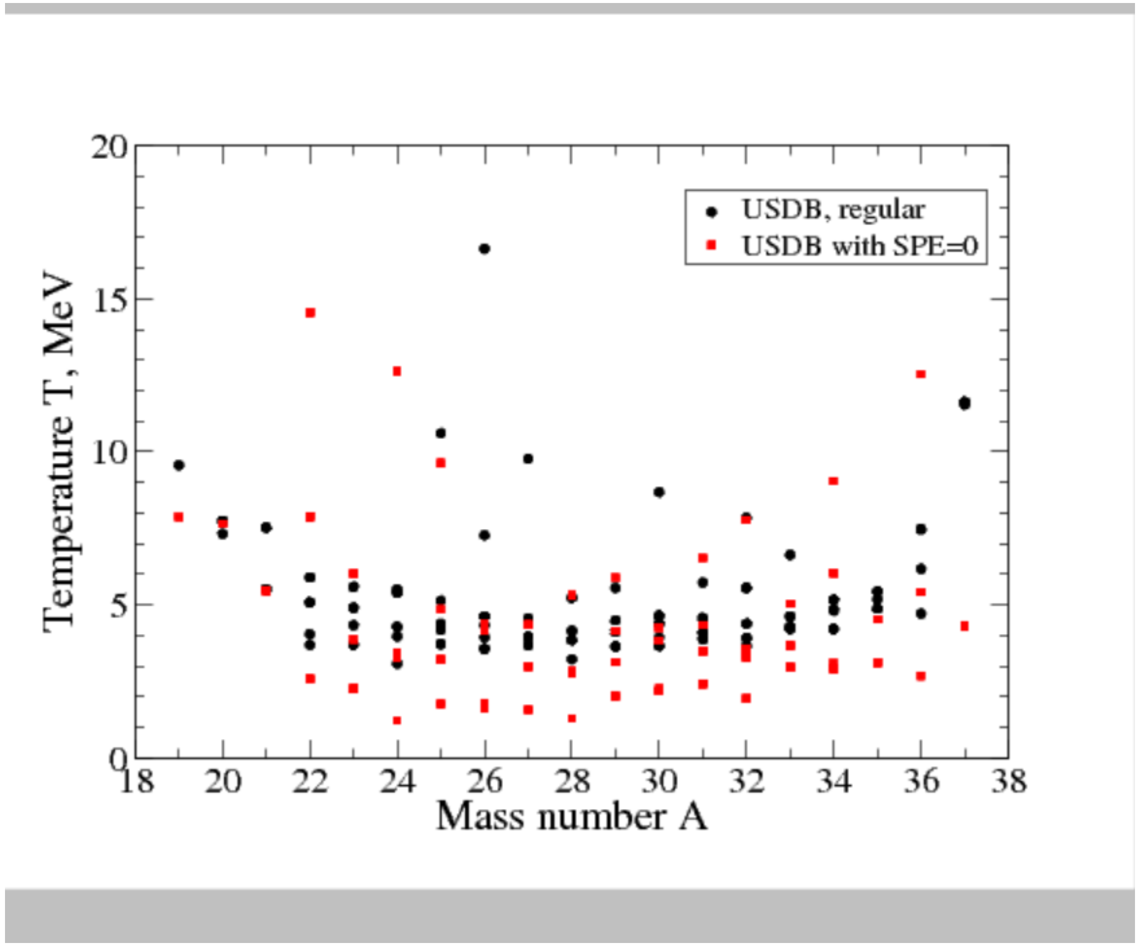
tabulated for all

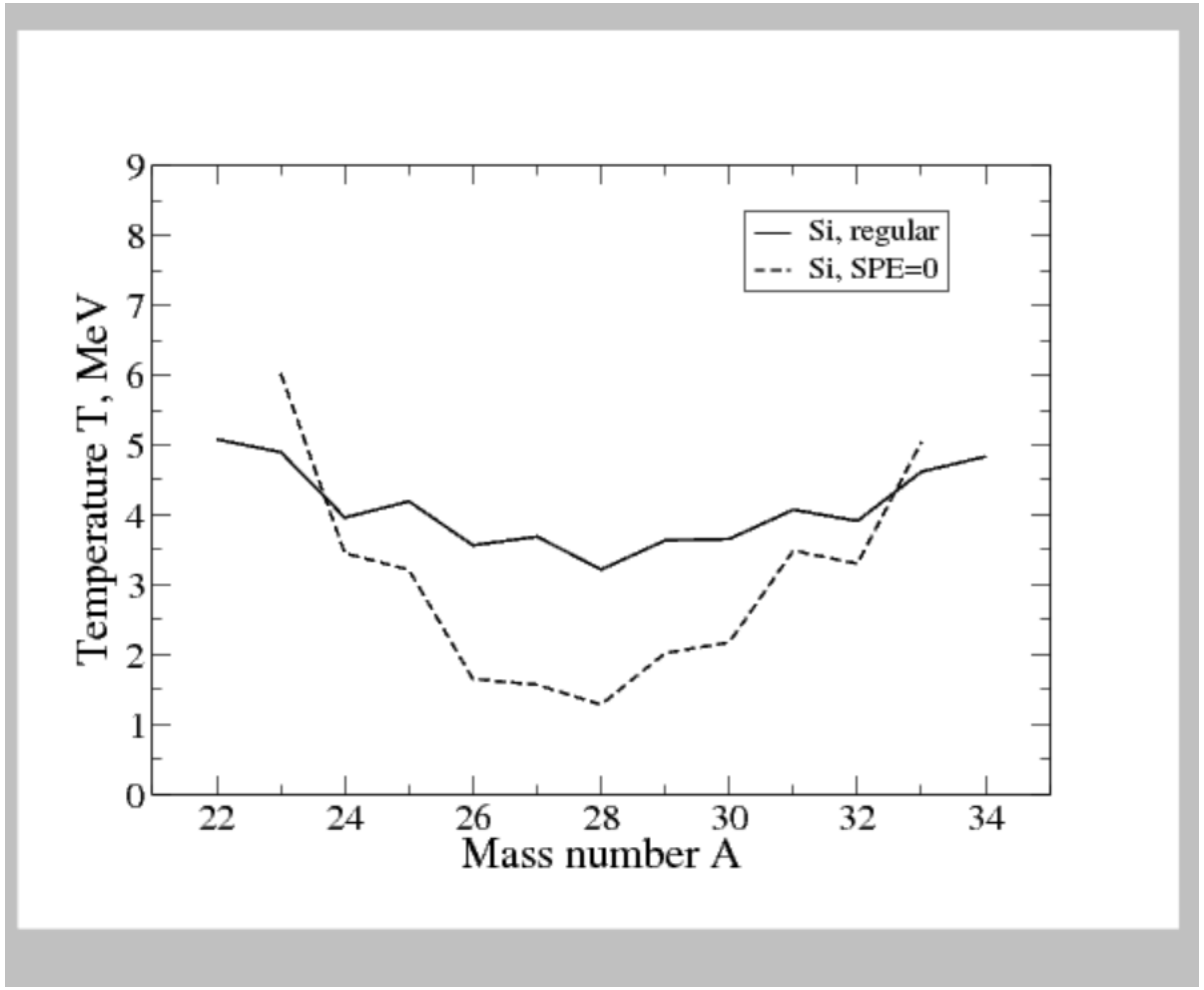
classes of spin

(ADNDT, 2018)

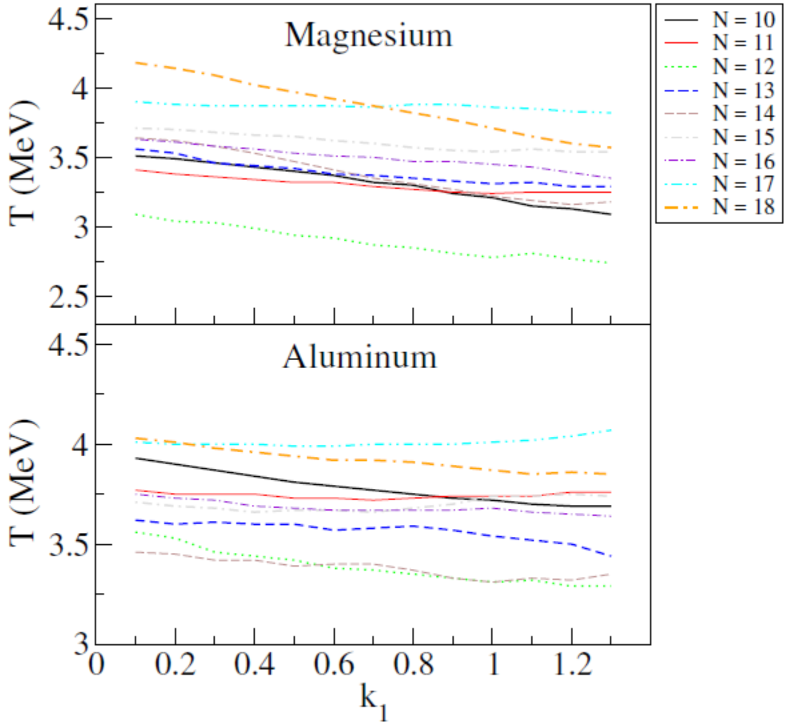


$_{14}$Si
T (MeV)
3.8
3.6
3.4
3.2
3
8
10
12
14
16
18
20
N

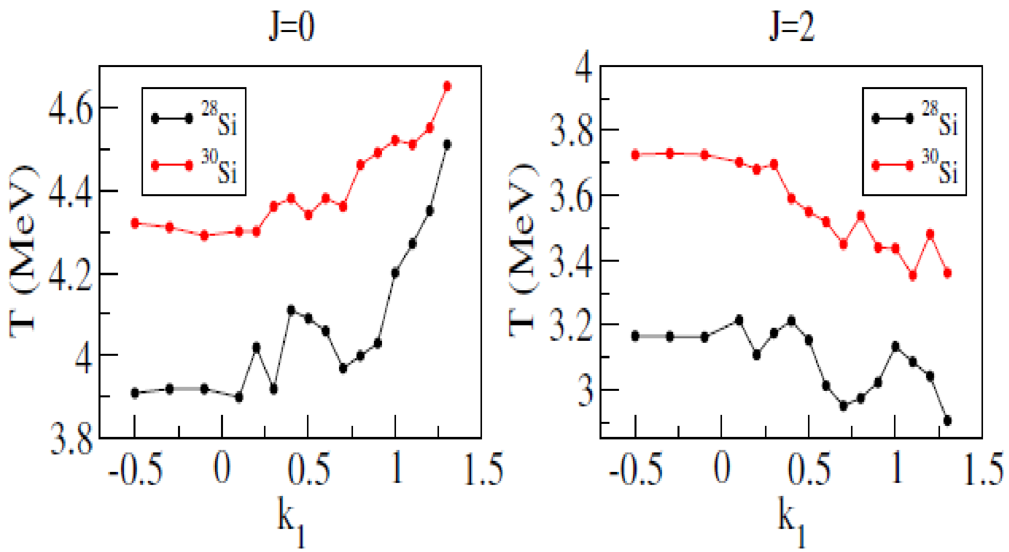




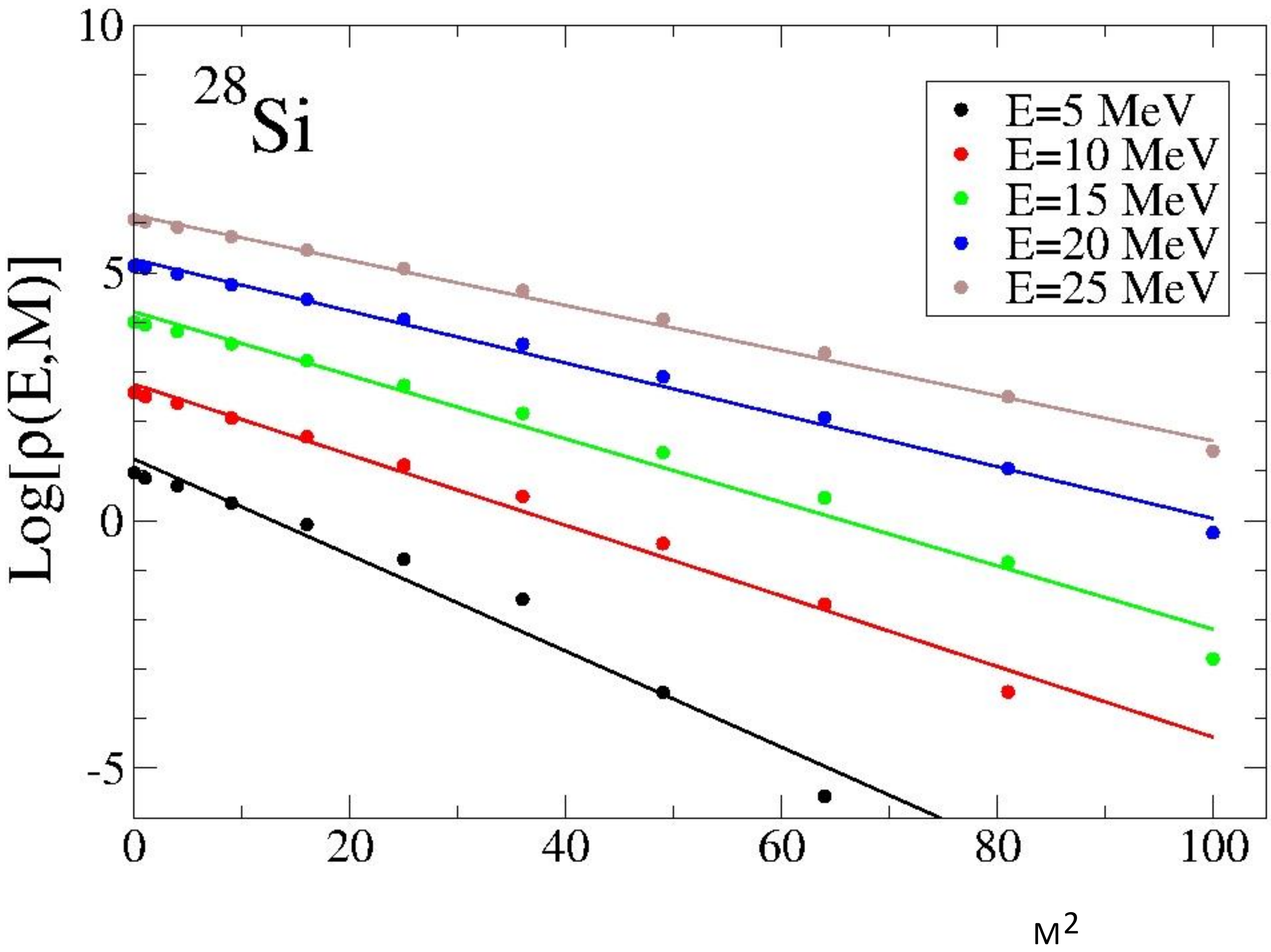
Degenerate single-particle levels – smaller T (faster chaotization)



Eliminating pairing interaction



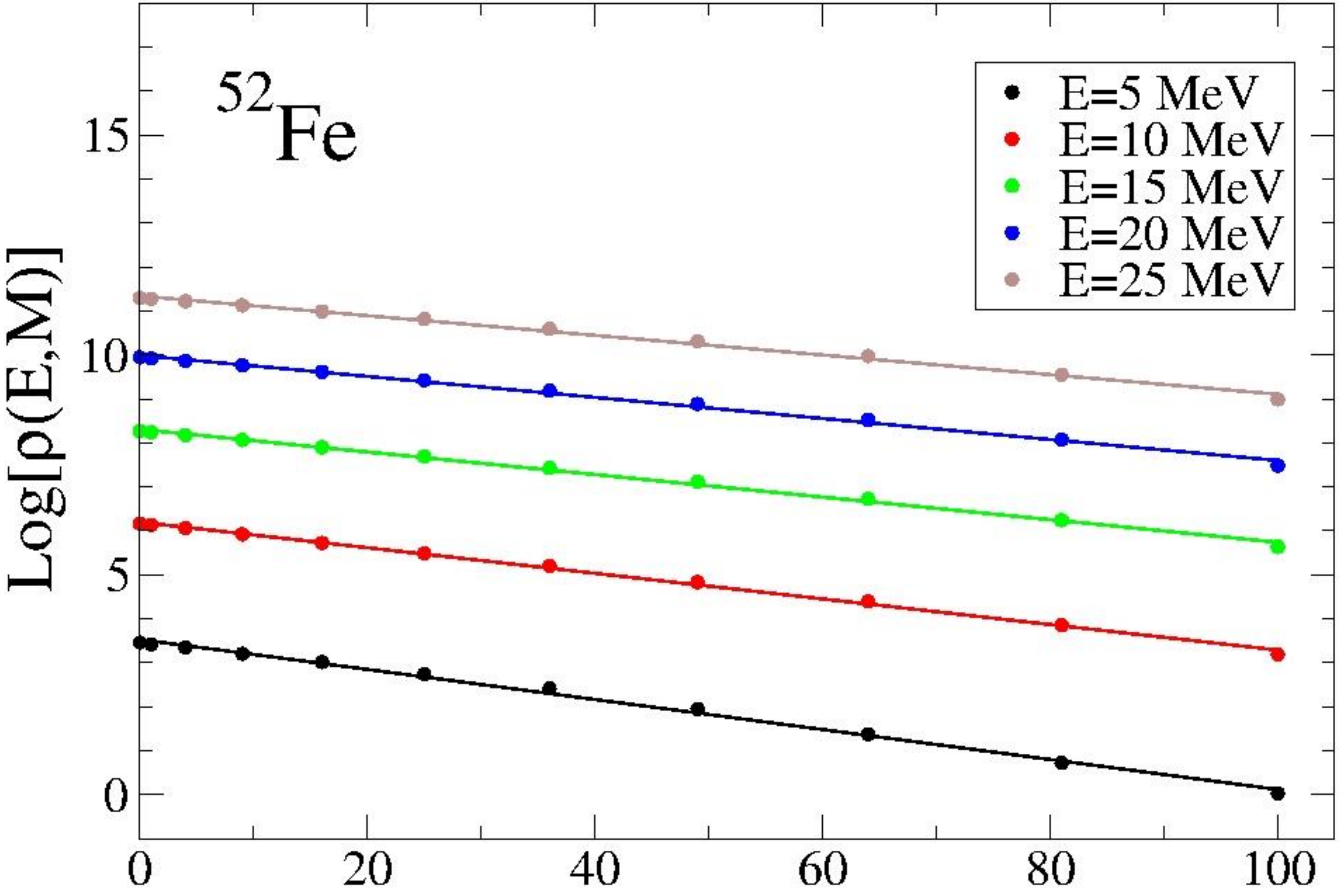
k(1) < 0 “antipairing”



