Nuclear reactions

Lecture 1

Aim of these lectures

- To discuss the nature and utility of nuclear reactions, especially in the context of ~~radioactive~~ ~~rare-isotope~~ exotic beams.
- What can reactions tell us about (exotic) nuclei?
- It is impossible to cover everything, so I will *not* discuss:
  - Intermediate energy (>100 MeV/nucleon) collisions aimed at equation-of-state physics, multi-fragmentation, etc.
  - Many other topics in Heavy-Ion reactions
  - A very formal exposition of nuclear reaction theory
  - A detailed description of existing reaction codes or their use

Here – “nuclear reaction” takes on a very narrow meaning: Reactions done in the context of learning about nuclear structure/exotic nuclei

A very rough outline

- What is a nuclear reaction, and why do one?
  - What are the basic characteristics that define a nuclear reaction?
  - What kinds of nuclear reactions are there?
  - What can we observe?
- What do the observables tell us about nuclear properties?
- How, technically, do we study nuclear reactions?

- Many things I show you will not involve RIBs, but *all* are *applicable* to RIBs.

Ask Questions!!

Basics (terminology, etc)

- Beam + Target → products
- Standard nomenclature for a 2-body reaction: *A(a,b)B*

  Where *A* is the target, *a* is beam, *b* is "ejectile", *B* is the "product"

- Q value: The amount of energy released in the collision. Q>0: energy is released, Q<0: energy is required to make the reaction go.

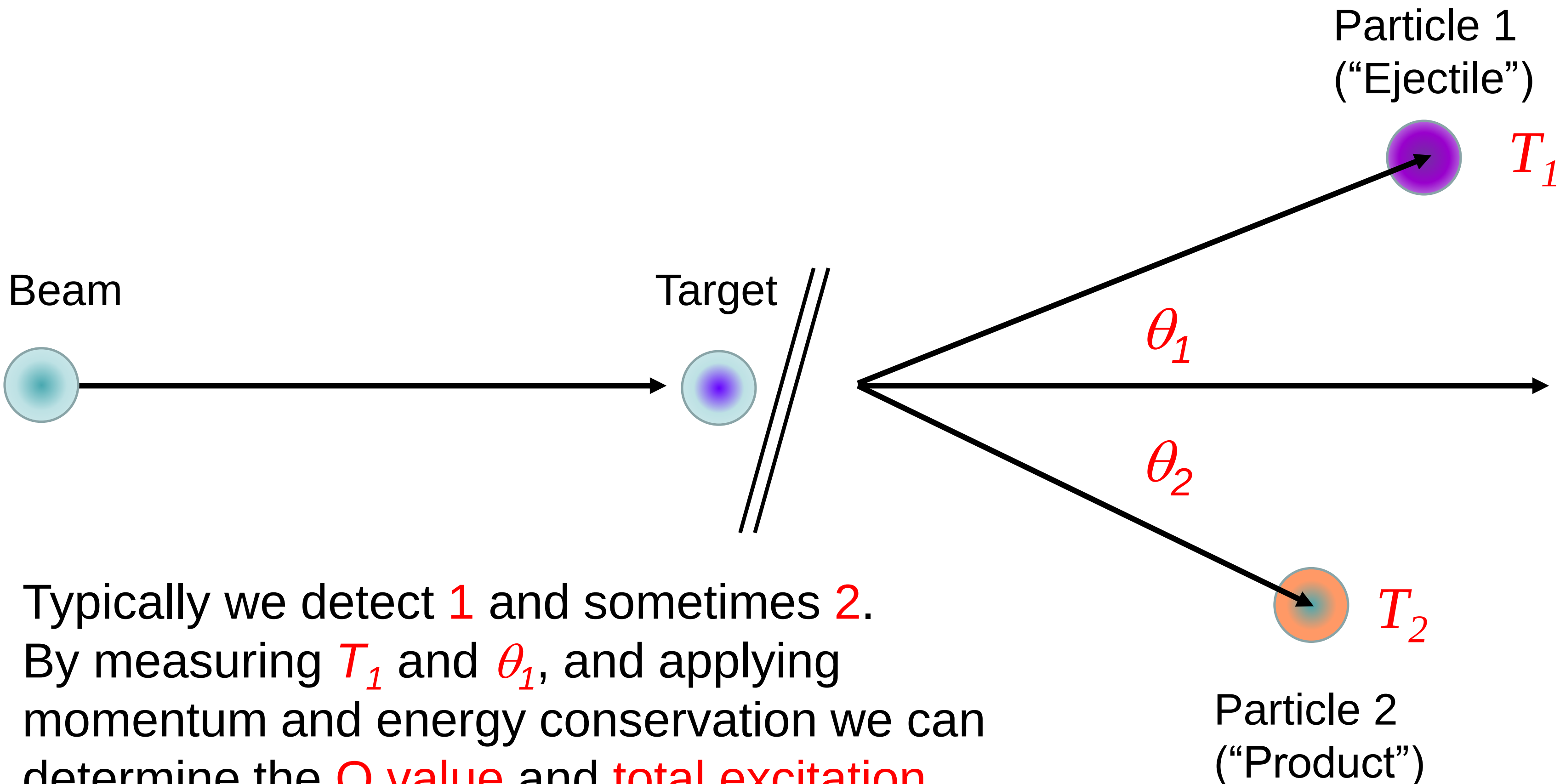
$$Q = m_a c^2 + m_A c^2 - m_b c^2 - m_B c^2 - E_X(b) - E_X(B)$$

- Center-of-mass energy: $E_{CM} = E_{BEAM} \times \frac{m_{TARGET}}{m_{BEAM} + m_{TARGET}}$

Things we can measure

- *Z,A* of emitted particles (by various means – *dE/dx*, time-of-flight)
  - Tells us what the reaction was, what was made
- Laboratory energies and angles of emitted particles
  - Can use to determine Q value and hence the excitation energies of the residual nuclei.
- Cross sections (probability of a reaction taking place) : σ, $\sigma(\theta)$ or $d\sigma/d\Omega$, $\sigma(E)$ or $d\sigma/dE$, $d^2\sigma/dEd\Omega$, etc.
  - The magnitude of the cross section can inform us about a variety of properties. The shapes of angular distributions can tell us about the reaction mechanism and properties of the residual nuclei – e.g. sizes, shapes, spins and parities of levels. The energy dependence also tells us about the reaction mechanism and can be used to identify resonances.
- Orientation of spins of emitted particles
  - Give more detailed information about the reaction mechanism and the properties of states in the residual nuclei

Reaction Kinematics



Typically we detect 1 and sometimes 2.
By measuring T_1 and θ_1, and applying momentum and energy conservation we can determine the Q value and total excitation energy.

Sometimes known as the "missing-mass" method.

Masses/excitation energies

(non-relativistic)

$$Q = Q_g - E_x = T_1\left(1 + \frac{m_1}{m_2}\right) + T_B\left(1 + \frac{m_B}{m_2}\right) - 2\sqrt{\frac{m_B m_1 T_B T_1}{m_2^2}} \times cos\theta_1$$

T_1: measured KE
T_B: Beam KE
θ_1: scattering angle

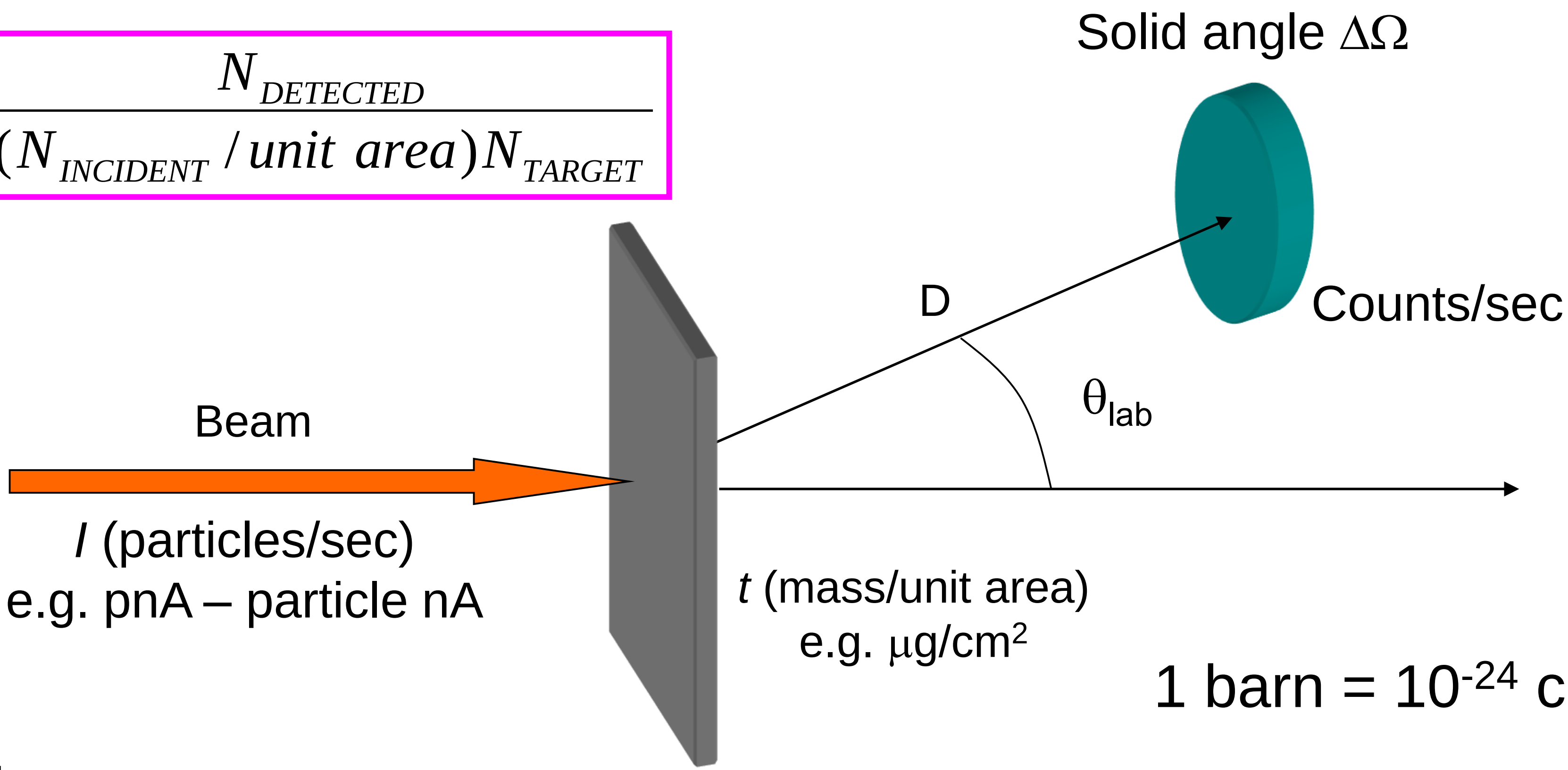
(relativistic)

