Nuclear reactions

Lecture 2

Non-elastic scattering

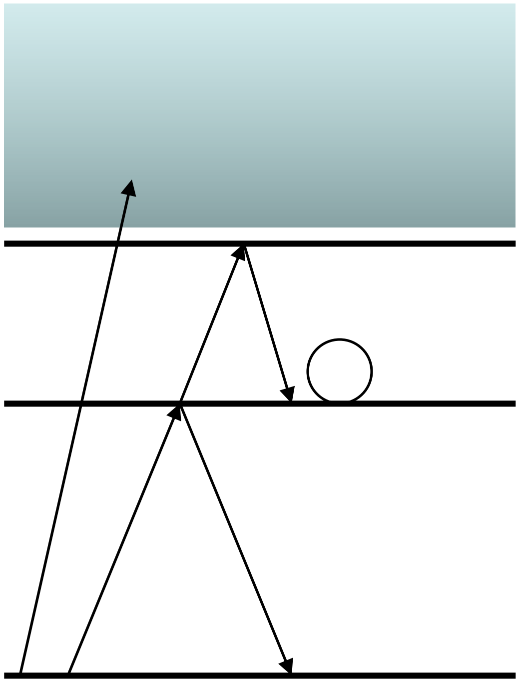
This is everything else.

Inelastic scattering

- $a+A \rightarrow a^* + A^*$: a and A retain their identity but are excited
- Change in both internal and external wave functions
- Inelastic effects can alter elastic scattering through channel coupling.
- Coupling can be to collective (rotational or vibrational), single-particle, or continuum degrees of freedom

Coupled-channels:

Explicit treatment of inelastic excitations
(Important for both elastic and inelastic scattering)



Discrete bound levels:

Instead of 1 equation, a system of coupled differential equations. More complicated but can reduce the uncertainty in the imaginary potential. "Coupled-channels" or "CC"

Continuum "levels":

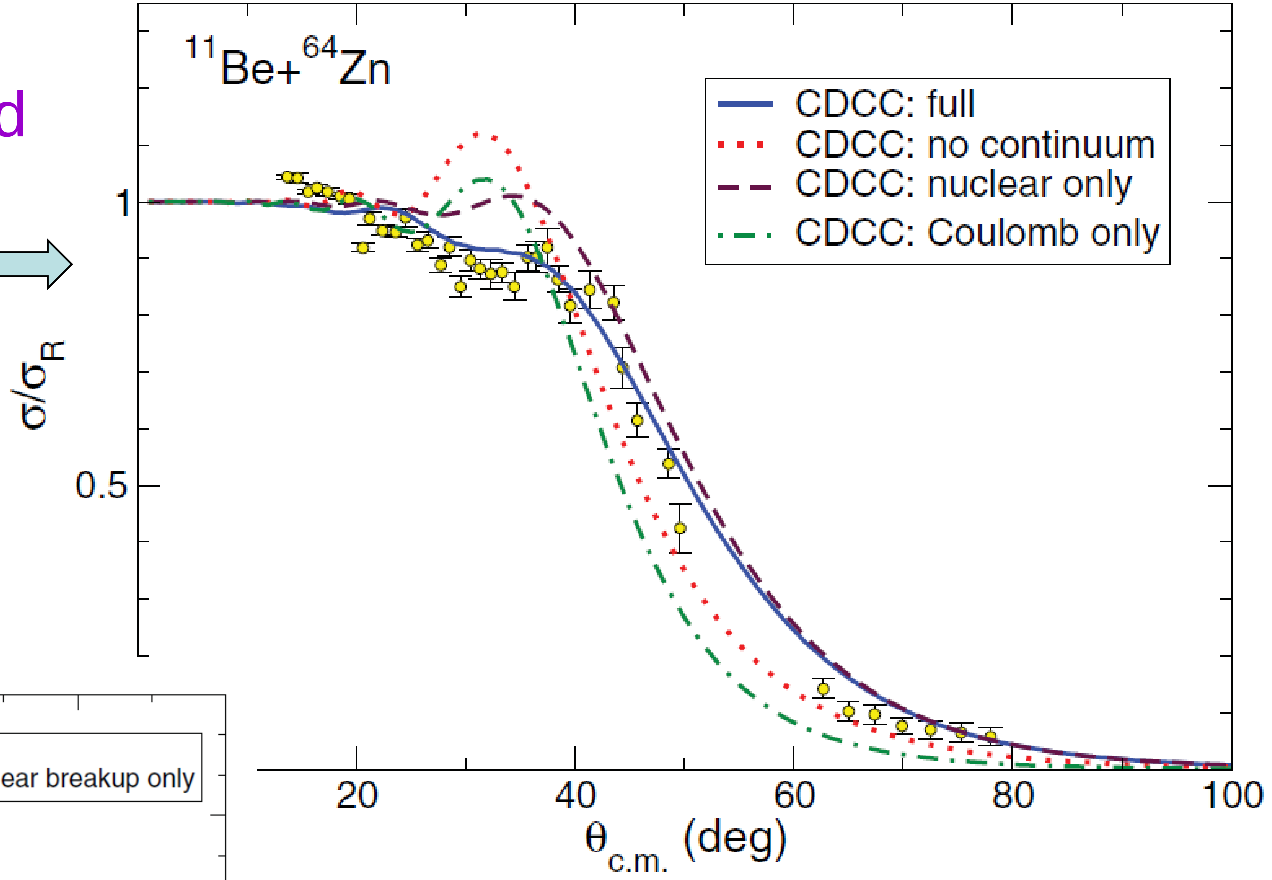
Artificially cut up continuum into small pieces – discretize. "Continuum Discretized Coupled Channels" or "CDCC*"

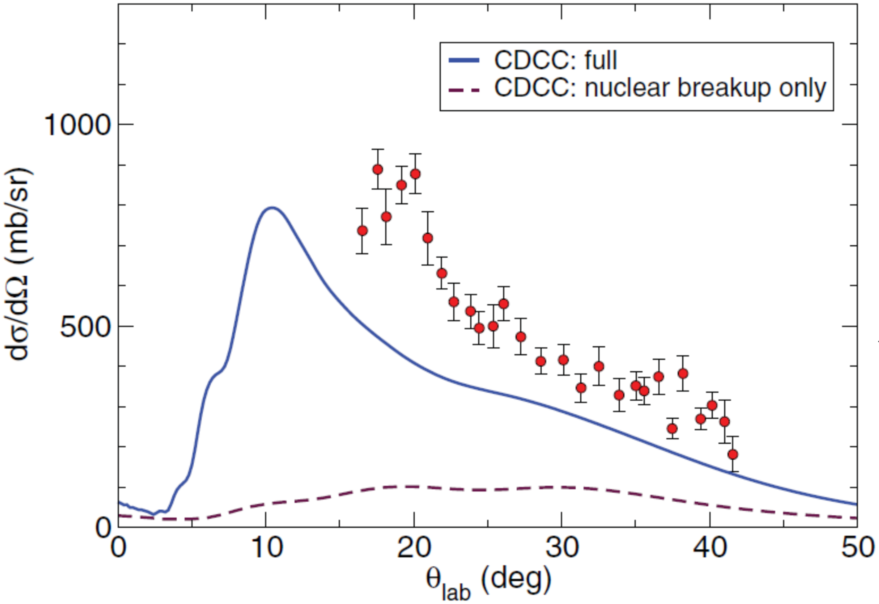
*M. Kamimura et al., Prog. Th. Phys. Suppl. 89, 1 (1986)

Recall $^{11}Be+^{64}Zn$

Elastic scattering and Coupled Channels

DiPietro et al., PRC 85 054607 (2012).





Breakup events and Coupled Channels

Inelastic scattering: Special cases

Optical potential

$$(E_\alpha - T_{\alpha l} - U_\alpha)u_\alpha^0 = 0$$

$$(E_{\alpha'} - T_{\alpha l} - U_\alpha)u_{\alpha'} = V_{\alpha\alpha'}u_\alpha^0$$

Coupling potential

Coupled differential equations

$$V_{\alpha\alpha'} \sim <\phi_{\alpha'}|V_{INEL}(r)|\phi_\alpha>$$

ϕ_α are the intrinsic states in some collective model, and V_{INEL} is a coupling potential

$$V_{VIB}(r) \sim R_0 \frac{dU}{dr}\alpha_{\lambda\mu}Y_{\lambda\mu}^*(\vec{r})$$ Vibrational model

$$V_{ROT}(r) \sim R_0 \frac{dU}{dr}\beta_L Y_{LM}(\vec{r})$$ Rotational model

These correspond to distortions of the nuclear surface.

The α's and β's tell us about the collectivity of the nuclei

The β_Ls in particular tell you the magnitude of different multipole deformations

Channel coupling really matters!

$\alpha + {}^{154}Sm$

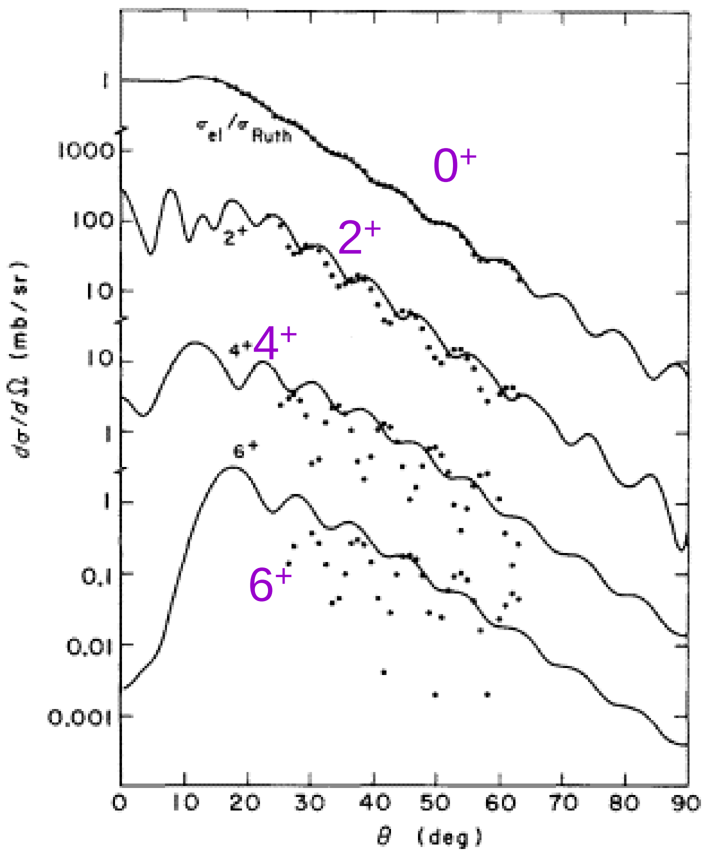


Fig. 13.3. Distorted-wave calculation with optical parameters that fit the elastic section as shown (listed in Chapter 4). $\beta_2 = 0.3$, $\beta_4 = 0.15$, $\beta_6 = 0.075$. DWBA, ^{154}Sm (Glendenning, 1969a).

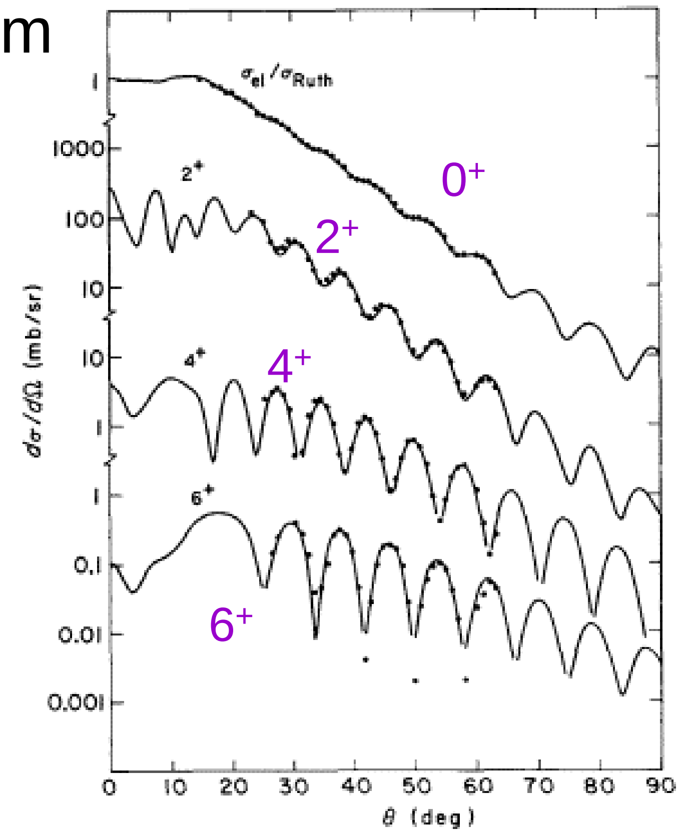


Fig. 13.4. Cross sections for 50-MeV alpha-excitation ground-state rotational band ^{154}Sm. Curves are coupled-channel calculation as described in text. The data were taken at t Berkeley 88-in. Cyclotron. $\beta_2 = 0.225$, $\beta_4 = 0.05$, $\beta_6 = -0.015$ (from Harvey *et al.*, 196 Hendrie *et al.* 1968; calculation by Glendenning 1969a).