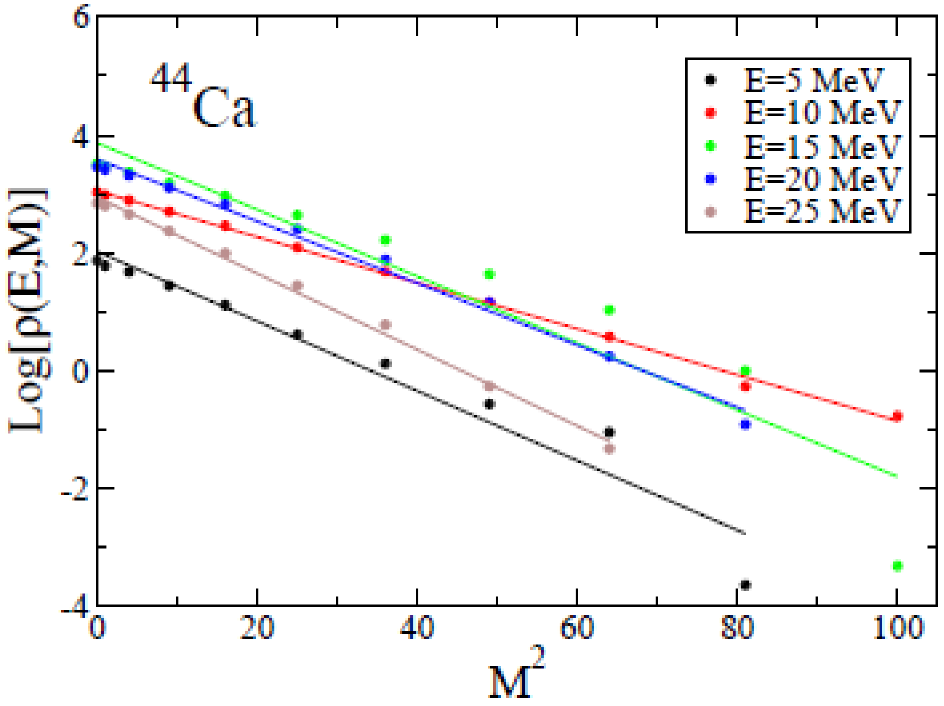
"Spin cut-off" parameter

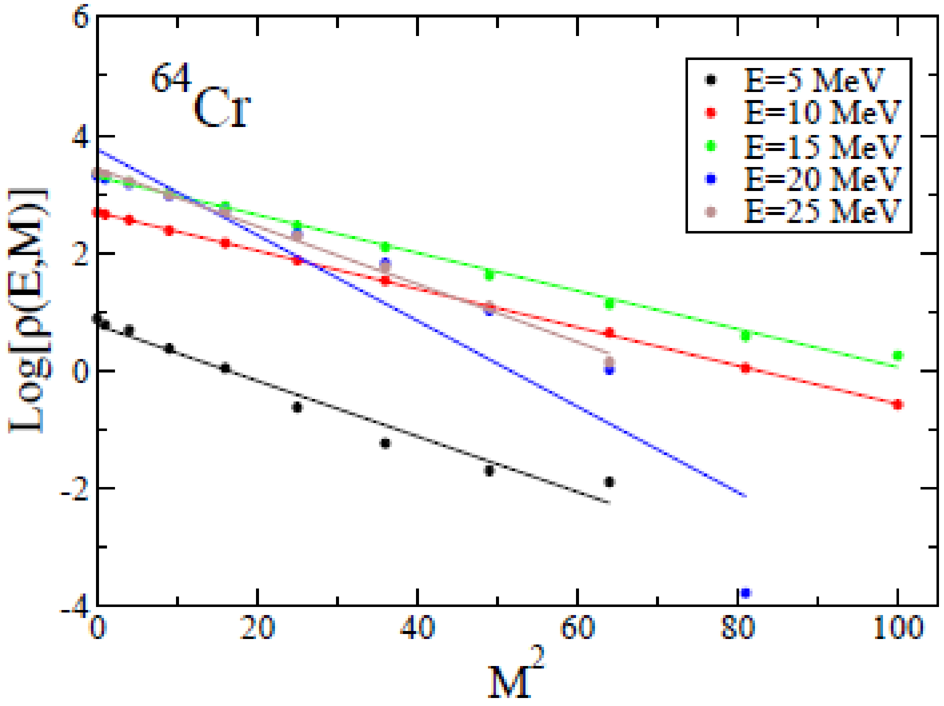
$$\frac{\rho(E,M)}{\rho(E)} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-M^2/2\sigma^2}$$

Markovian
random process
of angular momentum
coupling



$$\frac{\rho(E,M)}{\rho(E)} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-M^2/2\sigma^2}$$





4 valence neutrons

4 proton holes

Space – only T=2,
Two-body interaction through T=1 channel

$$\rho^{(0)}(E,J,0) = \rho(E,J,0)$$

$N\hbar\omega$ classification

Pure **Total** **(N=0)**

$$\rho^{(0)}(E,J,1) = \rho(E,J,1) - \sum_{J'=|J-1|}^{J+1} \rho(E,J',0)$$

(N=1)

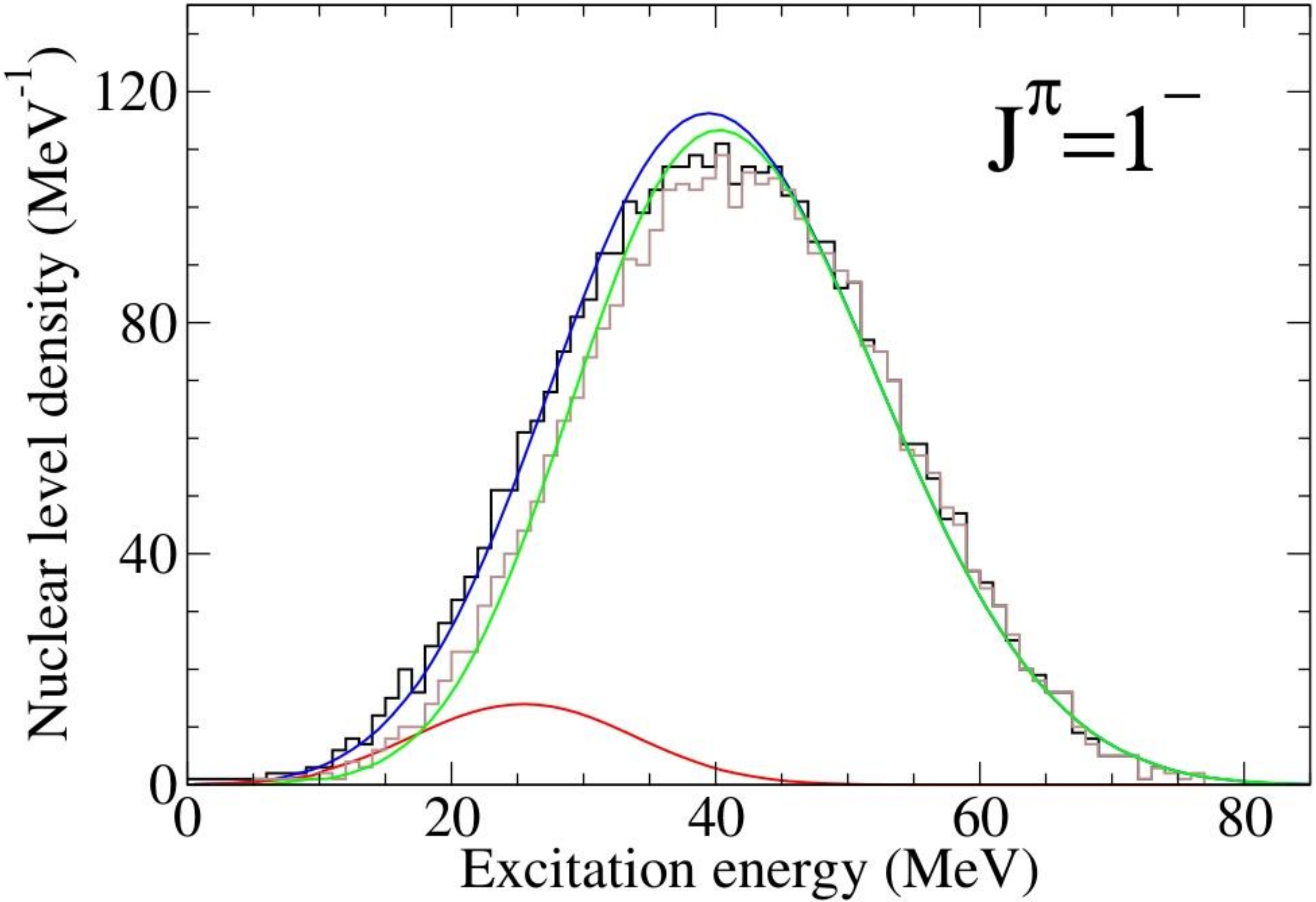
$$\rho^{(0)}(E,J,N) = \rho(E,J,N) -$$

$$-\sum_{K=1}^{N} \sum_{J_K=J_{\min}}^{N,\text{step }2} \sum_{J'=|J-J_K|}^{J+J_K} \rho^{(0)}(E,J',(N-K))$$

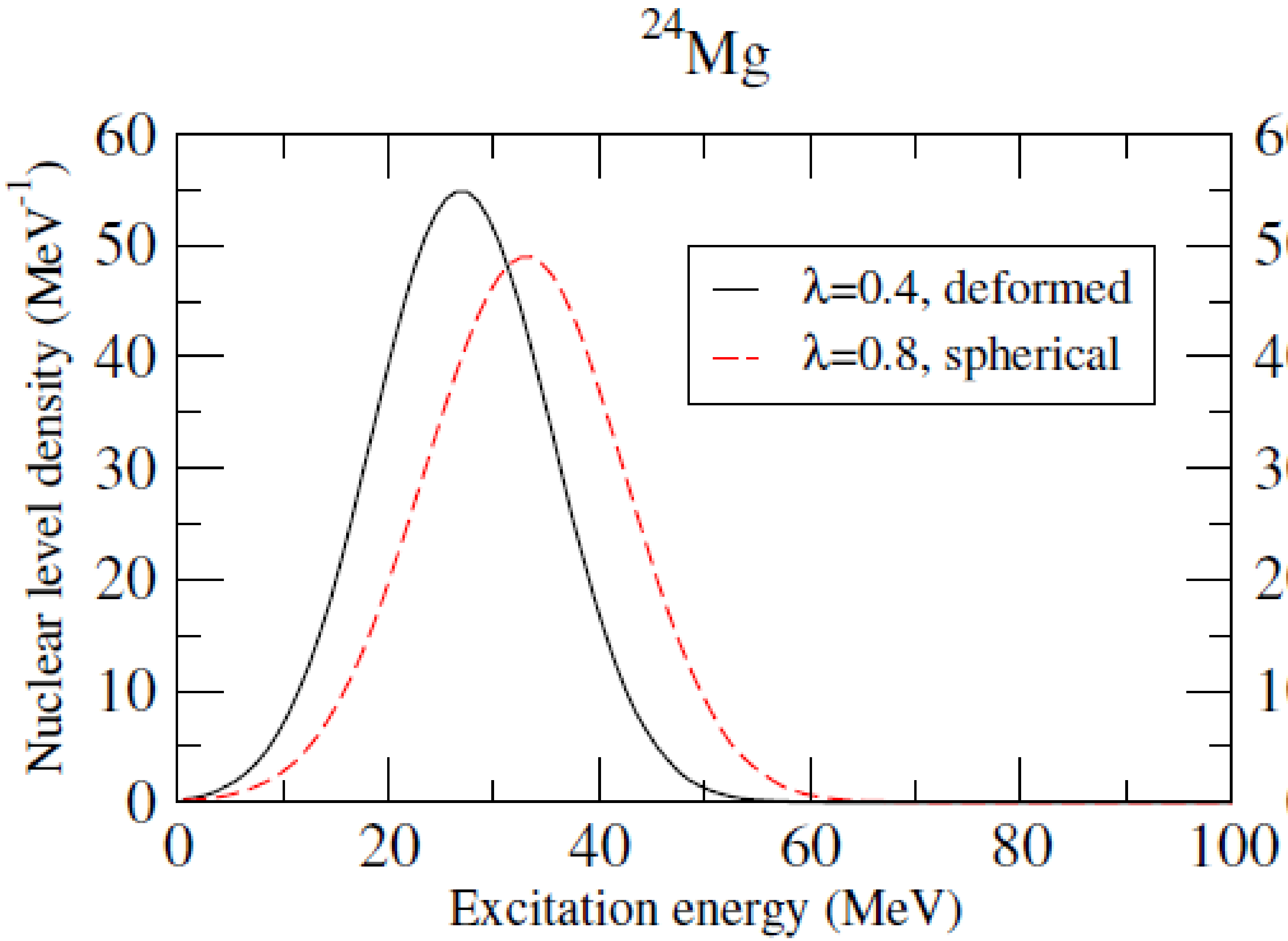
Recursive relation

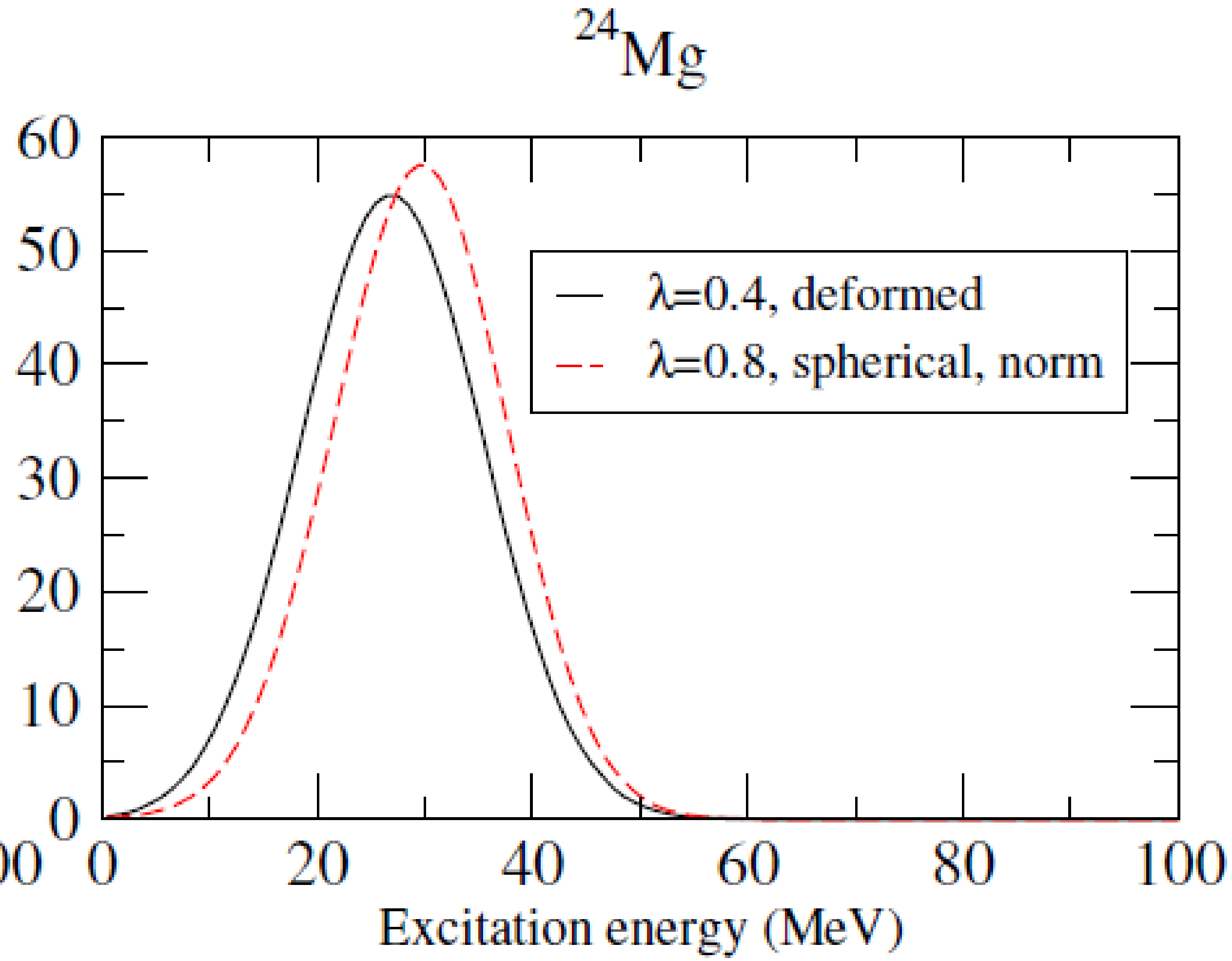
Exclusion of c.m. states

^{20}Ne (1ħω)



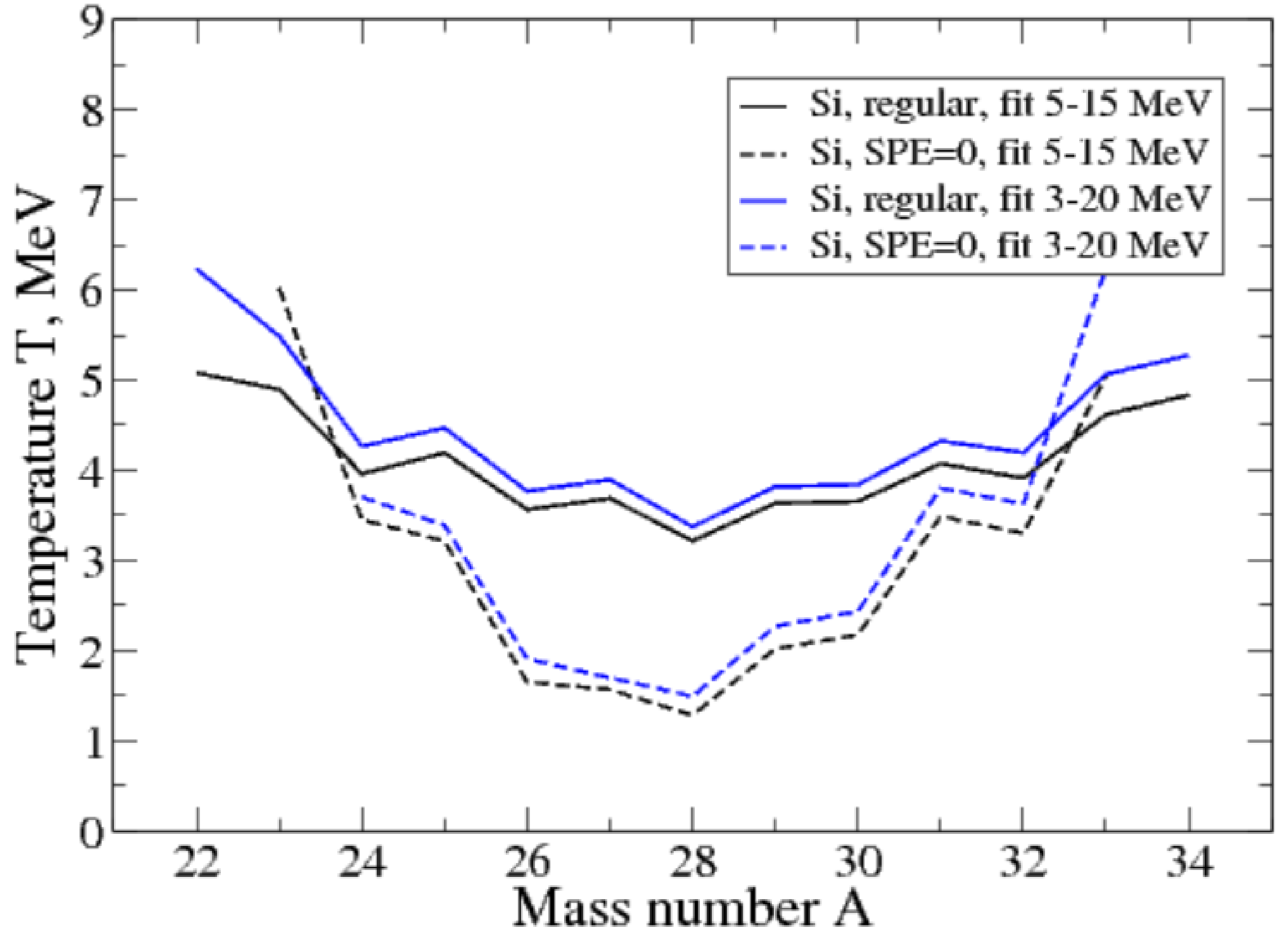
$J^{\pi}=1^{-}$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
40
80
120
0
20
40
60
80





Level density (0+) on two sides of deformation shape transition

/"collective enhancement"/



Sensitivity to the fit interval

What next?