$$m_2 = \sqrt{(E_0 - E_1)^2 - p_B^2 - p_1^2 + 2p_B p_1 \times cos\theta_1}$$

E_0: $T_B + m_B c^2 + m_{TGT} c^2$
E_1: $T_1 + m_1 c^2$ (Total energy of 1)
T_B: Beam KE
p_B, p_1: Beam and particle 1 momenta
θ_1: scattering angle

Cross sections:

$$\sigma = \frac{N_{DETECTED}}{(N_{INCIDENT} / unit\ area) N_{TARGET}}$$



1 barn = 10^{-24} cm^2

Numbers:

$$\frac{d\sigma}{d\Omega}\left(\frac{mb}{sr}\right) = \frac{Rate(counts/\sec) \times A_{TGT} \times 0.266}{I(pnA) \times t(\mu g/cm^2) \times \Delta\Omega_{LAB}(sr) \times J_{LAB-CM}}$$

$d\sigma/d\Omega$=10 mb/sr, I=10^5 pps, *A*=12, *t*=100μg/cm^2, ΔΩ=1 sr, *J*=1: Rate=.16/min

Very simple measurements can tell us important things

TABLE I. Interaction cross sections (σ_I) in millibarns.

Beam	Target Be	Target C	Target Al
^{6}Li	651 ± 6	688 ± 10	1010 ±
^{7}Li	686 ± 4	736 ± 6	1071 ±
^{8}Li	727 ± 6	768 ± 9	1147 ±
^{9}Li	739 ± 5	796 ± 6	1135 ±
^{11}Li		1040 ± 60	
^{7}Be	682 ± 6	738 ± 9	1050 ±
^{9}Be	755 ± 6	806 ± 9	1174 ±
^{10}Be	755 ± 7	813 ± 10	1153 ±

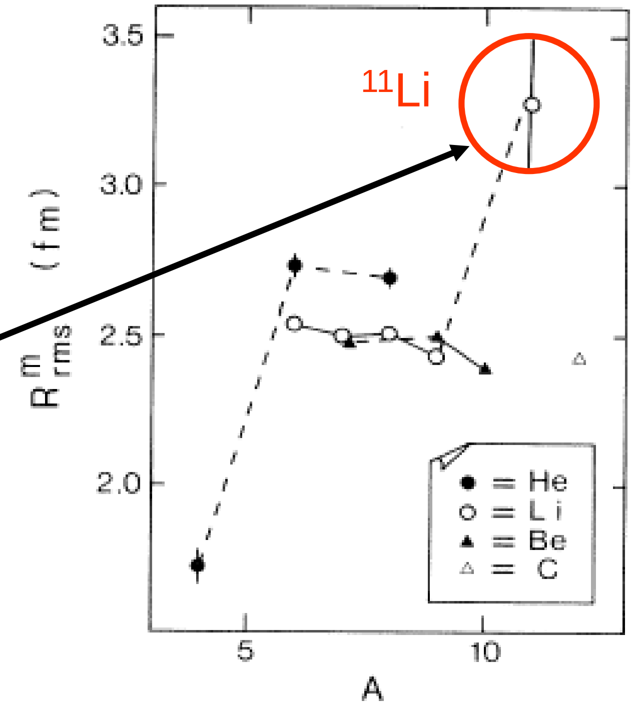
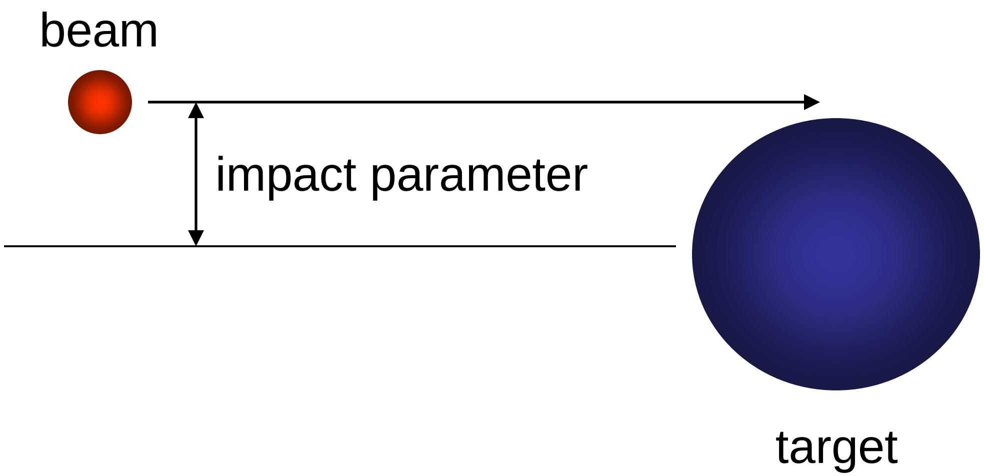


FIG. 3. Matter rms radius R^{m}_{rms}. Lines connecting isotopes are only guides for the eye. Differences in radii are seen for isobars with $A = 6, 8$, and 9. The ^{11}Li isotope has a much larger radius than other nuclei.

First experimental hint that ^{11}Li was special – simply a total interaction cross section

I. Tanihata et al, PRL **55**, 2676 (1985)

Characterizing nuclear collisions



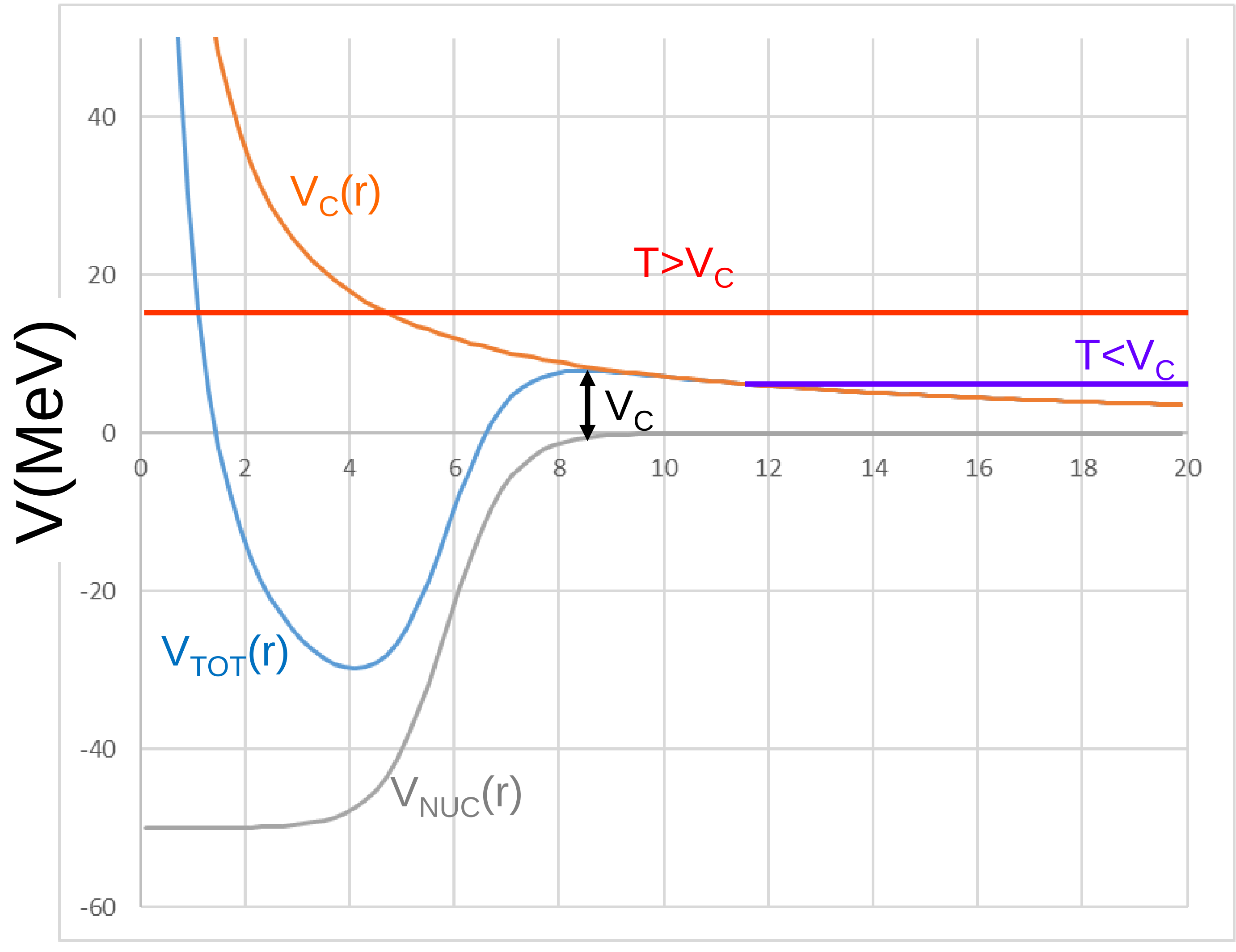
We can choose: (N,Z) for the beam, (N,Z) for the target, and the energy of the beam (and maybe the polarization state of the beam/target). Nothing else.

