Threshold Phenomena In Nuclei: From Decay To Clustering

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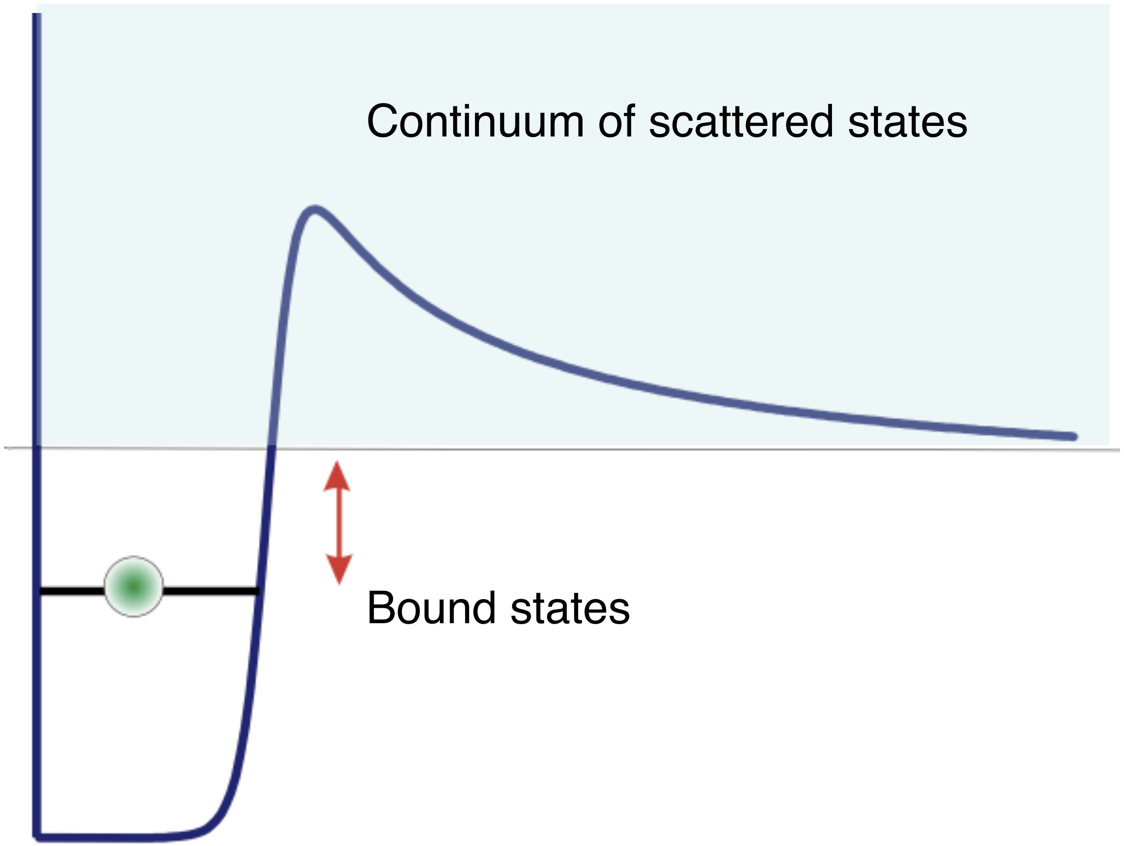
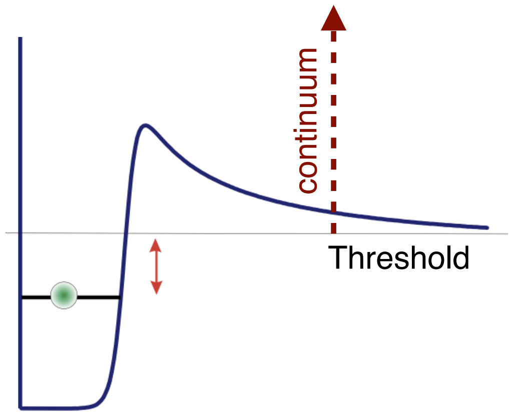
Supported by the US Department of
Energy Award number: DE-SC0009883

Perturbation from continuum



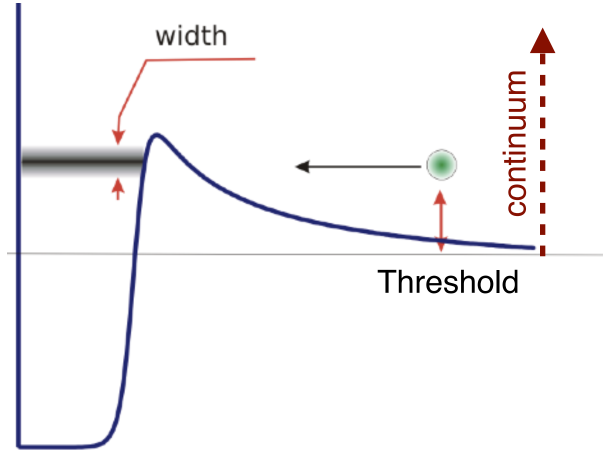
Perturbation from continuum

$$H'(\epsilon) = \int_0^\infty d\epsilon' \frac{|A(\epsilon')|^2}{\epsilon - \epsilon' + i0}.$$



Integration region involves no poles

$$H'(\epsilon) = \Delta(\epsilon) \qquad \Delta(\epsilon) = \int d\epsilon' \frac{|A(\epsilon')|^2}{\epsilon - \epsilon' + i0}$$

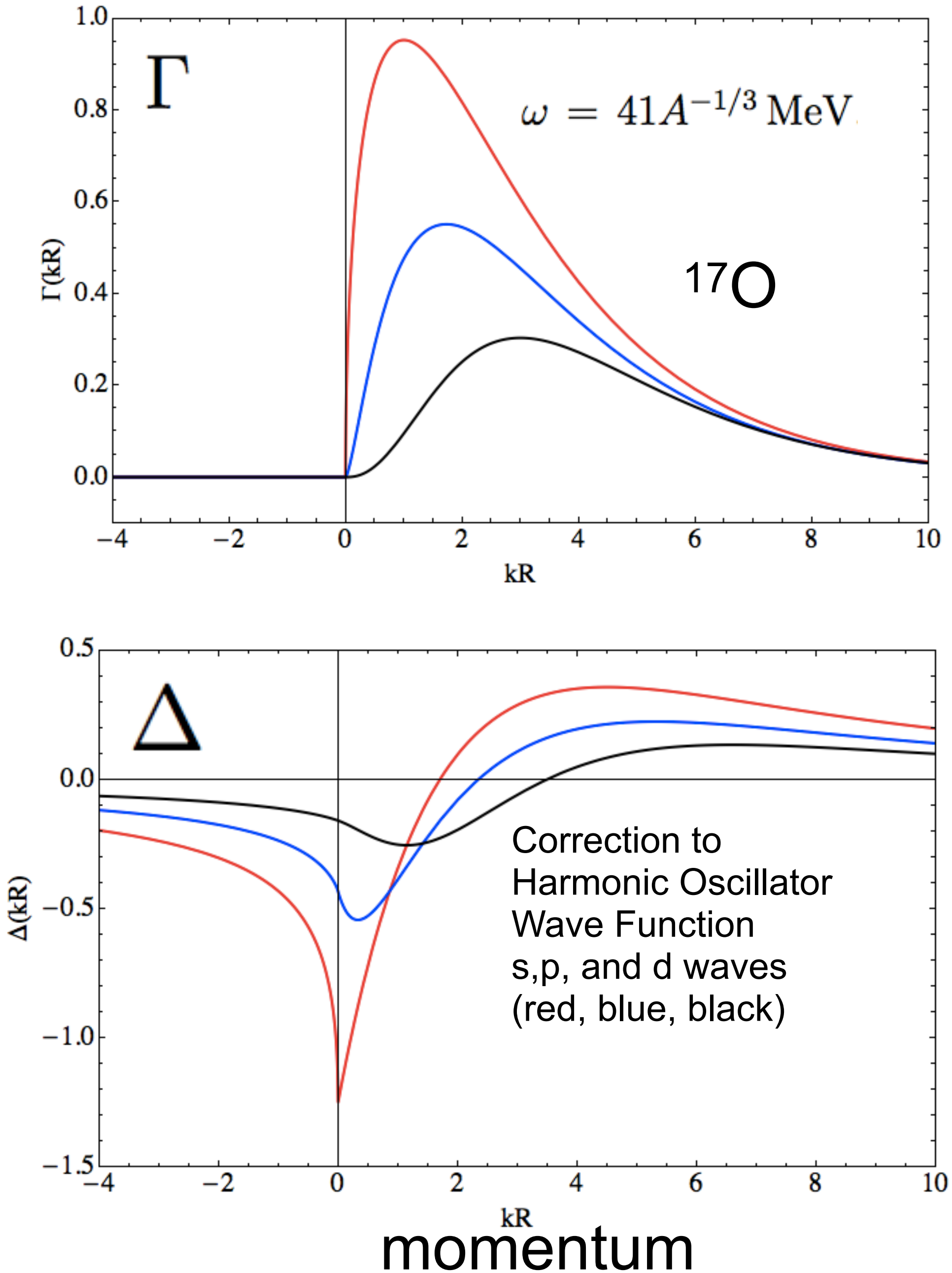


State embedded in the continuum

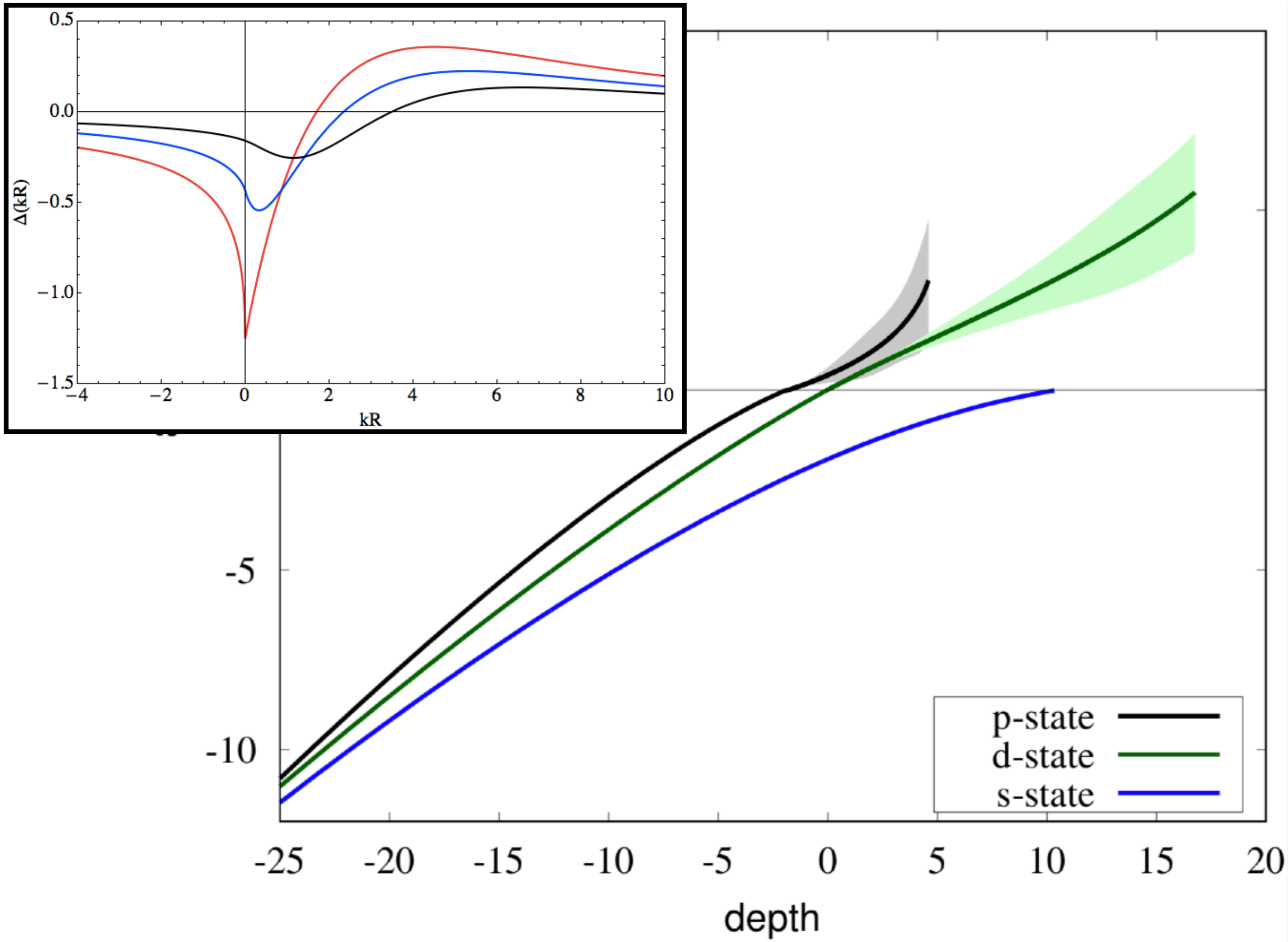
$$\frac{1}{x \pm i0} = \mathrm{p.v.}\frac{1}{x} \mp i\pi\delta(x)$$

$$H'(\epsilon) = \Delta(\epsilon) - \frac{i}{2}\Gamma(\epsilon) \quad \Gamma(\epsilon) = 2\pi\, |A(\epsilon)|^2$$