* Tables for **pf-shell** – and further?
* Comparison of phenomenological descriptions with “Constant temperature” model
* New methods - Lanczos algorithm
  - hybrid methods
  - **random interactions**
* **Mesoscopic applications (disordered solids)**
* **Can we analytically derive CTM?**
* Computational progress
* **Continuum effects, width distribution, overlapping resonances**
* Application to reactions
*
*

V. Z., B.A. Brown, N. Frazier and M. Horoi.
The nuclear shell model as a testing ground for many-body quantum chaos.
Phys. Reports 276 (1996) 315.

V. Z.. Quantum chaos and complexity in nuclei.
Annu. Rev. Nucl. Part. Sci. 46 (1996) 237.

A.Volya and V. Z.
Invariant correlational entropy as a signature of quantum phase transitions in nuclei.
Phys. Lett. B 574 (2003) 27.

V. Z. and A. Volya.
Nuclear structure, random interactions and mesoscopic physics.
Phys. Rep. 391 (2004) 311.

F. Borgonovi, F.M. Izrailev, L.F. Santos, and V.Z.
Quantum chaos and thermalization in isolated systems of interacting particles.
Physics Reports 626 (2016) 1.

V.Z. and A. Volya.
Chaotic features of nuclear structure and dynamics: Selected topics.
Physica Scripta 91 (2016) 033006.

V.Z. and M. Horoi. *Nuclear level density, thermalization, chaos and collectivity.*
Progress in Particle and Nuclear Physics, **105**, 180 (2019).

M. Horoi, J. Kaiser, and V. Z. *Spin- and parity-dependent nuclear level densities and the exponential convergence method.* Phys. Rev. C **67** (2003) 054309.
M. Horoi, M. Ghita, and V. Z. *Fixed spin and parity nuclear level density for restricted shell model configurations*. Phys. Rev. C **69** (2004) 041307(R).
M. Horoi and V. Z. *Exact removal of the center-of-mass spurious states from level densities*. Phys. Rev. Lett. **98** (2007) 262503.

R.A. Sen'kov, M. Horoi, and V.Z. *High-performance algorithm for calculating non-spurious spin- and parity-dependent nuclear level densities.* Phys. Lett. B **702** (2011) 413.

R.A. Sen'kov, M. Horoi, and V.Z. *A high-performance Fortran code to calculate spin- and parity-dependent nuclear level densities*. Computer Physics Communications **184** (2013) 215.
R.A. Sen'kov and V. Z. *Nuclear level density: Shell-model approach.* Phys. Rev. C **93** (2016) 064304.

S. Karampagia and V. Z. *Nuclear shape transitions, level density, and underlying interactions.* Phys. Rev. C **94** (2016) 014321.

S. Karampagia, A. Renzaglia, and V.Z. *Quantum phase transitions and collective enhancement of level density in odd-A and odd-odd nuclei*. Nucl. Phys. A962 (2017) 46.

S. Karampagia, R.A. Sen'kov, and V.Z. *Level density in the sd-nuclei - statistical shell model predictions.* ADNDT, 120, 1-120 (2018).

V.Z. and S. Karampagia. *Nuclear level density and related physics*. EPJ Web of Conferences, 194, 01001 (2028).

V. Z., S. Karampagia, and A. Berlaga. *Constant temperature model for nuclear level density*. Phys. Lett. B 783 (2018) 428 .

S. Karampagia and V.Z. *Nuclear shell model and level density.* Int. J. Mod. Phys. E 29 (2020) 2030005.

V. Z. and S. Karampagia. *Physics of thermalization and level density in an isolated system of strongly interacting particles*. Eur. Phys. J. Spec. Top. 230 (2021) 755.

MANY-BODY QUANTUM CHAOS AS AN INSTRUMENT

SPECTRAL STATISTICS – *signature of chaos*

- *missing levels*
- *purity of quantum numbers*
- ***level density without full diagonalization***
- *presence of time-reversal invariance*

EXPERIMENTAL TOOL – *unresolved fine structure*

- *width distribution (more work required)*
- *damping of collective modes*

NEW PHYSICS

- *statistical enhancement of weak perturbations (parity violation in neutron scattering and fission)*
- *mass fluctuations*
- *chaos on the border with continuum*

THEORETICAL CHALLENGES

- order out of chaos
- **chaos and thermalization**
- development of computational tools
- new approximations in many-body problem

INSIDE CHAOS

I. Percival, J. Phys. **B6** (1973) L229

- *DISORDERED* wave functions
- Any *SIMPLE* operator has matrix elements of the same order of magnitude between any two of these eigenfunctions
- All typical wave functions of roughly the same energy *LOOK ROUGHLY THE SAME* being spread over the large region of configuration space

Random matrix canonical ensembles – only as mathematical limit

Chaotic motion in mesoscopic systems

* Mean field (**one-body** chaos) - classical features
* Strong interaction (**many-body** chaos)
* High level density
* Mixing of simple configurations
* Destruction of quantum numbers,
  (in nuclei: conserved only energy; J,M; T,T3; parity)
* Local spectral statistics – **Gaussian Orthogonal Ensemble**
* Correlations between classes of states
* Coexistence with (damped) collective motion
* Thermal equilibrium – **without heat bath**
* Continuum effects – open system

CLOSED MESOSCOPIC SYSTEM

at high level density

Two languages: *individual stationary wave functions*
thermal excitation

* Mutually exclusive ?
* Complementary ?
* Equivalent ?

Answer depends on thermometer

CHAOS versus THERMALIZATION

L. BOLTZMANN – *Stosszahlansatz* = MOLECULAR CHAOS

N. BOHR - *Compound nucleus* = MANY-BODY CHAOS

N. S. KRYLOV - *Foundations of statistical mechanics*

L. Van HOVE – *Quantum ergodicity*

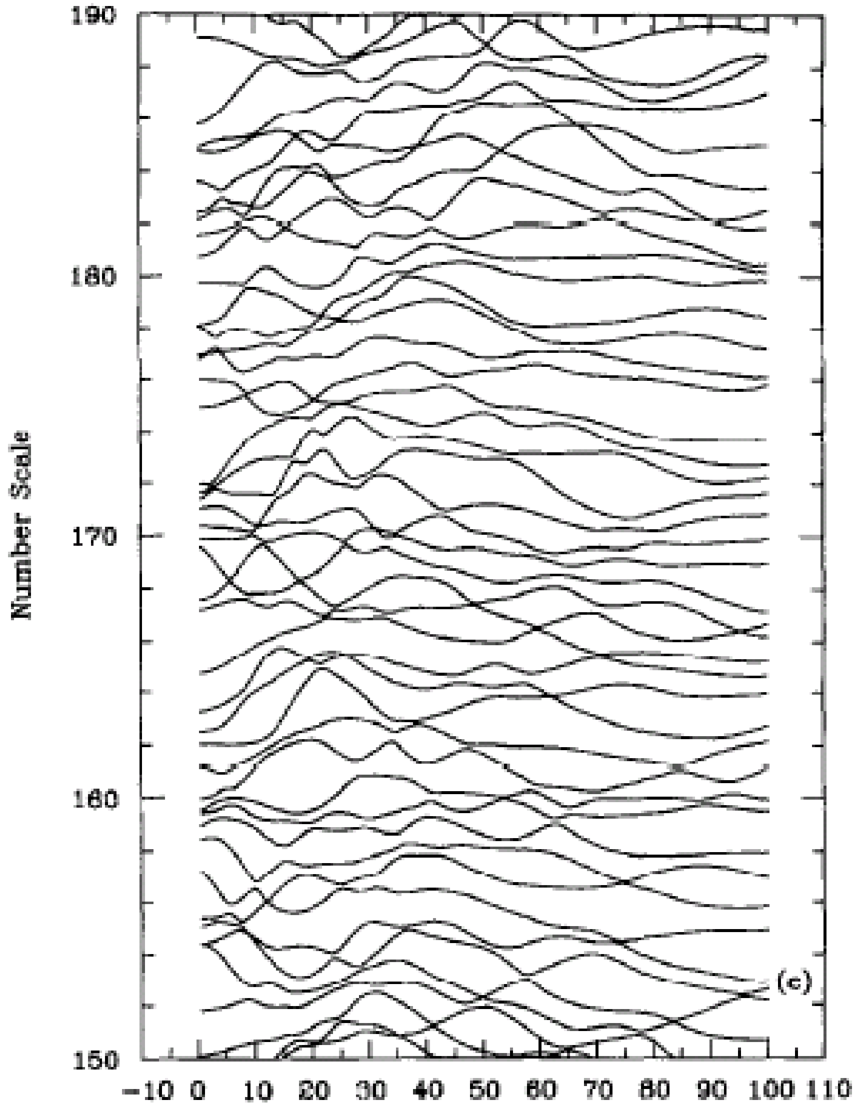
L. D. LANDAU and E. M. LIFSHITZ – "*Statistical Physics*"

Average over the equilibrium ensemble should coincide with the expectation value in a generic individual eigenstate of the same energy – the results of measurements in a closed system do not depend on exact microscopic conditions or phase relationships if the eigenstates at the same energy have similar macroscopic properties

Eigenstate Thermalization Hypothesis

TOOL: MANY-BODY QUANTUM CHAOS

LEVEL DYNAMICS



WAY to CHAOS:

MULTIPLE

AVOIDED

CROSSINGS

as a function

of interaction strength

(shell model of 24Mg
as a typical example)

Fraction (%) of realistic strength

From turbulent to laminar level dynamics

Chaos due to particle interactions at high level density

MEASURING COMPLEXITY

Eigenstate $|\alpha\rangle$ in a shell model basis $|k\rangle$

$|\alpha\rangle = \sum_k C_k^\alpha |k\rangle$

Information entropy

$S^\alpha = -\sum_k |C_k^\alpha|^2 \ln |C_k^\alpha|^2$

No mixing: $S^\alpha \to 0$

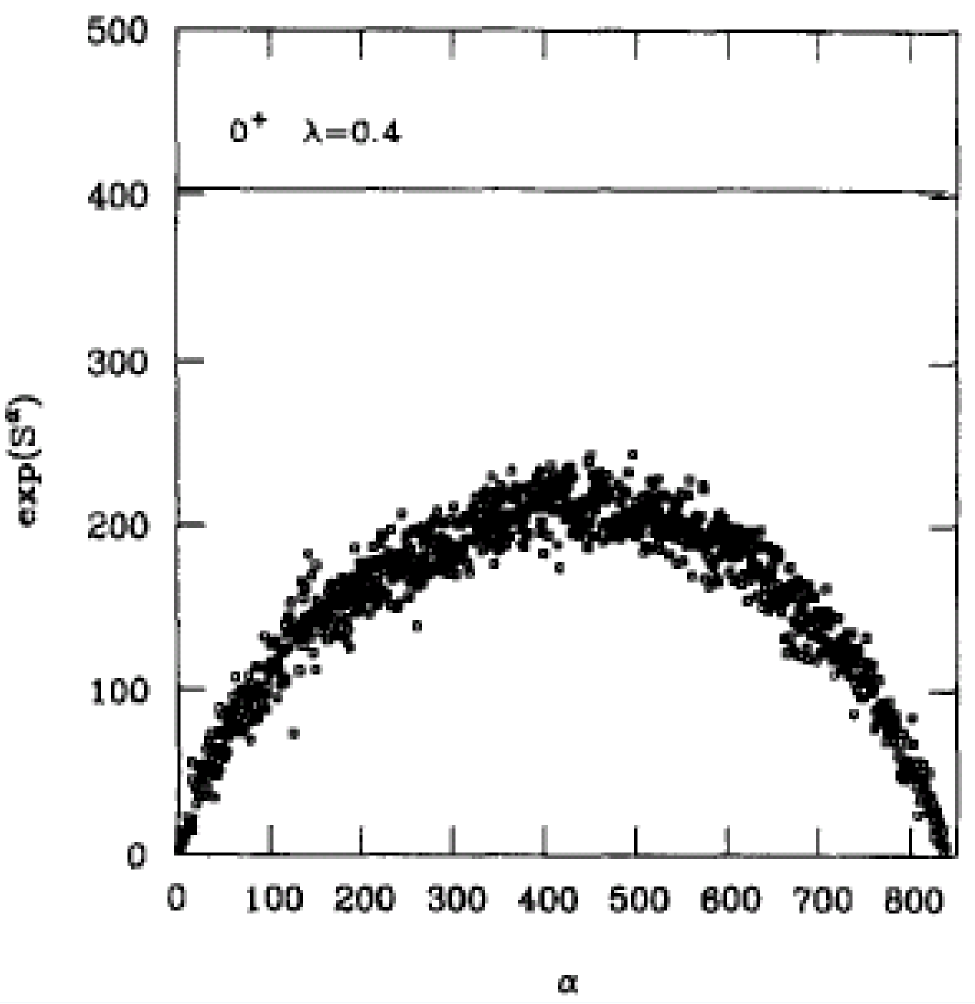
"Microcanonical" mixing: $S^\alpha \to \ln N$

GOE: $\overline{S^\alpha} = \ln(0.48N)$

Shannon entropy

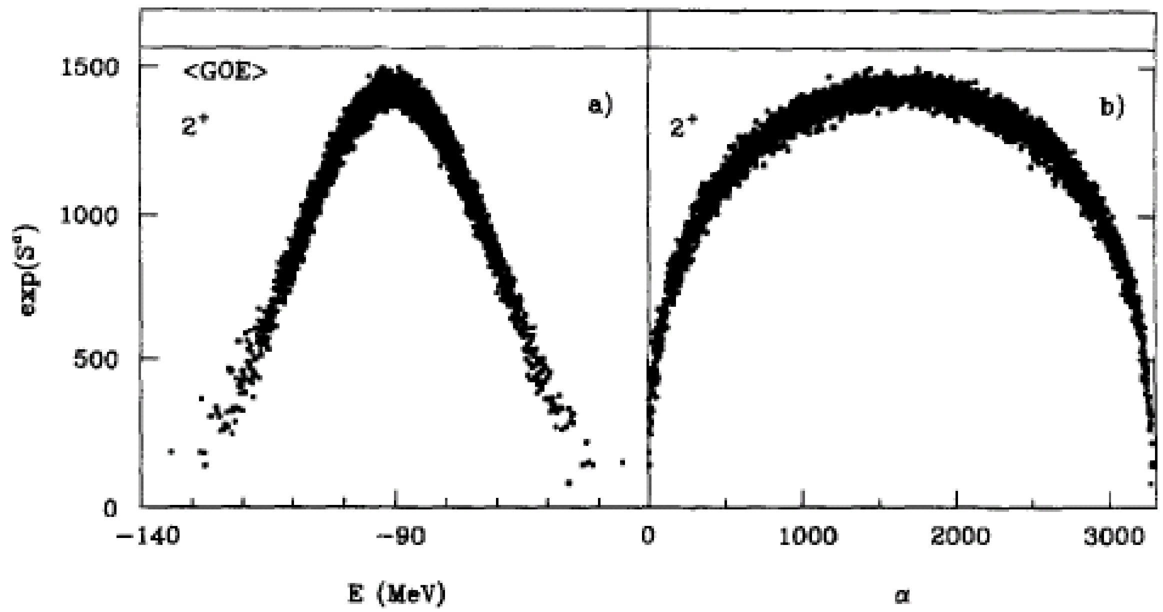
Information entropy is basis-dependent
- special role of mean field