Of course we can't pick the impact parameter, but we can deduce it later

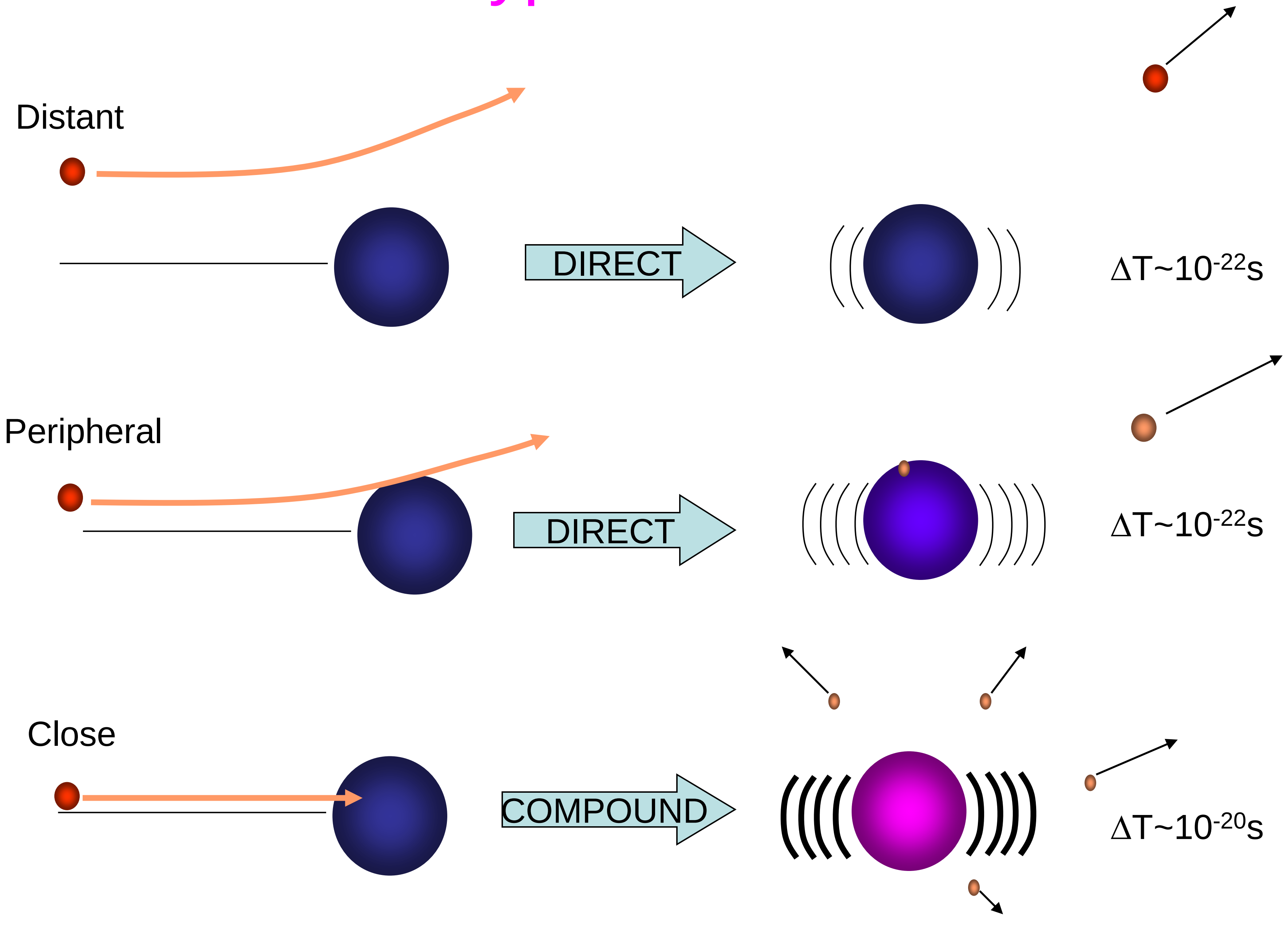
Important parameter is the Coulomb Barrier Energy: $V_C(MeV) = \frac{1.44 \times Z_1 \times Z_2}{r(fm)}$

where $r(fm) \sim 1.2(A_1^{1/3} + A_2^{1/3})$

Schematic nucleus-nucleus potential



Some types of collisions



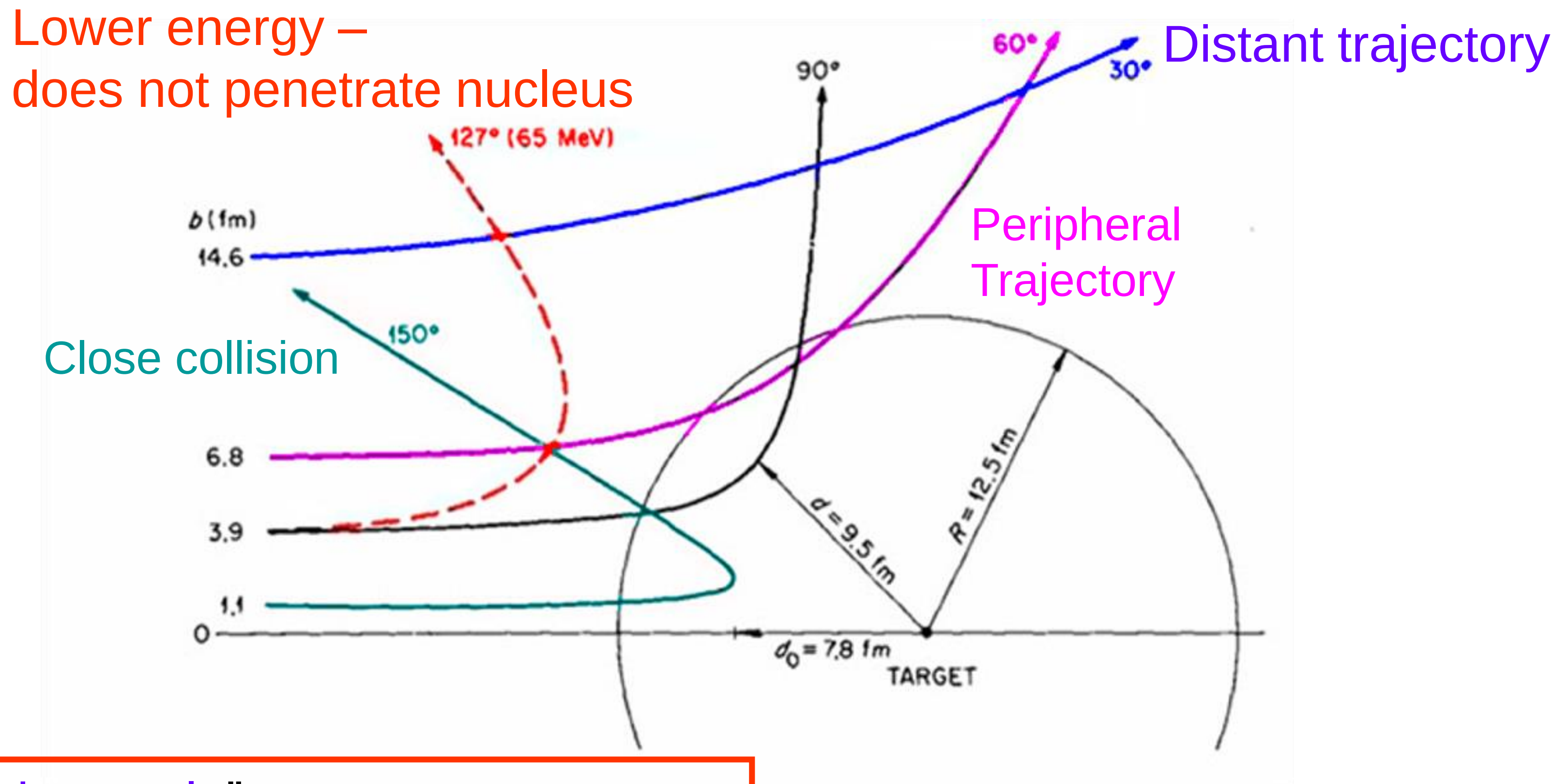
Some kinds of nuclear reactions

- Elastic scattering: $b=a$, $B=A$ (The nuclei are unchanged and unexcited)
- Inelastic scattering: $b=a^*$, $B=A^*$ (same values of N,Z but products can be in excited states; "*" means excited)
  - Coulomb excitation
- Transfer or rearrangement reactions: $b \neq a$, $B \neq A$ (N,Z changed, nucleons exchanged between a and A)
  - Pickup (remove nucleon(s) from target)
  - Stripping (add nucleon(s) from target)
  - charge exchange (change a p to n or n to p)
  - knock-out
- Deep inelastic scattering
  - Many nucleons may be exchanged, nuclei are strongly excited
- Compound nuclear fusion
  - Beam and target fuse (completely or incompletely)
  - High excitation energies and angular momenta are achieved, the resulting compound nucleus emits particles and gamma rays to remove energy