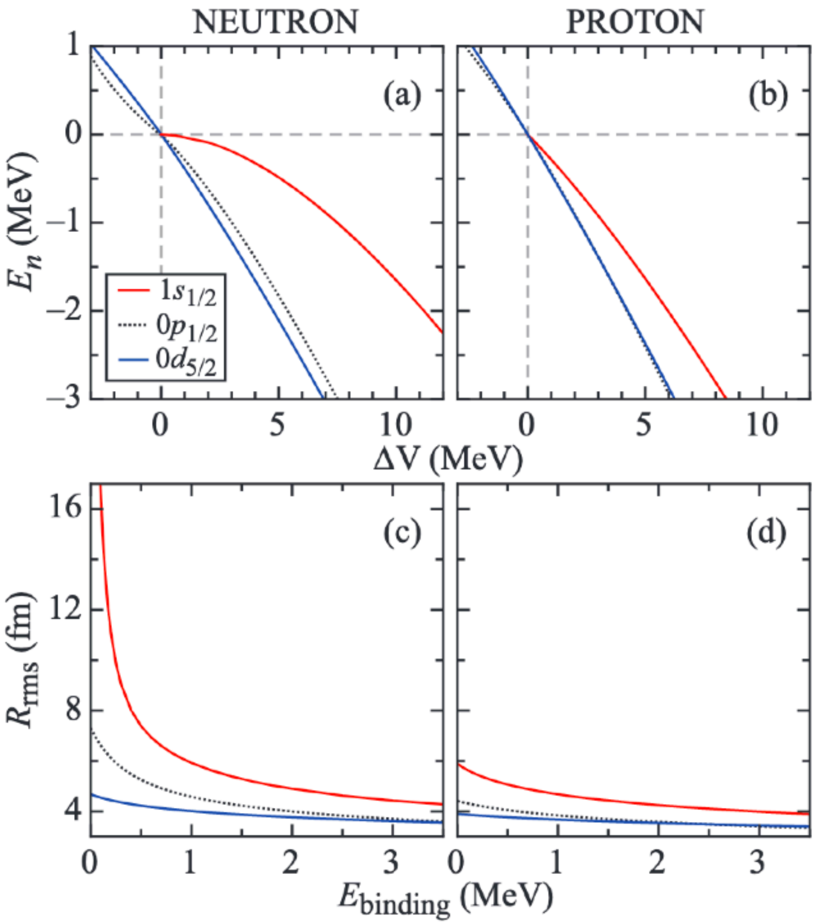
Self energy, interaction with continuum

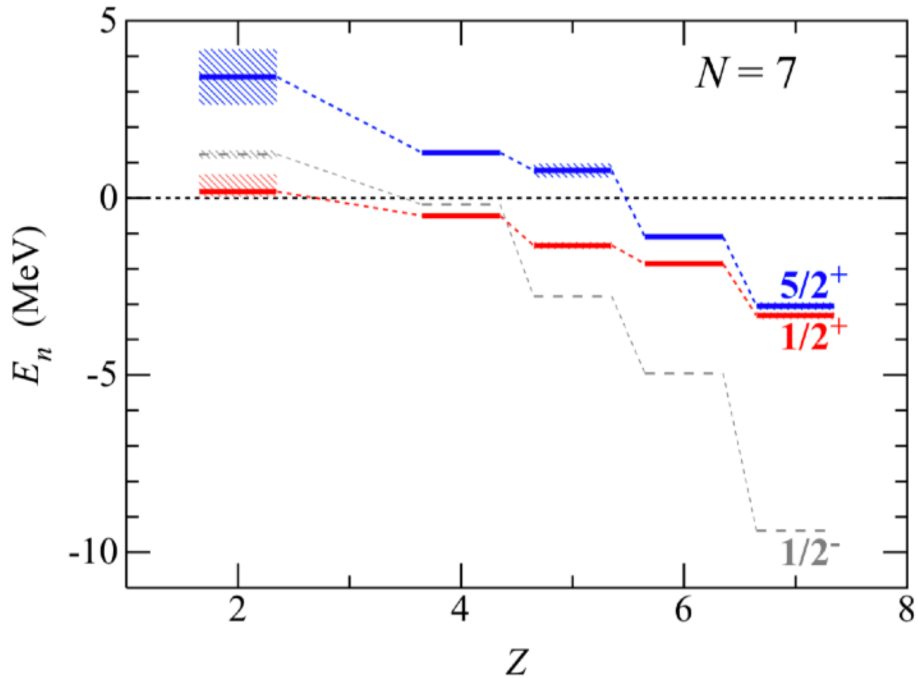


Effect of weak binding



Effect of weak binding



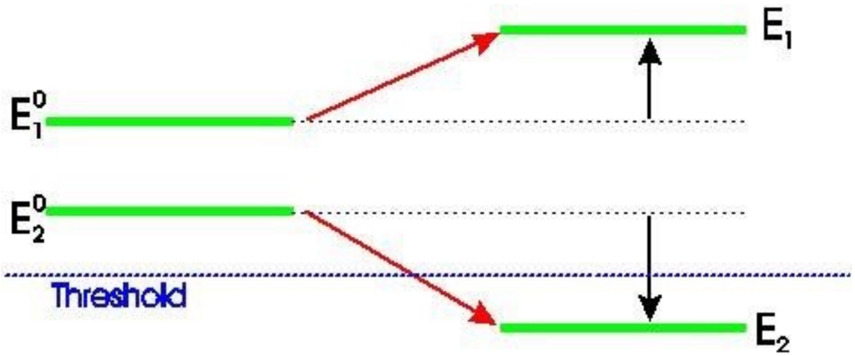


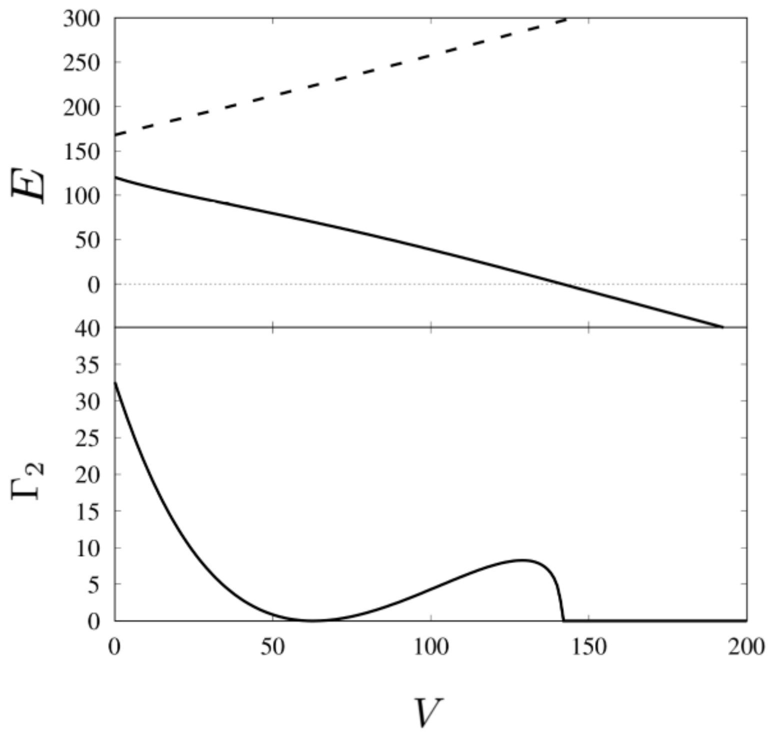
C. R. Hoffman, B. P. Kay, and J. P. Schiffer Phys. Rev. C 89, 061305(R)
B. P. Kay, C. R. Hoffman, and A. O. Macchiavelli Phys. Rev. Lett. 119, 182502

Wave function realignment
Superradiance

- Model $\mathcal{H}$

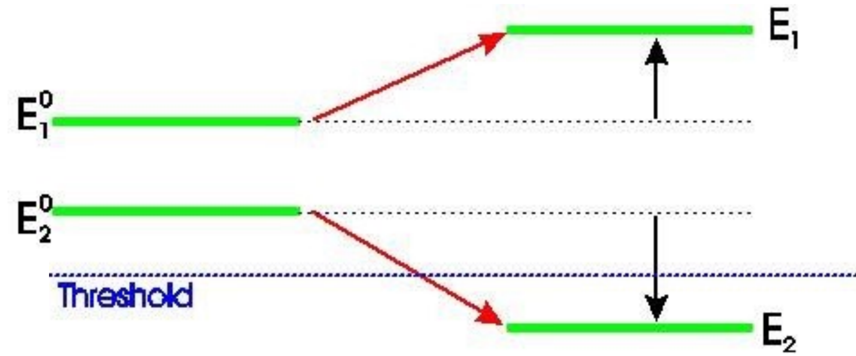
$$\mathcal{H}(E) = \begin{pmatrix} \epsilon_1 - \frac{i}{2}\gamma_1 & v - \frac{i}{2}A_1 A_2 \\ v - \frac{i}{2}A_1 A_2 & \epsilon_2 - \frac{i}{2}\gamma_2 \end{pmatrix}$$



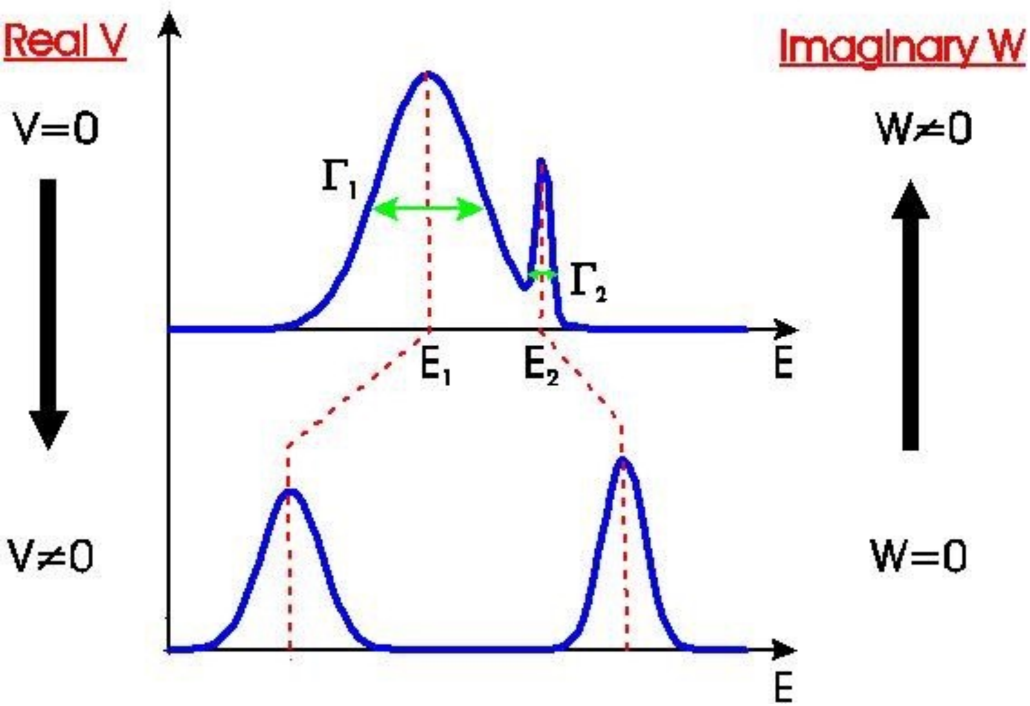


Example of interacting resonances

^{13}C, ^{13}N, ^{11}Li



$$\mathcal{H}=H^0+V - iW/2$$



Wave function realignment
Superradiance

$$H = \begin{pmatrix} \epsilon - \frac{i}{2}\Gamma & v \\ v & 0 \end{pmatrix} = H_0 - \frac{i\Gamma}{2} A^\dagger A \qquad A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Stationary system $\Gamma = 0$

Energies

$$E_{1,2} = \frac{1}{2}\left(\epsilon \pm \sqrt{\epsilon^2 + 4v^2}\right)$$

Spectroscopic Factors

$$\mathrm{SF}_{1,2} = \frac{1}{2}\left(1 \pm \frac{\epsilon}{\sqrt{\epsilon^2 + 4v^2}}\right)$$