28 Si Shell Model (artificially weak interaction)



INFORMATION ENTROPY AT WEAK INTERACTION

28 Si shell model
Realistic interaction
strength

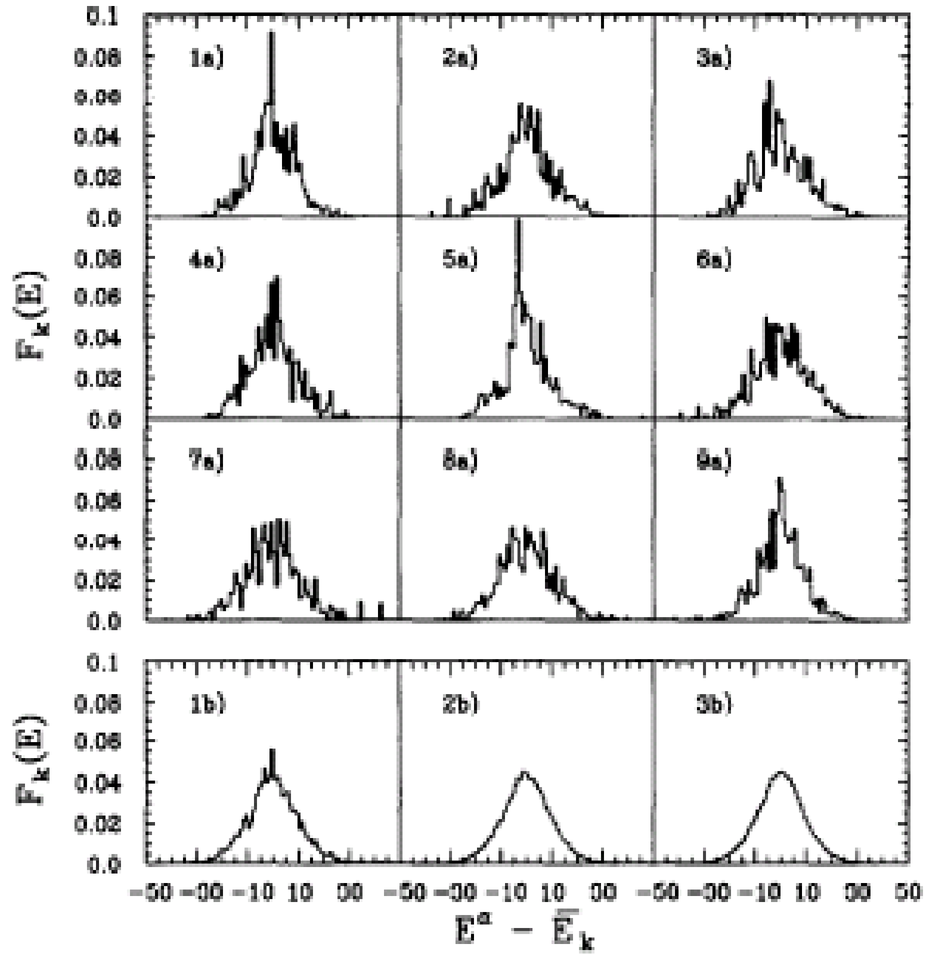


INFORMATION ENTROPY of EIGENSTATES

(a) function of energy; (b) function of ordinal number

ORDERING of EIGENSTATES of GIVEN SYMMETRY

SHANNON ENTROPY AS THERMODYNAMIC VARIABLE



9 INDIVIDUAL STATES

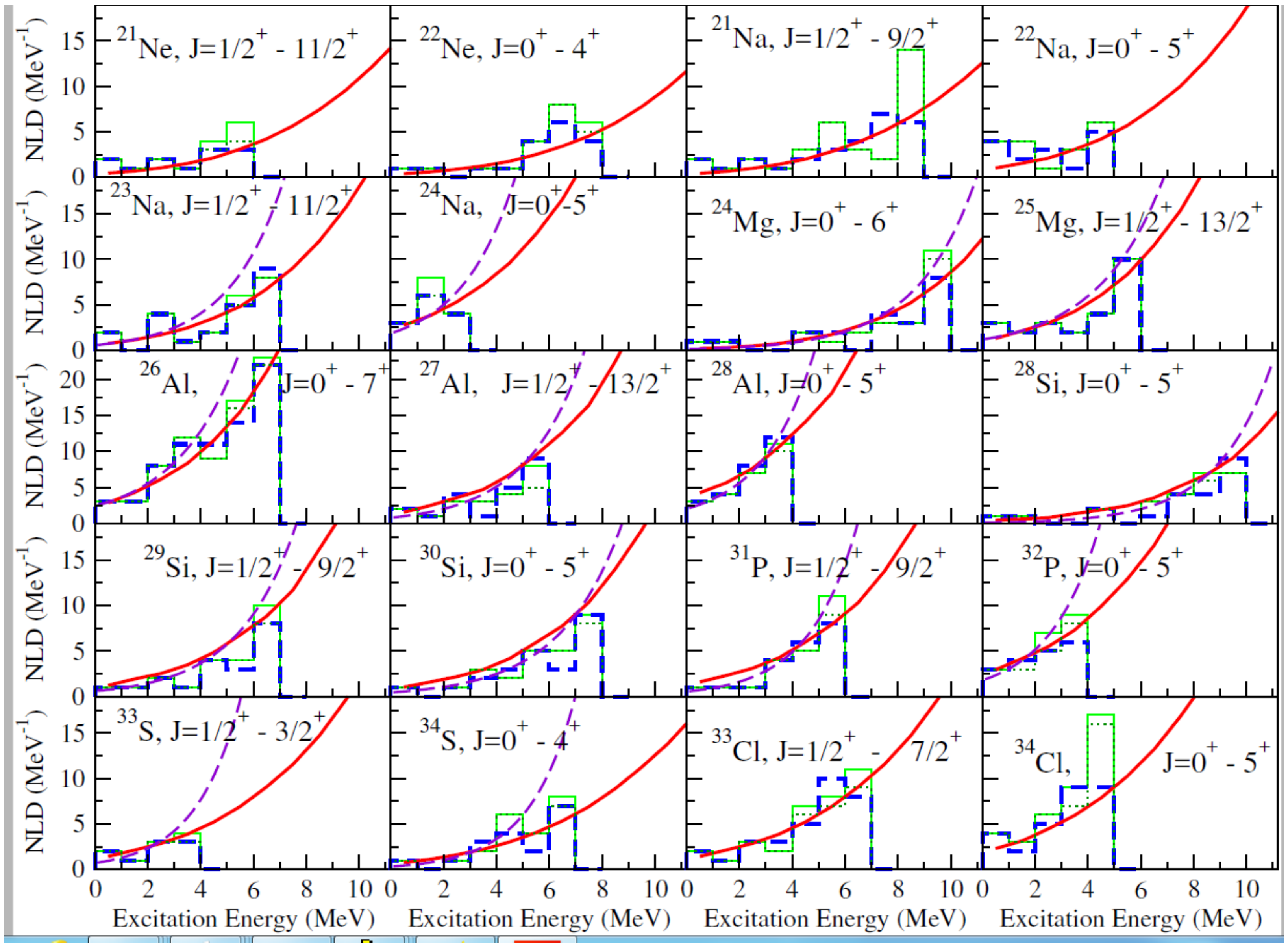
AVERAGE OVER

10, 100, 400 STATES

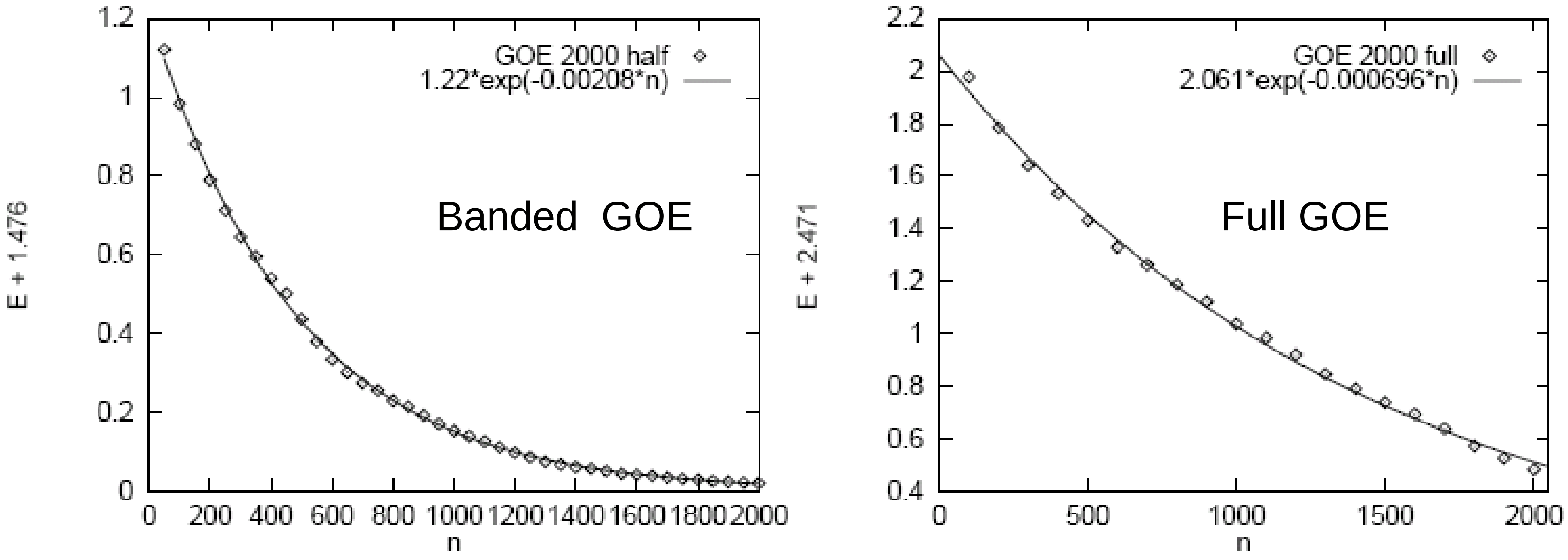
STRENGTH FUNCTION

$$F_k(E) = \Sigma_\alpha (C_k^\alpha)^2 \delta(E - E_\alpha)$$

Local density of states in condensed matter physics



^{21}Ne, J=1/2$^+$ - 11/2$^+$
^{22}Ne, J=0$^+$ - 4$^+$
^{21}Na, J=1/2$^+$ - 9/2$^+$
^{22}Na, J=0$^+$ - 5$^+$
^{23}Na, J=1/2$^+$ - 11/2$^+$
^{24}Na, J=0$^+$-5$^+$
^{24}Mg, J=0$^+$ - 6$^+$
^{25}Mg, J=1/2$^+$ - 13/2$^+$
^{26}Al, J=0$^+$ - 7$^+$
^{27}Al, J=1/2$^+$ - 13/2$^+$
^{28}Al, J=0$^+$ - 5$^+$
^{28}Si, J=0$^+$ - 5$^+$
^{29}Si, J=1/2$^+$ - 9/2$^+$
^{30}Si, J=0$^+$ - 5$^+$
^{31}P, J=1/2$^+$ - 9/2$^+$
^{32}P, J=0$^+$ - 5$^+$
^{33}S, J=1/2$^+$ - 3/2$^+$
^{34}S, J=0$^+$ - 4$^+$
^{33}Cl, J=1/2$^+$ - 7/2$^+$
^{34}Cl, J=0$^+$ - 5$^+$
NLD (MeV^{-1})
Excitation Energy (MeV)

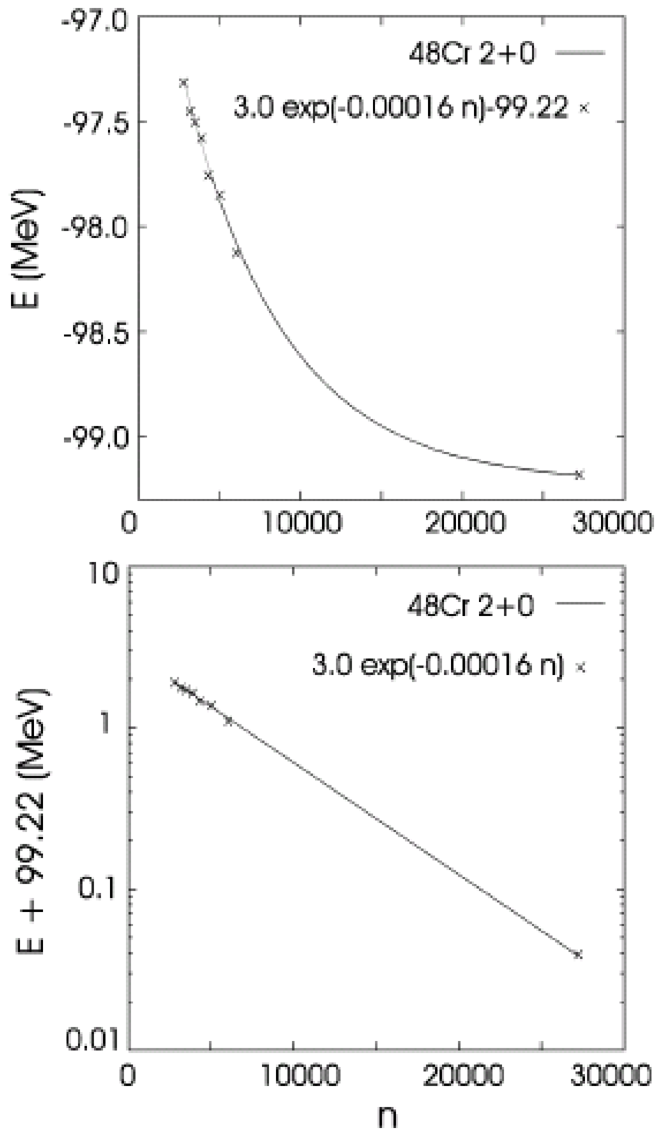


GROUND STATE ENERGY OF RANDOM MATRICES

EXPONENTIAL CONVERGENCE

SPECIFIC PROPERTY of RANDOM MATRICES ?

/The proof based on the Lanczos algorithm/



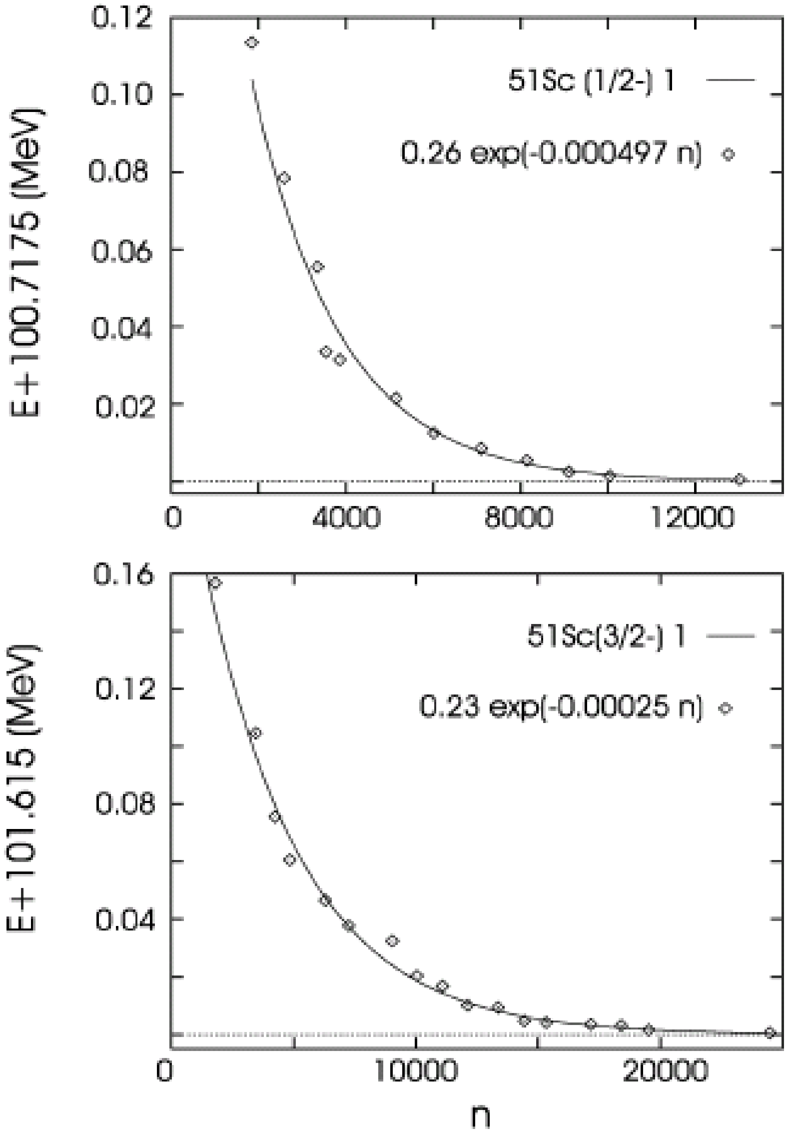
REALISTIC SHELL MODEL

48 Cr

Excited state
J=2, T=0

EXPONENTIAL CONVERGENCE !

E(n) = E + exp(-an)
n ~ 4/N



REALISTIC
SHELL
MODEL

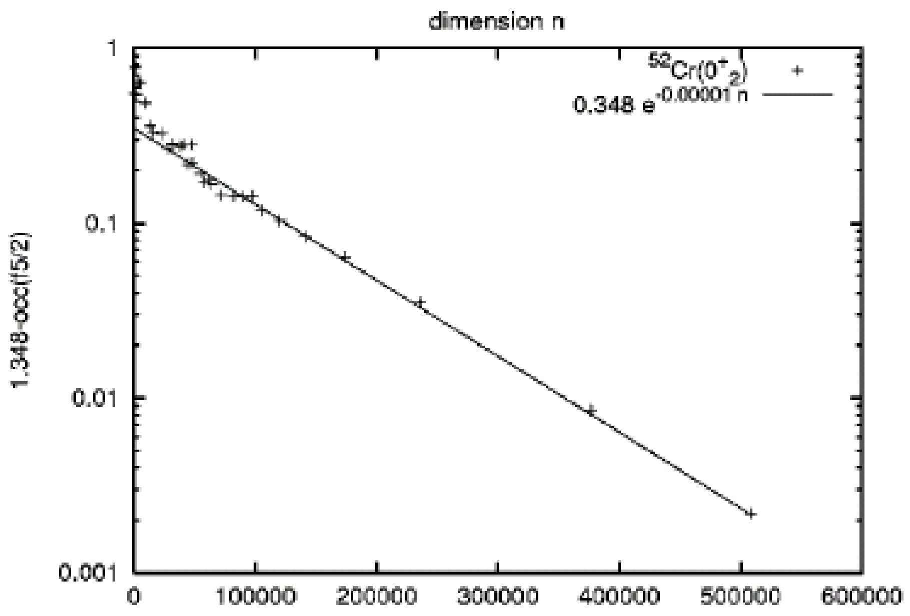
EXCITED STATES
51Sc

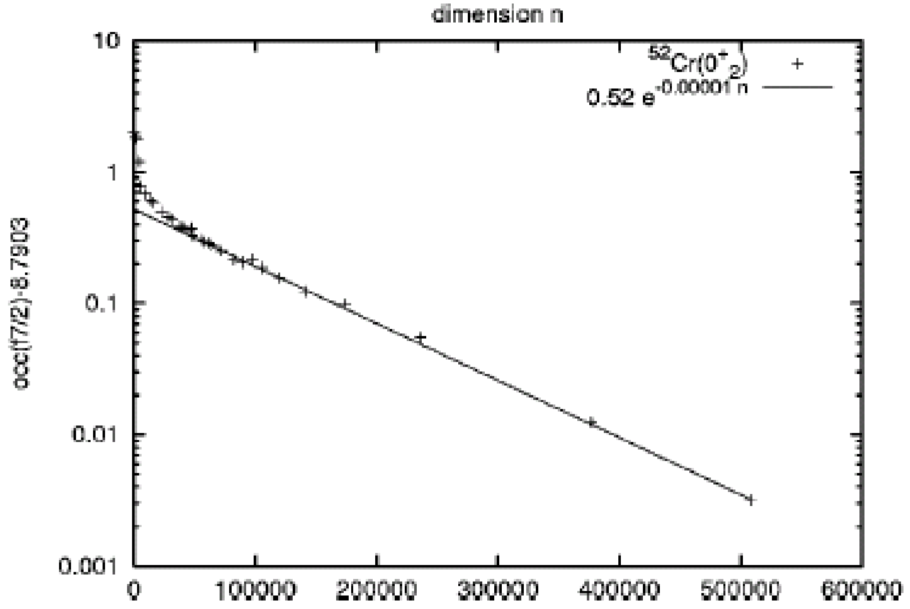
1/2-, 3/2-

Faster convergence:

$E(n) = E + \exp(-an)$

$a \sim 6/N$



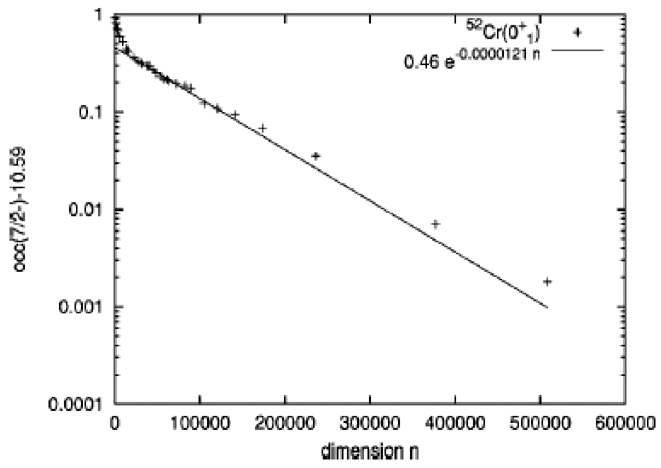


EXPONENTIAL
CONVERGENCE
OF SINGLE-PARTICLE
OCCUPANCIES

(first excited state J=0)

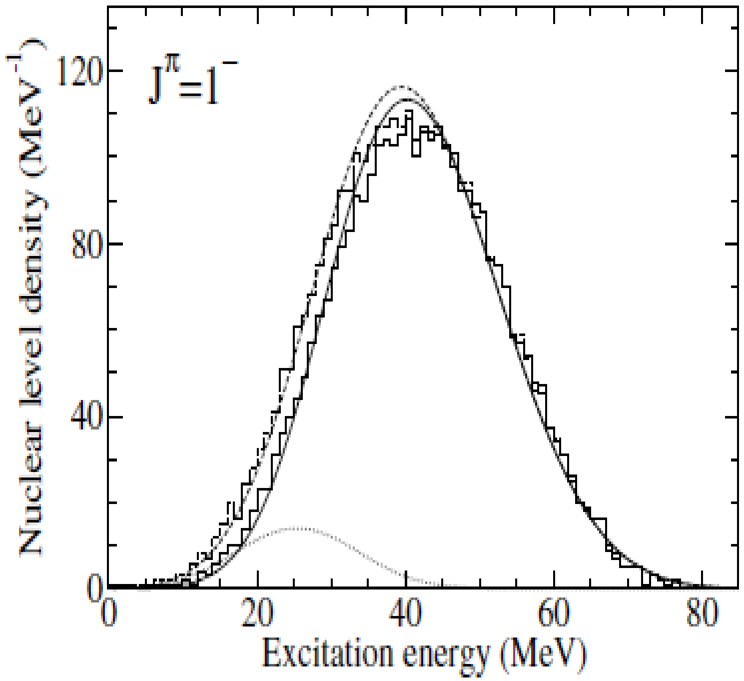
52

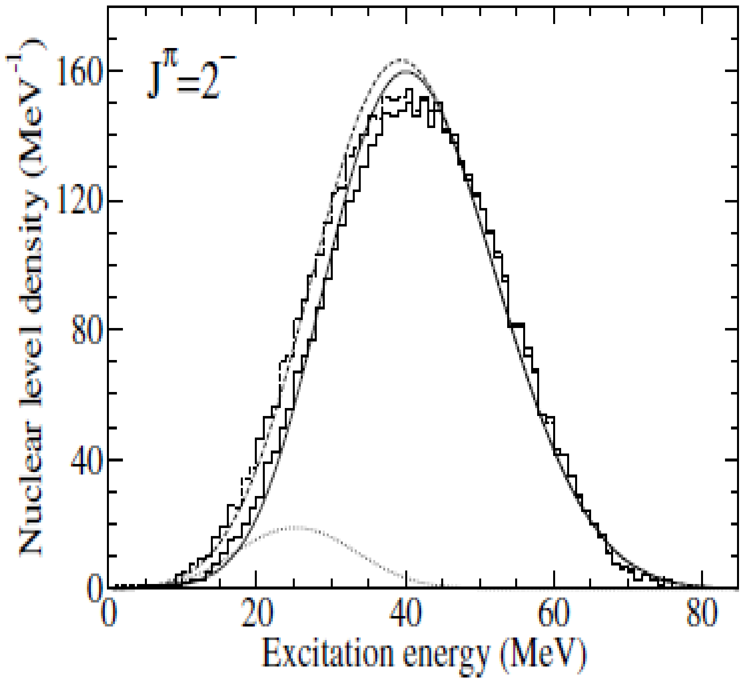
Cr



Fit with $\gamma' = \gamma$

20 Ne

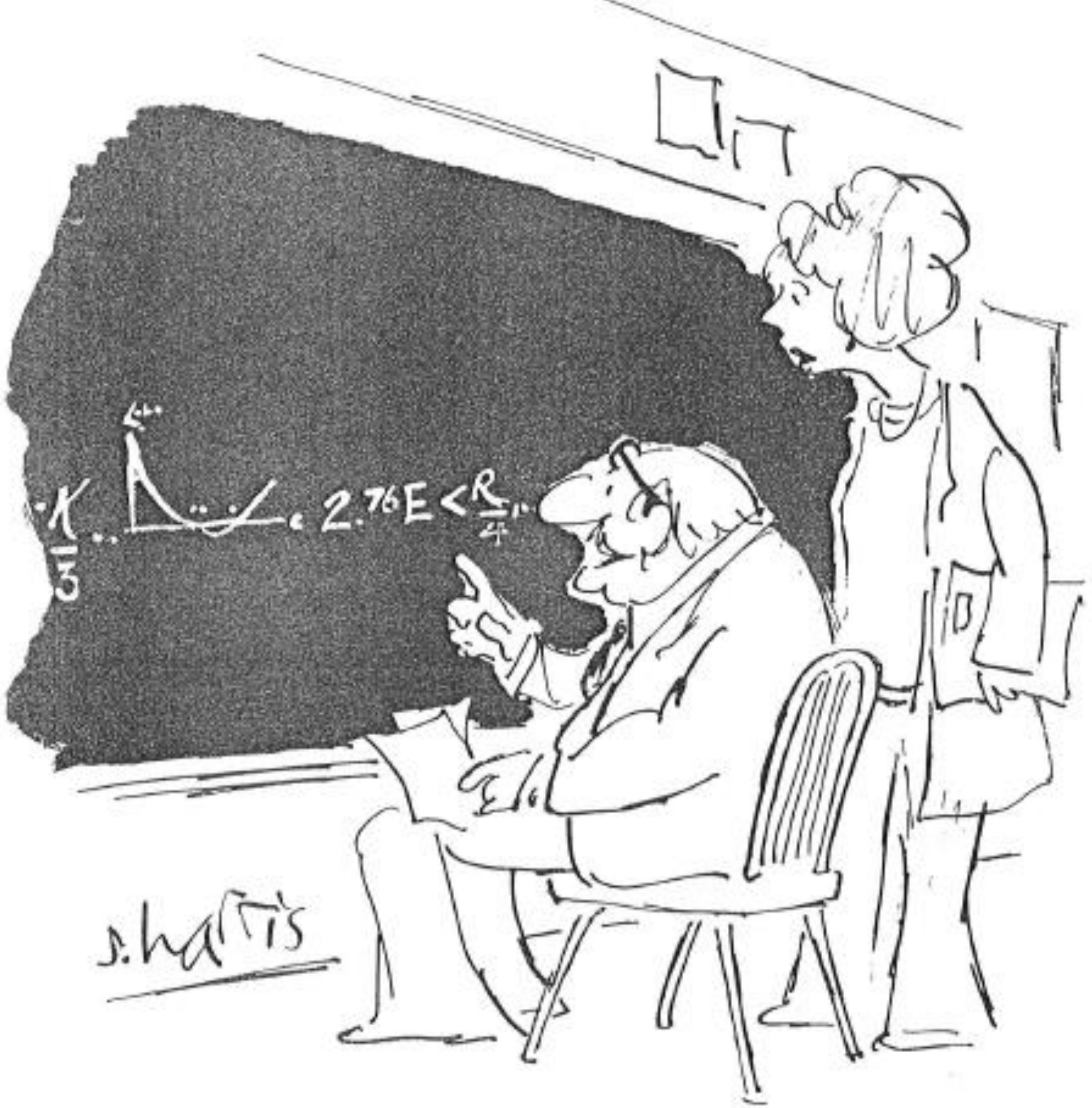




s + p + sd + pf shell space
WBT interaction,
negative parity

$1\hbar\omega$ subspace $H \to H' = H + \beta\left[\left(H_{CM} - \frac{3}{2}\hbar\omega\right)\frac{A}{\hbar\omega}\right]$

Exact shell model: stair-dashed (with CM) and stair-solid (no CM)
Method of moments: straight-dashed (with CM) and straight-solid (no CM)
Dotted line: spurious states



"THE BEAUTY OF THIS IS THAT IT IS ONLY OF THEORETICAL IMPORTANCE, AND THERE IS NO WAY IT CAN BE OF ANY PRACTICAL USE WHATSOEVER."

Analytical results for tridiagonal matrices

$$H = \begin{pmatrix} \epsilon_1 & V_2 & 0 & 0 & 0 & 0 \\ V_2 & \epsilon_2 & V_3 & 0 & 0 & 0 \\ 0 & V_3 & \epsilon_3 & V_4 & 0 & 0 \\ 0 & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & \dots & \dots & V_n \\ 0 & 0 & 0 & 0 & V_n & \epsilon_n \end{pmatrix}$$

Recurrence relation for determinants

$$D_n(E) = (\epsilon_n - E)D_{n-1}(E) - V_n^2 D_{n-2}(E)$$

Convergence is determined by

$$\lambda_n^2 = V_n^2/(\epsilon_n \epsilon_{n-1})$$

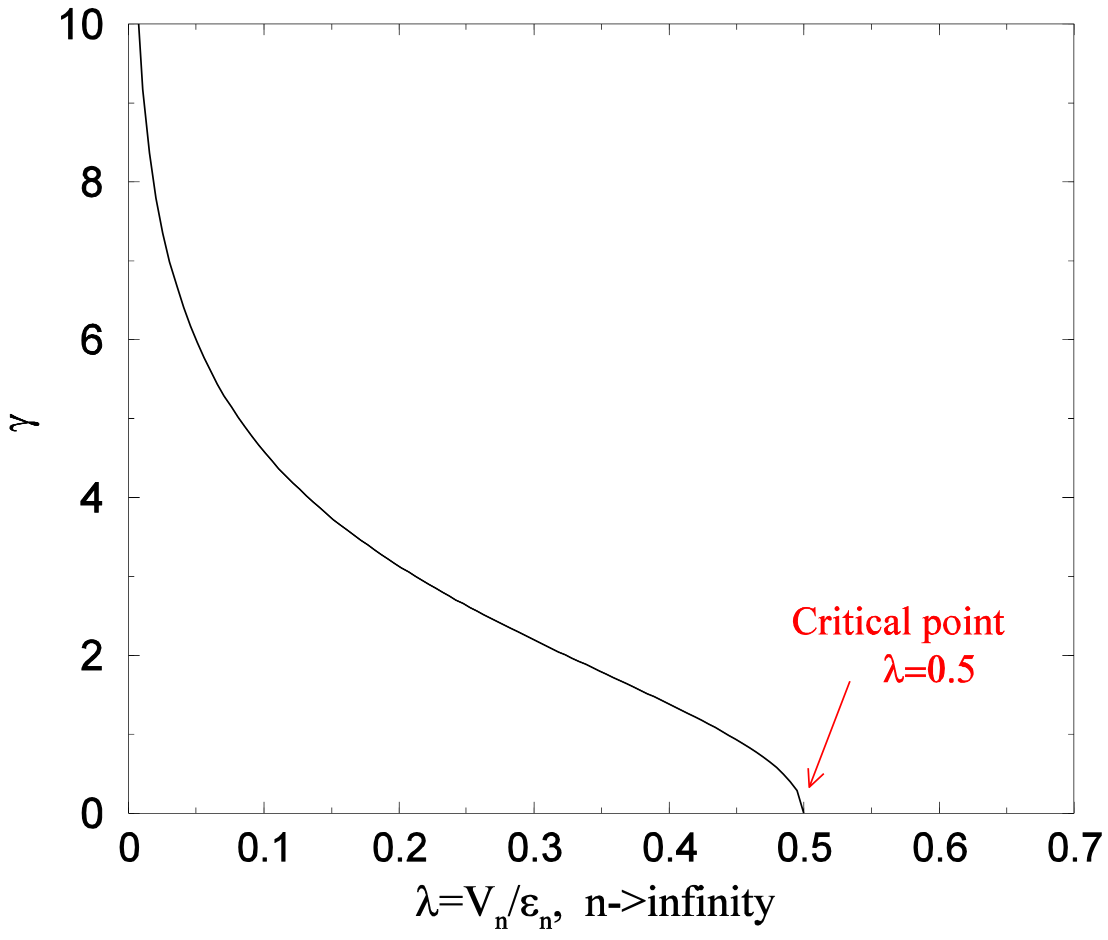
Assume existence of the limit

$$\lambda_n \Rightarrow \lambda$$

at

$$n \Rightarrow \infty$$

New method for shell-model level density /B.A. Brown, 2018/



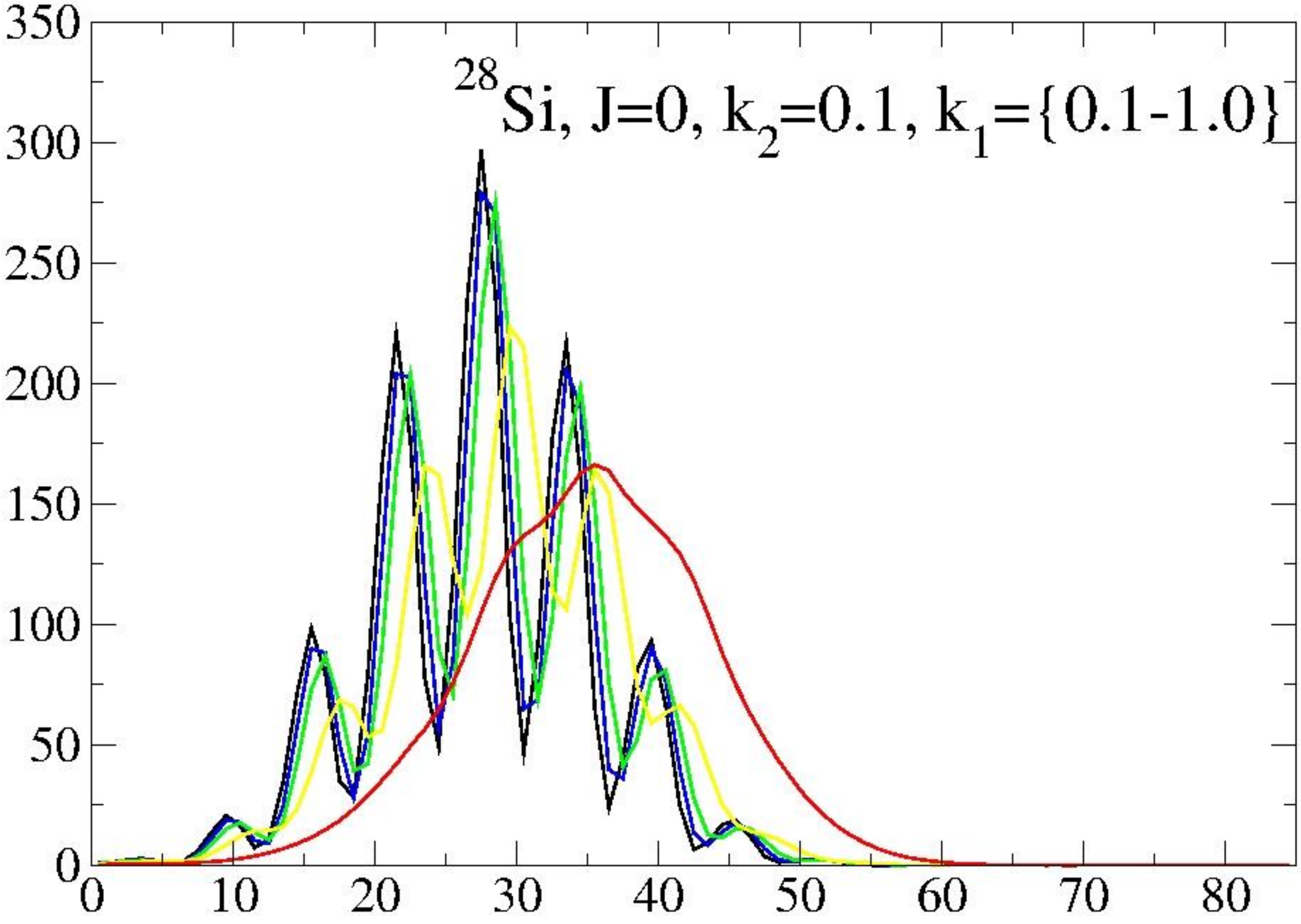
CONVERGENCE REGIMES

- $\lambda = 0$ — *Fast convergence*
- $0 < \lambda < 1/2$ — *Exponential convergence*
- $\lambda = 1/2$ — Power law
- $\lambda > 1/2$ — *Divergence*