Elastic scattering –
The simplest reaction

We need to understand this before we can do anything.

Coulomb trajectories

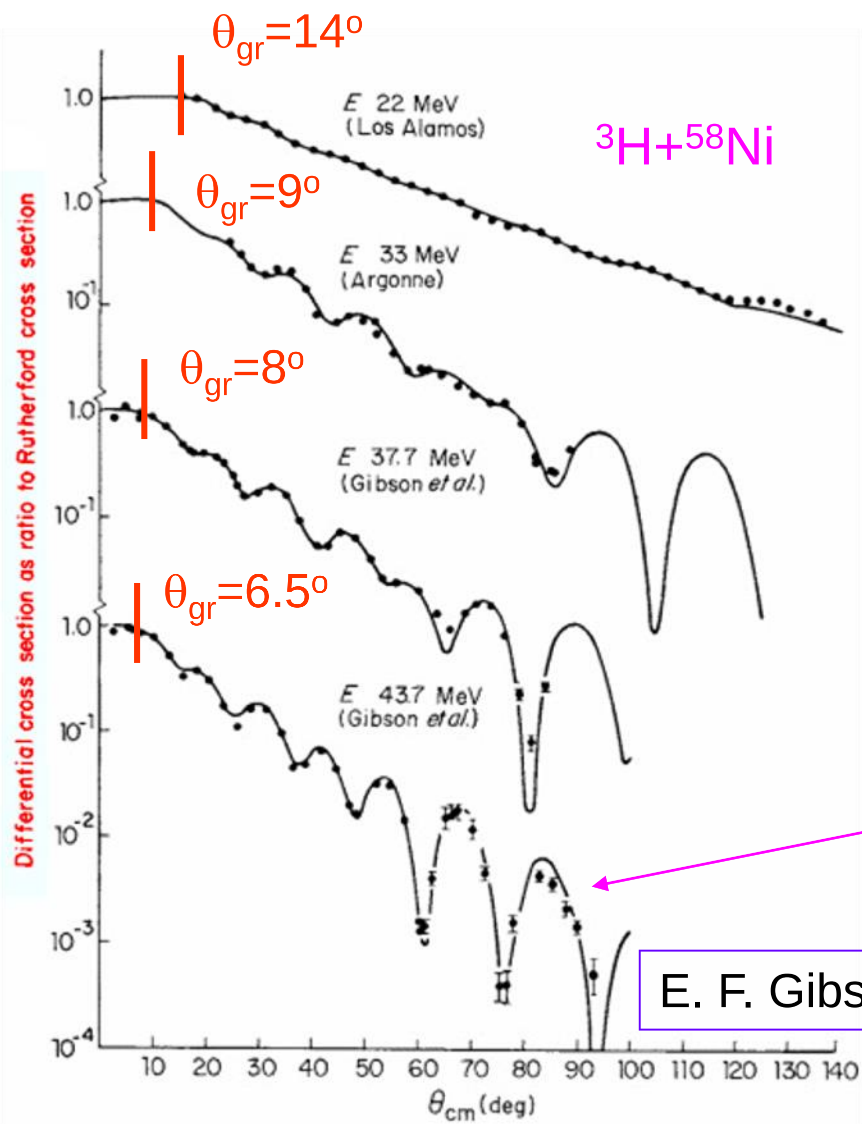


"Grazing angle":
angle corresponding to the trajectory for which the two nuclei just touch each other

$^{16}O+^{208}Pb$ $E(^{16}O)=130$ MeV, $V_C\sim93$ MeV

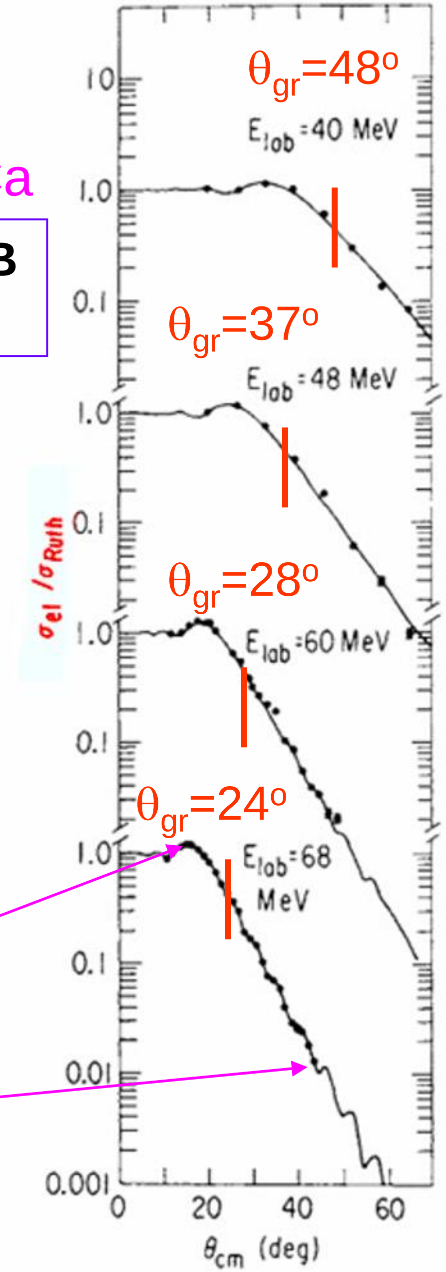
(Satchler 1980, pp 36)

Evolution of elastic-scattering angular distributions with energy



$^{13}C+^{40}Ca$

P. D. Bond, PL **47B** 231 (1971)



What's this??

What's this??

E. F. Gibson, pr **155**, 1208 (1967)

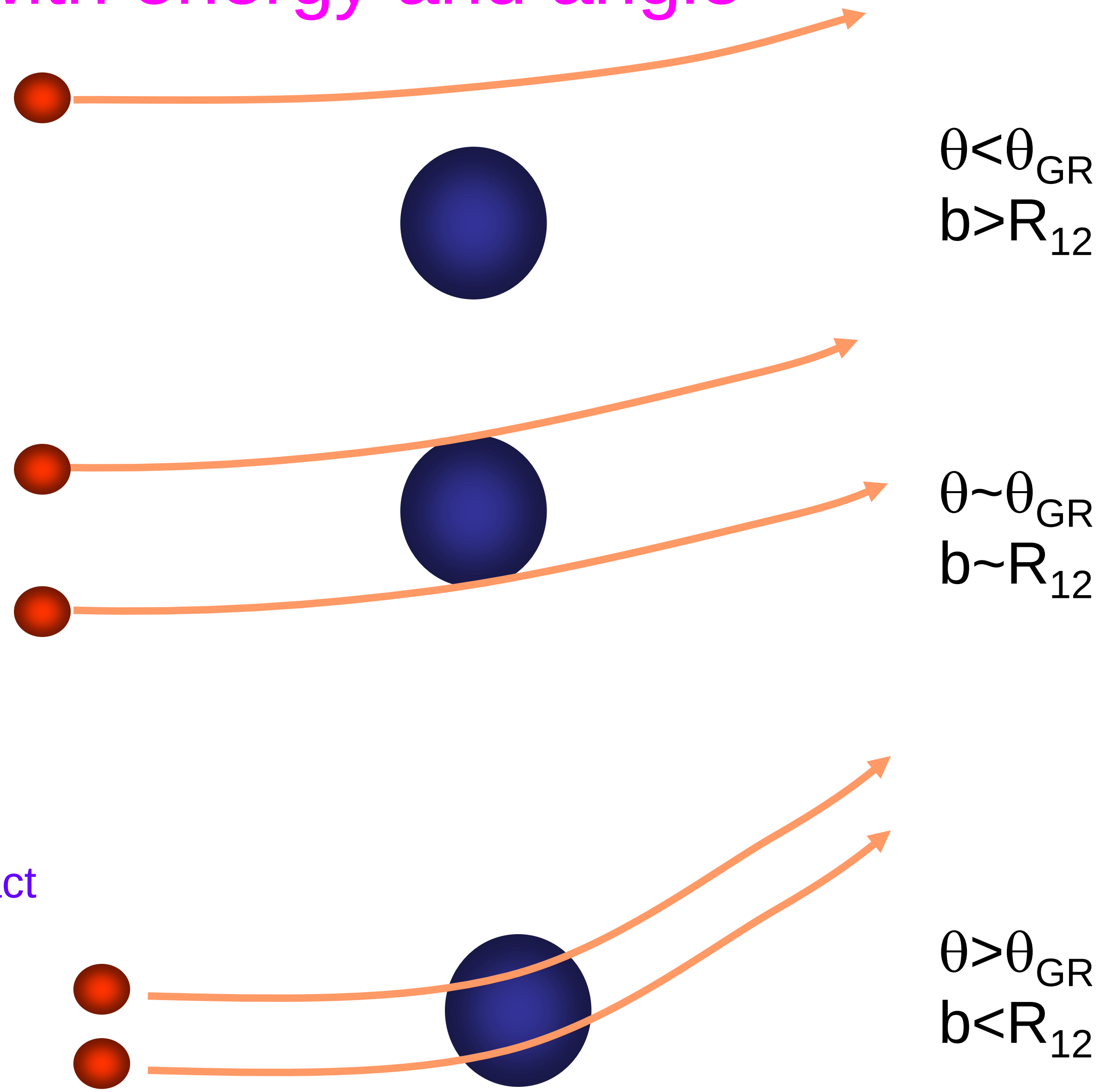
Figs. from Glendenning 2004, pp 38 and 39