Observing superradiance

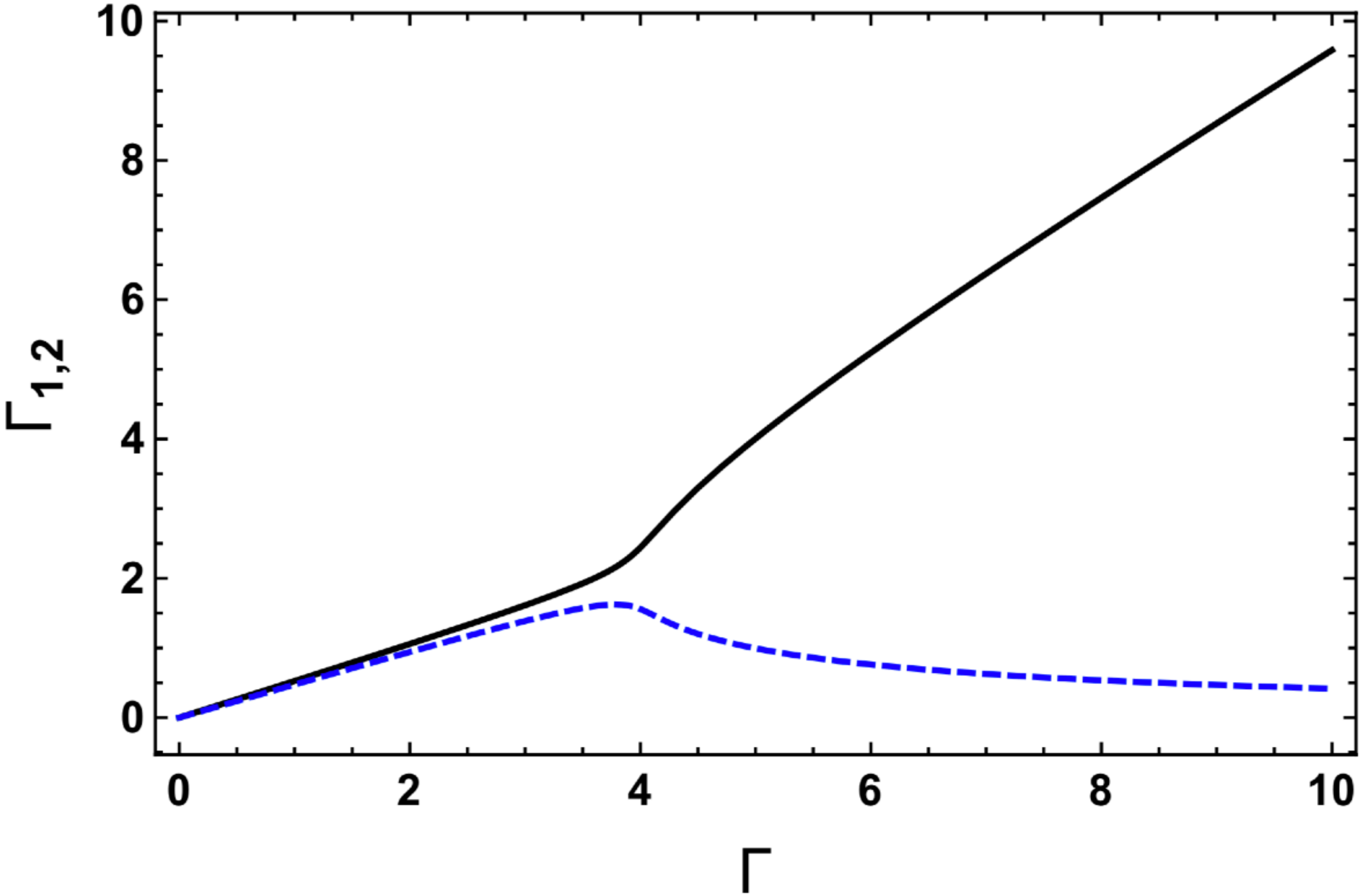
$$H = \begin{pmatrix} \epsilon - \frac{i}{2}\Gamma & v \\ v & 0 \end{pmatrix} = H_0 - \frac{i\Gamma}{2} A^\dagger A \qquad A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Energies $$\mathcal{E}_{1,2} = \frac{1}{2}\left(\epsilon - \frac{i}{2}\Gamma \pm \sqrt{\left(\epsilon - \frac{i}{2}\Gamma\right)^2 + 4v^2}\right)$$

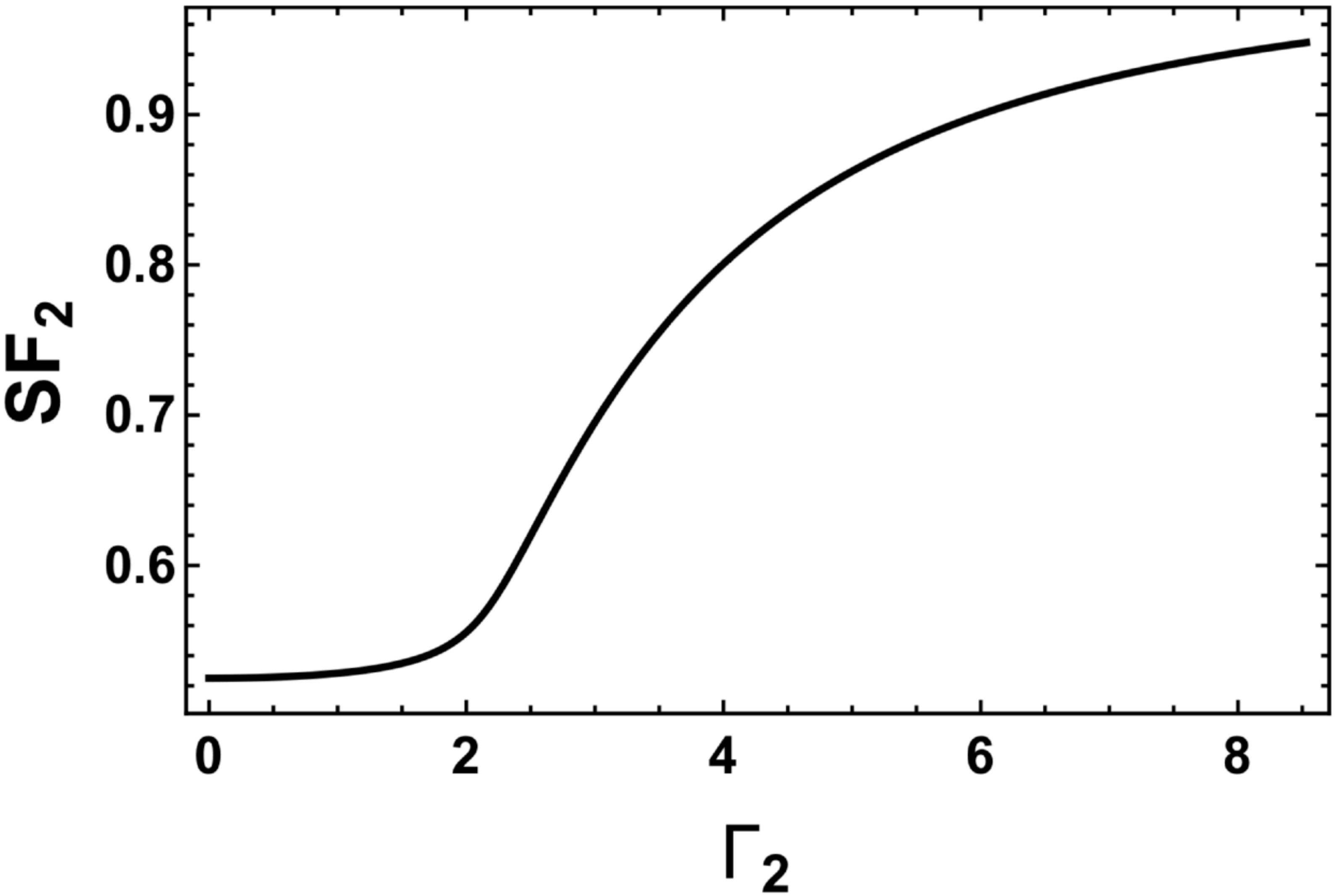
Width $$\Gamma_{1,2} = -2\,\mathrm{Im}\left(\mathcal{E}_{1,2}\right)$$