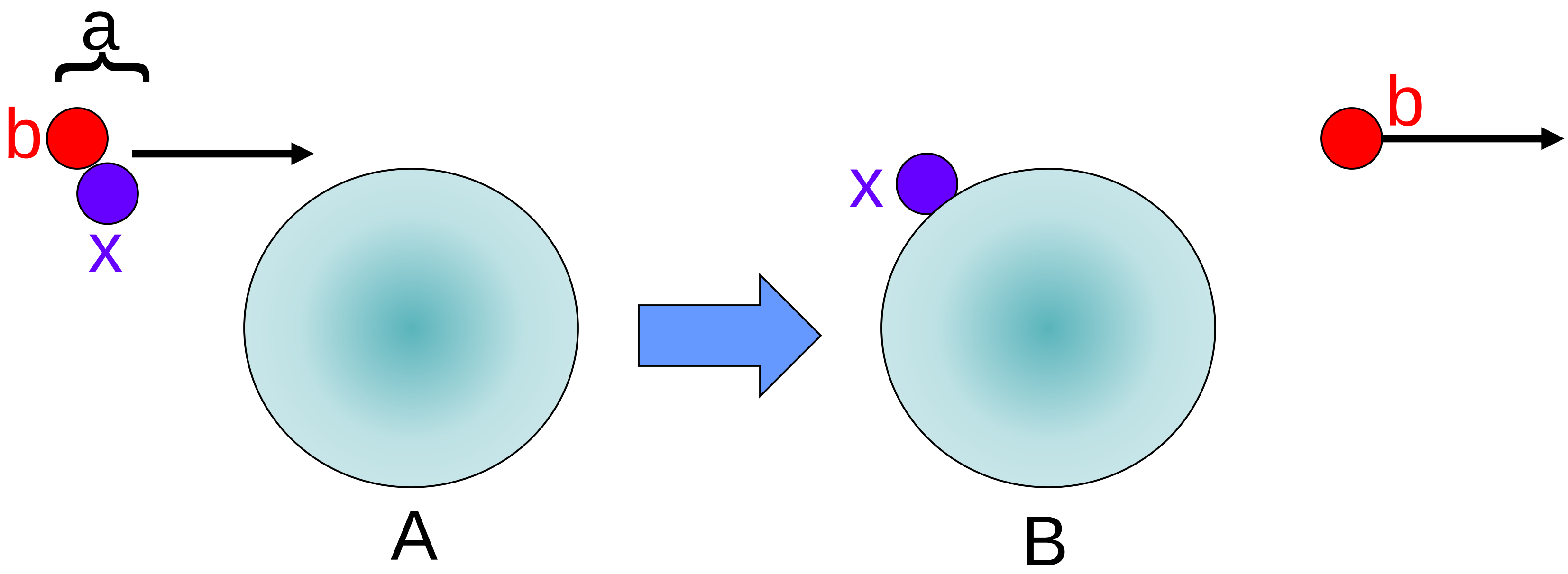
You can fit elastic scattering alone with an optical model...

...But you need channel-coupling to fit all the inelastic channels. Everything is treated simultaneously.

Re-arrangement reactions

- $a + A \rightarrow b + B$ or $A(a,b)B$
- Nuclei are transformed, nucleons are exchanged ($b \neq a$, $B \neq A$)
- We'll focus on simple processes – "Direct" reactions
- We need to use some of what we learned about elastic scattering.

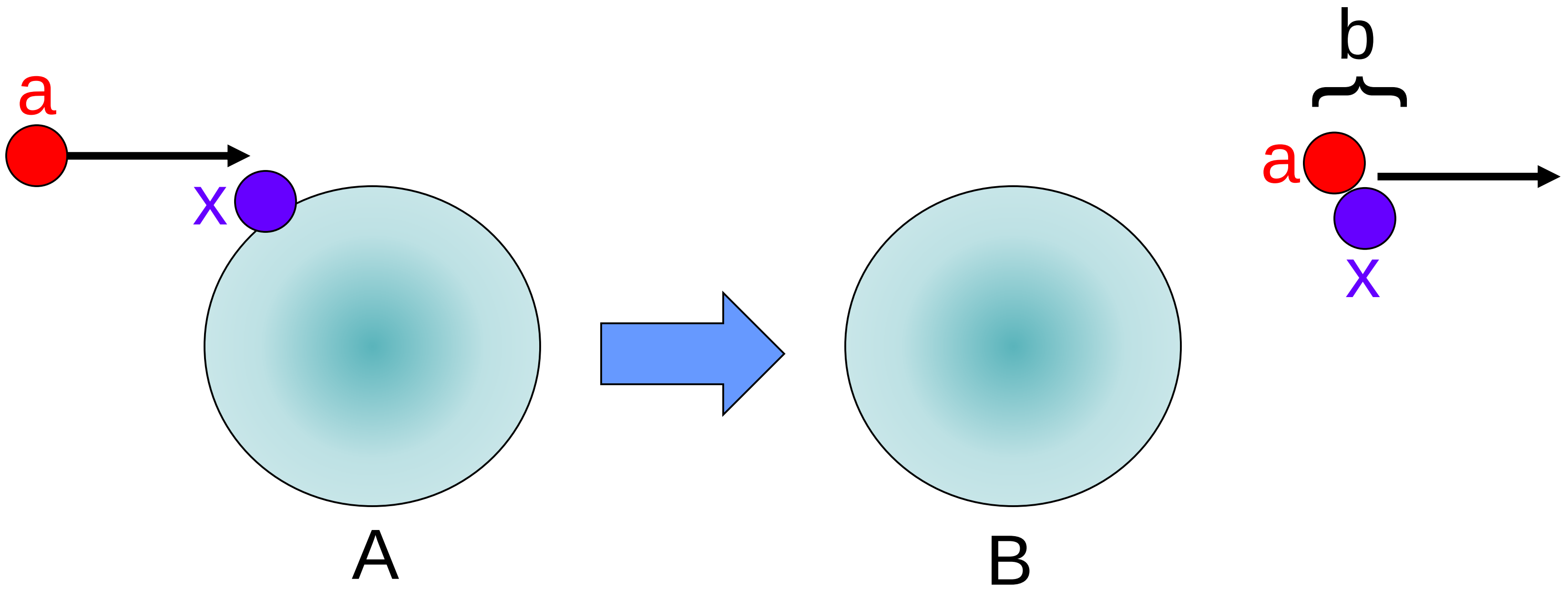
Direct transfer reactions



Adding nucleon(s) to A:
"x" is transferred from a to A, making B=A+x
and b=a-x

Known as "Stripping"
x can be one or more nucleons

Direct transfer reactions



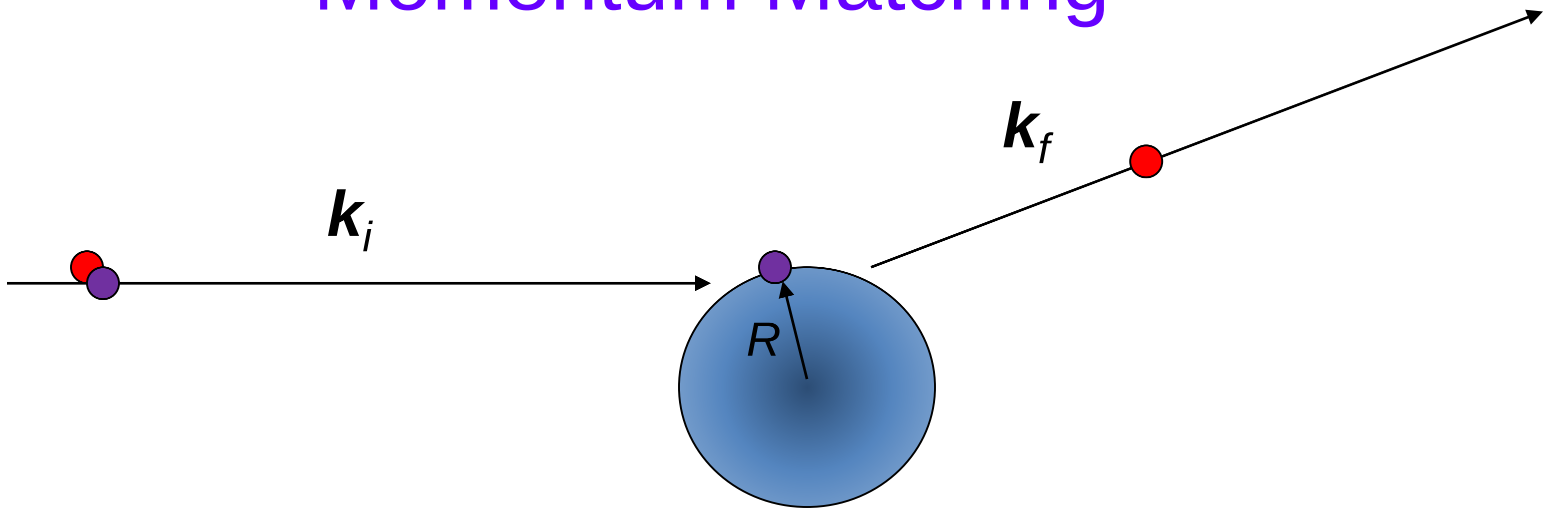
Removing nucleon(s) from A:
"x" is transferred from A to b, making B=A-x
and b=a+x

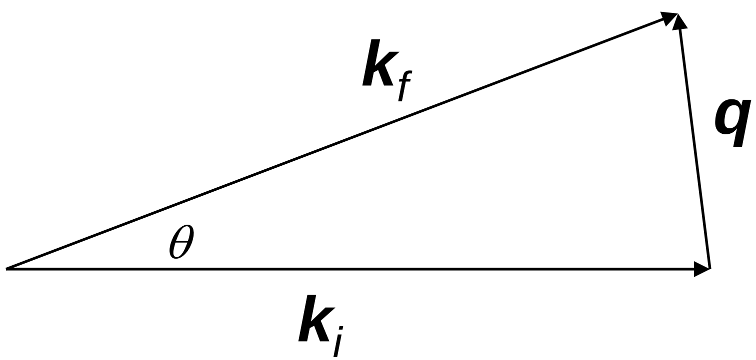
Known as "Pickup"
x can be one or more nucleons

Why do we like direct transfer?

- It is Selective
  - Single-nucleon transfer preferentially populates simple states with strong “single-particle” character
  - Important for understanding the nature of single-particle levels, especially interesting now in the era of “modified shell structure” in exotic nuclei
  - Different reactions probe different amplitudes
- It is “Easy” to understand
  - Reaction mechanism is relatively simple – a single-step transition between two states
- The cross sections tend to be “large”
  - 1 to 10s of mb/sr for single particle stripping & pickup
- In the old days it was “easy” to measure
  - Not so much any more...

Some simple considerations: Momentum Matching





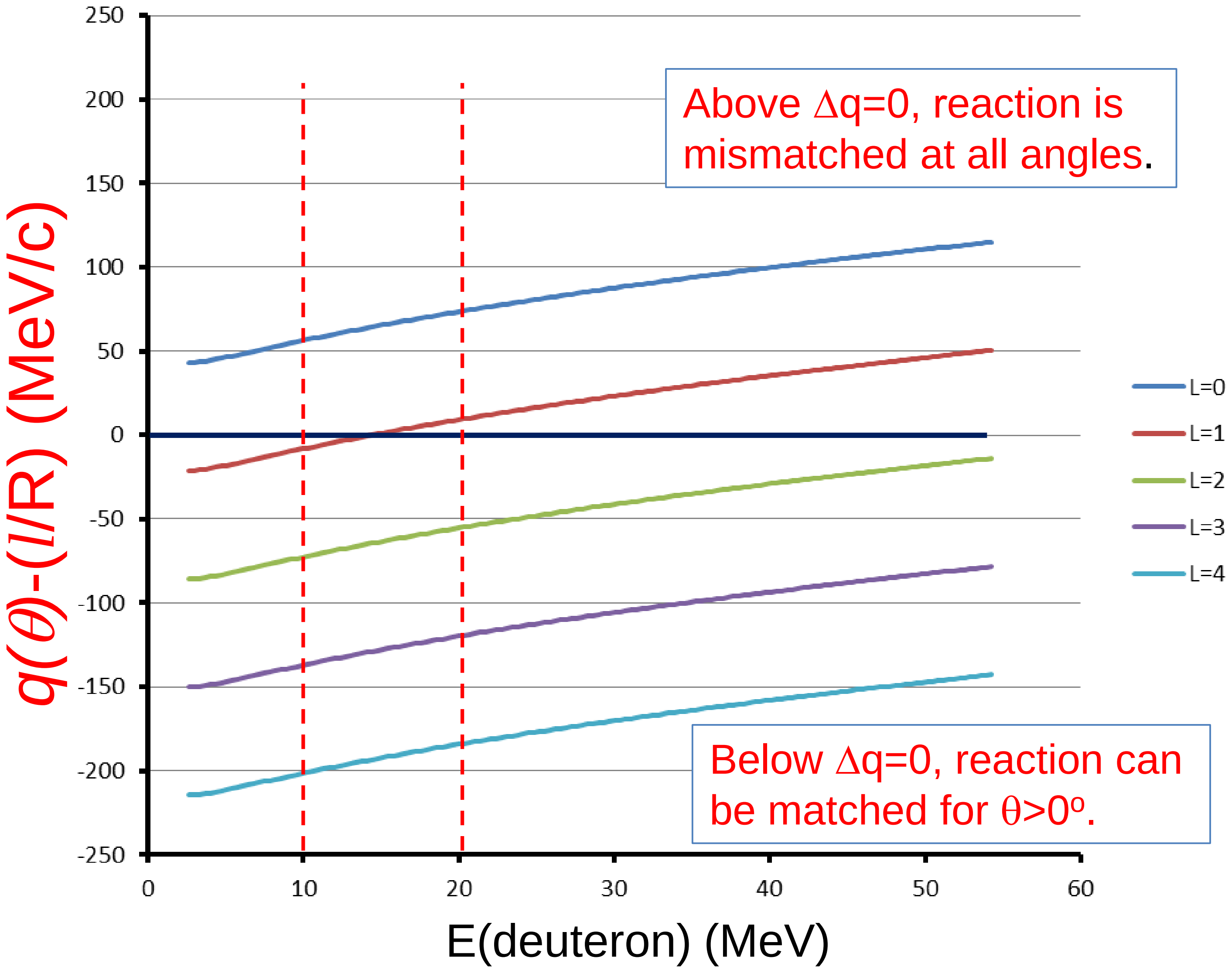
$$q^2 = k_i^2 + k_f^2 - 2k_i k_f \cos\theta$$

angular momentum of transferred particle = $qR = l$, or $q = l/R$

This roughly fixes the best angle for transfer:

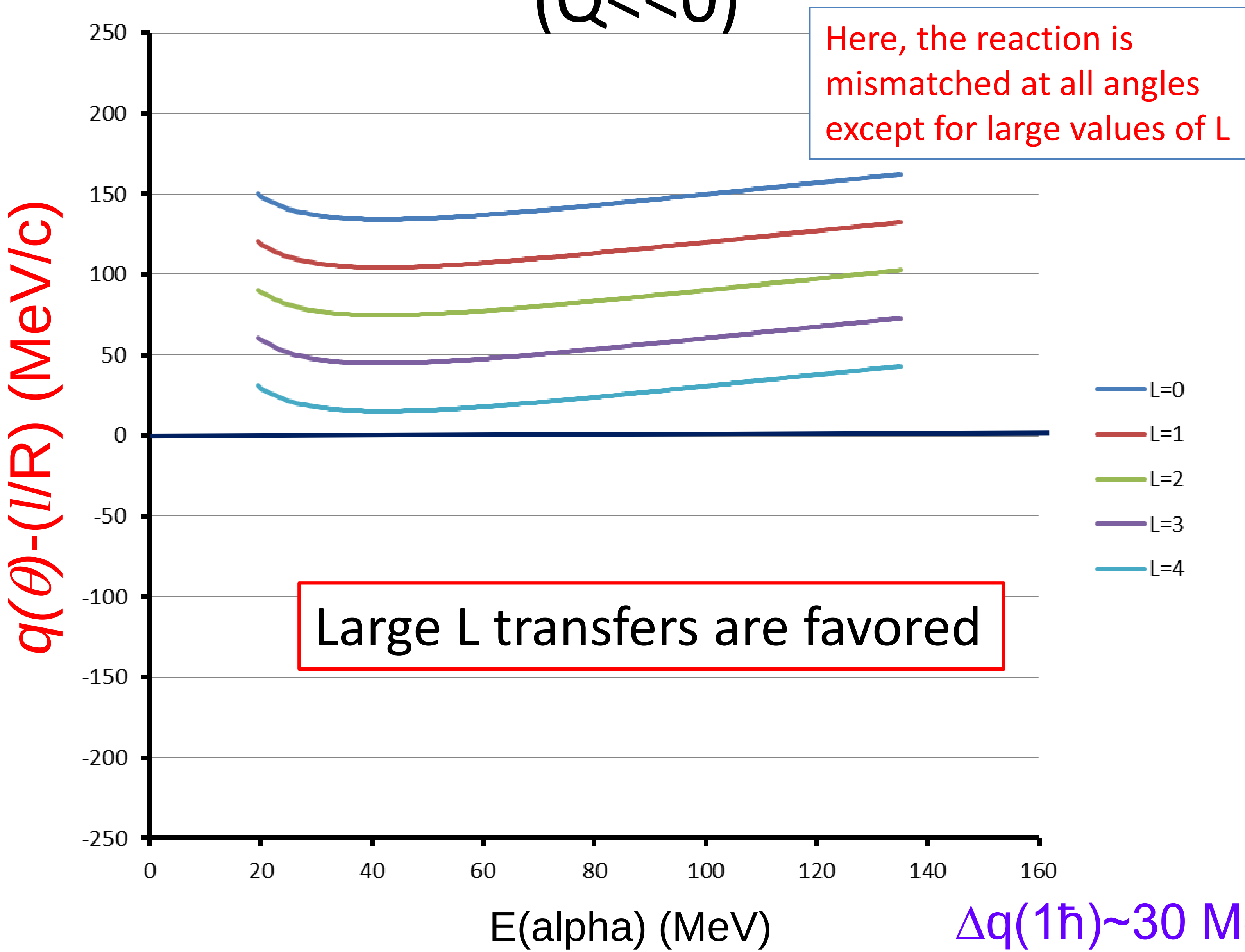
$$\theta_{\max} = \cos^{-1}\left(\frac{k_f^2 + k_i^2 - (l/R)^2}{2k_f k_i} \right)$$

(*d*,*p*) momentum mismatch at 0° (A_{tgt}=13) (Q~0)



Δq(1ħ)~65 MeV/c

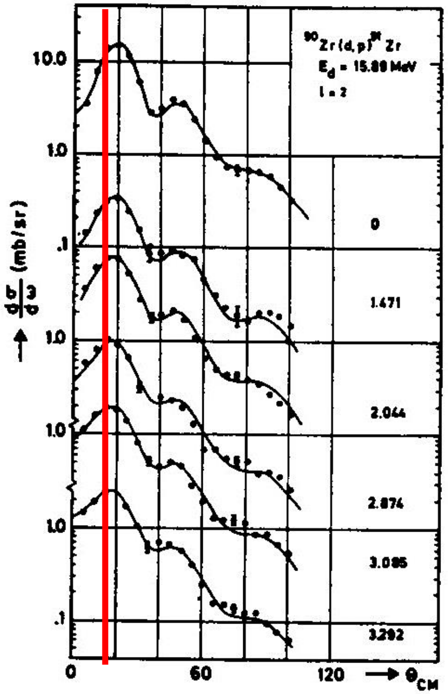
(α,t) momentum mismatch at 0° (A_{tgt}=132) (Q<<0)



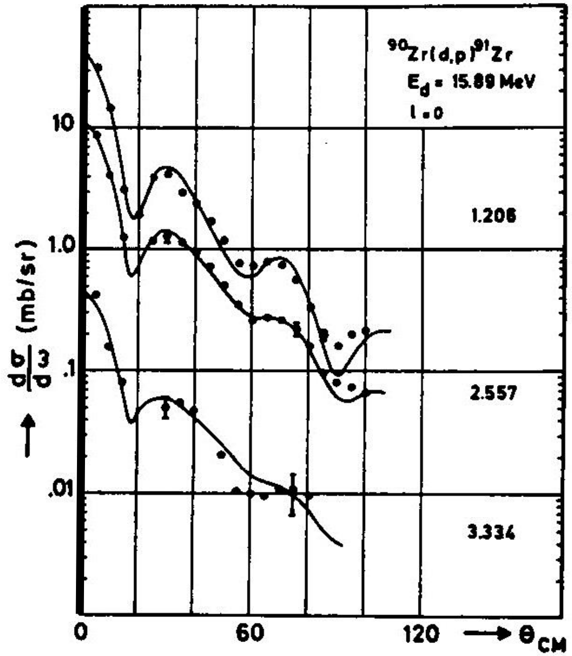
$\Delta q(1\hbar)$~30 MeV/c

Neutron stripping: $^{90}Zr(d,p)^{91}Zr$ (Q=4.97 MeV)

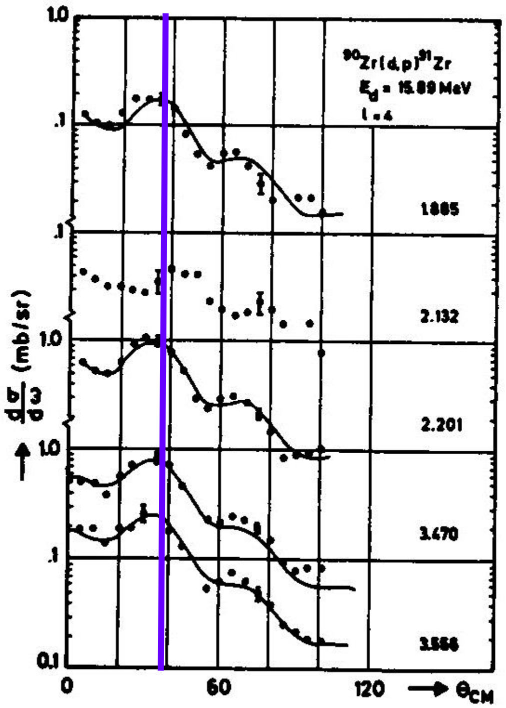
l=2



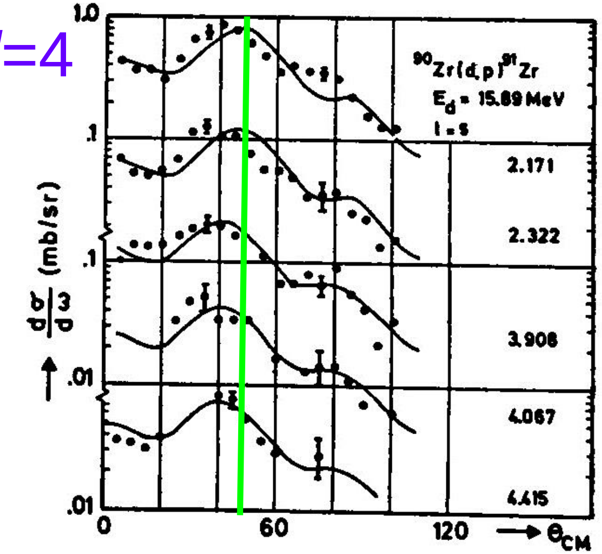
l=0



l=4



l=5



(*d,p*) is the prototypical direct-transfer reaction. ^{2}H is simple and loosely bound.

Lines indicate estimated θ_{max}

H. P. Block et al, NPA 273 142 (1976).

Early (*d,p*) theory and data from Phys. Rev. **80** (1950)

On Angular Distributions from (*d, p*) and (*d, n*) Nuclear Reactions

S. T. BUTLER*
Department of Mathematical Physics, University of Birmingham, Birmingham, England
October 30, 1950

$$\frac{d\sigma}{d\Omega} \propto \left| \int_{R_B}^{\infty} j_L(qr) u_{nl}(r) r dr \right|^2 \approx | j_L(qR_B) |^2$$

R_B is the "Butler radius"

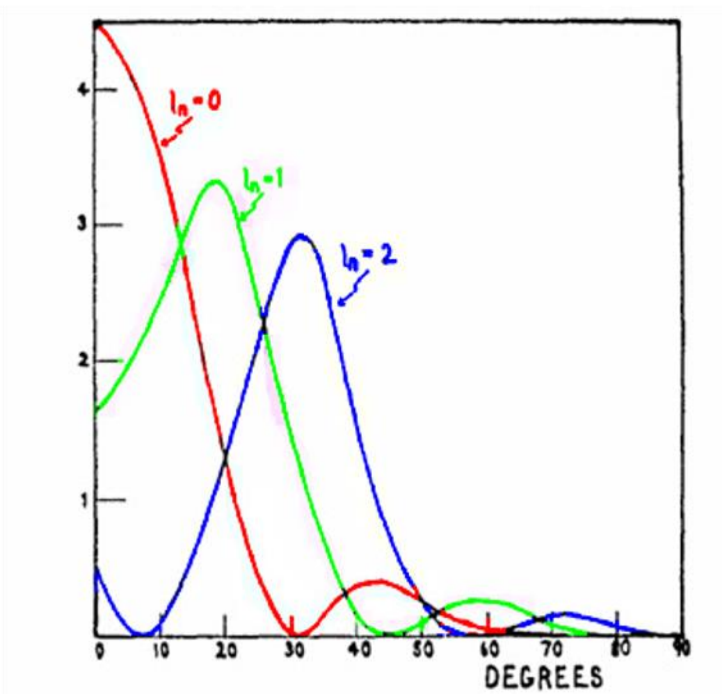


FIG. 1. Theoretical angular distributions for (*d, p*) and (*d, n*) reactions for different angular momentum transfers to the initial nucleus.

Angular Distributions of Protons from the Reaction $O^{16}(d,p)O^{17}$

HANNAH B. BURROWS
University of Liverpool, Liverpool, England
W. M. GIBSON
University of Bristol, Bristol, England
AND
J. ROTBLAT
Medical College of St. Bartholomew's Hospital, London, England
October 30, 1950

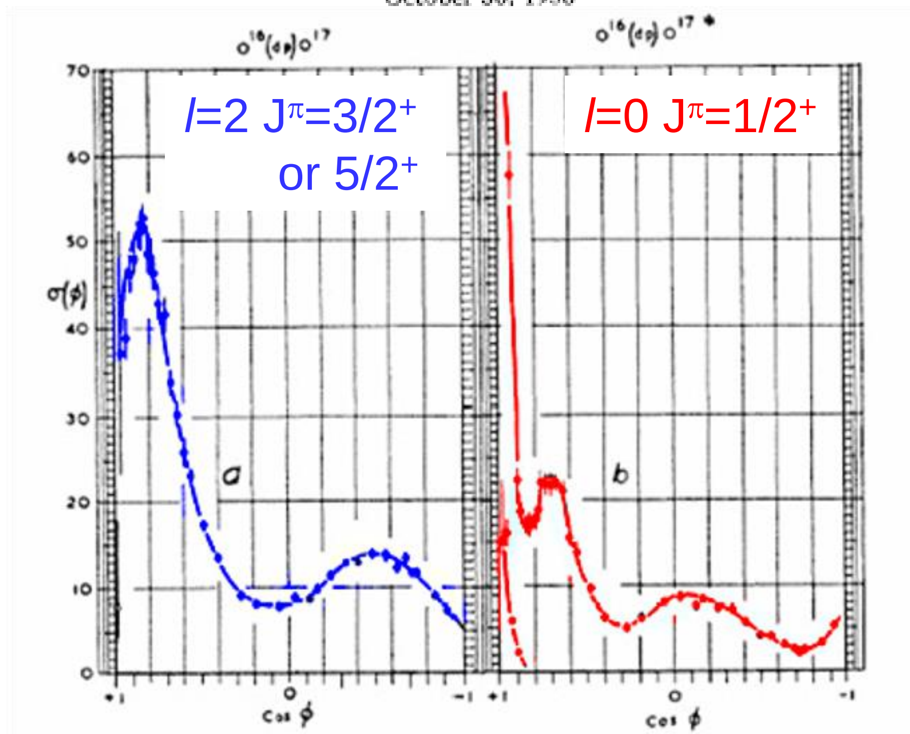


FIG. 1. $O^{16}(d, p)O^{17}$ angular distributions in the center-of-mass (c.m.) system: ϕ = c.m. angle, $\sigma(\phi)$ = c.m. differential cross section in arbitrary units. Curve *a* is for formation of O^{17} in the ground state, and curve *b* is for the 0.88-Mev excited state.

Early spin-parity assignments

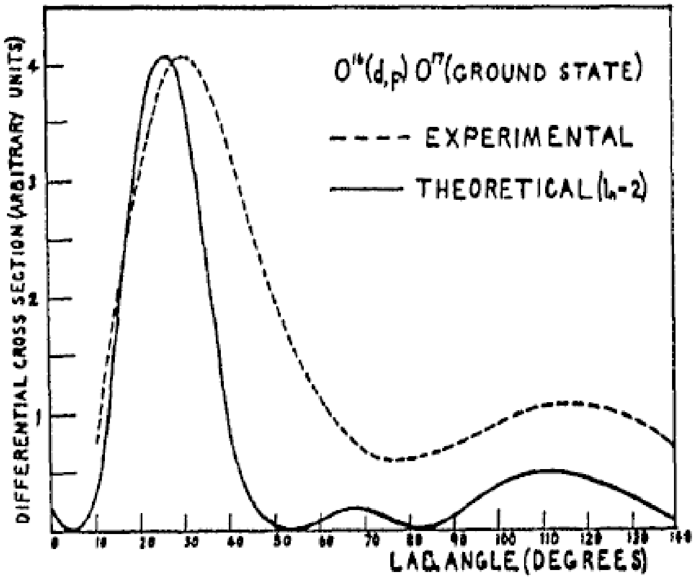


FIG. 2. Comparison of experimental and theoretical distributions for the ground-state transition of the reaction $O^{16}(d,p)O^{17}$ with 7.9-Mev incident euterons. The theoretical curve is that for $l_n=2$.