Schematic evolution of elastic scattering with energy and angle

Low energy/large impact parameter -
1 dominant trajectory
$d\sigma/d\Omega \sim d\sigma_R/d\Omega$

Grazing trajectories from either side interfere constructively – “Fresnel” or “Coulomb-nuclear” interference and $d\sigma/d\Omega > d\sigma_R/d\Omega$

Higher energy/small impact parameter:
absorption, 2 interfering trajectories:
$d\sigma/d\Omega << d\sigma_R/d\sigma$



Evolution of elastic-scattering angular distributions with energy

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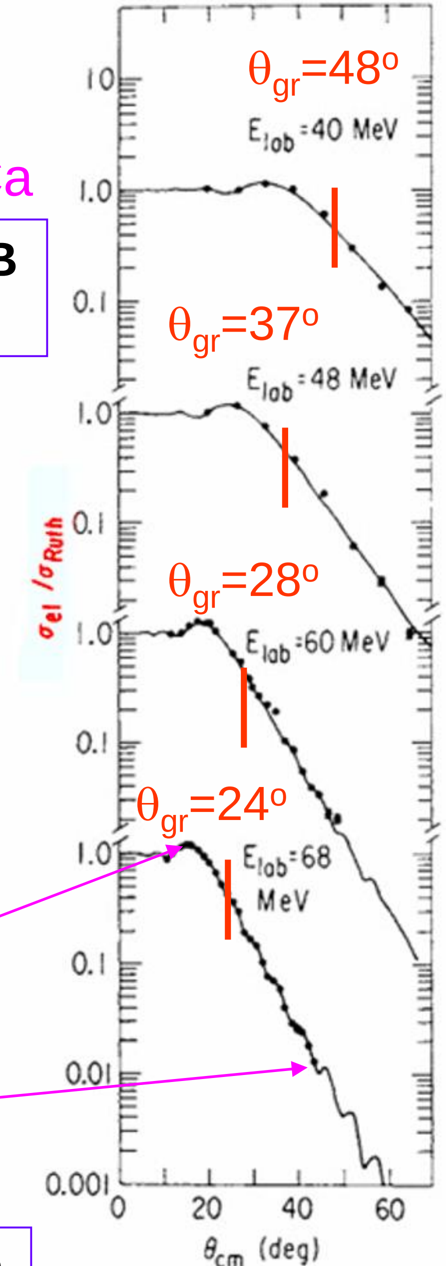
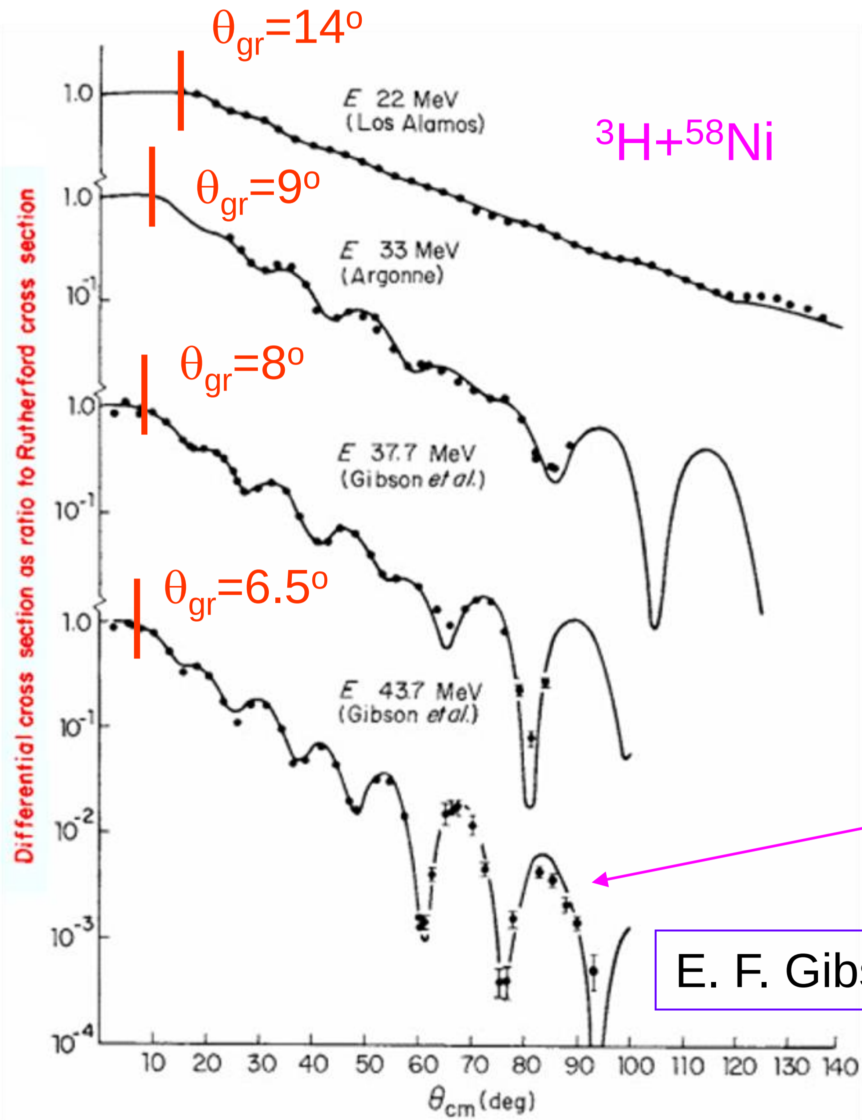
P. D. Bond, PL **47B** 231 (1971)





Fresnel peak

Interference Oscillations

E. F. Gibson, PR **155**, 1208 (1967)

Figs. from Glendenning 2004, pp 38 and 39

Where do those curves come from??

The Optical Model

- The optical model is a schematic model of nuclear scattering that sweeps all of the microscopic nuclear structure under the rug.

- It is called "optical" because it treats the incident and outgoing particles as waves scattered by some ~spherical region. Sometimes those waves can be absorbed ("cloudy ball") and we can lose flux (particles),reducing the elastic scattering cross section

- The combined effects of many complex states are averaged into a single nucleus-nucleus potential – called the Optical Potential

A bit of formalism

We need to solve Schrödinger's equation

$$-\frac{\hbar^2}{2m}\frac{d^2u_l(r)}{dr^2}+\left[U(r)+\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]u_l(r)=Eu_l(r)$$

The asymptotic solutions are waves and an approximate (Born) solution is:

$$\frac{d\sigma}{d\Omega}=\left|f(\theta,\phi)\right|^2; \quad f(\theta,\phi)=-\frac{1}{4\pi}\int \exp(-i\vec{k}'\cdot\vec{r}')U(r')\exp(i\vec{k}\cdot\vec{r}')dr'$$

A better treatment uses asymptotic solutions that are waves distorted by the Coulomb Potential (aka “Distorted Waves”).

U(r) is the Optical Potential

U(r) has several pieces...

Contributions to $U(r)$

- Coulomb part: $V_C(r) = Z_1Z_2e^2/r$
- Real Nuclear part: $V(r)$
  – Comes from the nuclear attraction
- Imaginary Nuclear Part (!): $W(r)$
  – Why? Other things can happen so we can lose elastic flux! There must be "absorption" of waves.
- Spin-Orbit (***l*** dot ***s***) part: $V_{SO}(r)$
  – Why? There is a spin-orbit component to the nuclear force so it seems natural to have one between nuclei. Also, it seems to be needed to explain polarization data!

$$U(r)=V_C(r) + V(r) + iW(r) + V_{SO}(r)$$

Only the real parts contribute to elastic scattering

What do the parts look like?