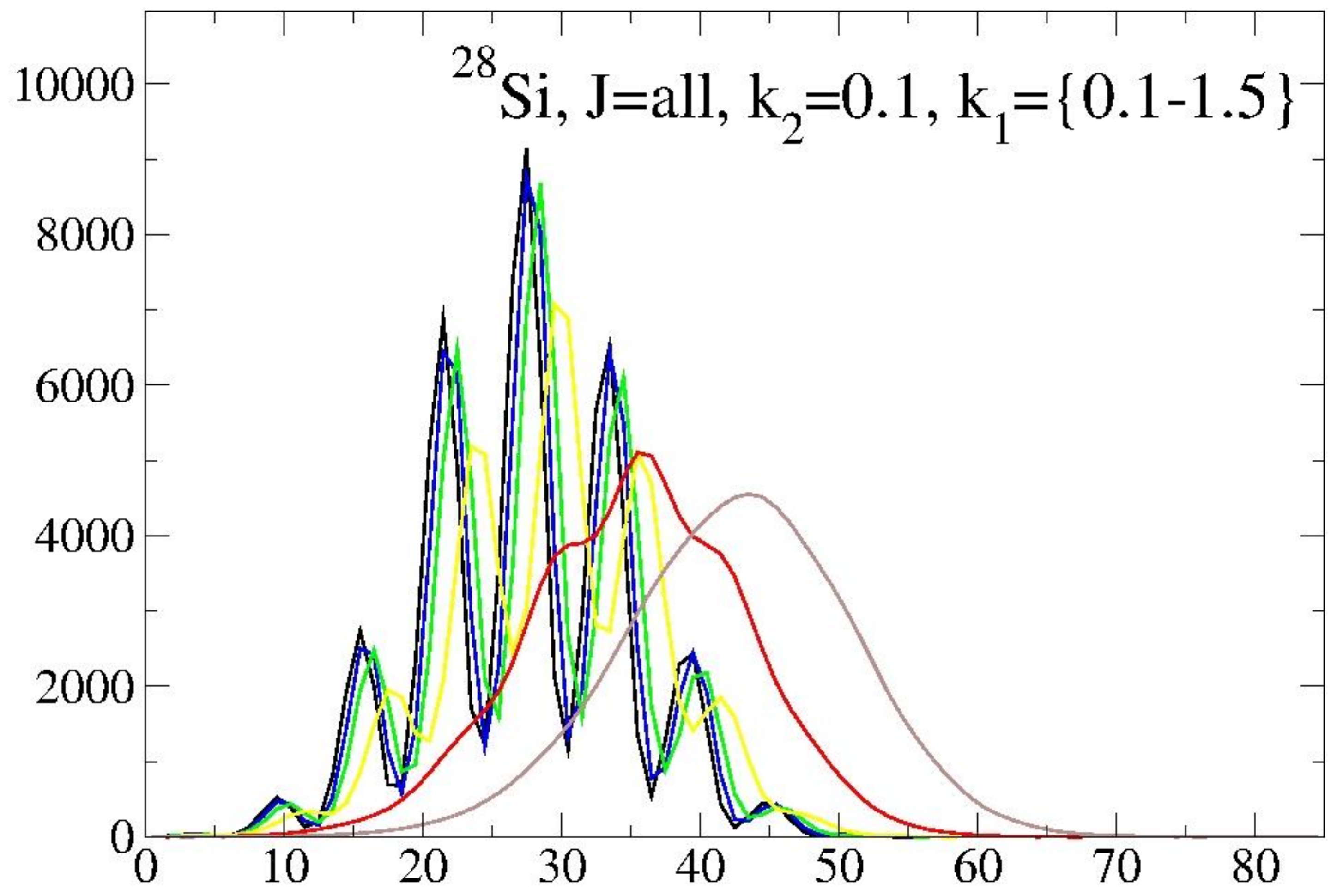
Exponential convergence

$\sim \exp(-\gamma n)$

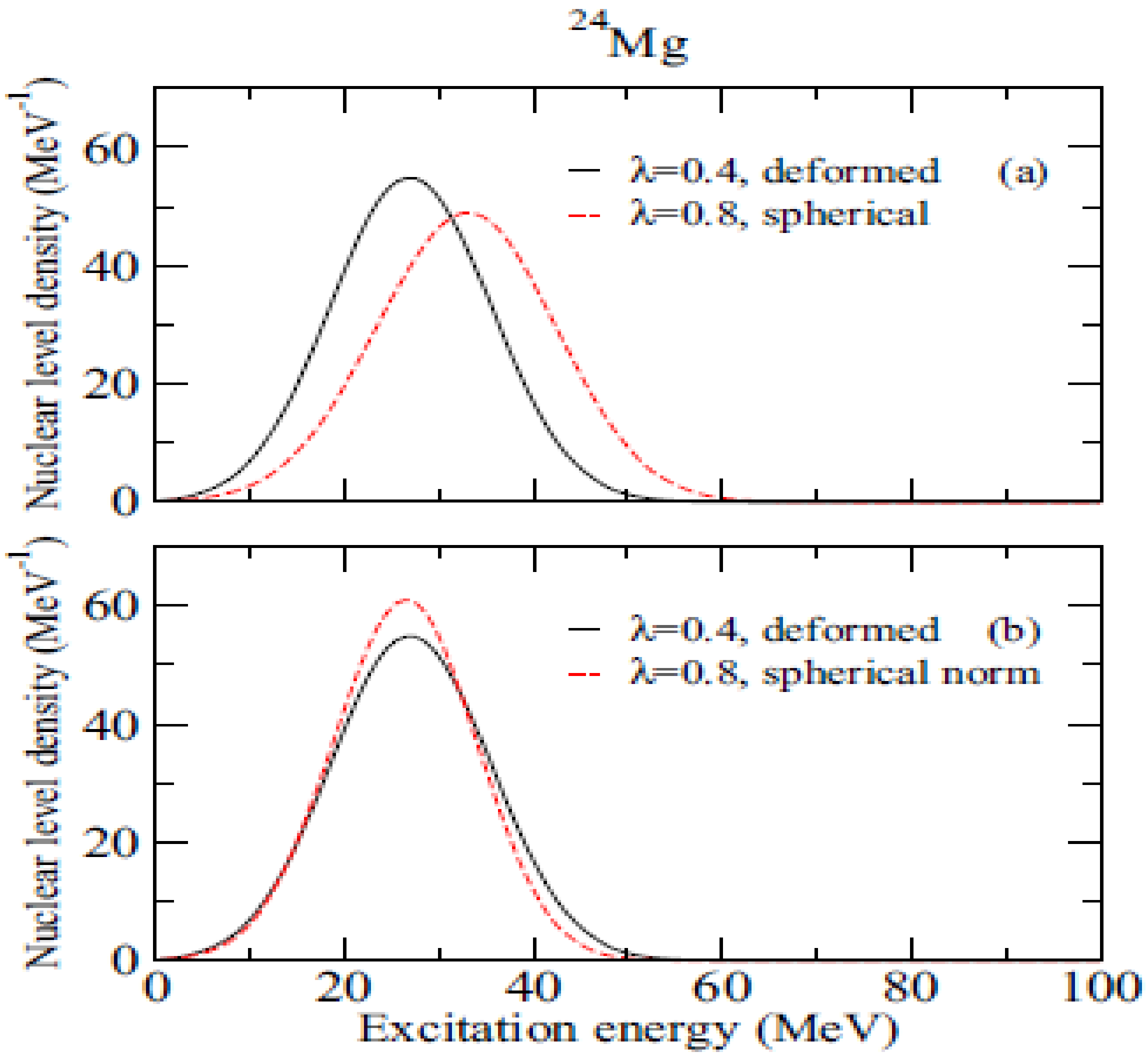
$$\gamma = -\ln\left\{\frac{1}{2\lambda^2}\left(1 - 2\lambda^2 - \sqrt{1-4\lambda^2}\right)\right\}$$



^{28}Si, J=0, k_2=0.1, k_1={0.1-1.0}
0
50
100
150
200
250
300
350
0
10
20
30
40
50
60
70
80



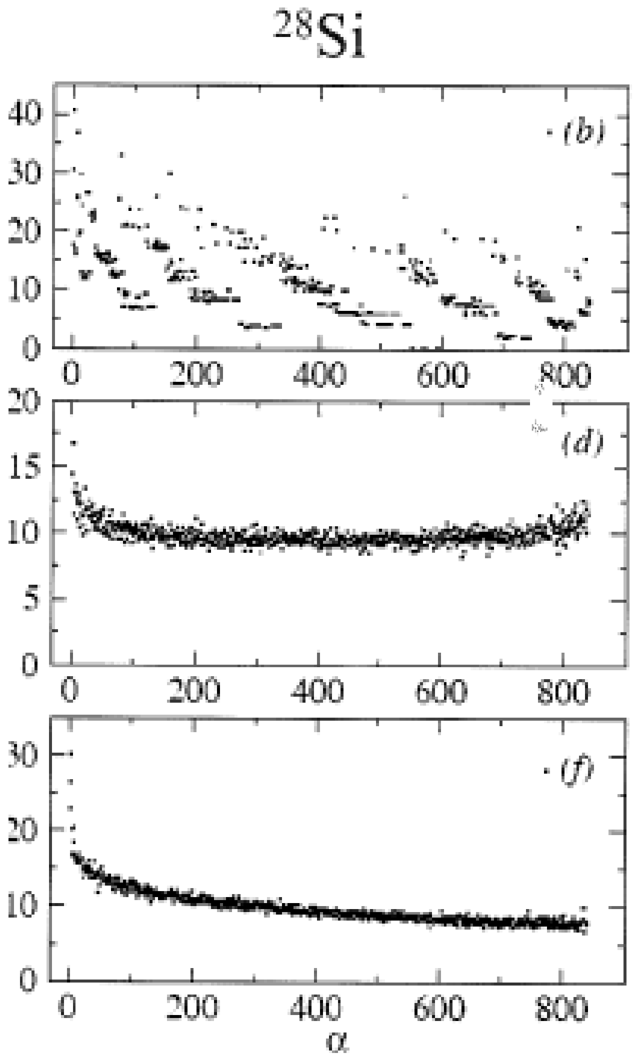
^{28}Si, J=all, k_2=0.1, k_1={0.1-1.5}
10000
8000
6000
4000
2000
0
0
10
20
30
40
50
60
70
80



Level density (0+) on two sides of deformation shape transition

/"collective enhancement"/

States J=0



PAIR CORRELATOR

(b) Only pairing

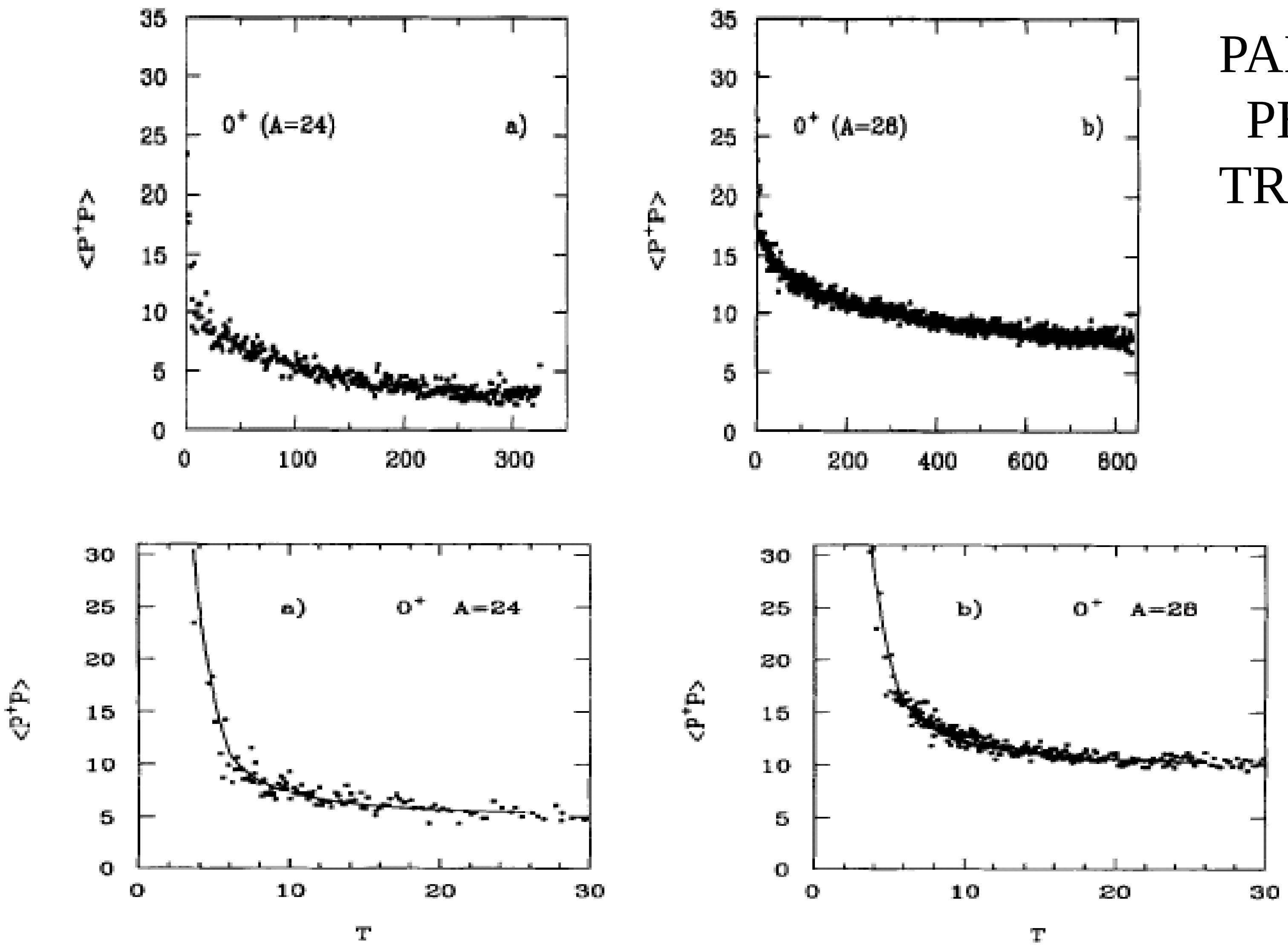
(d) Non-pairing interactions

(f) All interactions

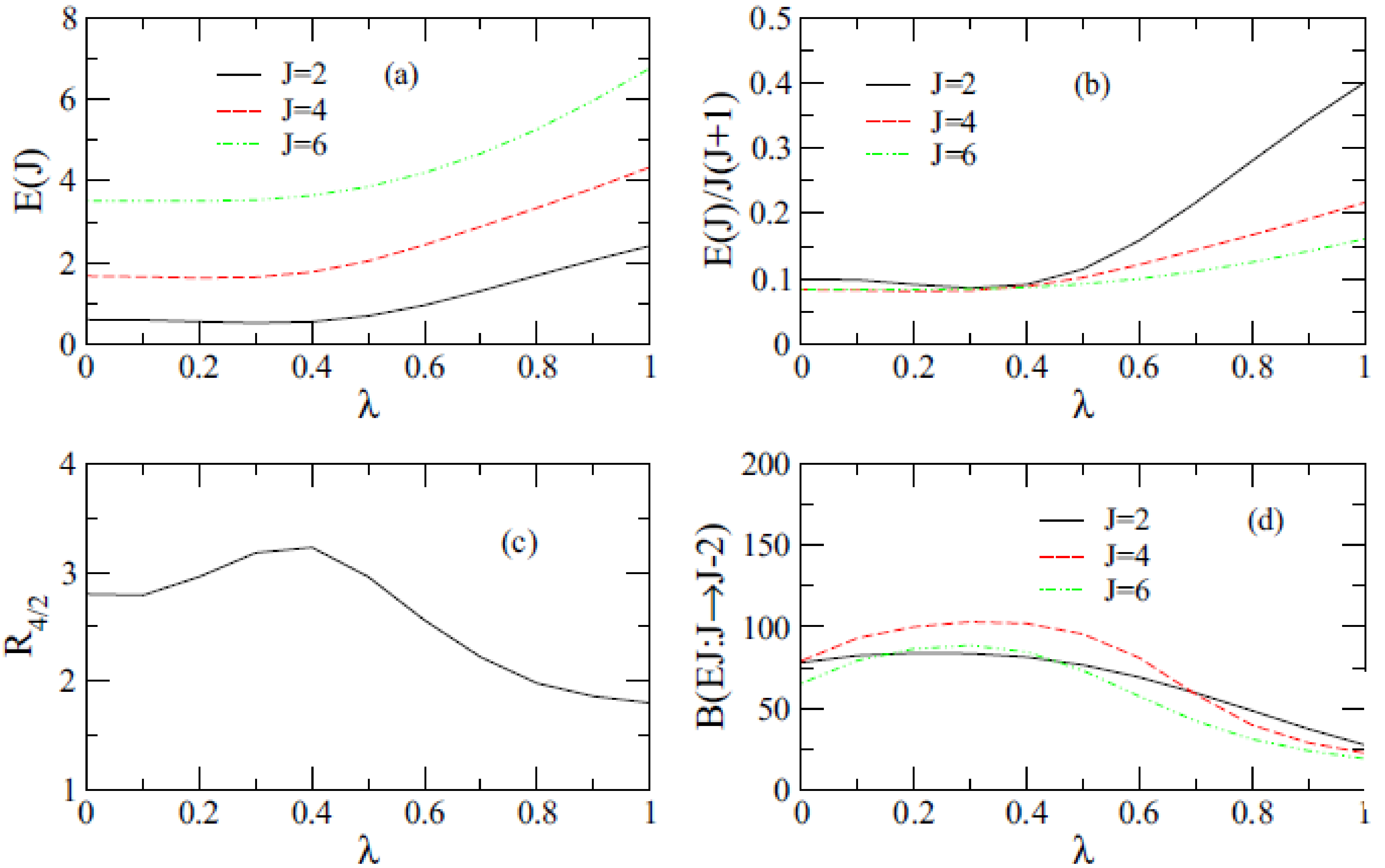
$$\mathcal{H}_P = \sum_{t=0,\pm1} P_t^\dagger P_t$$

$$P_t = \frac{1}{\sqrt{2}} \sum_j [a_j a_j]_{J=0,T=1,T_3=t}$$

PAIRING
PHASE
TRANSITION



PAIR CORRELATOR as a THERMODYNAMIC FUNCTION



24 Mg

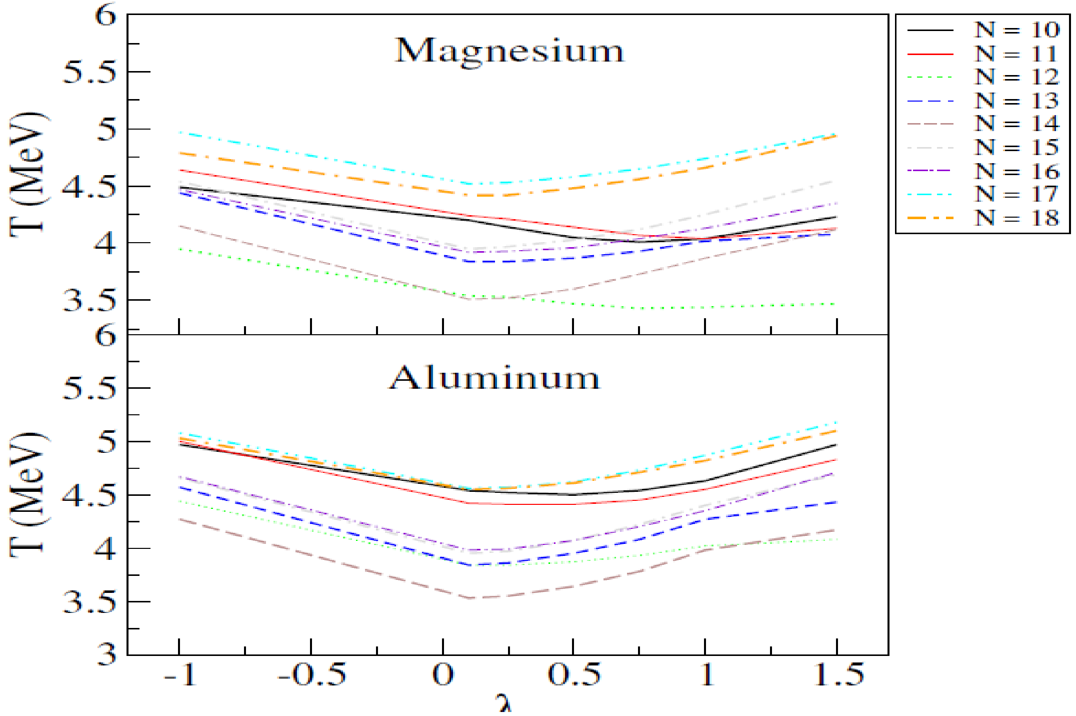
Low-lying levels
in absolute (a)
and rotational (b)
units;

Ratio E(4)/E(2) (c)