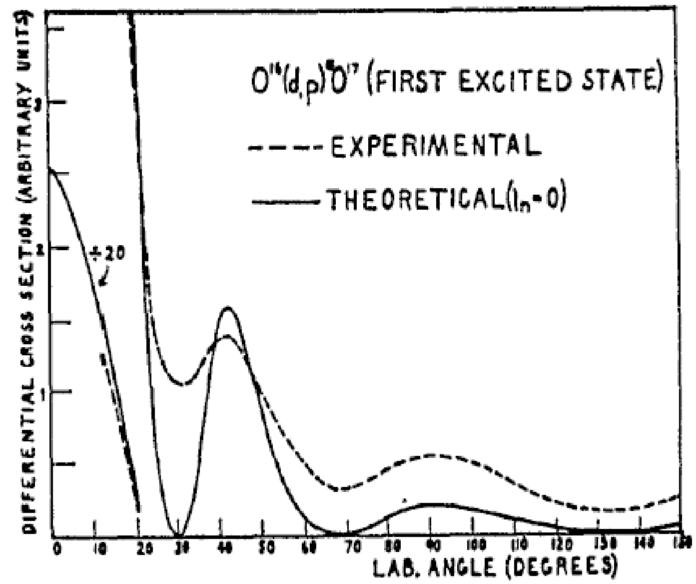


FIG. 3. Comparison of experimental and theoretical distributions for the transition to the 0.88-Mev excited state of O^{17} in the reaction $O^{16}(d,p)O^{17}$ with 7.9-Mev incident deuterons. The theoretical curve is that for $l_n=0$.

TABLE I. Spin and parity assignments.

Reaction	Ground state initial nucleus	Final nucleus: Ground state	Final nucleus: First excited state
$O^{16}(d,p)O^{17}$ [a]	0+	(5/2 or 3/2)+	1/2+
$N^{14}(d,p)N^{15}$ [a]	1+	(1/2, 3/2, or 5/2)−	
$C^{12}(d,p)C^{13}$ [a]	0+	(1/2 or 3/2)−	
$Al^{27}(d,p)Al^{28}$ [b]	5/2+	(2 or 3)+	(0, 1, 4, or 5)+

Butler, Phys. Rev. **50** (1950)

The shape tells you *l* – what about the rest?

> I have calculated angular distributions resulting from such a stripping process by equating, at the nuclear surface, the exact wave function for a particle outside the nucleus to the interior wave function. After some simplification the resulting boundary equations can be solved in such a way that unknown properties of the nuclear wave functions affect the important parts of the distributions merely as a constant multiplying factor. The re-

(Butler, 1950)

...Known today as the "spectroscopic factor"

This contains the nuclear structure information

What does it mean and How do we get it?

Interpretation of S

- *S* reflects the overlap between the initial and final states; $d\sigma/d\Omega \propto S$
- *S* “measures” orbital vacancies (# of holes) for stripping, or orbital occupancies (# of particles) for pickup.
- McFarlane and French (RMP **32**, 1960):
  - #Holes=$\Sigma C^2 S_i\,(2J_F+1)/(2J_I+1)$ (adding or “stripping”)
  - #Particles=$\Sigma C^2 S_i$ (removing or “pickup”)
  - Sum is over all states that could have a particle in the orbital of interest
- Connection to resonances: $S_i = \gamma^2{}_i/\gamma^2{}_{SP}$ (“Schiffer’s anzatz”)

How do we "measure" *S* ??

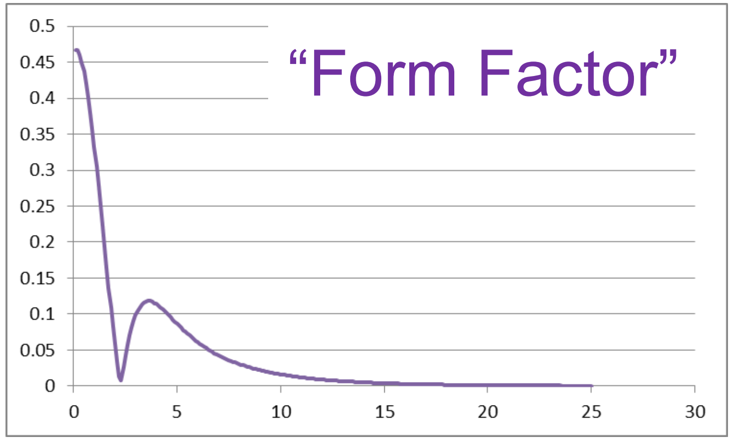
- *S* is *not* an experimental observable, so you *cannot* "measure" it.
- Does that mean *S* is meaningless, as some might claim?
- I think no – meaningful values of *S* can be deduced from comparisons between measured cross sections and the predictions of nuclear reaction models. (Typical is the Distorted Wave Born Approximation or DWBA).
- But then – *S* is *model dependent*, so *caveat emptor*.
- We can try to deduce *absolute* or *relative* values of *S*.

What more can spectroscopic factors tell us?

- They tell us about the occupancy of nuclear shells
- By knowing the energies, spins, parities, and spectroscopic factors of levels we can estimate the energies of the single-particle orbitals
- Knowing how the strength is distributed between different states can tell us about the residual interaction, and help to tune shell-model calculations.
- We can investigate effects that come about through terms in the NN interaction such as the tensor force

But – we need a theory to describe the reaction:
"Distorted-wave Born approximation" or DWBA

One-page summary of the DWBA

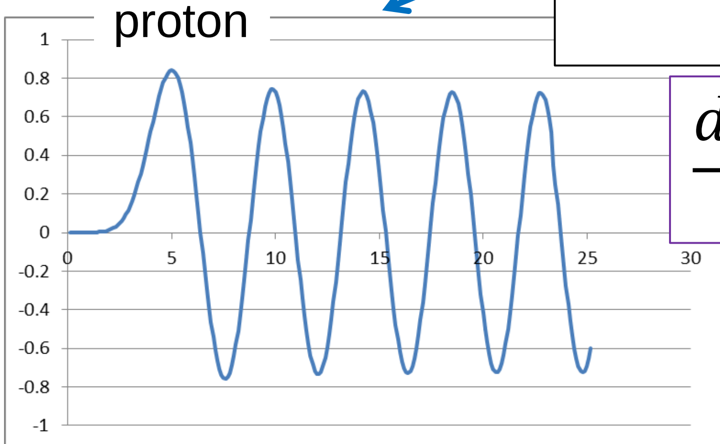


(or single-particle overlap for $B=A+x$)

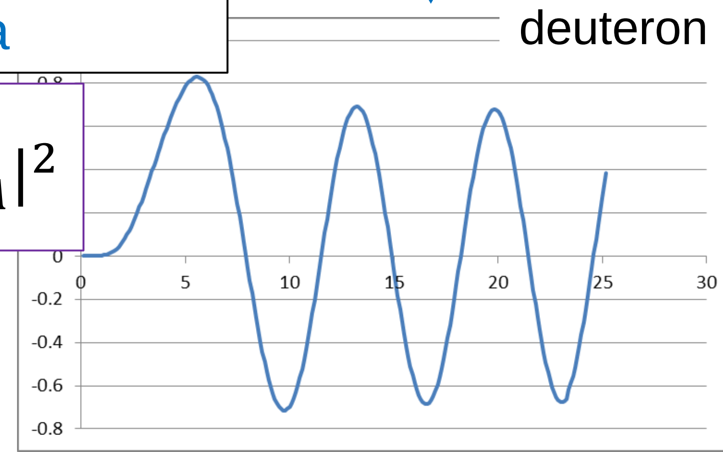
Matrix element with nuclear structure

$$T_{DWBA} = J \int d^3 r_b \int d^3 r_a \, \chi^-(\vec{k_f}, \vec{r_b}) < bB|V|aA > \chi^+(\vec{k_i}, \vec{r_a})$$

Distorted waves from OM – many different incident and outgoing angular momenta



$$\frac{d\sigma_{DWBA}}{d\Omega} = |T_{DWBA}|^2$$



Compare data to DWBA:

$$\frac{d\sigma_{EXP}}{d\Omega} = C^2 S \times \frac{d\sigma_{DWBA}}{d\Omega}$$

$C^2S = C^2S(b + x \rightarrow a)\ C^2S(A + x \rightarrow B)$

(for stripping)

Can often calculate these:
e.g. $d \rightarrow p + n$
or $^3He \rightarrow d + p$

$C^2S = C^2S(a + x \rightarrow b)\ C^2S(B + x \rightarrow A)$

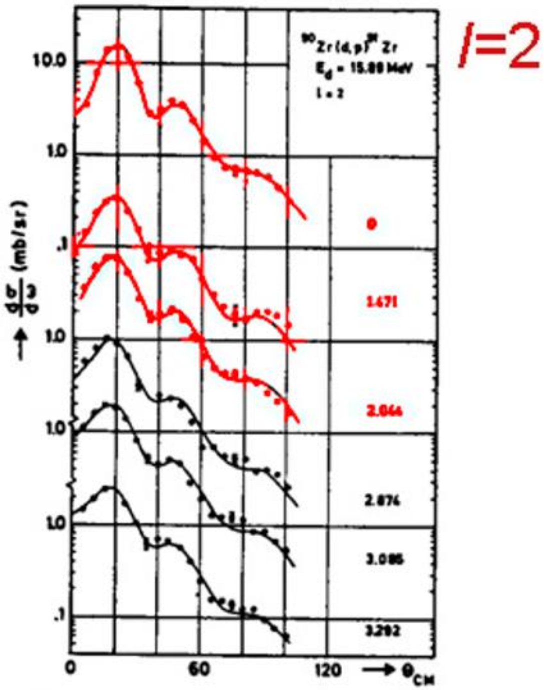
(for pickup)

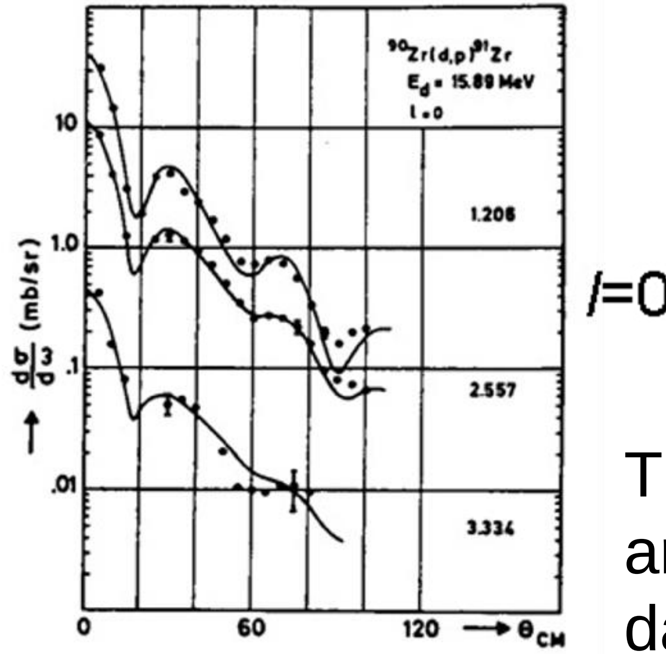
C's are Isospin Clebsch-Gordan coefficients:

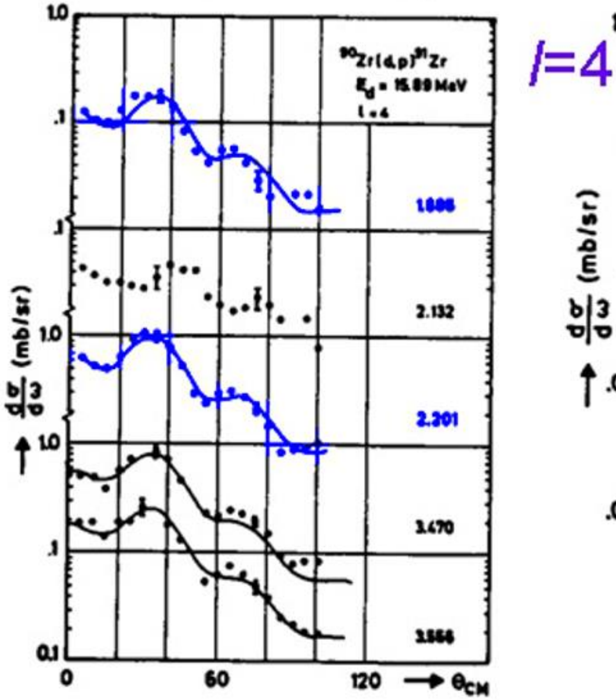
$C_{bx} = (T_b m_b T_x m_x \mid T_a\ m_a)$

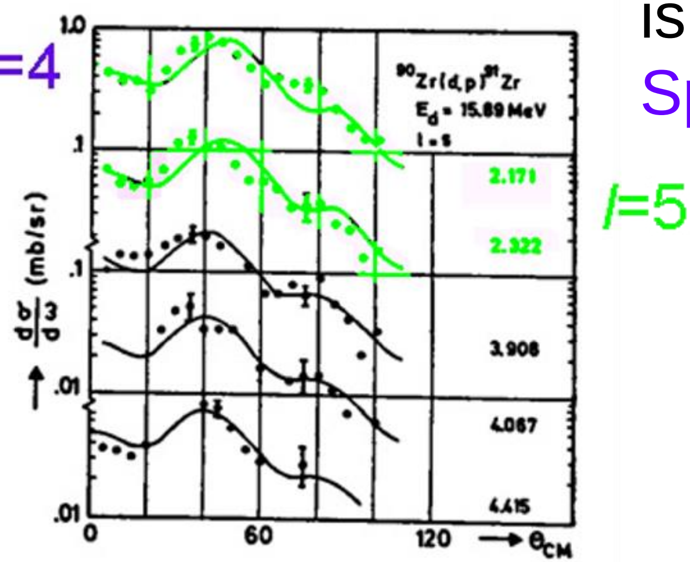
$C_{Ax} = (T_A m_A T_x m_x \mid T_B\ m_B)$