The form follows our rough understanding of the density profile of nuclei:

"Woods-Saxon" or "Saxon-Woods" parameterization

$$V(r) = \frac{-V_0}{1+e^{(r-R_R)/a_R}}$$

"Volume" terms: parameters are V_0, R_R, a_R, W_0, R_I, a_I

$$W(r) = \frac{-W_0}{1+e^{(r-R_I)/a_I}}$$

Note the minus signs. Often we write: $V(r) = -V_0\, g(r)$

There can also be "Surface" terms that look like:

$V_S(r)=V_S\, dg(r)/dr$ and $W_S(r)=W_S\, dg(r)/dr$.

They deal with processes restricted to the nuclear surface

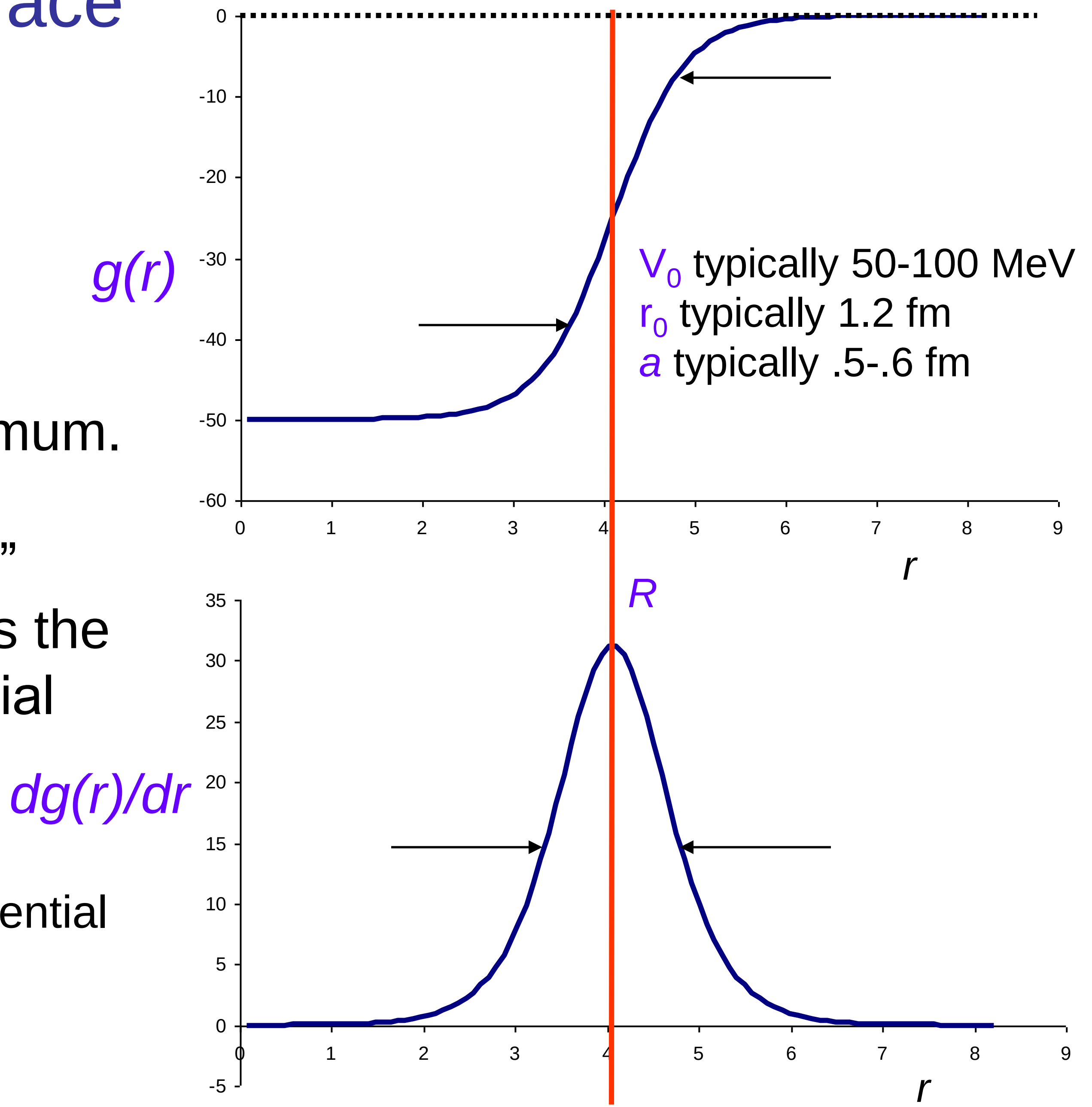
Volume and surface potentials

$R = r_0(A_1^{1/3} + A_2^{1/3})$ is the radius where the potential is ½ its maximum.

"a" is the "diffuseness" parameter. It describes the "spread" of the potential about R.

Typically, the spin-orbit potential is described as

$$V_{SO} = \frac{C}{r}\frac{dg(r)}{dr}\vec{l}\cdot\vec{s}$$



V_0 typically 50-100 MeV
r_0 typically 1.2 fm
a typically .5-.6 fm

Somewhat Unsatisfying...

6-10 parameters, you'd think you could fit anything!

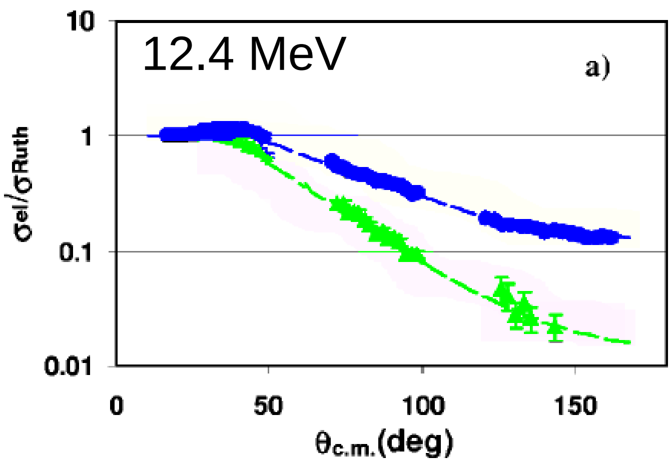
Do these parameters have any meaning?

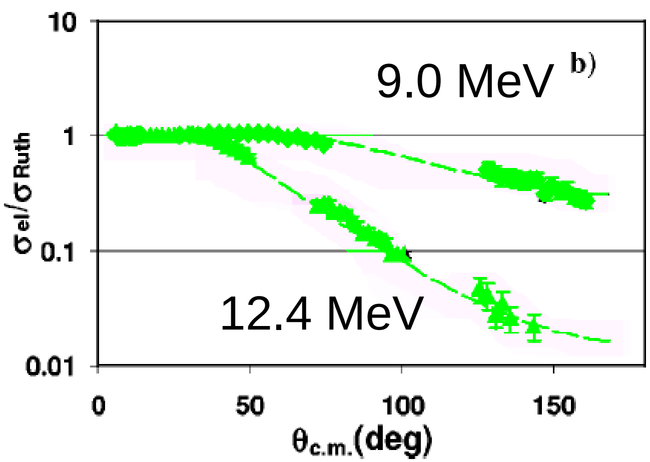
Can their extraction from a measurement give you any insight into the underlying nuclear structure??

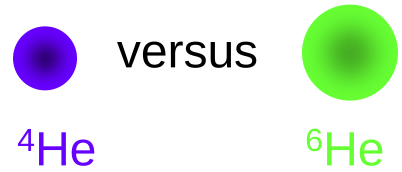
One hopes so...