Spectroscopic Factors $$\mathrm{SF}_{1,2} = \Gamma_{1,2}/\Gamma$$

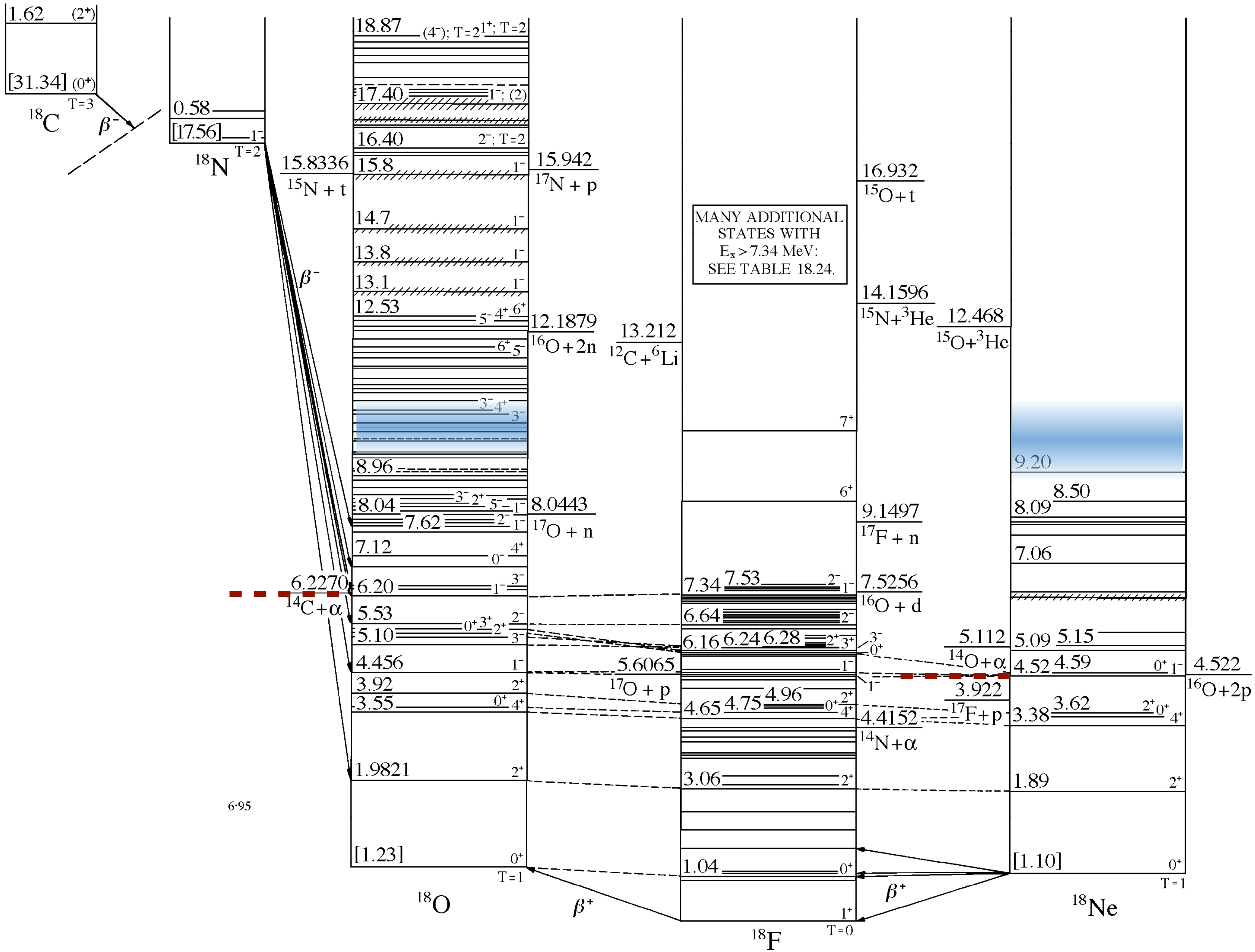
Observing superradiance



Spectroscopic factor for superradiant state

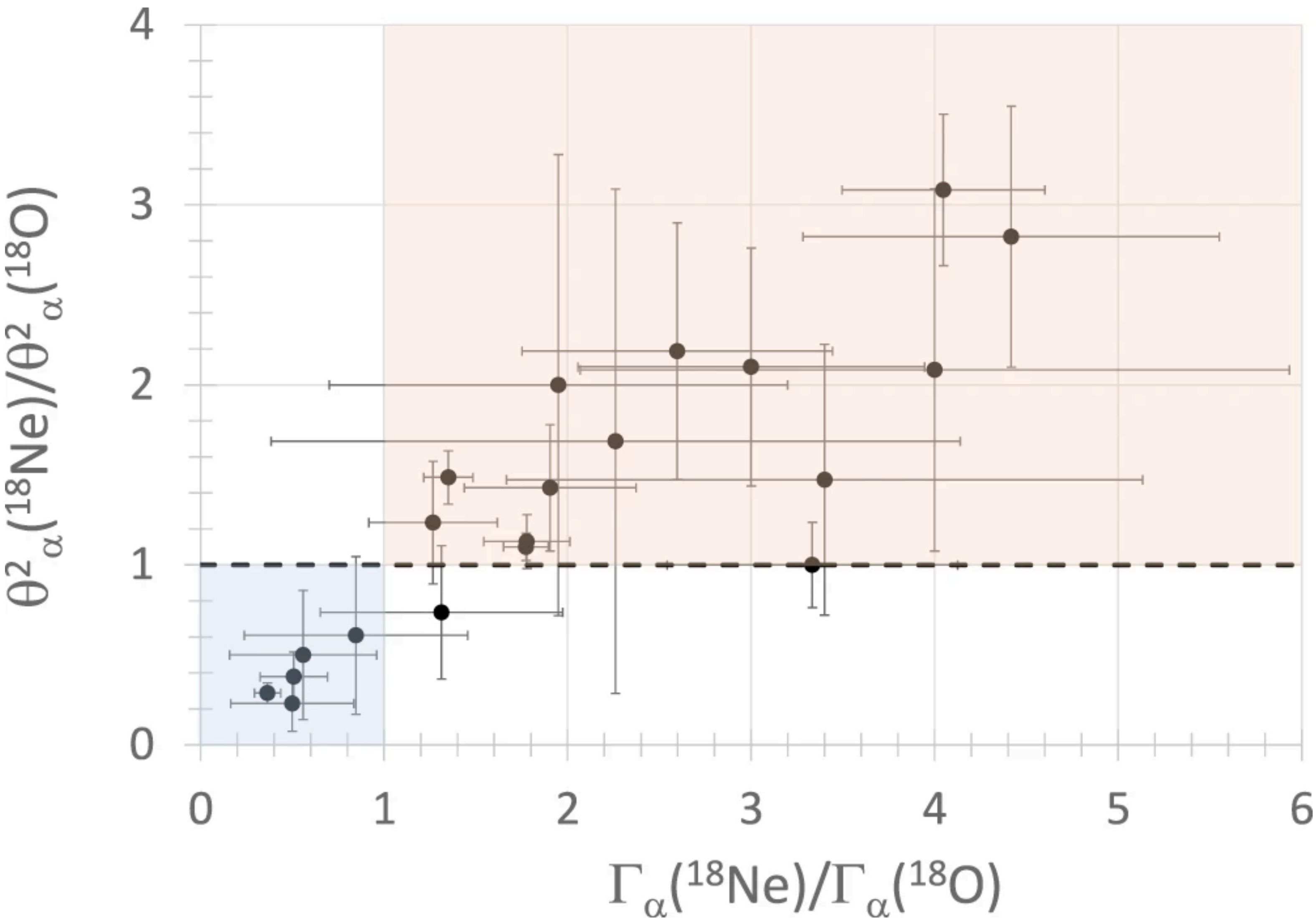


Isospin symmetry



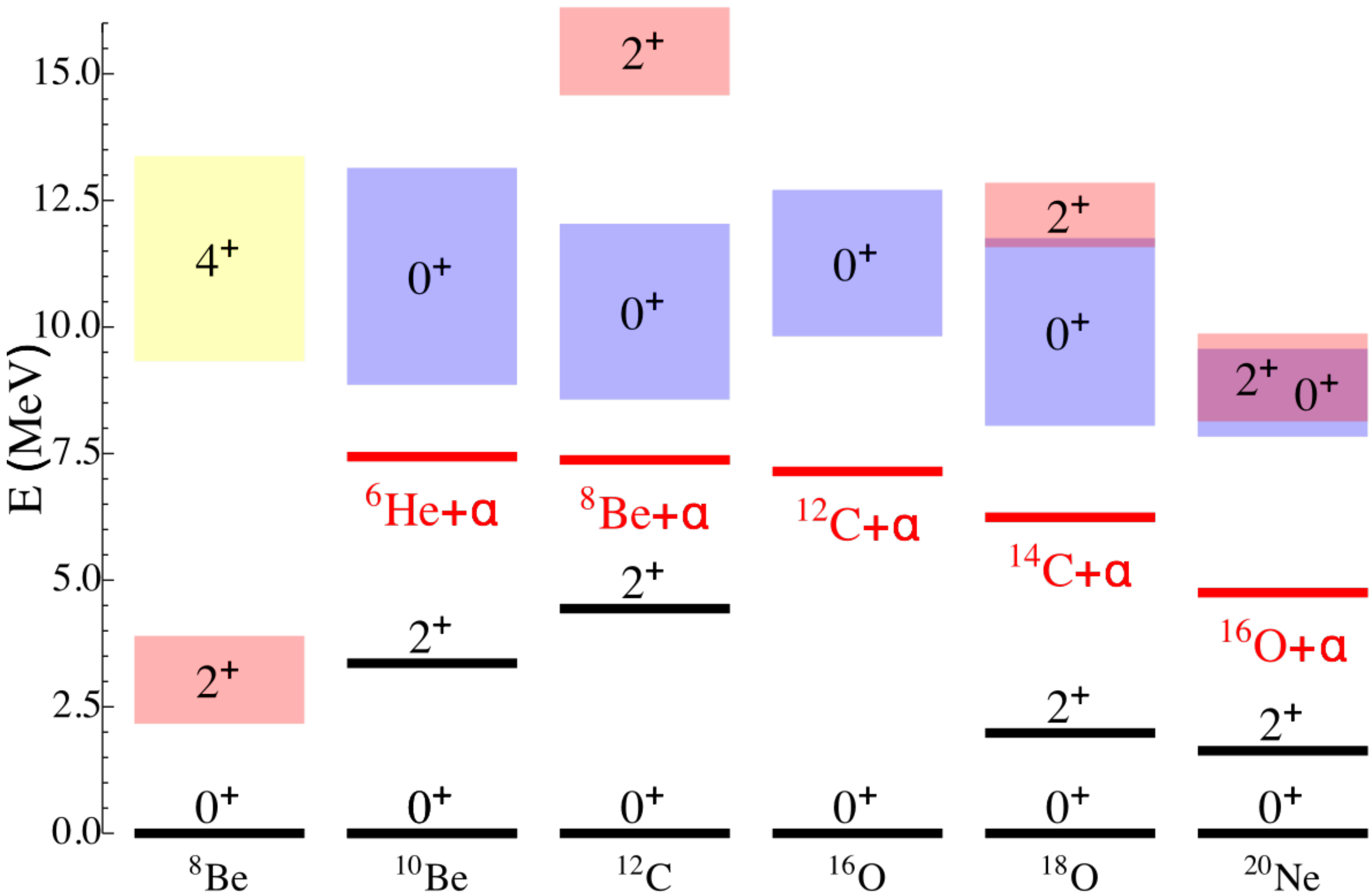
	^{18}Ne			^{18}O		
J	E (MeV)	*Γ* (keV)	SF	E (MeV)	*Γ* (keV)	SF
1^-	9.08(1)	357	**0.21(1)**	9.19(2)	200	**0.20(1)**
1^-	9.57(1)	1062	**0.51(5)**	9.76(2)	630	**0.46(4)**
1^-	10.58(4)	416	**0.15(5)**	10.8(3)	630	**0.29(4)**
1^-	13.730(2)	780	**0.2(1)**	14.3(3)	400	**0.10(4)**
2^+	9.19(3)	265	**0.21(2)**	9.79(6)	90	**0.10(3)**
2^+	10.94(6)	1302	**0.52(3)**	12.21(8)	1000	**0.37(9)**
2^+	13.4 (2)	1755	**0.45(8)**	12.8(3)	4800	**1.56(13)**
2^+	16.9(2)	1515	**0.3(2)**			
3^-				8.29(6)	2.9	**0.18(1)**
3^-	8.77(8)	419	**1.0(4)**	9.35(2)	110	**0.48(13)**
3^-	11.0(1)	497	**0.28(7)**	11.95(1)	300	**0.17(2)**
3^-	12.7(2)	2025	**0.7(2)**	12.98(4)	770	**0.32(5)**
3^-	14.8(2)	3967	**1.0(2)**	14.0(2)	2100	**0.7(1)**
4^+	8.16	31	**0.8(3)**	7.11*		
4^+	13.3(3)	845	**0.37(4)**	13.46(2)	210	**0.12(1)**
4^+	14.15(21)	375	**0.14(10)**	14.77(5)	680	**0.28(2)**
5^-	11.31(4)	15	0.03(2)	11.63(1)	30	**0.13(1)**
5^-	12.9(2)	532	**0.48(12)**	13.08(1)	120	**0.17(1)**
5^-	13.79(8)	219	**0.14(10)**	14.1(1)	260	**0.23(2)**
5^-	14.6(7)	521	**0.27(20)**	14.7(1)	230	**0.16(6)**
6^+	11.8(2)	54	**0.30(7)**	11.69(5)	12	**0.23(1)**
6^+	12.4(2)	167	**0.56(26)**	12.57(1)	50	**0.38(8)**

M. Barbui *et al.*, "α-cluster structure of ^{18}Ne," *Phys. Rev. C*, vol. 106, no. 5, p. 054310, Nov. 2022, doi: 10.1103/PhysRevC.106.054310.



A. Volya, M. Barbui, V. Z. Goldberg, and G. V. Rogachev, "Superradiance in alpha clustered mirror nuclei," *Commun Phys*, vol. 5, no. 1, Art no. 1, Dec. 2022, doi: [10.1038/s42005-022-01105-9](https://doi.org/10.1038/s42005-022-01105-9).

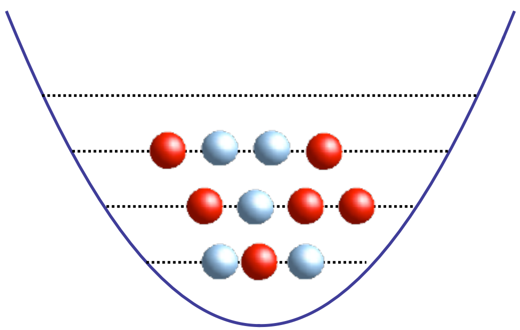
Clustering and continuum



Configuration interaction approach and clustering

Traditional shell model configuration m-scheme

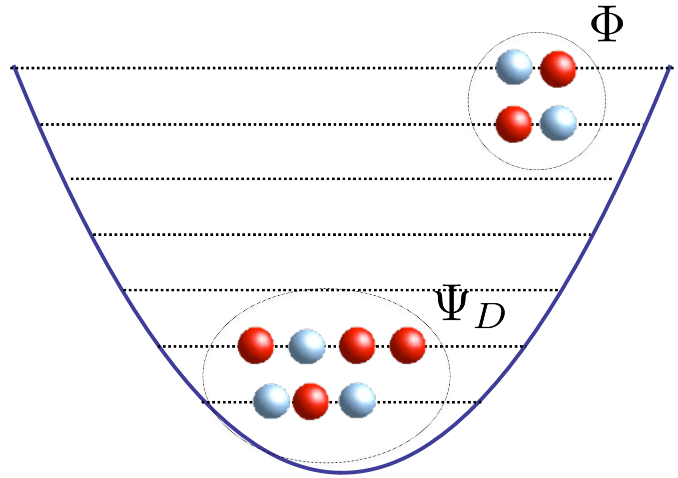
$$|\Psi\rangle = \Psi^\dagger|0\rangle \sim a_1^\dagger a_2^\dagger \dots a_A^\dagger|0\rangle$$



+

Cluster configuration

$$|\text{channel}\rangle \sim |\Phi\Psi_D\rangle \equiv \Phi^\dagger \Psi_D^\dagger|0\rangle$$



$|\Psi\rangle$ + $\Phi^\dagger|\Psi_D\rangle$

Translational invariance and Center of Mass (CM)

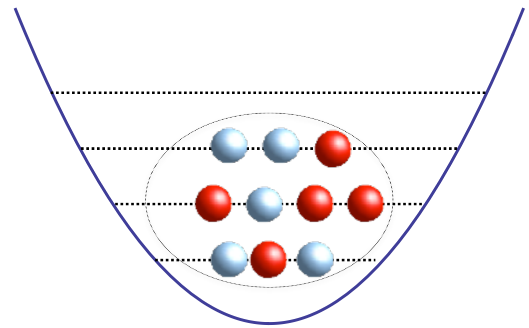
Shell model, Glockner-Lawson procedure

$$\Psi_D = \phi_{000}(\mathbf{R}_D)\,\Psi'_D$$

SM state

Center-of-mass vibration

Intrinsic state



Controlling CM with operator $\mathbf{R}$

Control only CM quanta

$$D_\mu = \sqrt{\frac{4\pi}{3}} R_\mu \qquad R_\mu = \sqrt{\frac{\hbar}{2Am\omega}}(\mathcal{B}^\dagger_\mu + \mathcal{B}_\mu)$$

K. Kravvaris and A. Volya, "Study of nuclear clustering from an ab initio perspective," *Phys. Rev. Lett.*, vol. 119, no. 6, p. 062501, 2017.

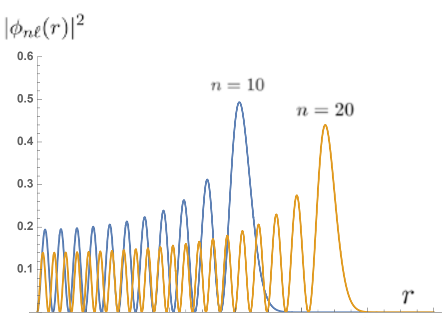
K. Kravvaris and A. Volya, "Clustering in structure and reactions using configuration interaction techniques," *Phys. Rev. C*, vol. 100, no. 3, p. 034321, Sep. 2019, doi: 10.1103/PhysRevC.100.034321.