Back to ^{91}Zr





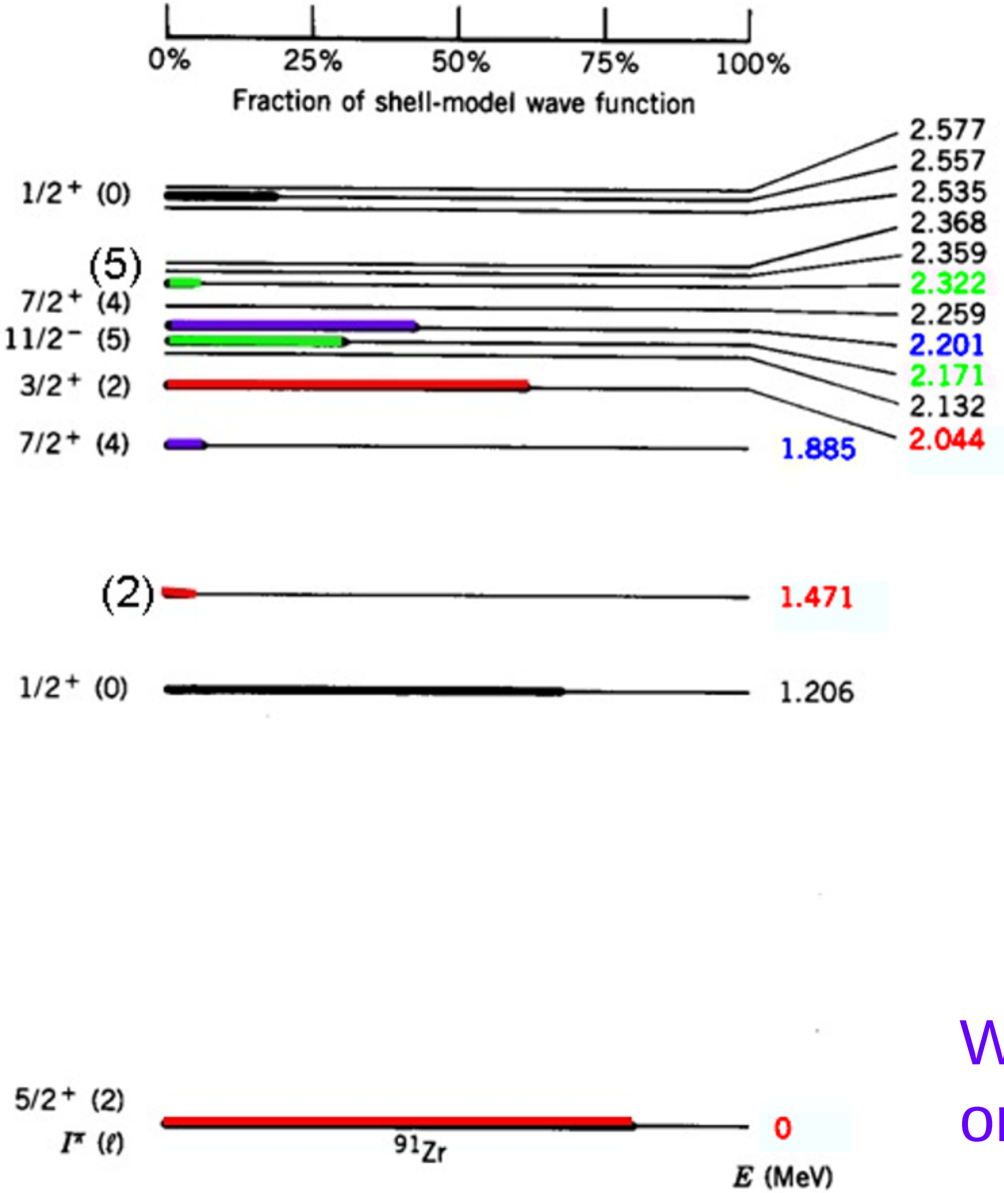




The heights of curves are adjusted to fit the data – the normalization is interpreted as the Spectroscopic Factor

H. P. Block et al, NPA 273 142 (1976).

Extracted S.F. for ^{91}Zr



Neutron orbitals of interest:

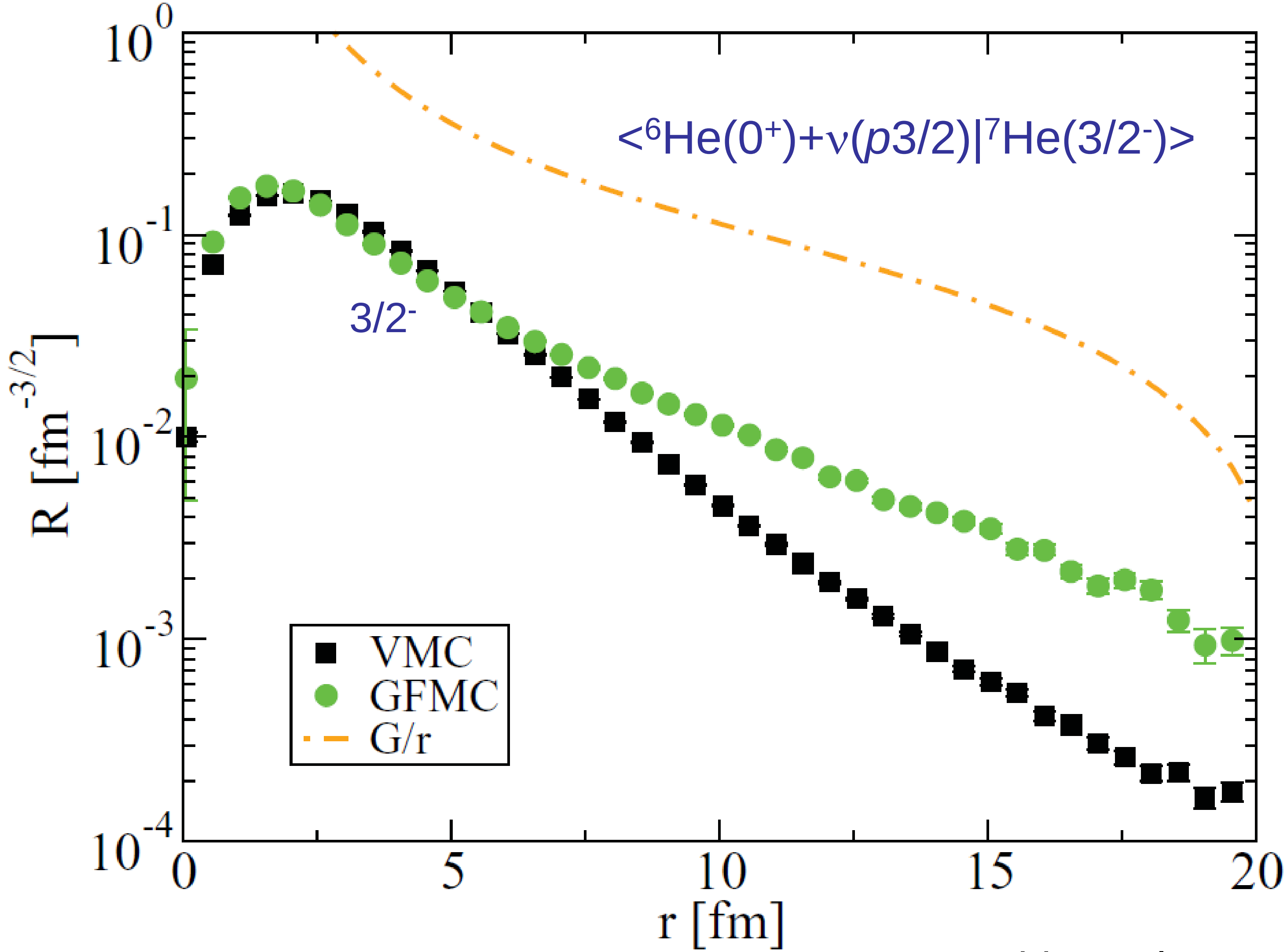
1g7/2 : l=4
2d5/2 : l=2
2d3/2 : l=2
3s1/2 : l=0
1h11/2: l=5

We can use this to deduce the order of single-particle orbitals

Caveat Emptor

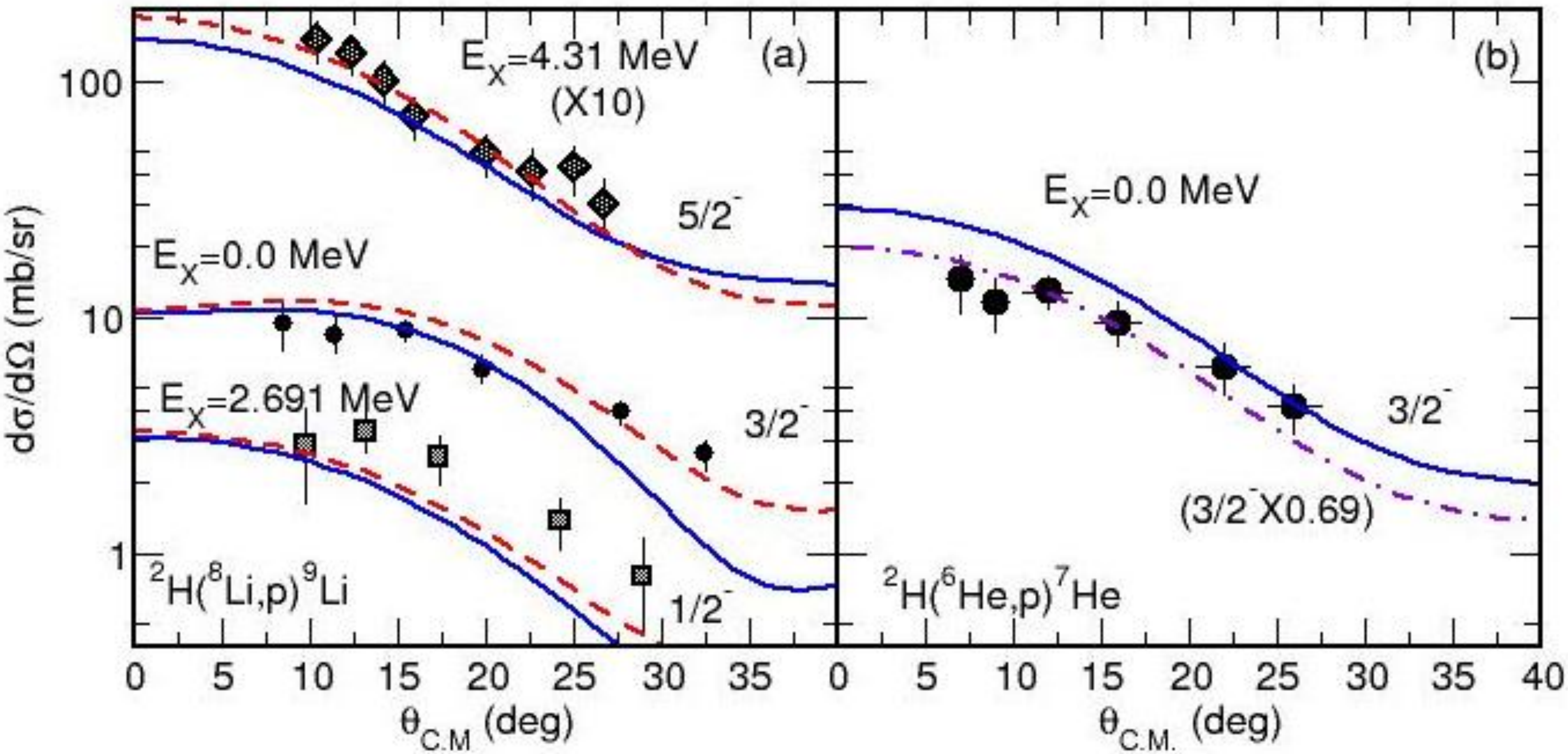
- Limitations:
  - Arbitrary normalization to peak σ is unsatisfying
  - Approach is model dependent (potential parameters)
  - May miss important physics (*d* breakup, for instance)
  - Limited predictive power
  - (In) Consistency of optical potentials. For an excellent survey, see J. Lee et al., PRC **75**, 064320 (2007).
  - The energy must be high enough to be Direct: CN contributions can occur below 2-4 MeV/u (!)
- Improvements:
  - Use global potentials, folding model or CC/CDCC to zero in on elastic scattering (χ's) and inelastic contributions
  - We can use other modern methods to try to *predict* the Form Factor (AKA the nuclear structure information) and S, for example:
  - "Quantum Monte Carlo", "No-Core Shell Model" : so-called "*ab-initio*" methods can be done for light nuclei.

^{7}He->^{6}He+n Overlap from VMC/GFMC



I. Brida et al, PRC 84 (2011)

(*d,p*) with ^{8}Li, ^{6}He: No Fitting Allowed



n+^{8}Li, *n*+^{6}He: from QMC (good)

d+^{8}Li, *d*+^{6}He: OMP from old *p*-shell work (ok for ^{8}Li, *not* so good for ^{6}He)

p+^{9}Li, *p*+^{7}He: ditto!

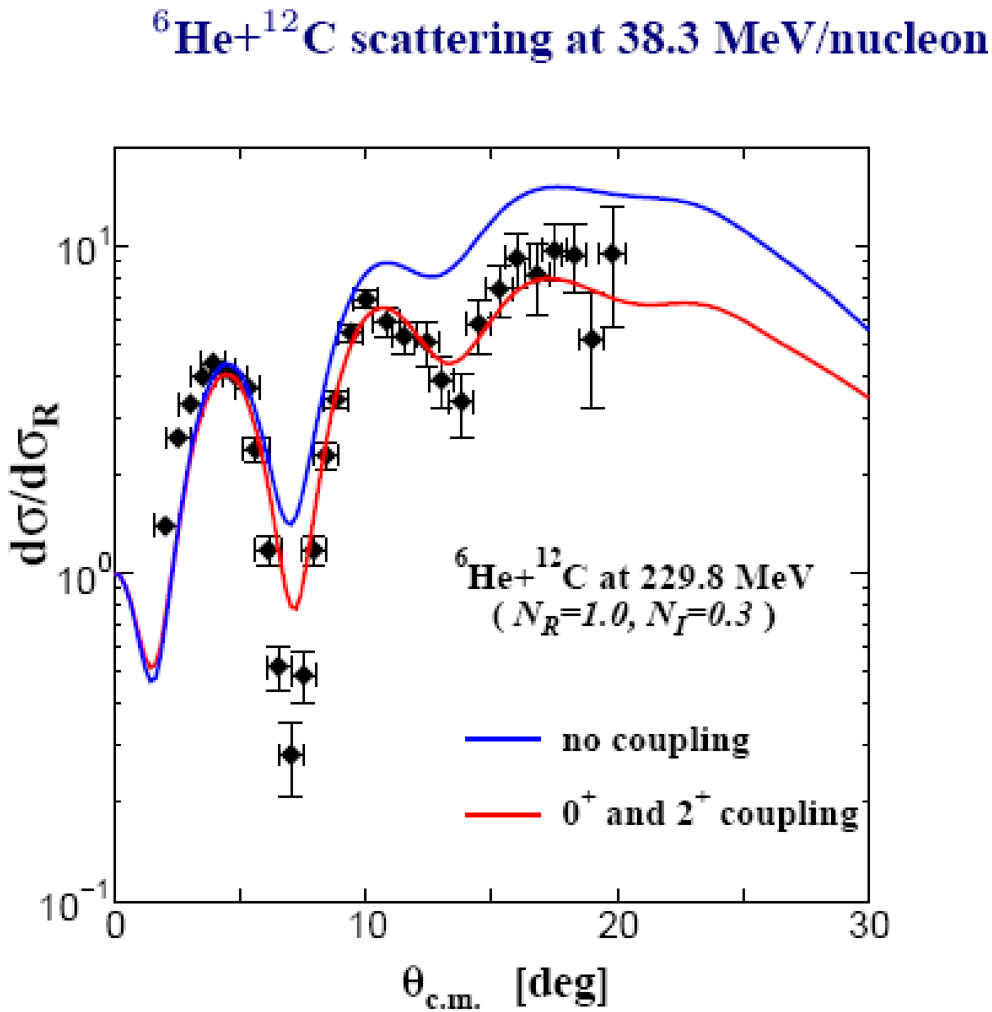
And no channel coupling for ^{6}He or ^{7}He (also maybe not so good!)