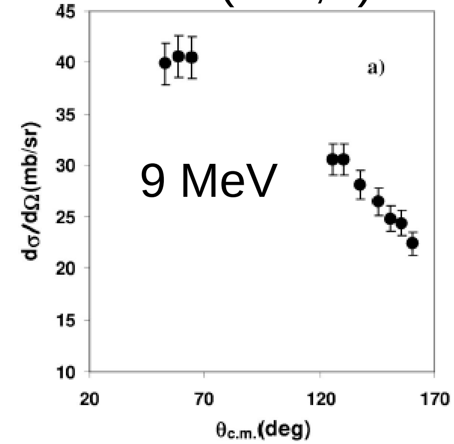
^{4,6}He+^{64}Zn scattering

Elastic scattering









These (breakup) events are lost to elastic scattering

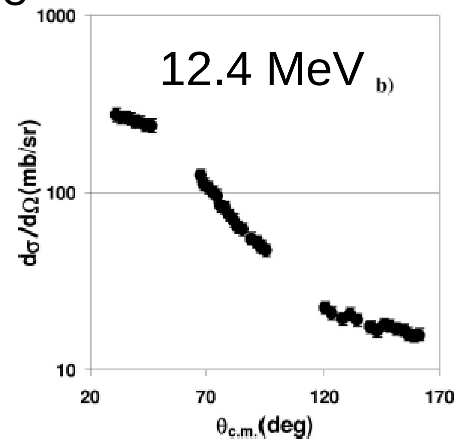
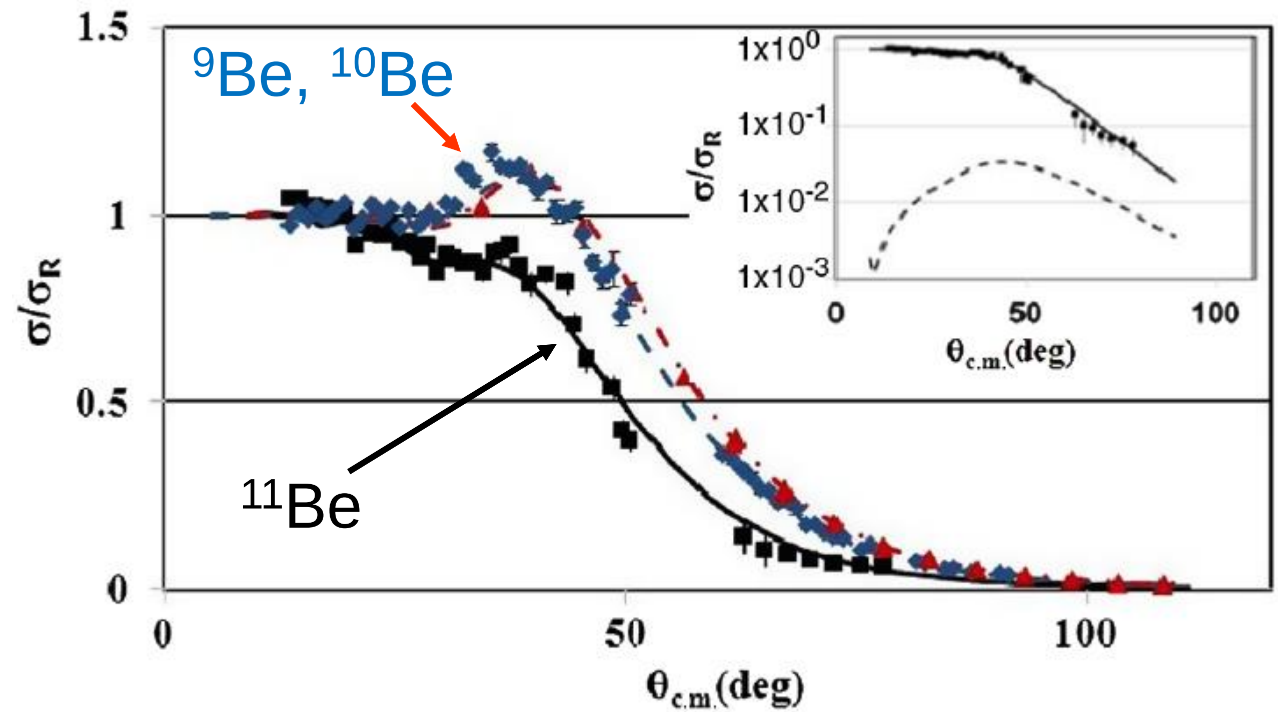


TABLE I. Optical model parameters.

Reaction	$E_{c.m.}$(MeV)	V	r_0	a	W	r_i	a_i
^{4}He+^{64}Zn	12.4	123	1.2	0.43	20.4	1.05	0.43
^{6}He+^{64}Zn	12.4	104	1.2	0.6	38.9	1.2	0.85
^{6}He+^{64}Zn	9.1	47.4	1.2	0.6	10.7	1.2	0.85

A. Di Pietro et al, PRC **69** 044613 (2004)

$^{11}Be+^{64}Zn$



PRL **105,** 022701 (2010) PHYSICAL REVIEW LETTERS week ending 9 JULY 2010

TABLE I. WS optical potentials obtained from the fit of the experimental data. The real potential radius parameter is $r_0 = 1.1$ fm and the imaginary one is $r_i = 1.2$ fm, where $R_{0,i,si} = r_{0,i,si}(A_p^{1/3} + A_t^{1/3})$. The Coulomb radius parameter is $r_C = 1.25$ fm.

Reaction	V (MeV)	a (fm)	V_i (MeV)	a_i (fm)	V_{si} (MeV)	r_{si} (fm)	a_{si} (fm)	J_V (MeV fm^3)	J_W (MeV fm^3)
$^9Be + ^{64}Zn$	126	0.6	17.3	0.75				295	53
$^{10}Be + ^{64}Zn$	86.2	0.7	43.4	0.7				193	124
$^{11}Be + ^{64}Zn$	86.2	0.7	43.4	0.7	0.151	1.3	3.5	193	129

^{11}Be is a "neutron-halo" nucleus with an extended surface

Improvements: Global potentials

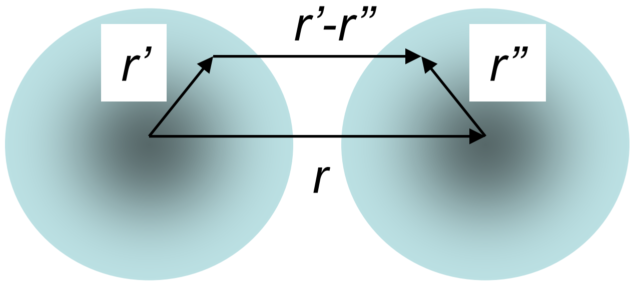
- By studying elastic scattering for many, many systems at many, many energies, one can develop a "Global" parametrization of optical-model parameters.
- Potential depths are energy dependent (as is the nucleon-nucleon force)
- Geometrical parameters can be mass dependent but are typically not energy dependent
- This eliminates some of the "arbitrariness" of optical-potential analyses

Deuterons: An and Cai, PRC **73**, 054605 (2006)
^{3}H/^{3}He: Pang et al., PRC **79**, 024615 (2009)

Improvements: Folding potential

"Double-Folding model" :

$$U(r) = N \int \rho(r')d^3r' \int \rho(r'')v_{nn}(r'-r'')d^3r''$$



N is a normalization, we can have N_R and N_I

v_{nn} is a nucleon-nucleon interaction – there are many on the market.

In principle this can fix the volume terms in *U(r)* and describe their energy dependence.

Summary

- Much of what we need to know to understand basic nuclear reactions is not very complicated
- We should not forget what has been learned by studying reactions involving stable beams
- Even the simplest measurements can tell us many useful things – especially important if all you can do is the simplest measurement!
- We can derive much guidance just by looking at elastic scattering.

Tomorrow: excitations and moving nucleons around – (Direct) re-arrangement reactions and nuclear structure

Some useful references

- *Introduction to Nuclear Reactions,* G. R. Satchler, Wiley 1980.
- *Direct Nuclear Reactions*, G. R. Satchler, Oxford University Press 1983.
- *Direct Nuclear Reactions*, N. Glendenning, World Scientific 2004.
- *Introductory Nuclear Physics*, K. Krane, Wiley 1987.
- *Introductory Nuclear Physics*, P. E. Hodgson, E. Gadioli and E. Gadioli Erba, Oxford University Press 1997.
- *Nuclear Physics of Stars*, C. Iliadis, Wiley 2007.

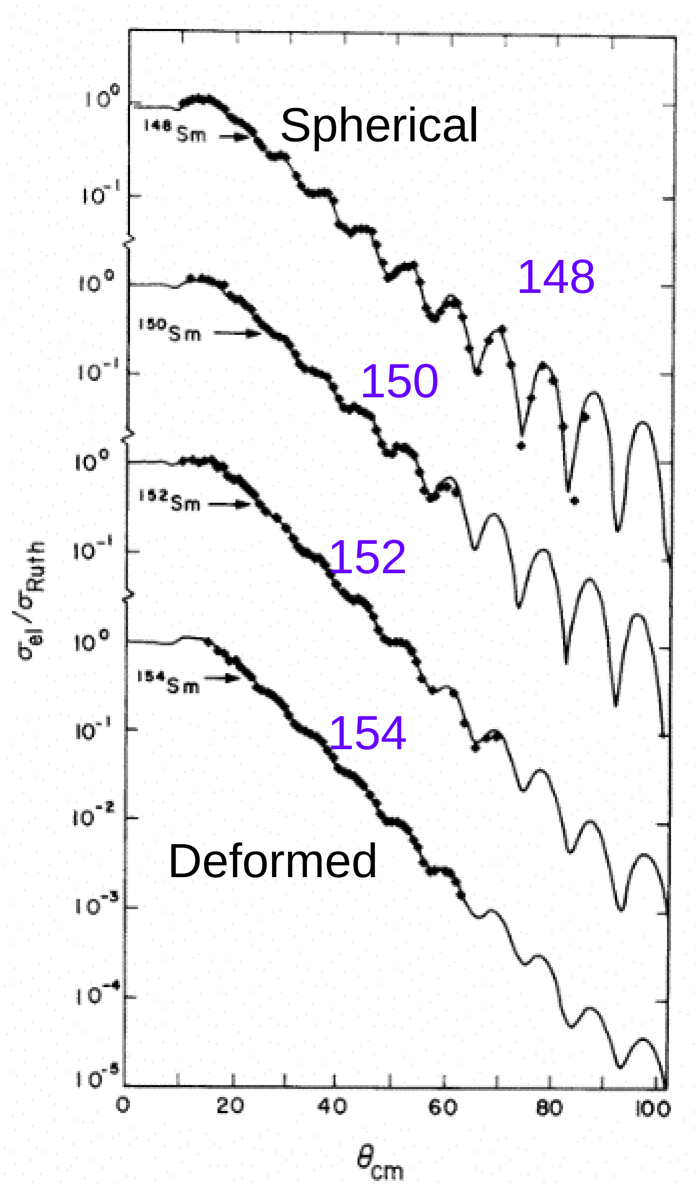
Sensitivity to quadrupole deformation

Optical-Model Parameters for 50-MeV Alpha Particles

Isotope	V	W	$r_v = r_w$	$a_v = a_w$	r_c
^{148}Sm	−65.5	−29.8	1.427	0.671	1.4
^{154}Sm	−34.6	−29.4	1.404	0.819	1.4

The oscillations at large angles get washed out – the deformed nucleus is effectively more diffuse. The weaker interference pattern suggests a weaker real potential *V*.