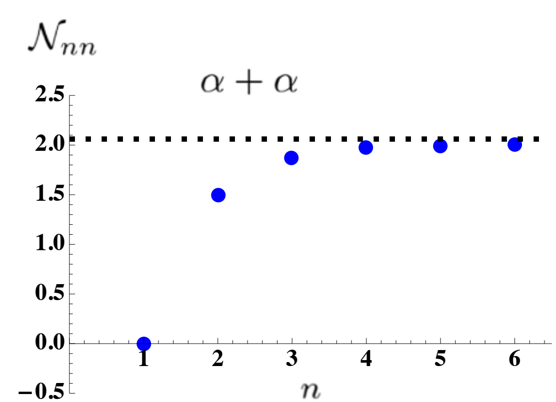
Resonating group method and reactions

$$\sum_n \mathcal{H}^{(\ell)}_{nn'} \chi_{n'} = E \sum_n \mathcal{N}^{(\ell)}_{nn'} \chi_{n'}$$

$$\begin{pmatrix} \mathcal{H}_{00} & \dots & \mathcal{H}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{H}_{n0} & \dots & \mathcal{H}_{nn} & T_{nn+1} & 0 & \vdots \\ 0 & 0 & T_{n+1n} & T_{n+1n+1} & T_{n+1n+2} & 0 \\ 0 & \dots & 0 & T_{n+2n+1} & T_{n+2n+2} & \ddots \\ 0 & \dots & \dots & 0 & \ddots & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix} = E \begin{pmatrix} \mathcal{N}_{00} & \dots & \mathcal{N}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{N}_{n0} & \dots & \mathcal{N}_{nn} & 0 & 0 & \vdots \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \dots & 0 & 0 & 1 & 0 \\ 0 & \dots & \dots & 0 & 0 & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix}$$

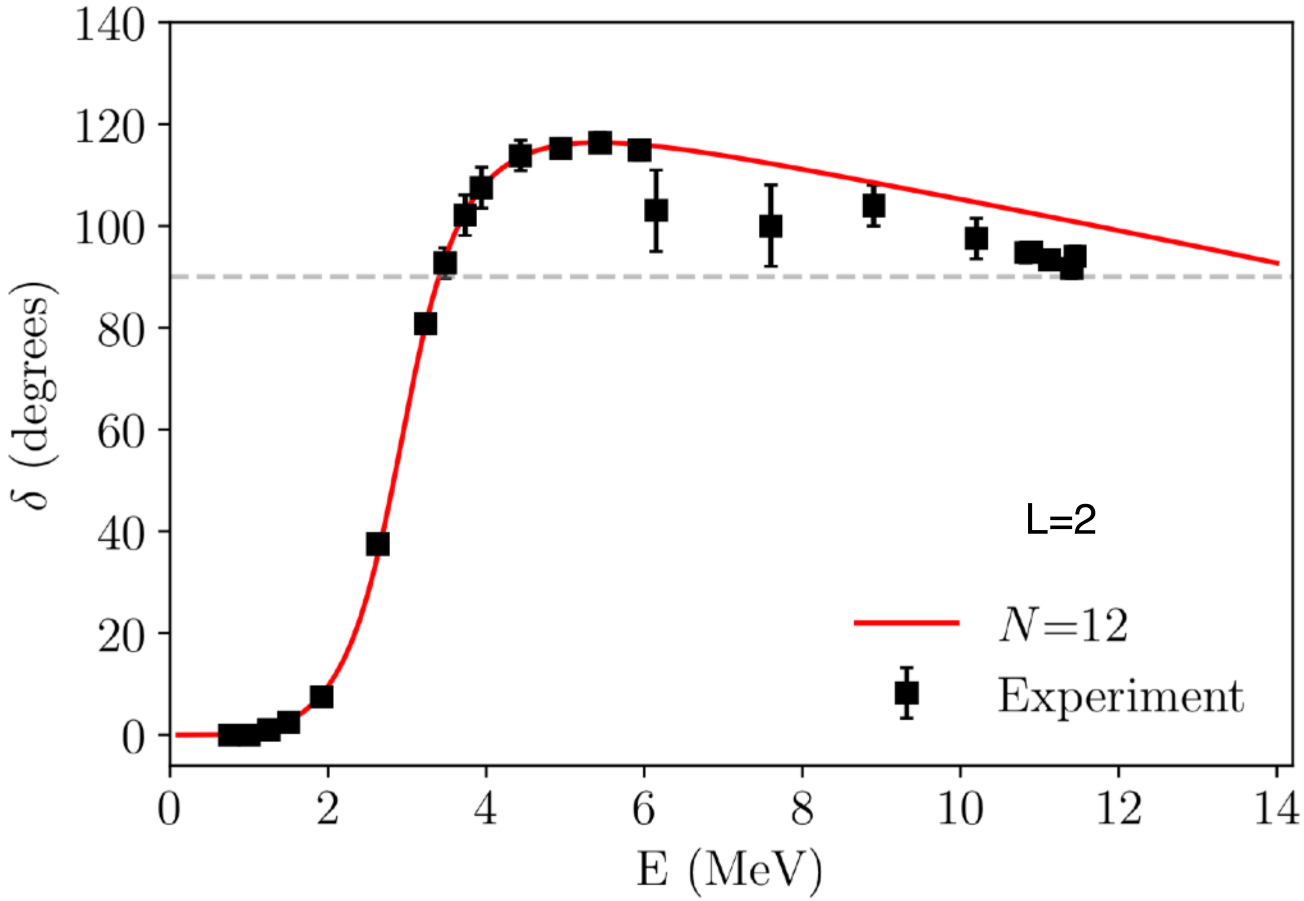
Asymptotic solution with phase shift





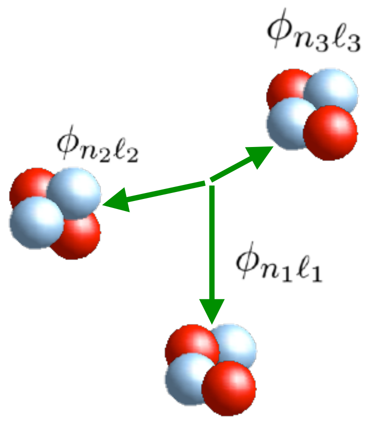
J-matrix (or HORSE) method: J. M. Bang, Annals of Physics **280**, 299 (2000)
Experimental data: Phys. Rev. 168, 1114 (1968); Nucl. Phys. **A287**, 317 (1977)

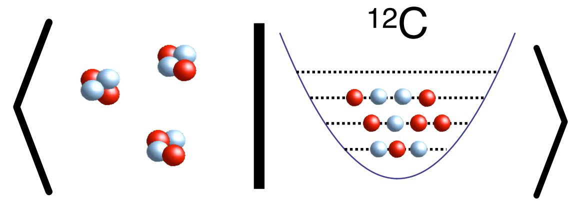
alpha+alpha scattering phase shifts



Experimental data from S. A. Afzal, A. A. Z. Ahmad, and S. Ali, Rev. Mod. Phys. 41, 247 (1969).

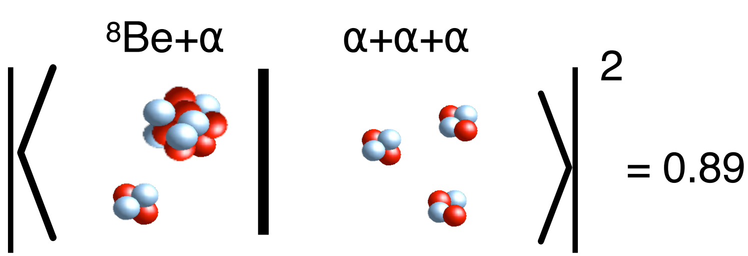
Ttriple-alpha RGM





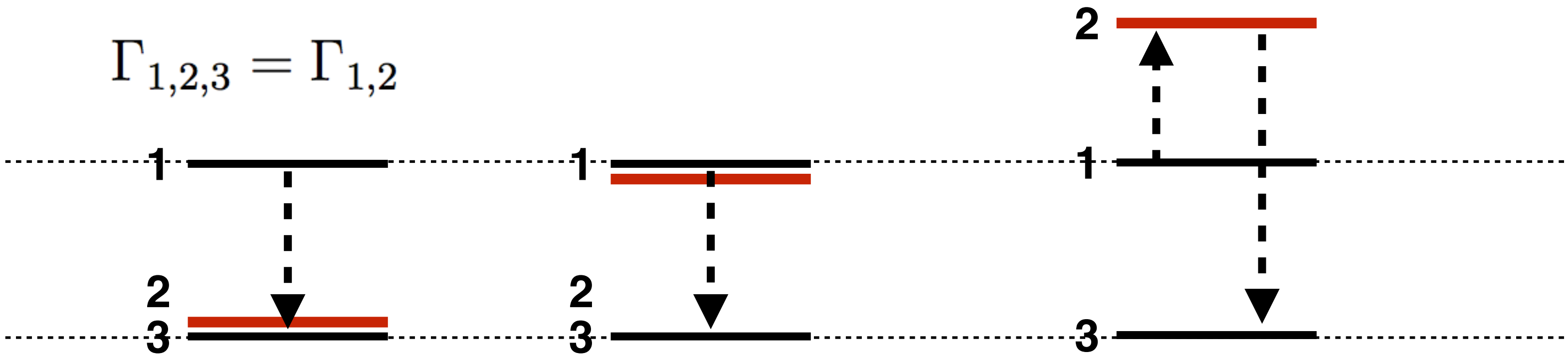
parent	channel	overlap
$^{12}\mathrm{C}[4](0_1^+)$	$\alpha[0]+\alpha[0]+\alpha[0]$	0.841
$^{12}\mathrm{C}[4](0_2^+)$	$\alpha[0]+\alpha[0]+\alpha[0]$	0.229