Results still seem to be ok to the 20-30% level

Conclusions

- Scattering and transfer reactions can tell us a lot about nuclear structure.
- We have to combine information from many different places to gain understanding.
- We must not forget that much of what we “know” we actually don’t – we surmise in the context of models, so we should be careful about our claims.
- Next time: some concrete examples

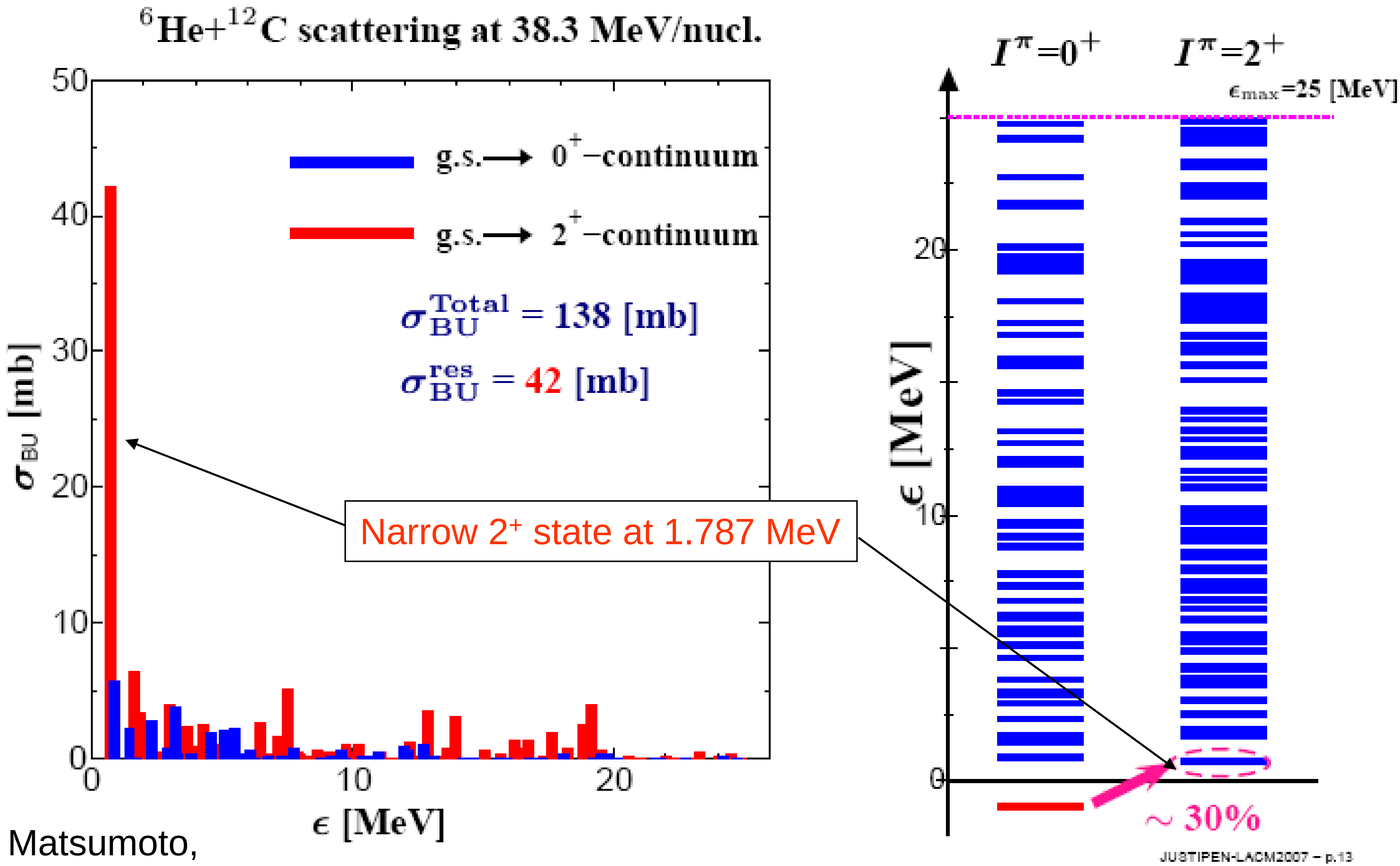
Continuum coupling important for diffuse, loosely bound nuclei



V. Lapoux *et al.*, Phys. Rev. C 66, 034608 (2002).

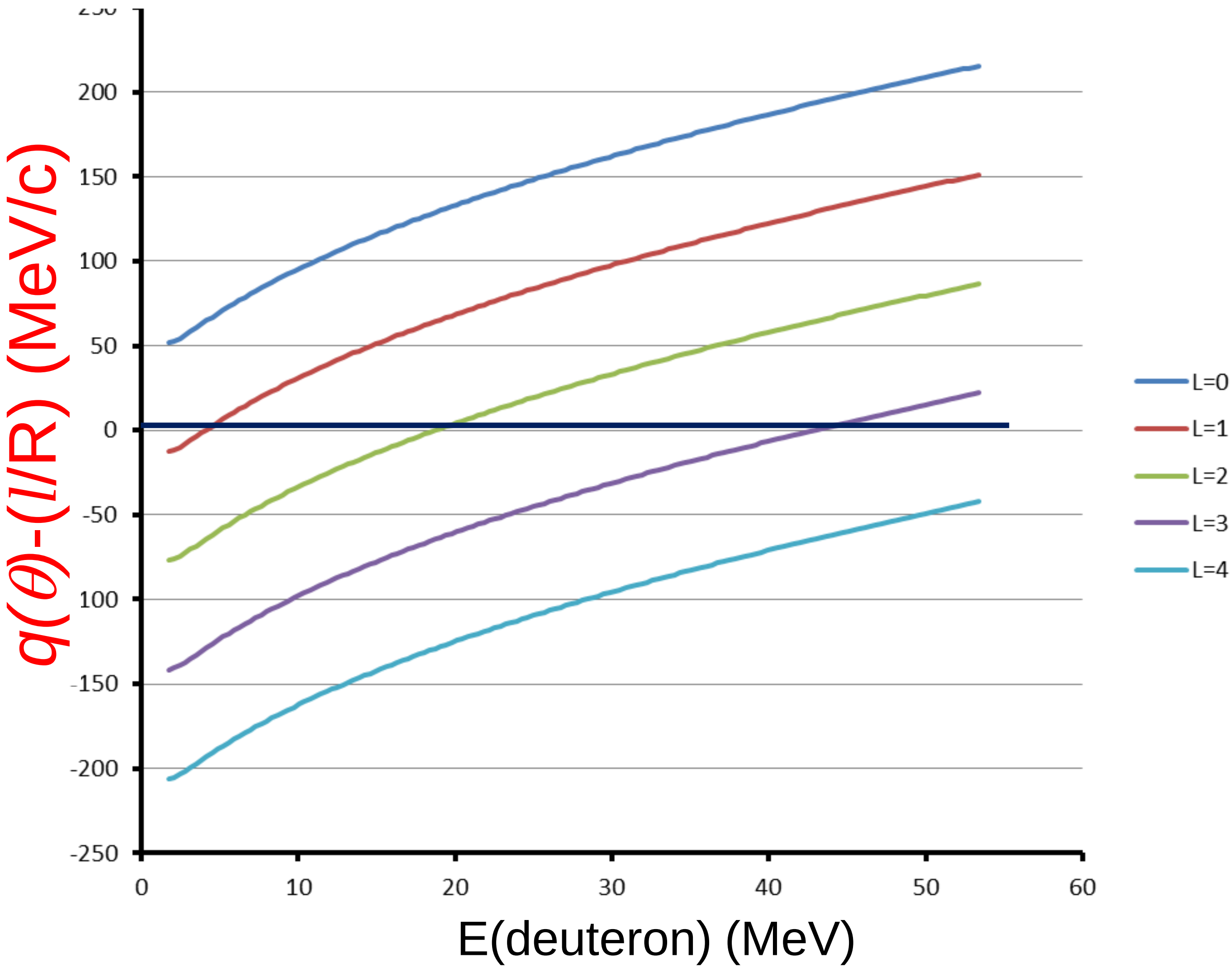
T. Matsumoto,
Joint JUSTIPEN-LAMC workshop, 2007

Where does that flux go?



T. Matsumoto,
Joint JUSTIPEN-LAMC workshop, 2007

(d,p) momentum mismatch at 30^o (A_{tgt}=13) (Q~0)



$\Delta q(1\hbar)$~65 MeV/c

Formalism (start with stripping)

Probability of transition:

$T_{fi} = \langle \chi^{-}(\mathbf{k}_b,\mathbf{r}_b)\,\psi_b\,\psi_B \mid V \mid \chi^{+}(\mathbf{k}_a,\mathbf{r}_a)\,\psi_a\,\psi_A \rangle$

where

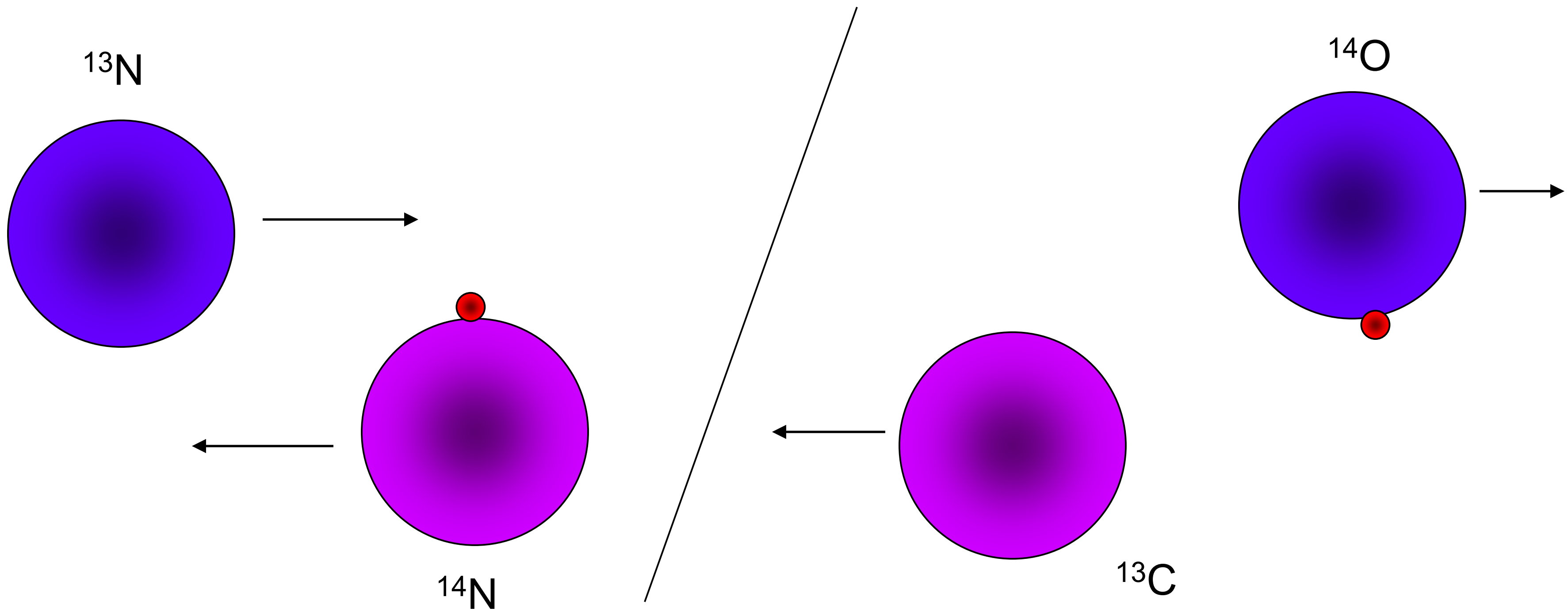
$V = V_{bx} + V_{bA} + U_{bB}(r)$

(recall x is the transferred particle)

The $\chi^{+-}(\mathbf{k},\mathbf{r})$ are Optical Model solutions to elastic scattering in the a+A and b+B channels.
The $\psi_{b,B,a,A}$ are the internal wave functions of the particles in the exit and entrance channels.

We need: Optical model potentials for the entrance *and* exit channels (so we should *measure* elastic scattering for *both* a+A and b+B if possible, or use Global potentials)
And – we need a calculation of the bound state of A+x.