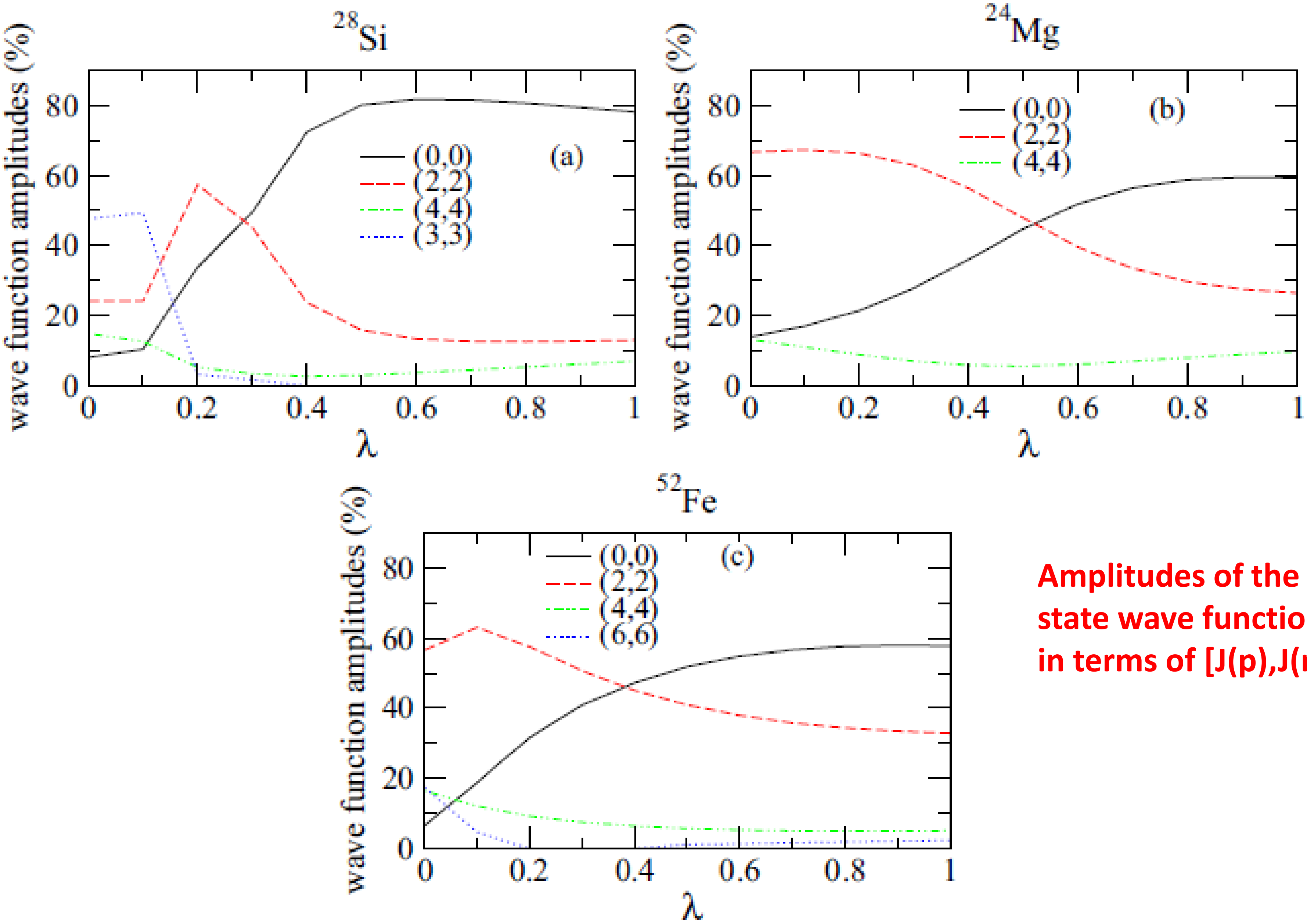
Transition rates (d)

$$H = h + (1-\lambda)V_1 + \lambda V_2$$

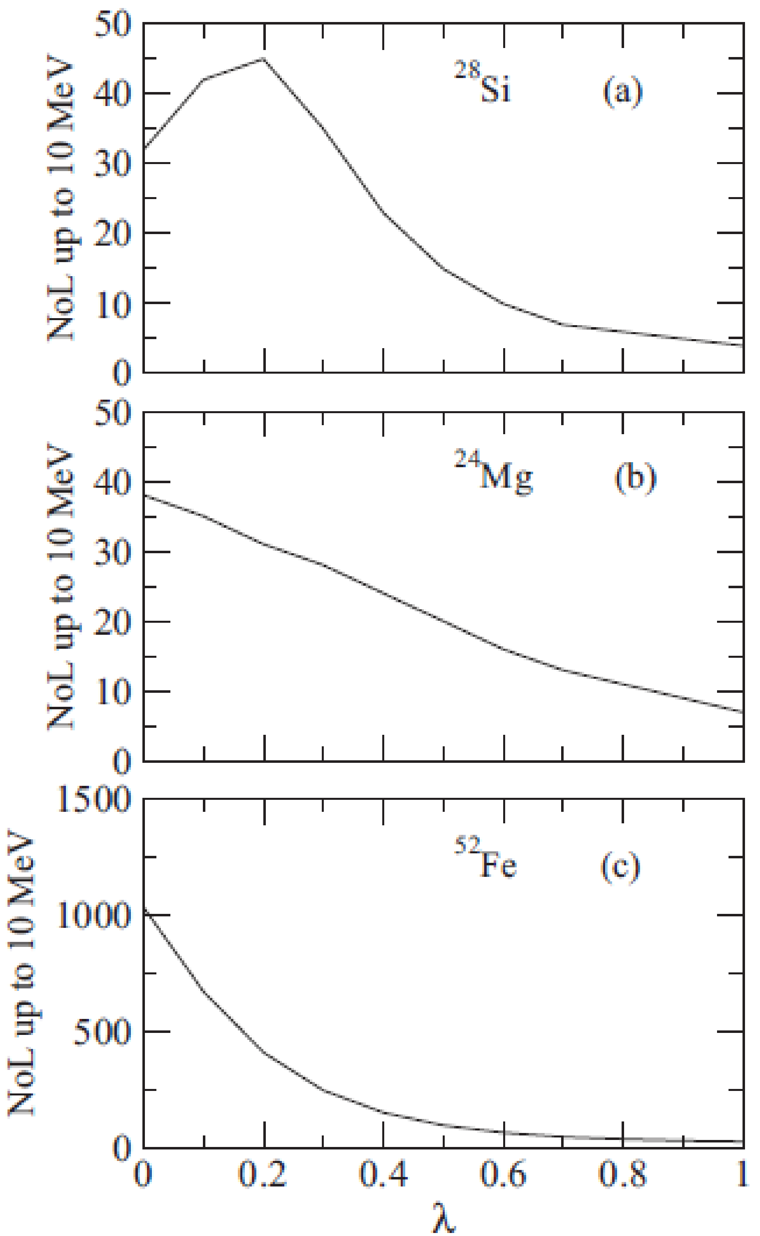
V(1) = matrix elements of the two-body interaction
with change of orbital momentum of one particle
by 2 units (the same parity) – way to deformation



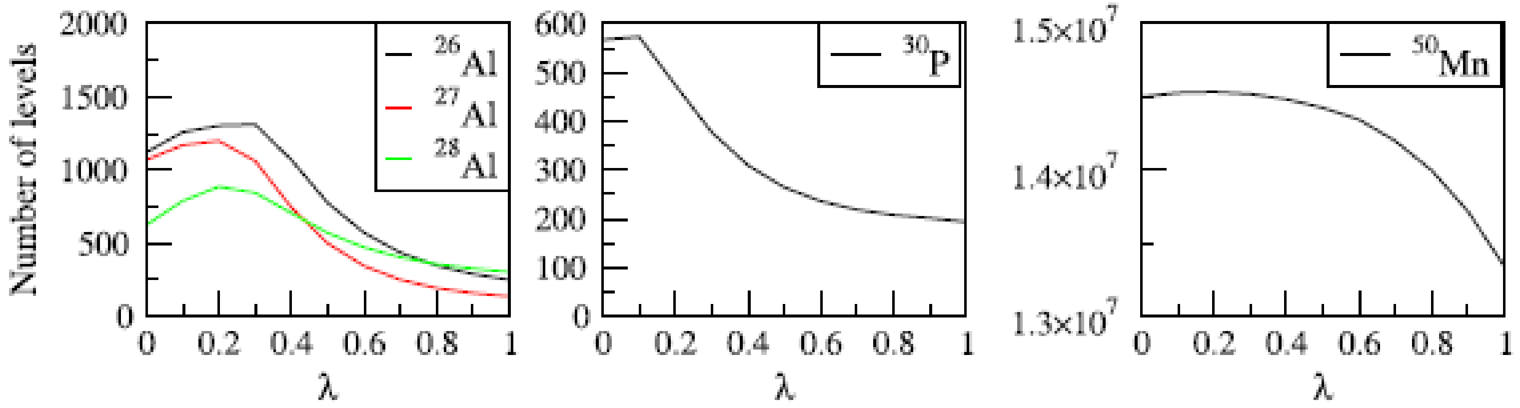
V(1) = matrix elements of the two-body interaction
with change of orbital momentum of one particle
by 2 units (the same parity) – way to deformation



Amplitudes of the ground state wave functions in terms of [J(p),J(n)]



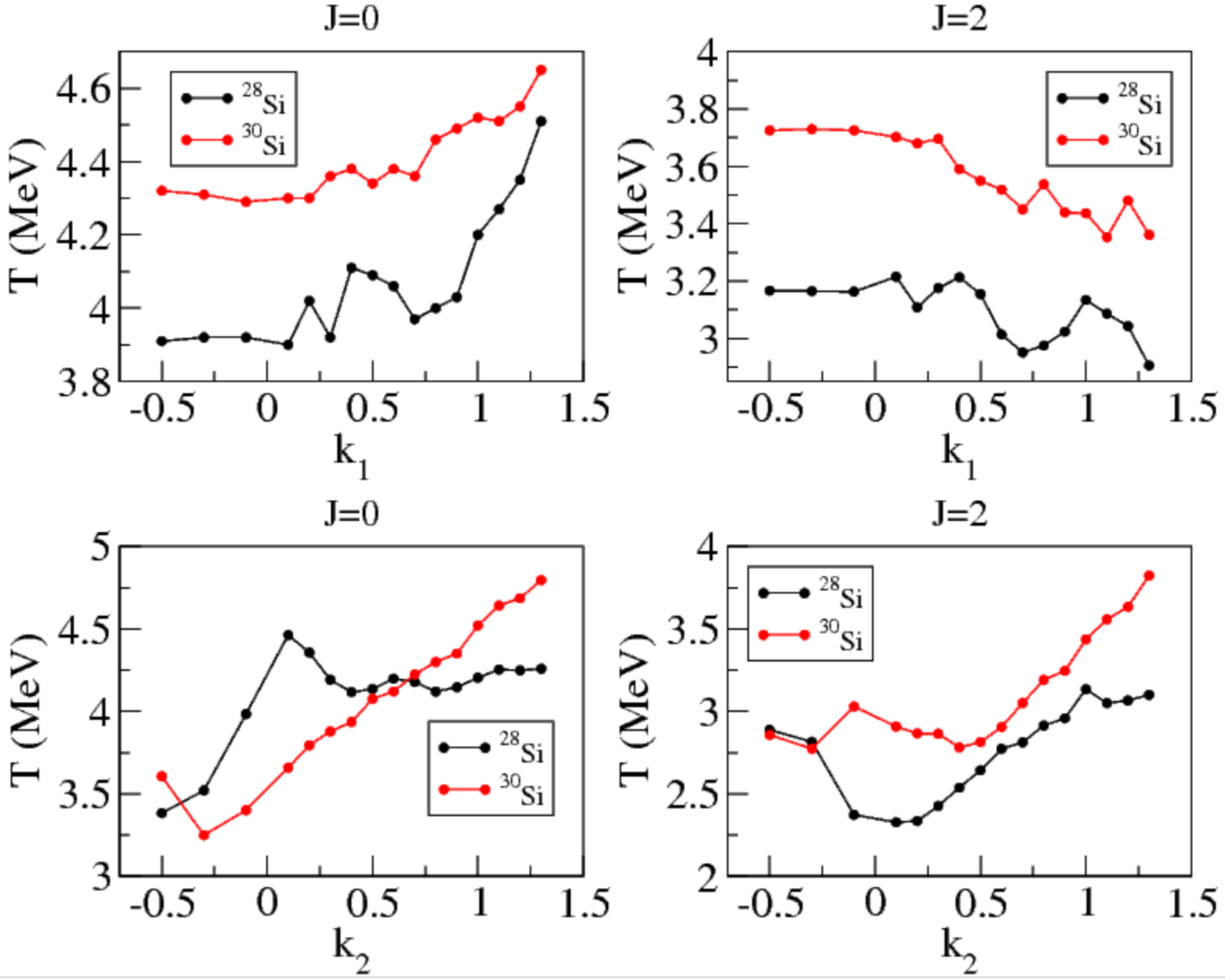
Number of 0+ levels up to energy 10 MeV



J=0 – 10 for 26 Al, 28 Al, 30 P (up to 10 MeV)

J=1/2 – 21/2 for 27 Al (up to 10 MeV)

J=0 – 10 for 50 Mn (up to 60 MeV)

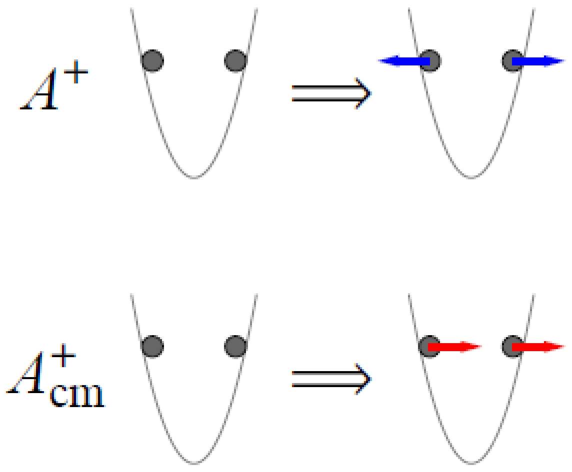


J=0
J=2
J=0
J=2
T (MeV)
k1
k2
28Si
30Si

Removal of the center-of-mass spurious states

Harmonic oscillator:

$$\mathcal{N}_{spur}(K\hbar\omega) \sim \sum_{K'=1}^{K} \mathcal{N}_{pure}((K-K')\hbar\omega),$$



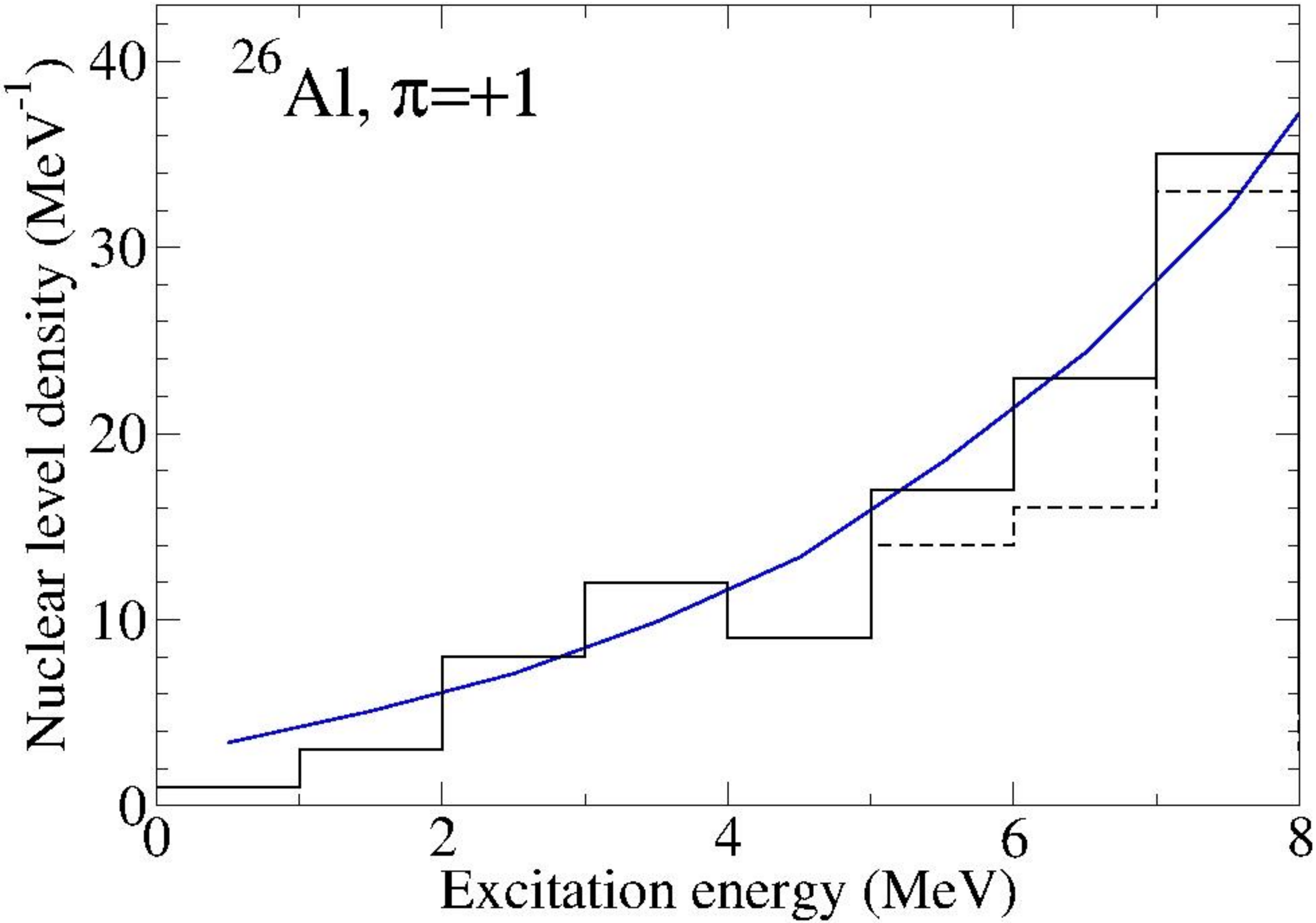
where K' presents how many times we act with $A^{\dagger}_{cm}$

P. Van Isacker, Phys. Rev. Lett. **89**, 262502 (2002)

Nuclear level density. Recursive method:

$$\rho_{pure}(E,J,K) = \rho(E,J,K) - \sum_{K'=1}^{K} \sum_{J_{K'}=J_{min}}^{K,step\ 2} \sum_{J'=|J-J_{K'}|}^{J+J_{K'}} \rho_{pure}(E,J',K-K')$$

M. Horoi and V. Zelevinsky, Phys. Rev. Lett. **98**, 262503 (2007)



^{26}Al, π=+1
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
40
0
2
4
6
8

TABLE III: Cumulative Number of Levels (NoL) of $J = 0$ up to energy 10 MeV for different (k_1, k_2) combinations for ^{28}Si, ^{24}Mg and ^{52}Fe found with the moments method. The column NoL corresponds to the calculation of the moments method, while the column Renorm corresponds to the renormalized level density (NoL up to 0.4).