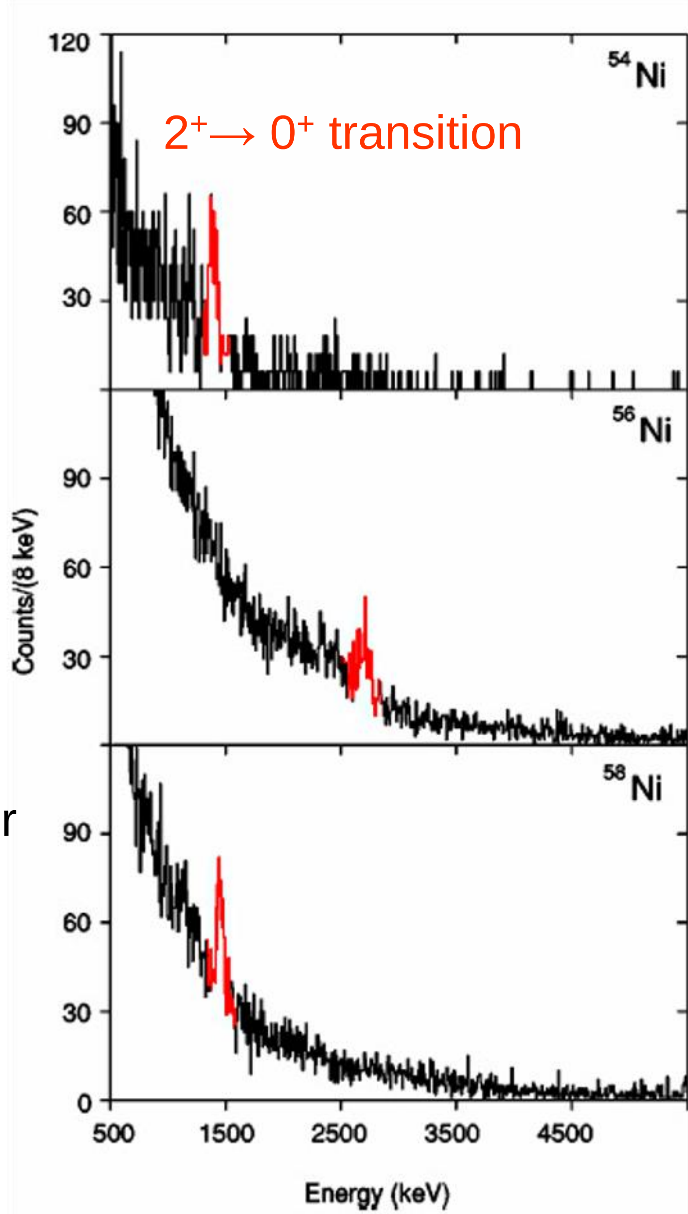
α+^{A}Sm elastic scattering

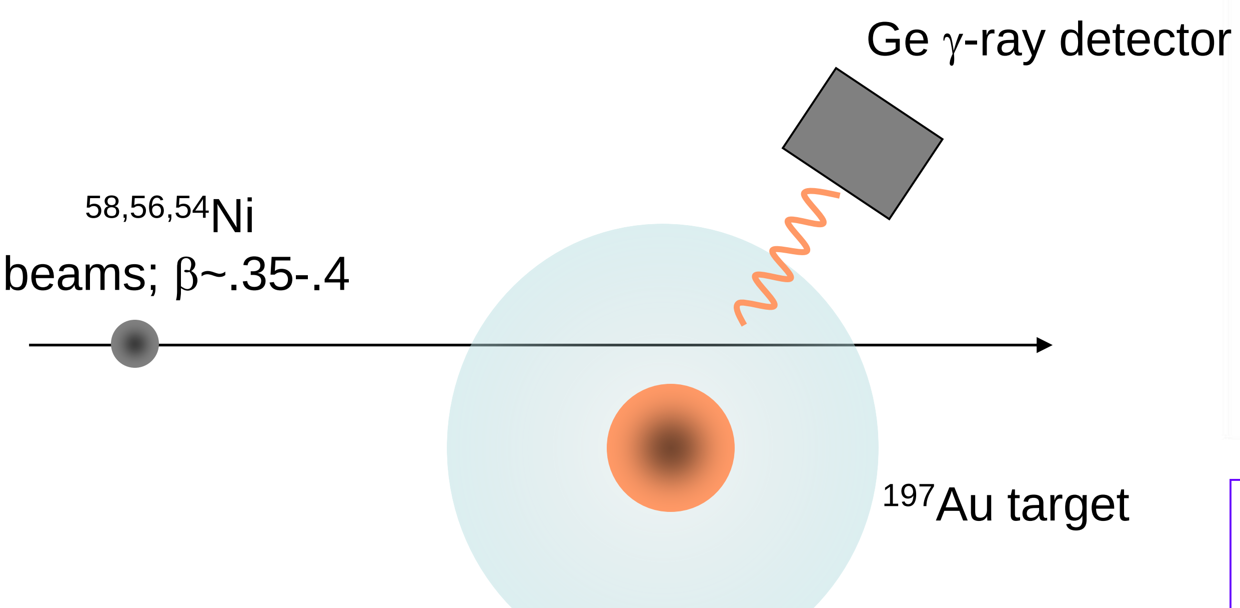


Glendenning 2004, pp 34

Coulex as a spectroscopic tool

	$^{58}Ni^{a}$	^{56}Ni	^{54}Ni
	Experimental results		
E_γ (keV)	1453(8)	2695(15)	1396(9)
σ (mb)	175(36)	107(26)	134(36)
v/c (midtarget)	0.373	0.391	0.346
θ_{lab}^{max} (degrees)	3.2	2.9	3.5
b_{min} (fm)	13.9	14.3	16.2
$B(E2\uparrow)$ (e^2 fm^4)	707(145)	494(119)	626(169)
	Adopted values		
E_γ (keV)	1454.28(10)	2700.6(7)	
$B(E2\uparrow)$ (e^2 fm^4)	695(20)	600(120)	





K. L Yurkewicz et al, PRC **70**, 054319 (2004)

A textbook example

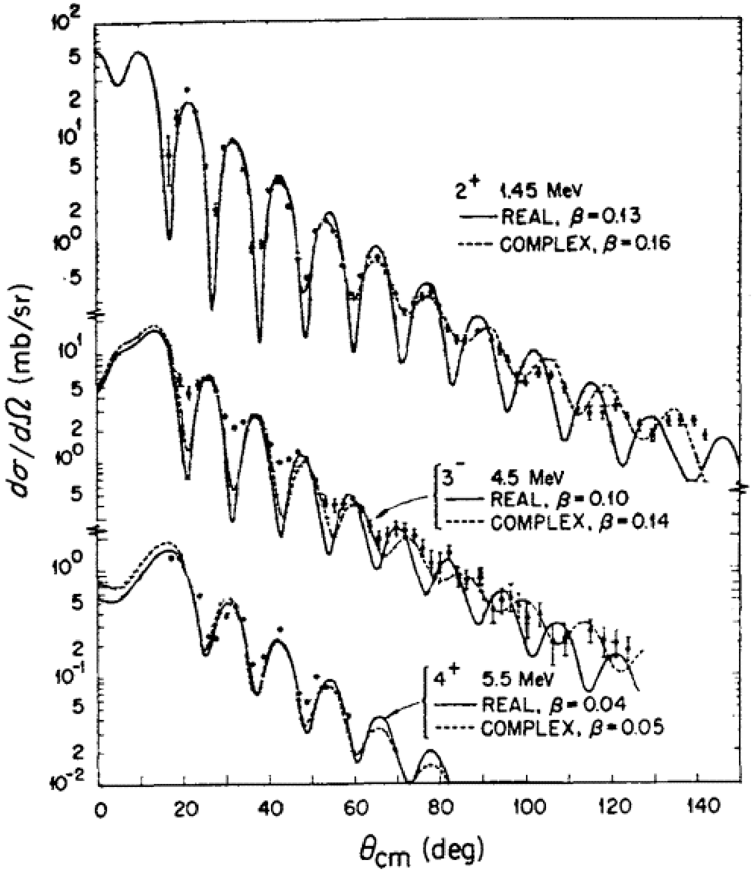


Fig. 7.4. Cross sections for collective states calculated as vibrational levels for the reaction $^{58}Ni(\alpha, \alpha')$ with $E_\alpha = 43$ MeV. Two calculations for each level correspond to the use of a purely real and a complex form factor. (From Broek *et al.*, 1965.)

Explicit coupled-channels treatment of inelastic alpha-particle scattering on ^{58}Ni.

Both real and imaginary couplings are necessary