Heavy-ion transfer reactions

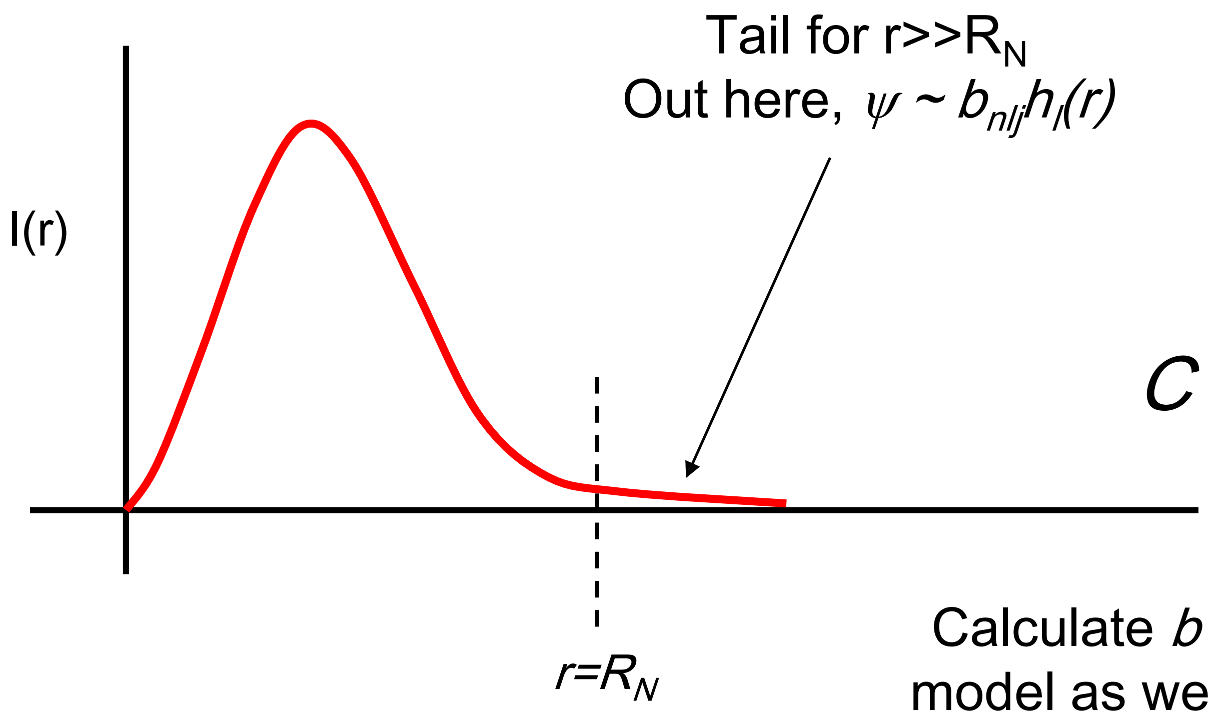


Peripheral collision between heavy ions
"Asymptotic Normalization Coefficient" or ANC takes the place of the spectroscopic factor.
Sample the tails of the nuclear wave functions.

Heavy-Ion transfer and ANC’s

- Peripheral collisions – close or head-on collisions lead to more complex processes
- Samples the tail of the wave functions- “Asymptotic Normalization Coefficients” or “ANCs”
- Why do it this way? Many astrophysical processes occur at very low energies and are extremely peripheral.
- Analyze in a very similar way.
- Are the approaches consistent?...

ANCs - schematically



$$C^2_{lj} = S_{nlj} b^2_{nlj}$$

Calculate *b* from a single-particle model as we did for S.F. from (*d,p*)

S is the overlap integral for all *r*

C^2 is the overlap only in the asymptotic region

See D. Y. Pang et al, PRC **75** 024601 (2007) for a nice review of the connection between SFs and ANCs

$^{14}N(^{13}N,^{14}O)^{13}C$: Application to $^{13}N(p,\gamma)^{14}O$

- Want to learn about $^{13}N+p \rightarrow {}^{14}O$
  - Interesting for the CNO cycle
- Need to understand:
  - $^{13}C+p \rightarrow {}^{14}N$ (you can take both p1/2 and p3/2 protons from ^{14}N)
  - $^{14}N+^{13}N$ and $^{14}O+^{13}C$ elastic scattering
- Introduce the "Asymptotic Normalization Coefficient" or "ANC"

(Many) Pieces you need:

Want to determine this

Have previously measured these

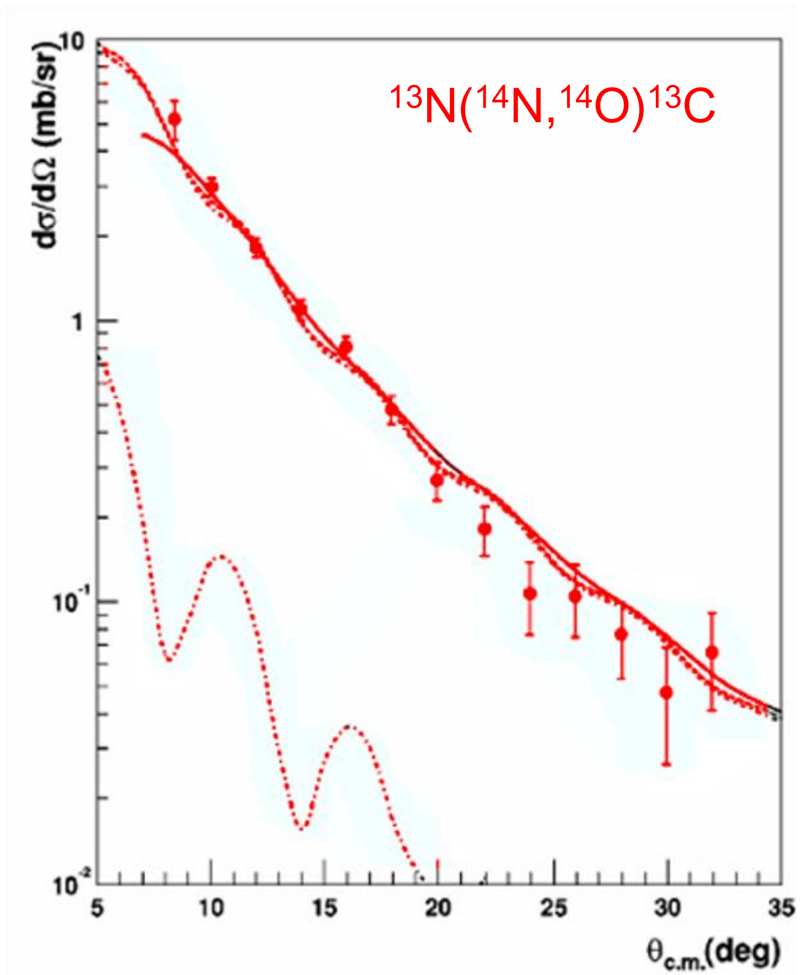
$$\sigma_{EXP} = \left(C^{^{14}O}_{p1/2}\right)^2 \left[\left(\frac{C^{^{14}N}_{p1/2}}{b^{^{14}O}_{p1/2} b^{^{14}N}_{p1/2}} \right)^2 \sigma^{DWBA}_{1/2;1/2} + \left(\frac{C^{^{14}N}_{p3/2}}{b^{^{14}O}_{p1/2} b^{^{14}N}_{p3/2}} \right)^2 \sigma^{DWBA}_{1/2;3/2} \right]$$

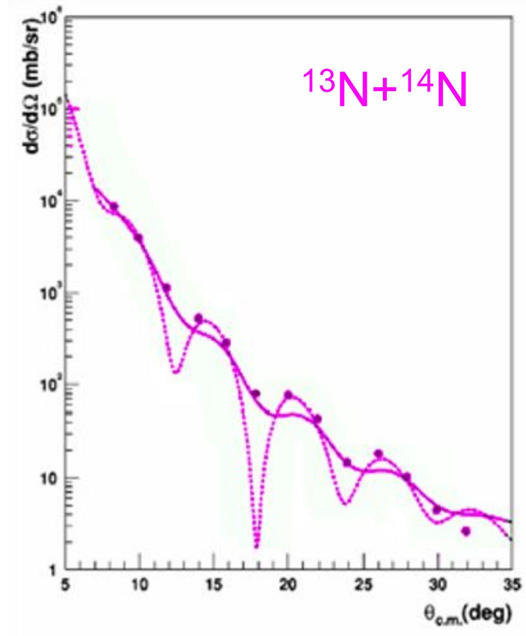
Calculate these with a single-particle model

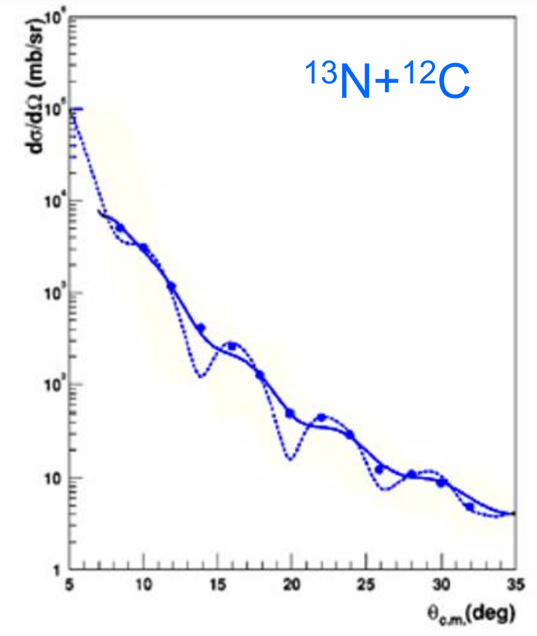
Calculate these with a reaction code

Asymptotic Normalization Coefficients are the Cs
(Not the same as the isospin Clebsch-Gordan coefficients)

The measurement...







$^{13}N+^{14}N$ elastic scattering determines the optical potential. Using the same parameters reproduces $^{13}N+^{12}C$ so it should be ok for $^{14}O+^{13}C$.

Knowing $C(p+^{13}N \rightarrow ^{14}O)$, you can understand
proton capture and it's influence on the CNO cycle in novae