shape	case	nucleus	$R_{4/2}$	NoL	Renorm
deformed	$k_1 = 1.0, k_2 = 0.4$	^{28}Si	3.31	22	60
deformed	$k_1 = 1.0, k_2 = 0.5$	^{28}Si	3.33	17	54
deformed	$k_1 = 1.0, k_2 = 0.6$	^{28}Si	3.21	13	49
spherical	$k_2 = 1.0, k_1 = 0.9$	^{28}Si	2.12	5	34
deformed	$k_1 = 1.0, k_2 = 0.5$	^{24}Mg	3.20	10	24
deformed	$k_1 = 1.0, k_2 = 0.6$	^{24}Mg	3.21	8	21
spherical	$k_2 = 1.0, k_1 = 0.3$	^{24}Mg	2.03	6	18
deformed	$k_1 = 1.0, k_2 = 0.4$	^{52}Fe	3.07	236	6516
spherical	$k_2 = 1.0, k_1 = 0.0$	^{52}Fe	2.25	30	2617

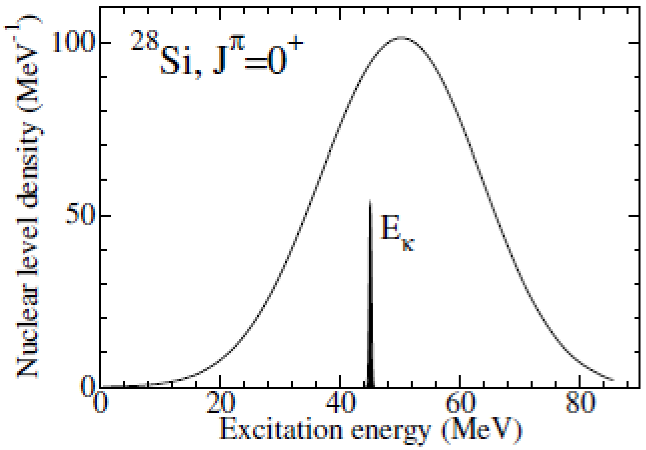
H = k(1)V(1) + k(2)V(2)

V(1) – matrix elements of

single-particle transfer

Statistical approach to Nuclear Level Density (cont.)

$$\rho(E,\beta) = \sum_{\kappa} D_{\beta\kappa} \cdot G(E - E_{\beta\kappa}, \sigma_{\beta\kappa})$$



$G(x,\sigma)$ - Gaussian distribution

$\beta = \{n, J, T_z, \pi\}$ - quantum numbers

κ - configurations

κ	$d\frac{5}{2}$	$s\frac{1}{2}$	$d\frac{3}{2}$
1	6	0	0
2	5	1	0
3	5	0	1
4	4	2	0
...	...	...	...
15	0	2	4

$D_{\beta\kappa}$ - number of many-body states with given β that can be built for a given configuration κ

Moments of H for each configuration κ:

$$E_{\beta\kappa} = \mathrm{Tr}^{(\beta\kappa)}[H]/D_{\beta\kappa}$$

$$\sigma^2_{\beta\kappa} = \mathrm{Tr}^{(\beta\kappa)}[H^2]/D_{\beta\kappa} - \left(\mathrm{Tr}^{(\beta\kappa)}[H]/D_{\beta\kappa}\right)^2$$

M. Horoi, M. Ghita, and V. Zelevinsky, PRC **69** (2004) 041307(R)

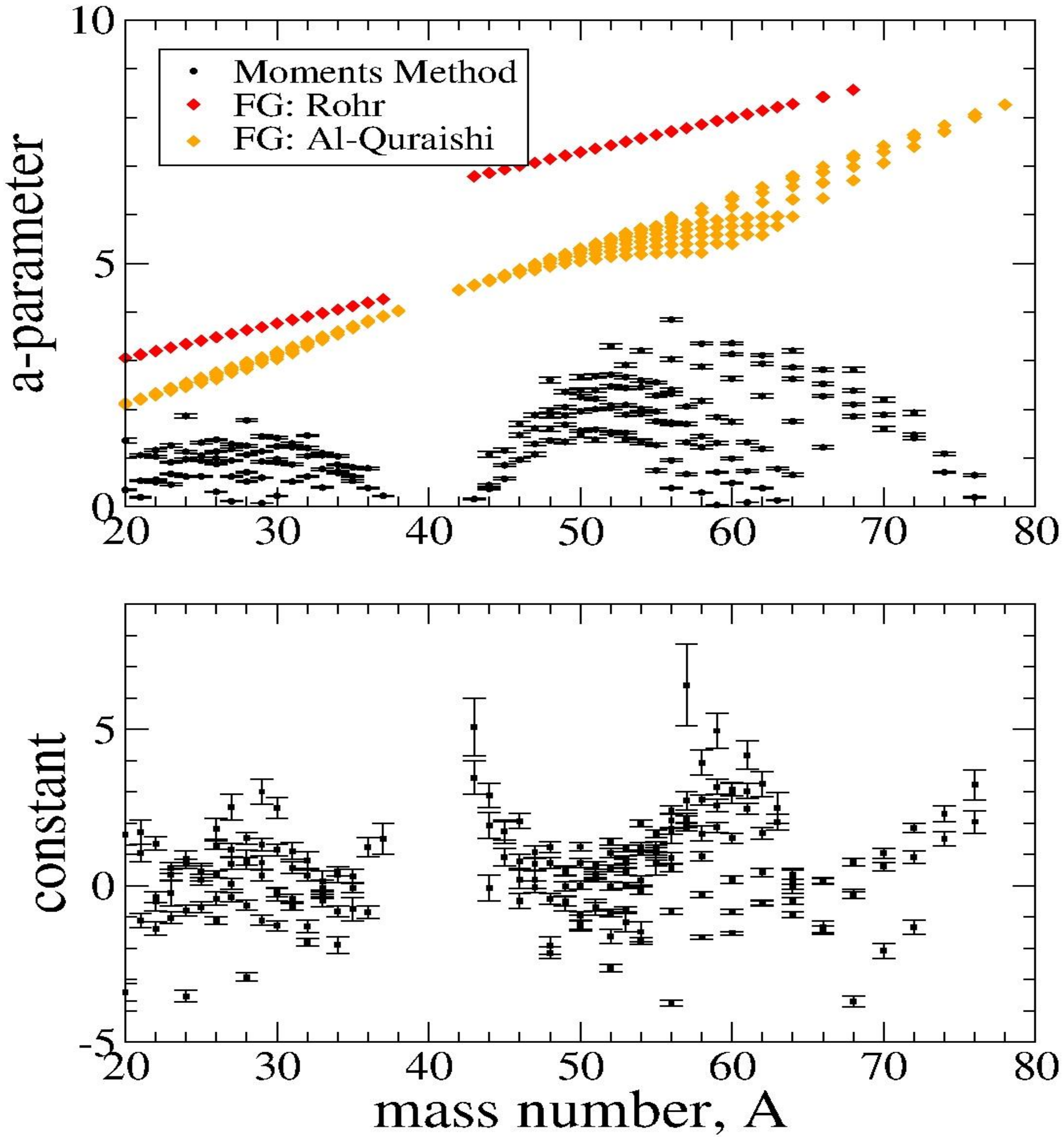
No diagonalization required

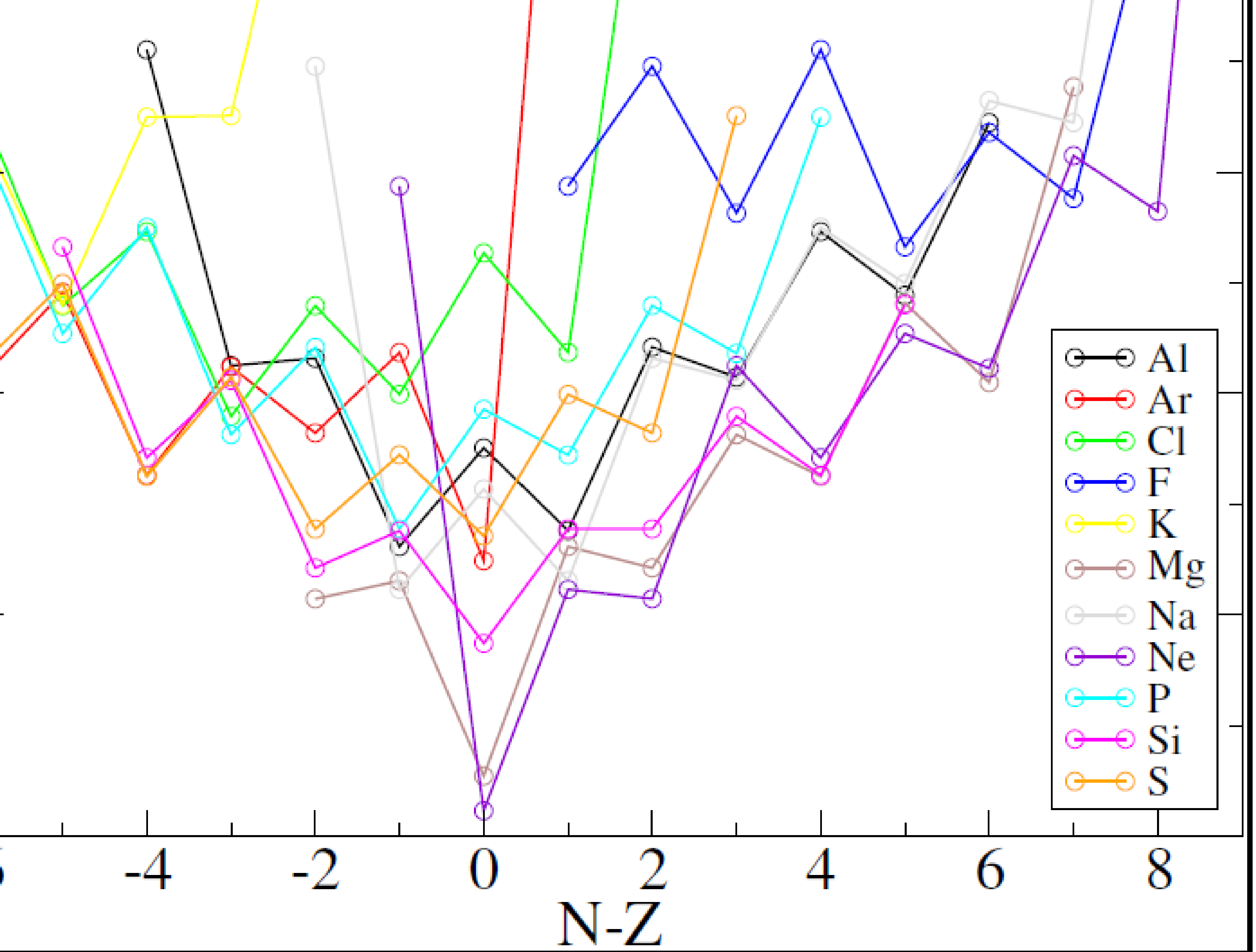
****

Neutron resonances

****

Low-lying levels

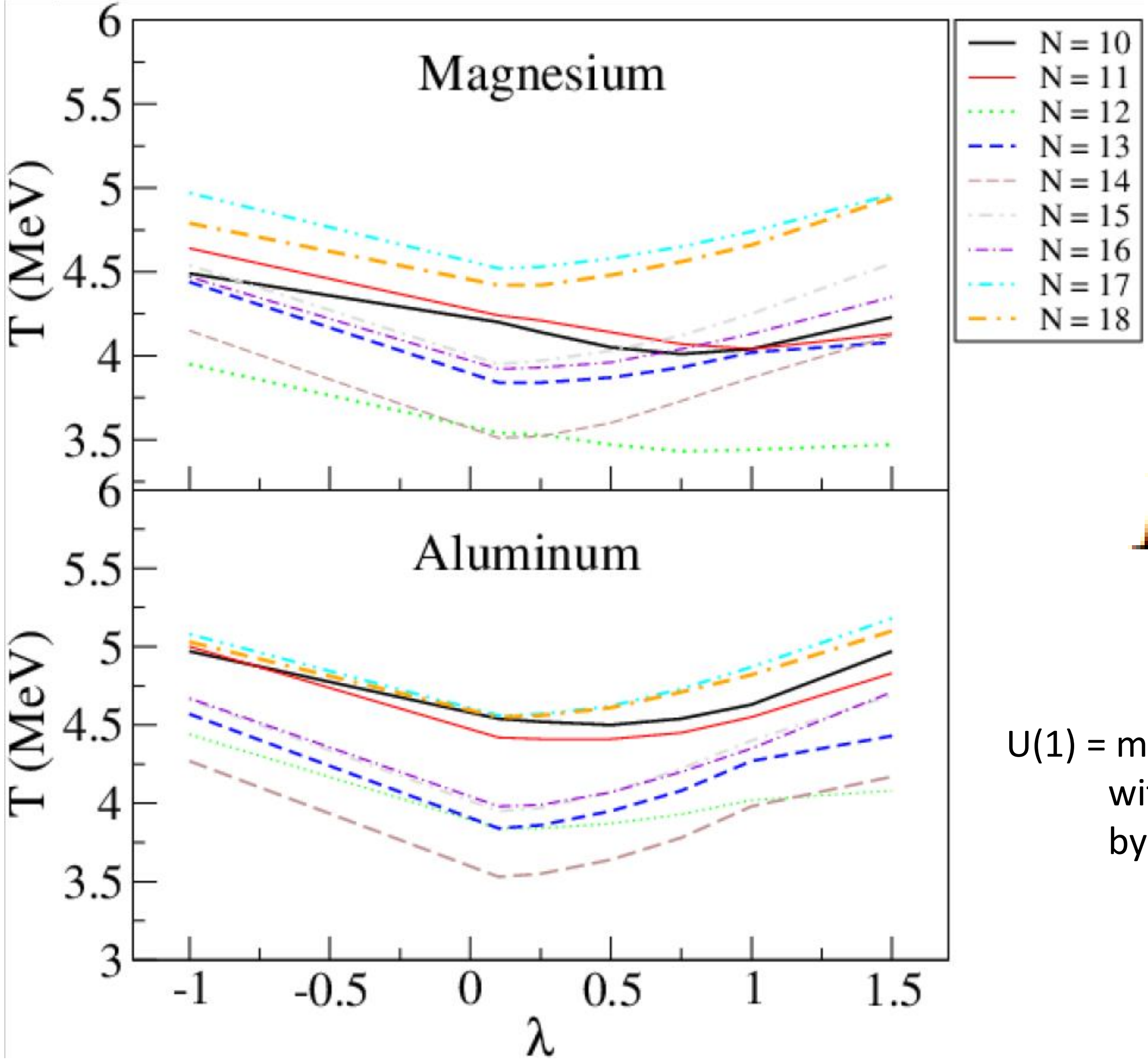




Effective temperature for the level density at low energy (up to 6 – 8 Mev)

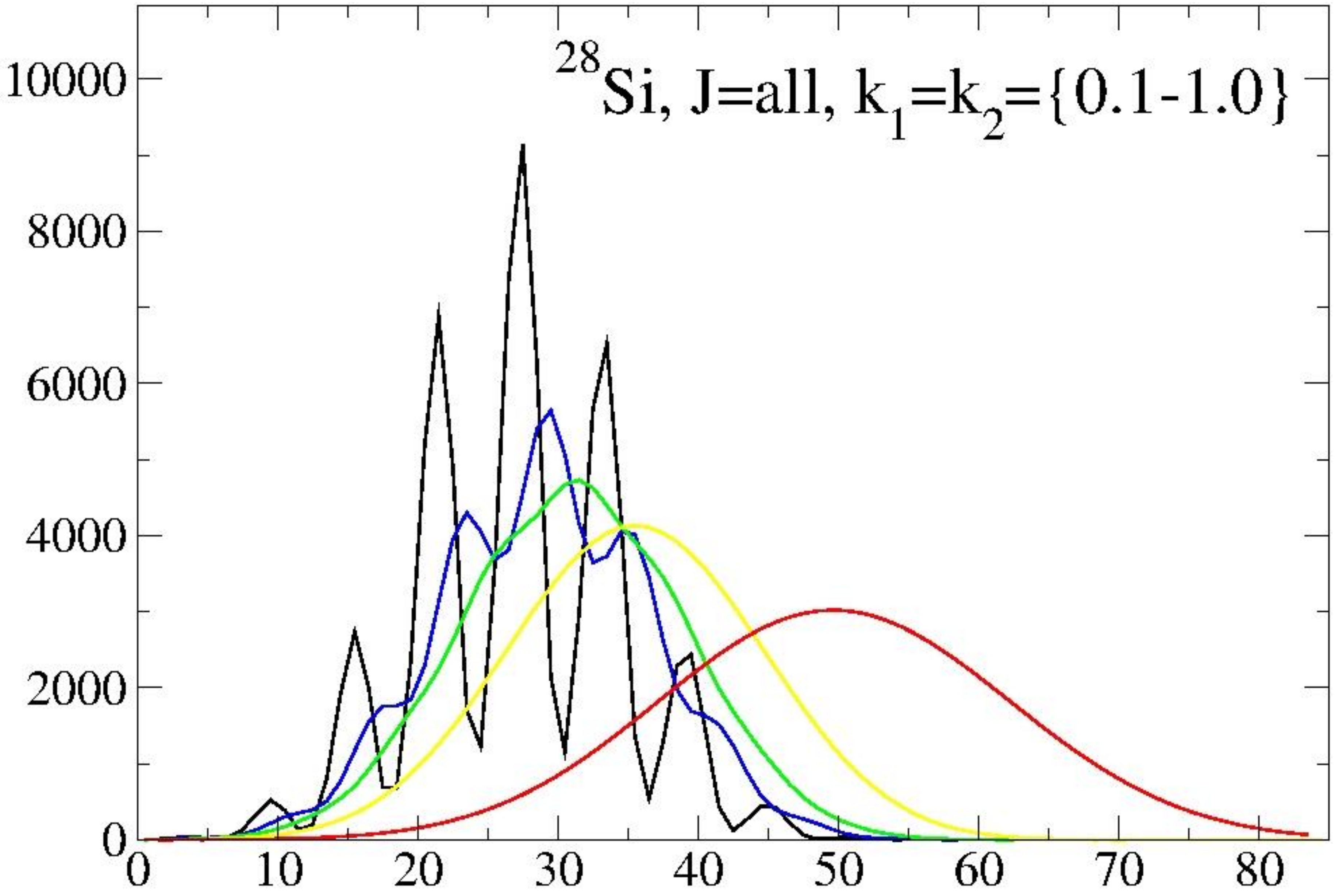
Even-odd staggering

Clear minima in the vicinity of N=Z



$$H = h + \lambda U_1 + U_2.$$

U(1) = matrix elements of the two-body interaction
with change of orbital momentum of one particle
by 2 units (the same parity) – way to deformation



^{28}Si, J=all, k_1=k_2={0.1-1.0}
10000
8000
6000
4000
2000
0
0
10
20
30
40
50
60
70
80