N_{max}(rel)=12



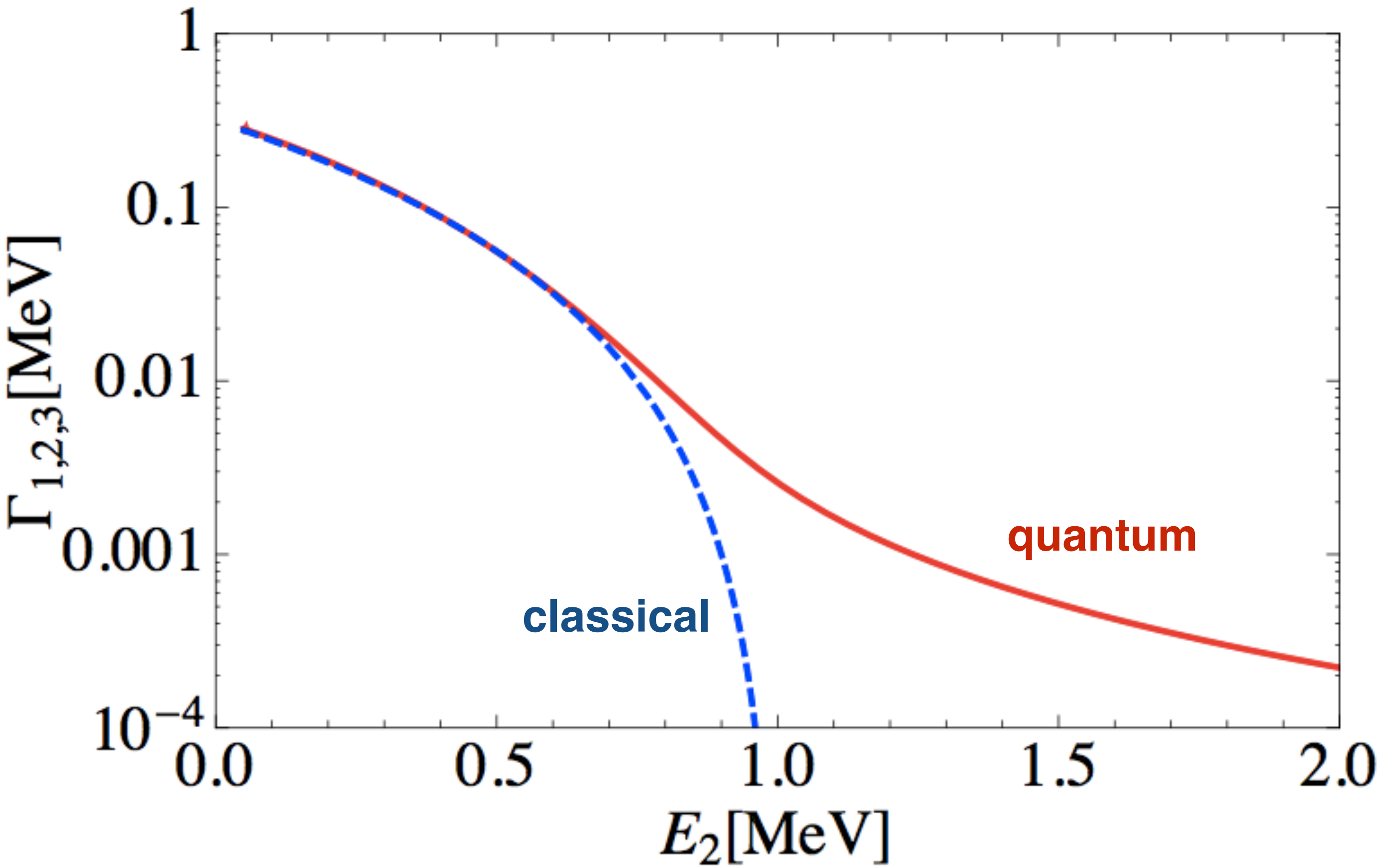
Sequential mechanism

$\Gamma_{1,2,3} = \Gamma_{1,2}$

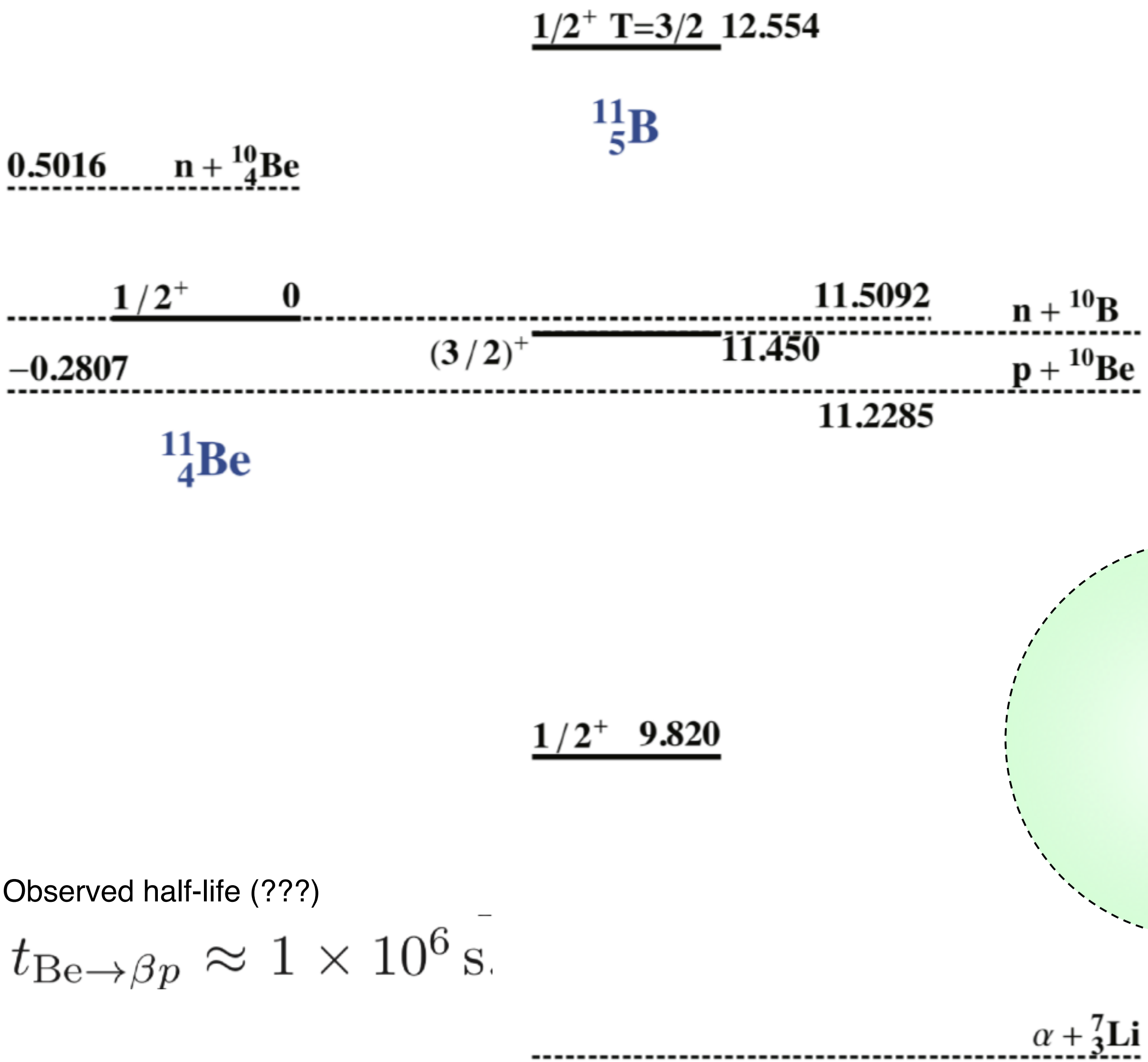


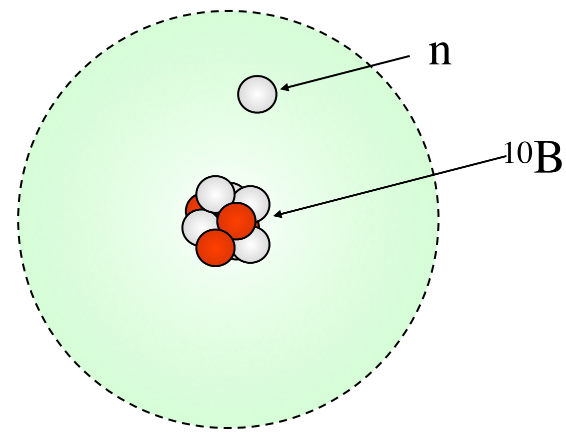
Classical limit

Virtual process



11Be beta-delayed proton decay

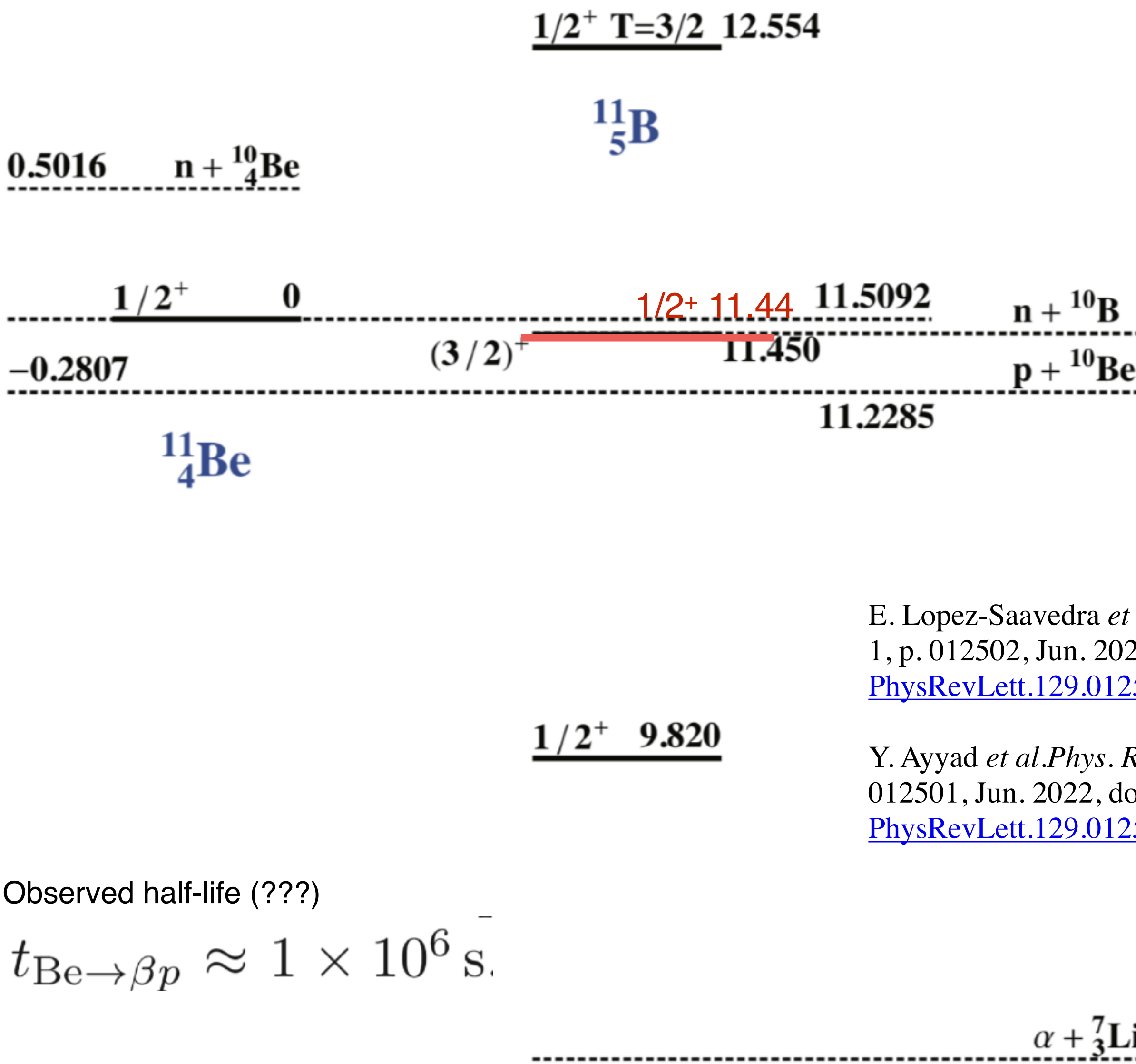




Observed half-life (???)

$$t_{Be \to \beta p} \approx 1 \times 10^6 \text{ s.}$$

11Be beta-delayed proton decay

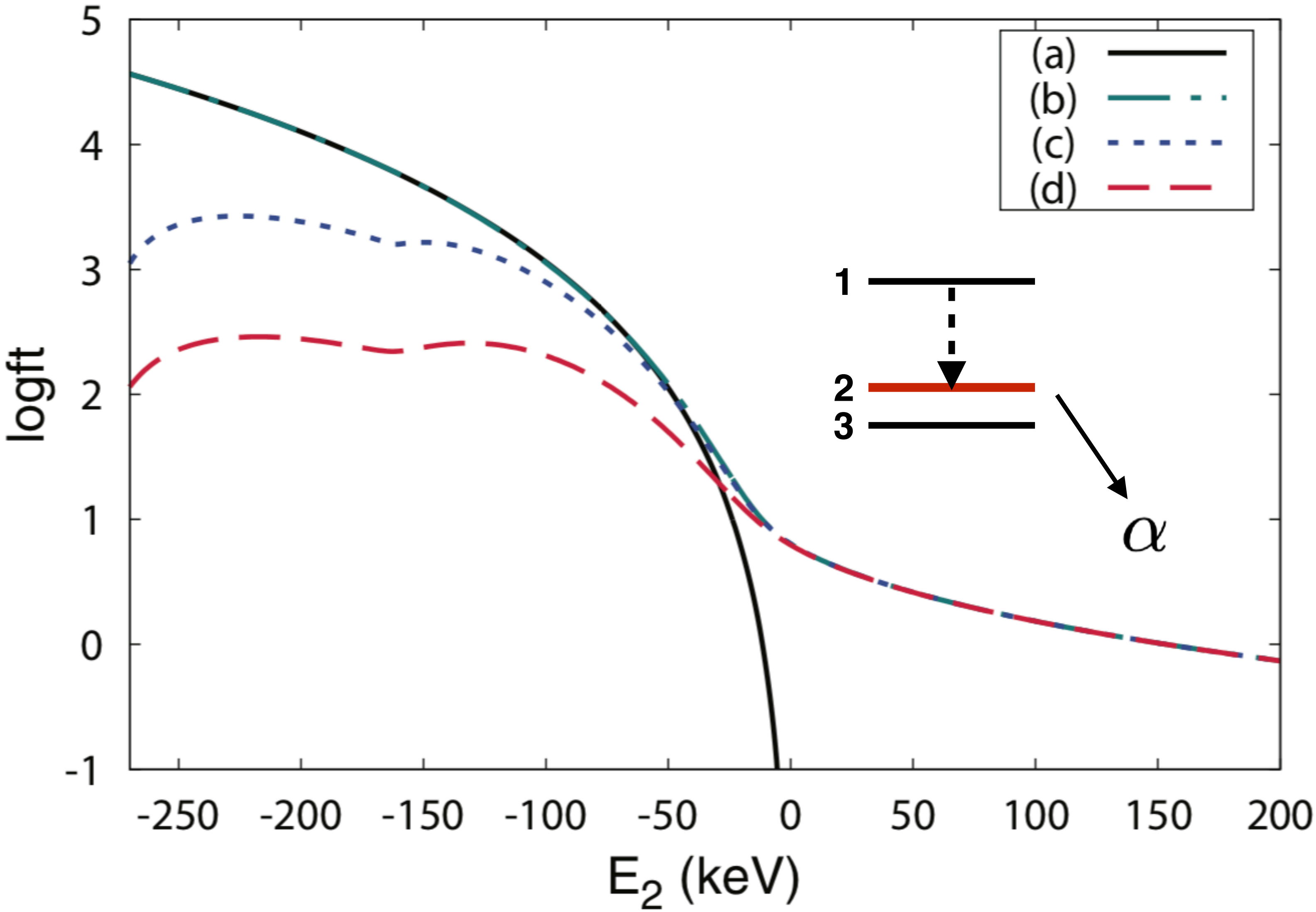


E. Lopez-Saavedra *et al*., *Phys. Rev. Lett*., vol. 129, no. 1, p. 012502, Jun. 2022, doi: 10.1103/PhysRevLett.129.012502.

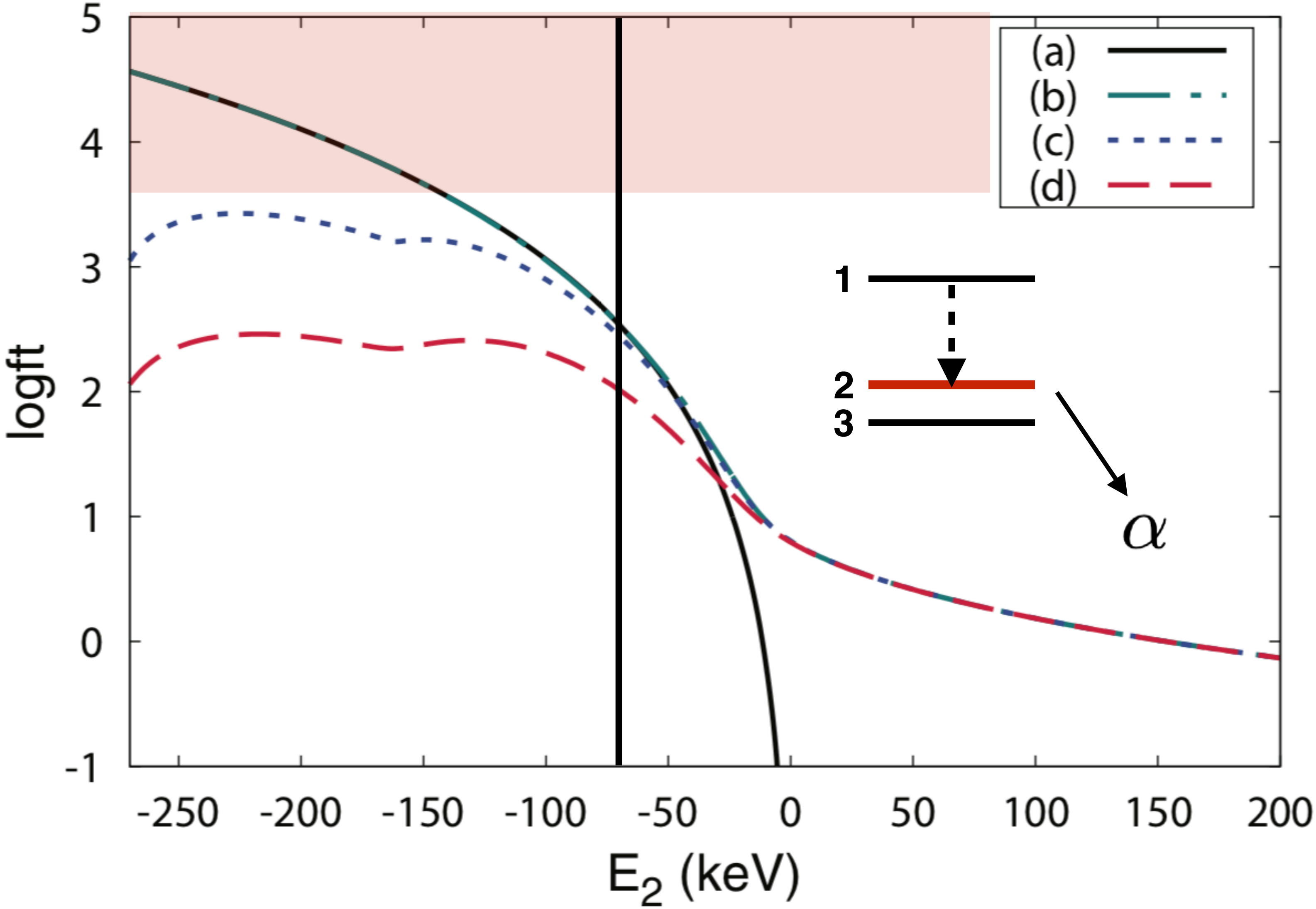
Y. Ayyad *et al*.*Phys. Rev. Lett*., vol. 129, no. 1, p. 012501, Jun. 2022, doi: 10.1103/PhysRevLett.129.012501.

Observed half-life (???)

$$t_{\mathrm{Be}\to\beta p} \approx 1 \times 10^{6}\ \mathrm{s}.$$



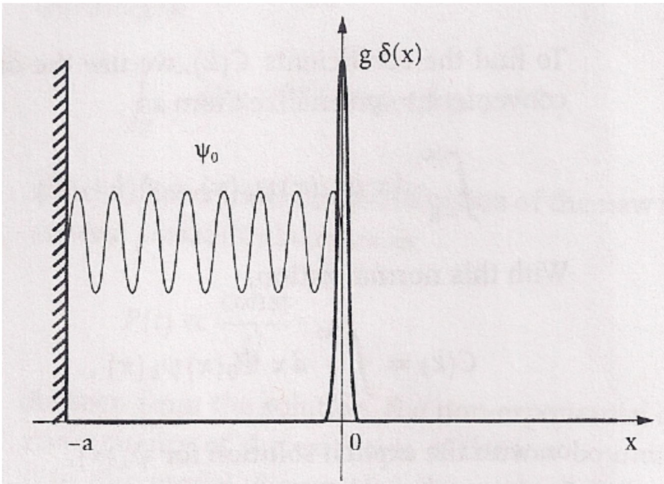
5
4
3
2
1
0
-1
logft
-250
-200
-150
-100
-50
0
50
100
150
200
E_2 (keV)
(a)
(b)
(c)
(d)
1
2
3
α

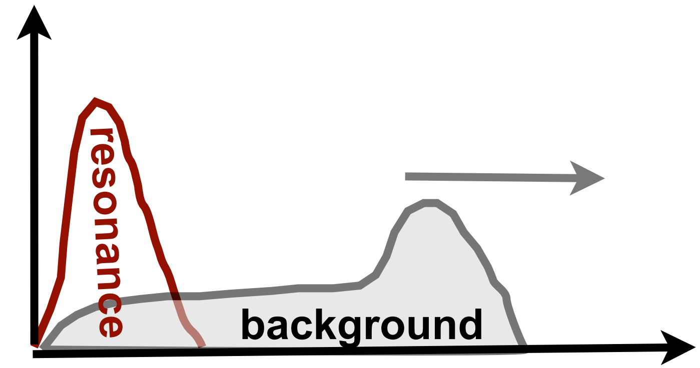


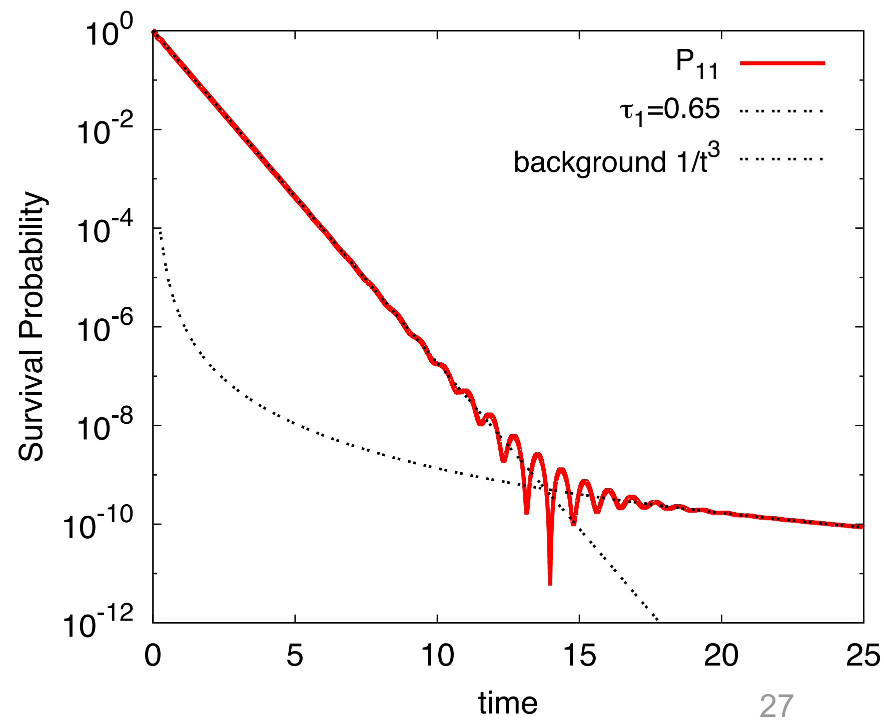
A. Volya, "Assessment of the beta-delayed proton decay rate of 11Be," *EPL*, vol. 130, no. 1, p. 12001, 2020, doi: [10.1209/0295-5075/130/12001](https://doi.org/10.1209/0295-5075/130/12001).

Time dependence of decay

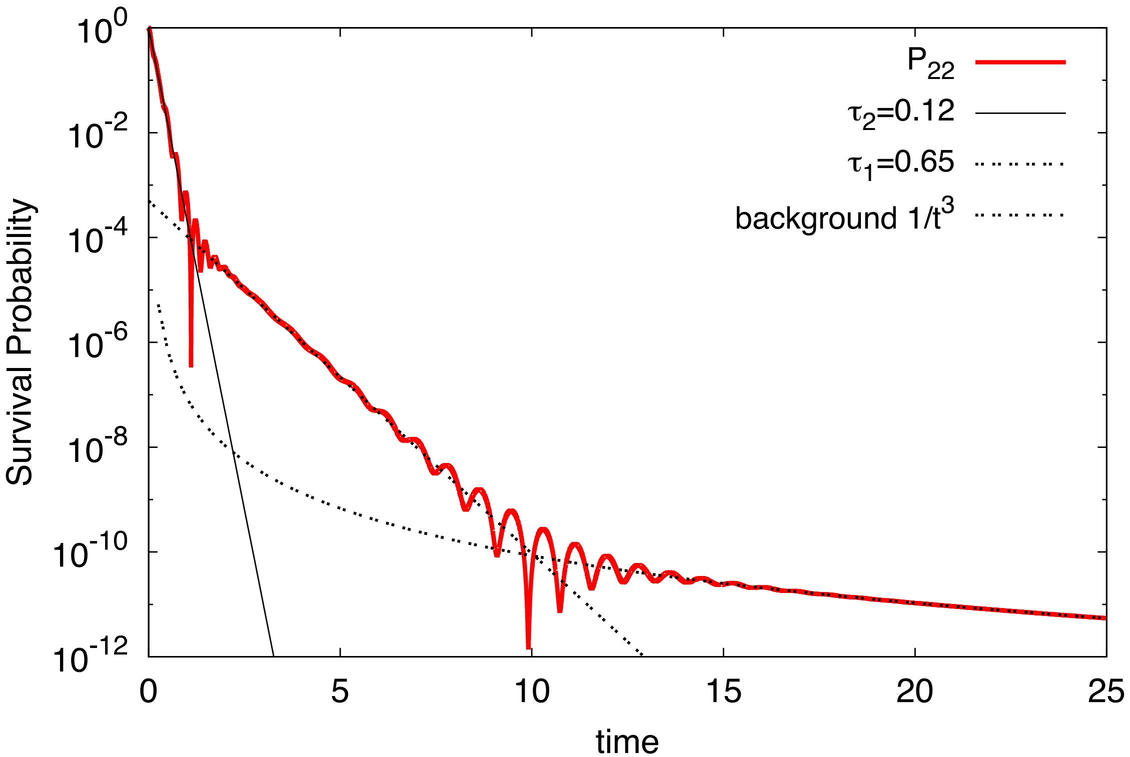
Winter, Phys. Rev., ***123****,1503 1961.*

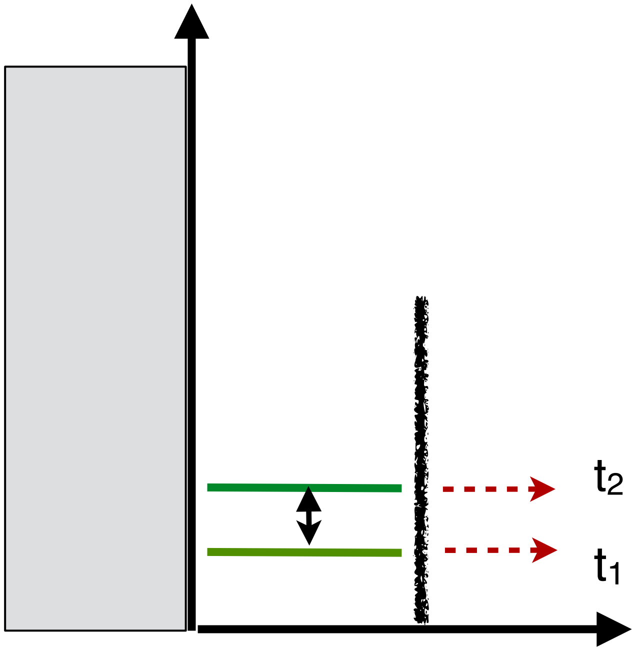






Internal dynamics in decaying system
Winter's model





M. Peshkin, A. Volya, and V. Zelevinsky, "Non-exponential and oscillatory decays in quantum mechanics," *EPL*, vol. 107, no. 4, p. 40001, 2014.

Time-dependent
Continuum Shell Model Approach

- Expand Using evolution operator in Chebyshev polynomials

$$\exp(-iHt) = \sum_{n=0}^{\infty} (-i)^n (2-\delta_{n0}) J_n(t) T_n(H)$$

- Chebyshev polynomial $T_n[\cos(\theta)] = \cos(n\theta)$
- Use iterative relation and matrix-vector multiplication to generate

$$|\lambda_n\rangle = T_n(H)|\lambda\rangle$$

$$|\lambda_0\rangle = |\lambda\rangle, \quad |\lambda_1\rangle = H|\lambda\rangle \;\; |\lambda_{n+1}\rangle = 2H|\lambda_n\rangle - |\lambda_{n-1}\rangle$$

$$\langle\lambda'|T_{n+m}(H)|\lambda\rangle = 2\langle\lambda'_m|\lambda_n\rangle - \langle\lambda'|\lambda_{n-m}\rangle, \quad n \geqslant m$$

- Use FFT to find return to energy representation

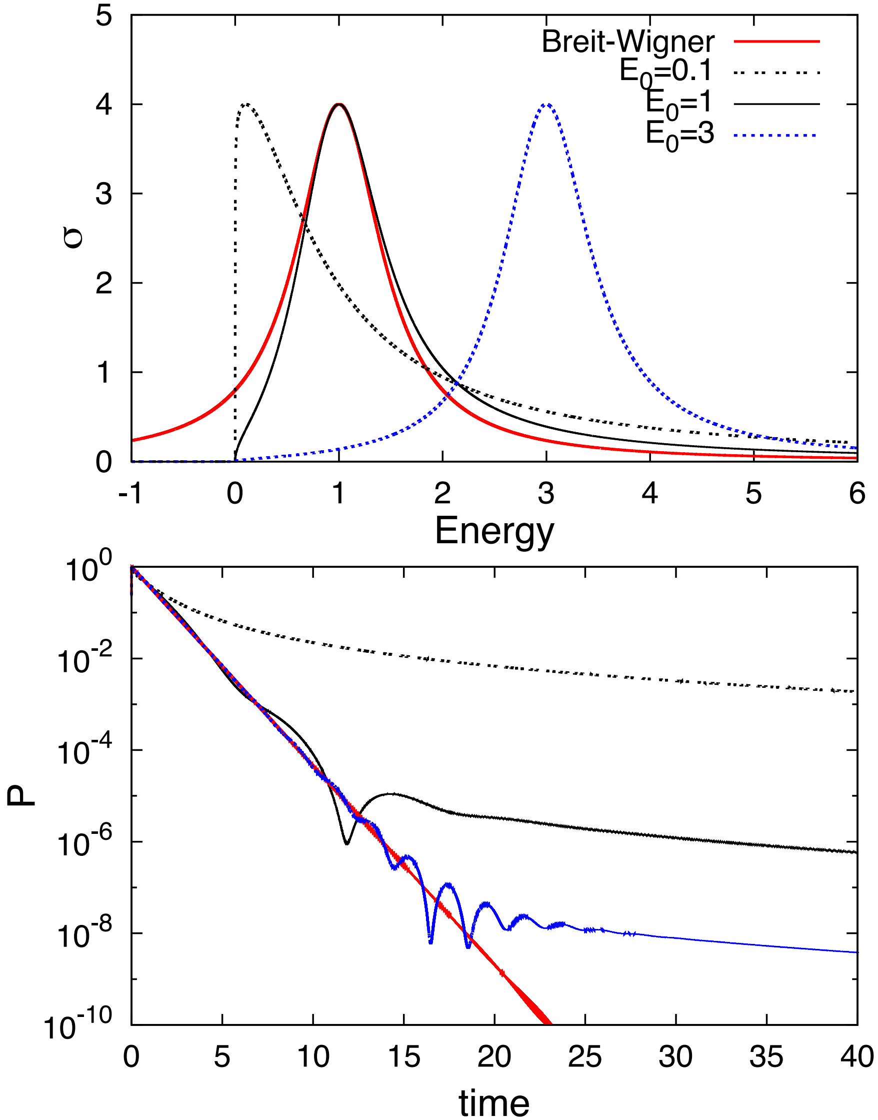
A. Volya, Phys. Rev. C 79, 044308 (2009).

Probing the Non-exponential Decay Regime in Open Quantum Systems

- Broad threshold resonance (^{9}N,^{9}He)
  - Pronounced non-exponentiality
  - Very short half-life
- Three-body decay (^{6}Be, ^{13}Li, ^{16}Be)
  - Nucleon-nucleon correlations
  - Energy dependence
- Overlapping resonances (^{13}C, ^{13}N)
  - Interference, pronounced non-exponentiallity
  - Superradiance

S. M. Wang, W. Nazarewicz, A. Volya, and Y. G. Ma, "Probing the Non-exponential Decay Regime in Open Quantum Systems." arXiv, Nov. 21, 2022. doi: 10.48550/arXiv.2211.11619.

Time-dependent picture



$$\mathcal{G} = \frac{1}{E - E_o + i/2\,\Gamma(E)}$$

$$\Gamma(E) \propto \sqrt{E}$$

Power-law remote decay rate!

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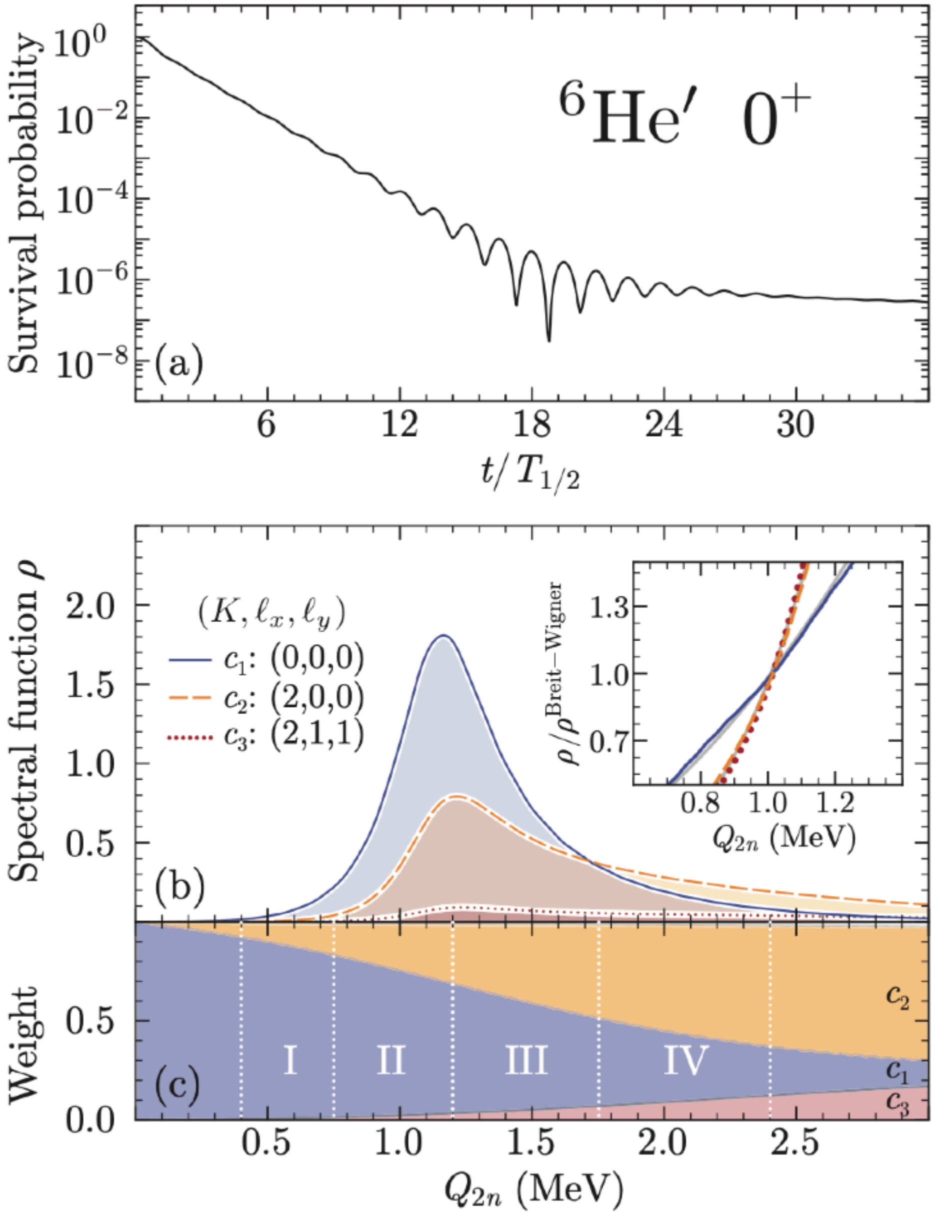
Time-dependent picture
Two-neutron decay



S. M. Wang, W. Nazarewicz, A. Volya, and Y. G. Ma, "Probing the Non-exponential Decay Regime in Open Quantum Systems." arXiv, Nov. 21, 2022. doi: 10.48550/arXiv.2211.11619.

Probing the Non-exponential Decay Regime in Open Quantum Systems

- Broad threshold resonance (^{9}N,^{9}He)
  - Pronounced non-exponentiality
  - Very short half-life
- Three-body decay (^{6}Be, ^{13}Li, ^{16}Be)
  - Nucleon-nucleon correlations
  - Energy dependence
- Overlapping resonances (^{13}C, ^{13}N)
  - Interference, pronounced non-exponentiallity
  - Superradiance

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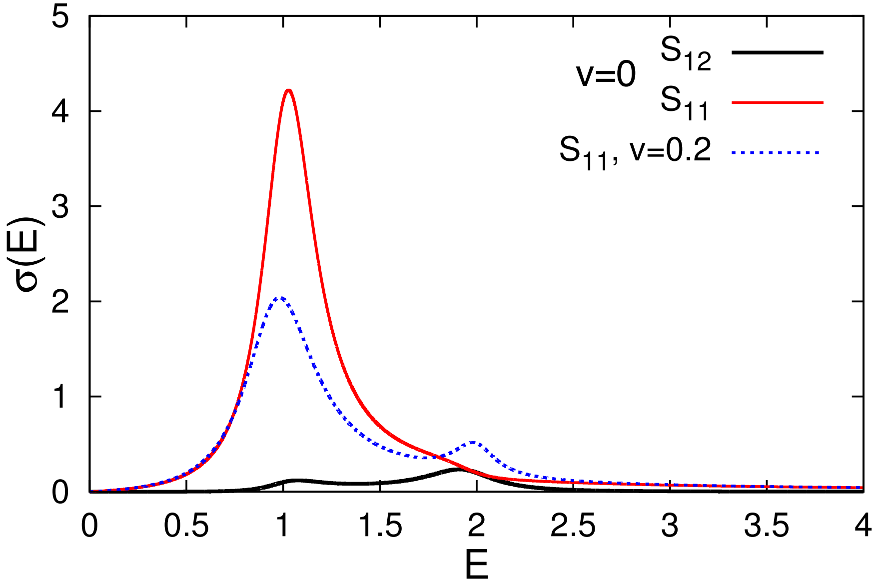
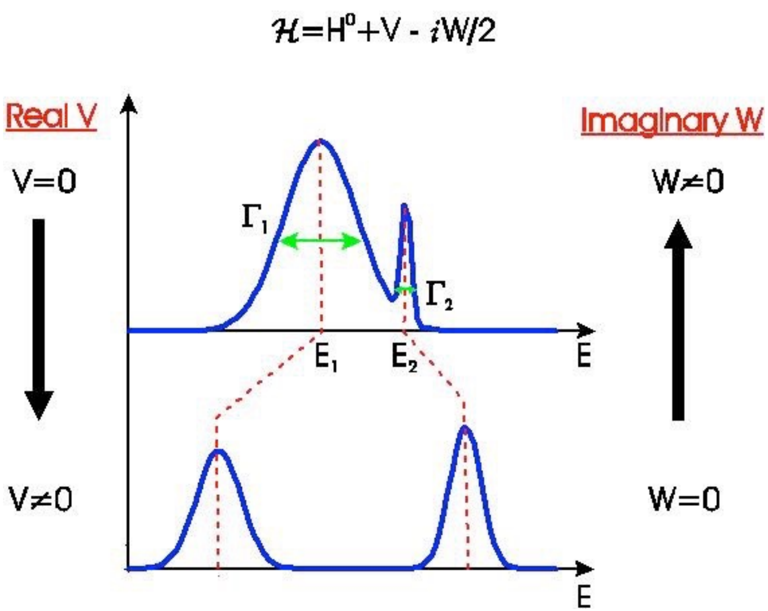
Two-level system

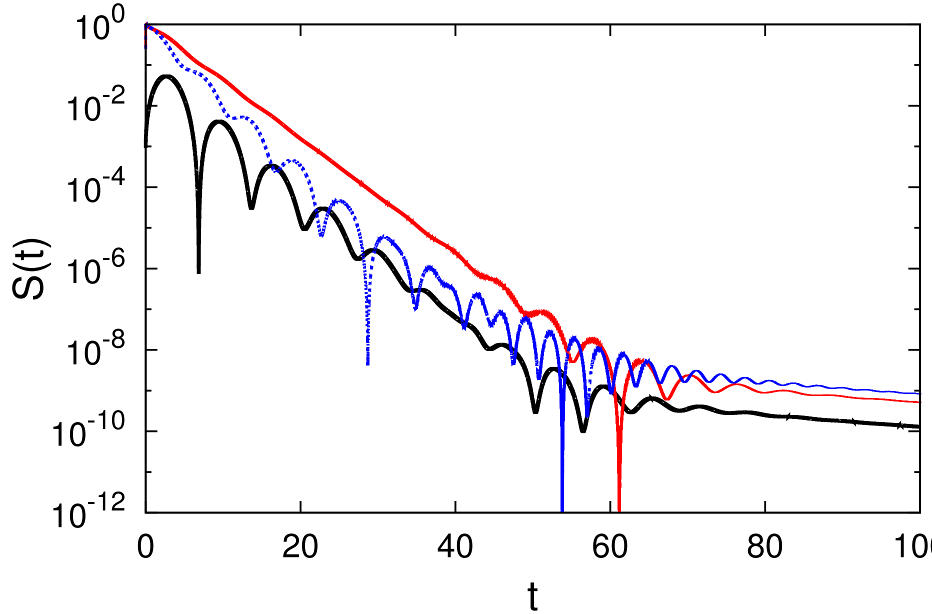
$$\mathcal{H} = \begin{pmatrix} \epsilon_1 - (i/2)\Gamma_1 & v - (i/2)A_1A_2 \\ v - (i/2)A_1A_2 & \epsilon_2 - (i/2)\Gamma_2 \end{pmatrix}$$

$$\Gamma_1 = A_1^2, \quad \Gamma_2 = A_2^2,$$

$$S(t) = |\langle\Psi(0)|\Psi(t)\rangle|^2$$







Other topics

- Collectivization by decay and many-body complexity
- Competition between decay and other collective modes (pairing, etc)
- Eigenstate thermalization hypothesis and universal relaxation of observables
- From open nuclear systems to quantum information and fundamental physics.

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Funding Acknowledgements:
Funding: U.S. DOE contract DE-SC0009883.