X. D. Tang et al, PRC 69, 055807 (2004)

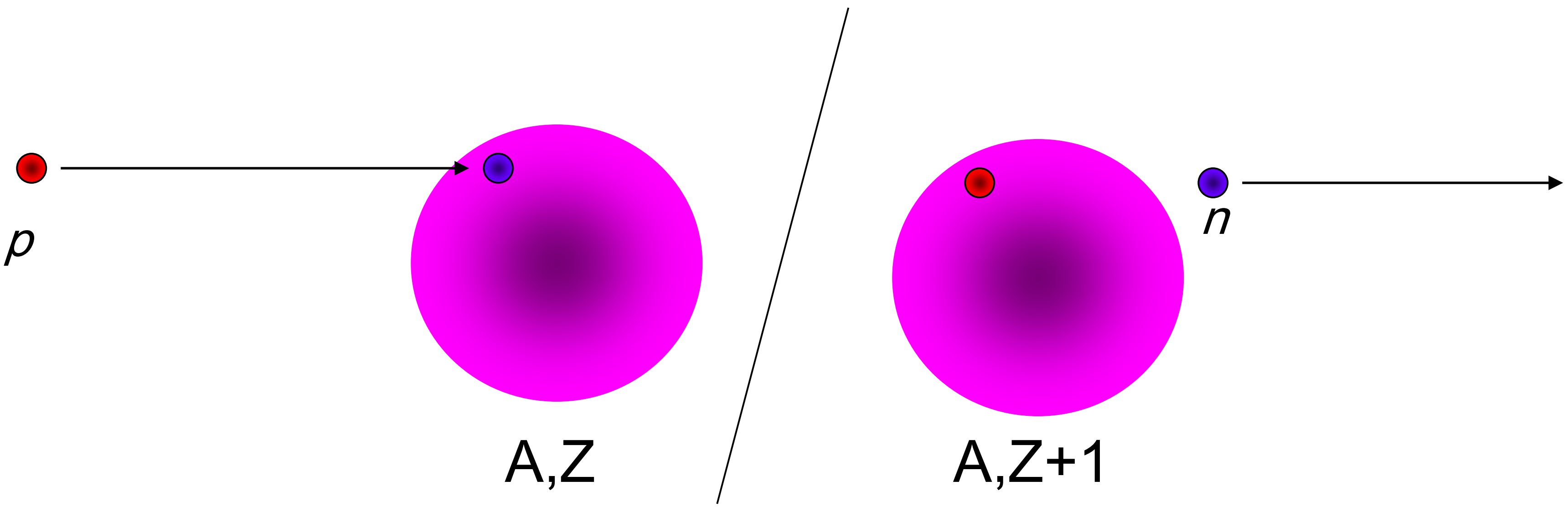
Two other direct processes

Charge Exchange

and

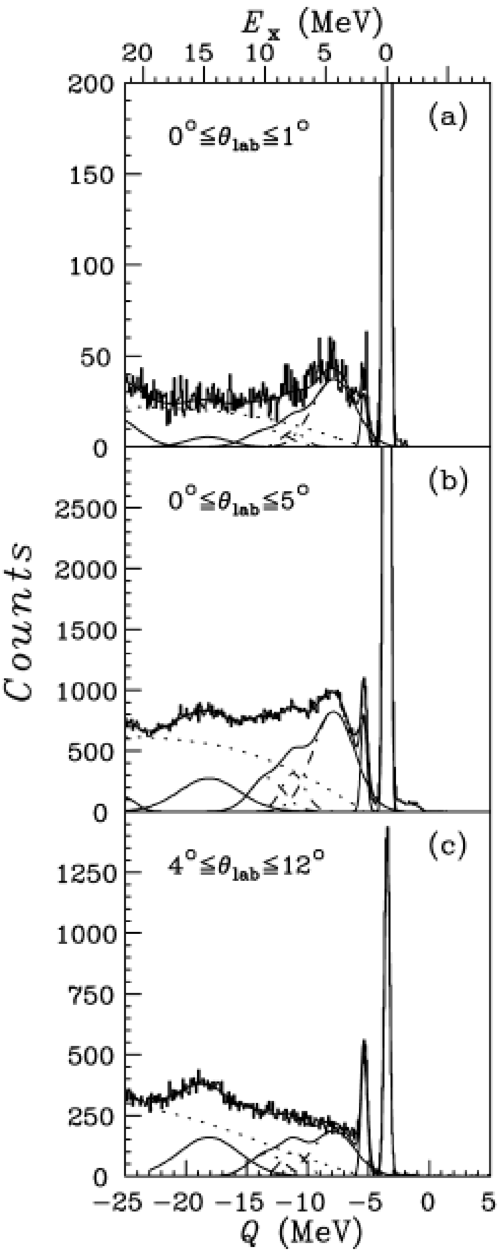
Knock Out

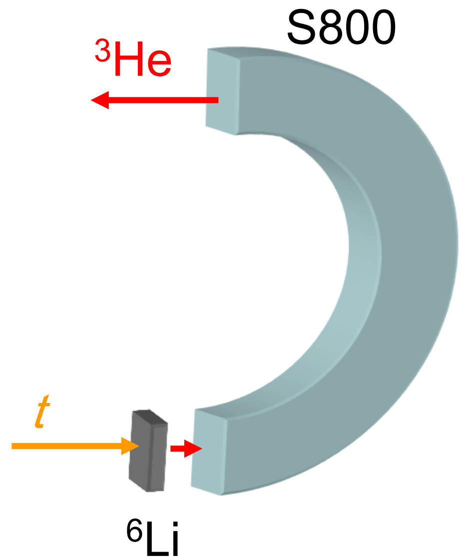
Charge exchange

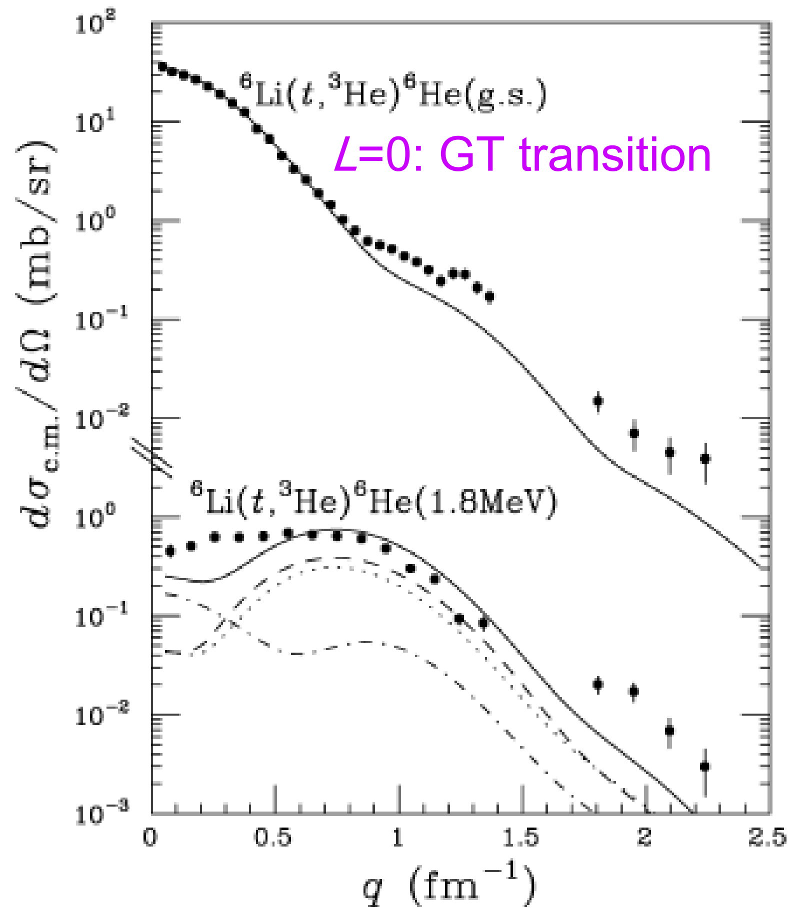


- Like β decay, changes a neutron into a proton or vice-versa (a good probe of Gamow-Teller strength: ΔL=0,ΔT=1,ΔS=1)
- Some examples are (p,n), (^{3}He,t), (t,^{3}He), (d,^{2}He)
- Strongly populates "Isobaric Analog States"

^{6}Li(t,^{3}He)^{6}He charge exchange



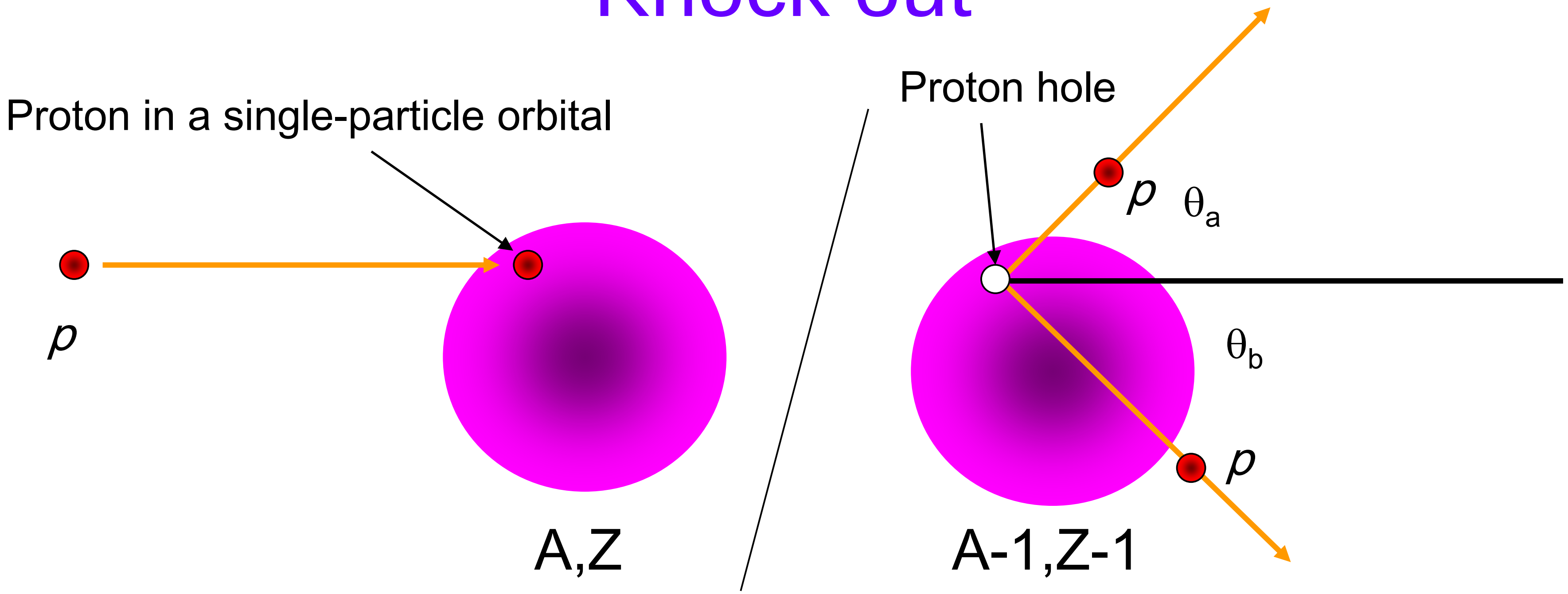




High-resolution measurement with the S800

T. Nakamura et al, PLB **493**, 209 (2000)

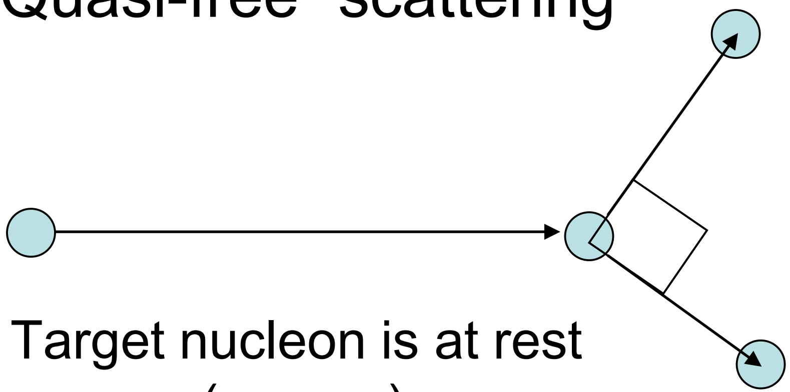
Knock out



- Examples: (*e,e'p*), (*p,2p*), (*p,pn*)
- Need enough energy to overcome proton or neutron binding, and to be approximately single step
- Samples the structure of the target in a way similar to pickup reactions (you can measure a spectroscopic factor)
- Good for studying single-hole (instead of single-particle) states

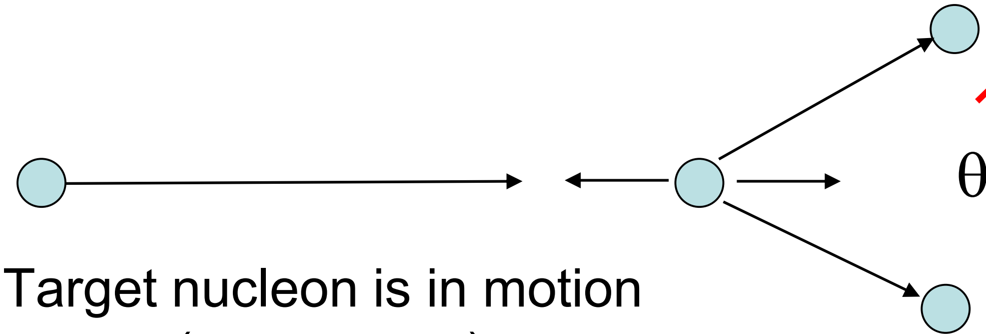
Spectroscopy with knock out: $^{6}Li(p,2p)$

"Quasi-free" scattering



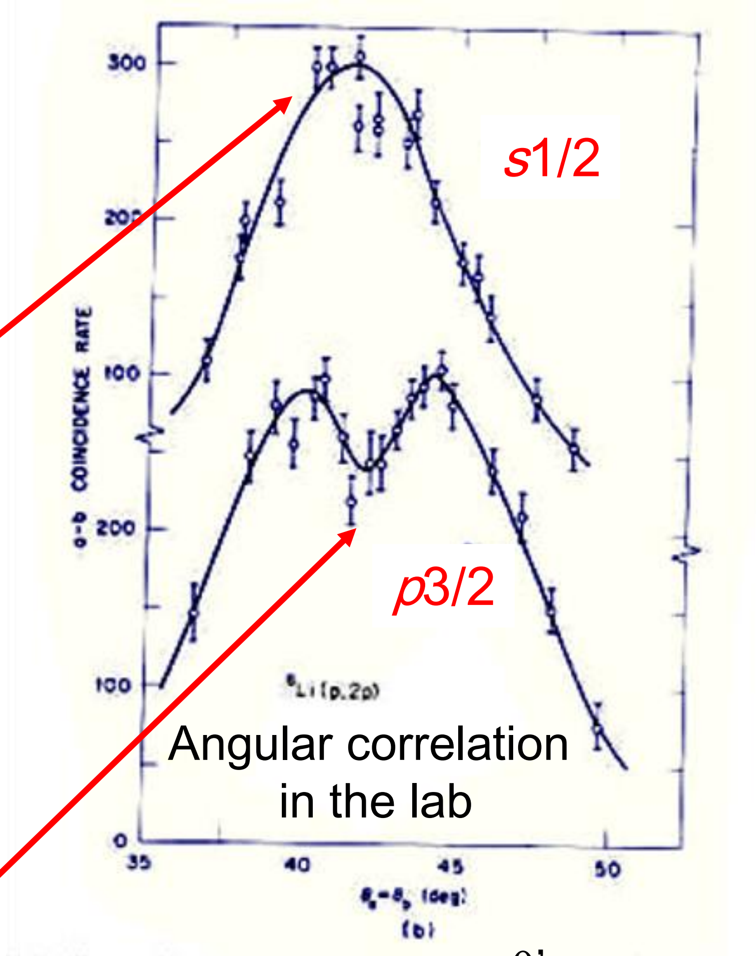
Target nucleon is at rest
(*s*-wave)

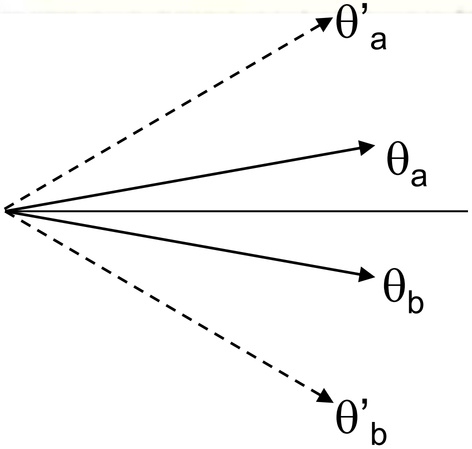
$\theta_a+\theta_b=90^o$



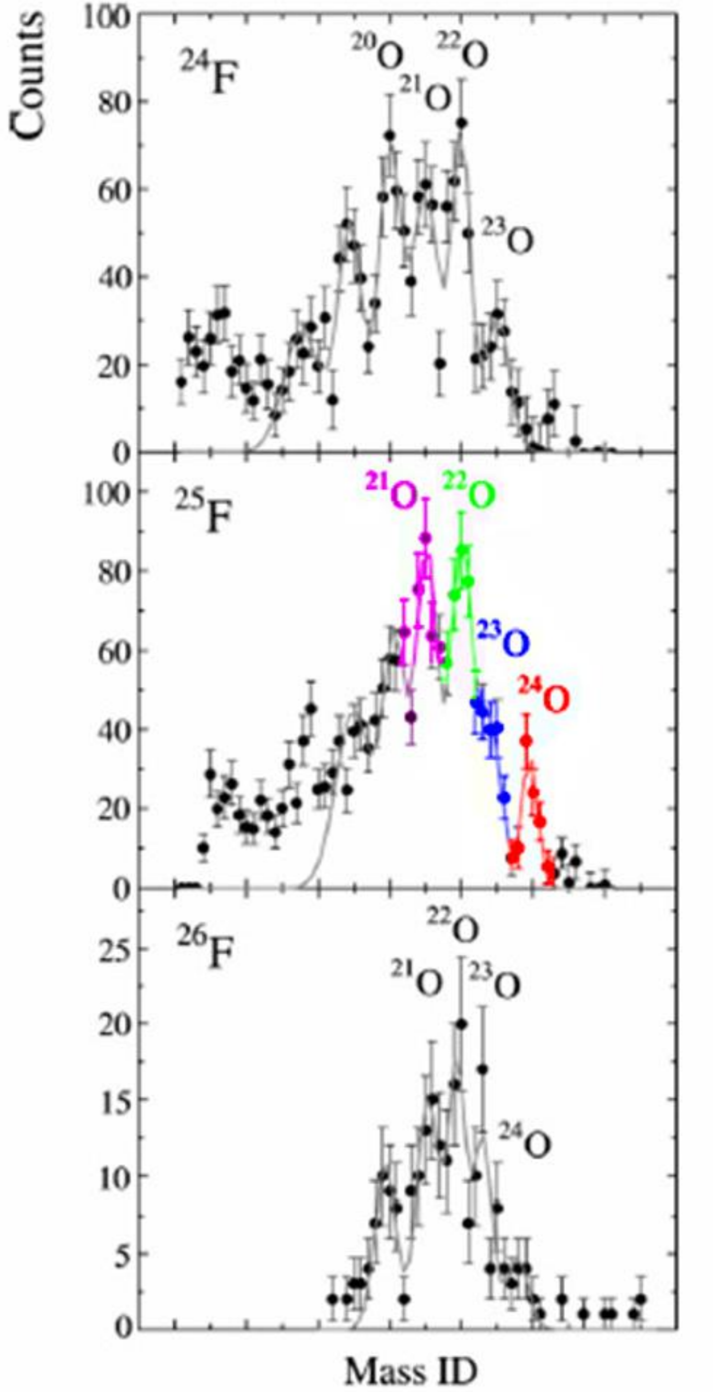
Target nucleon is in motion
(e.g. *p*-wave)

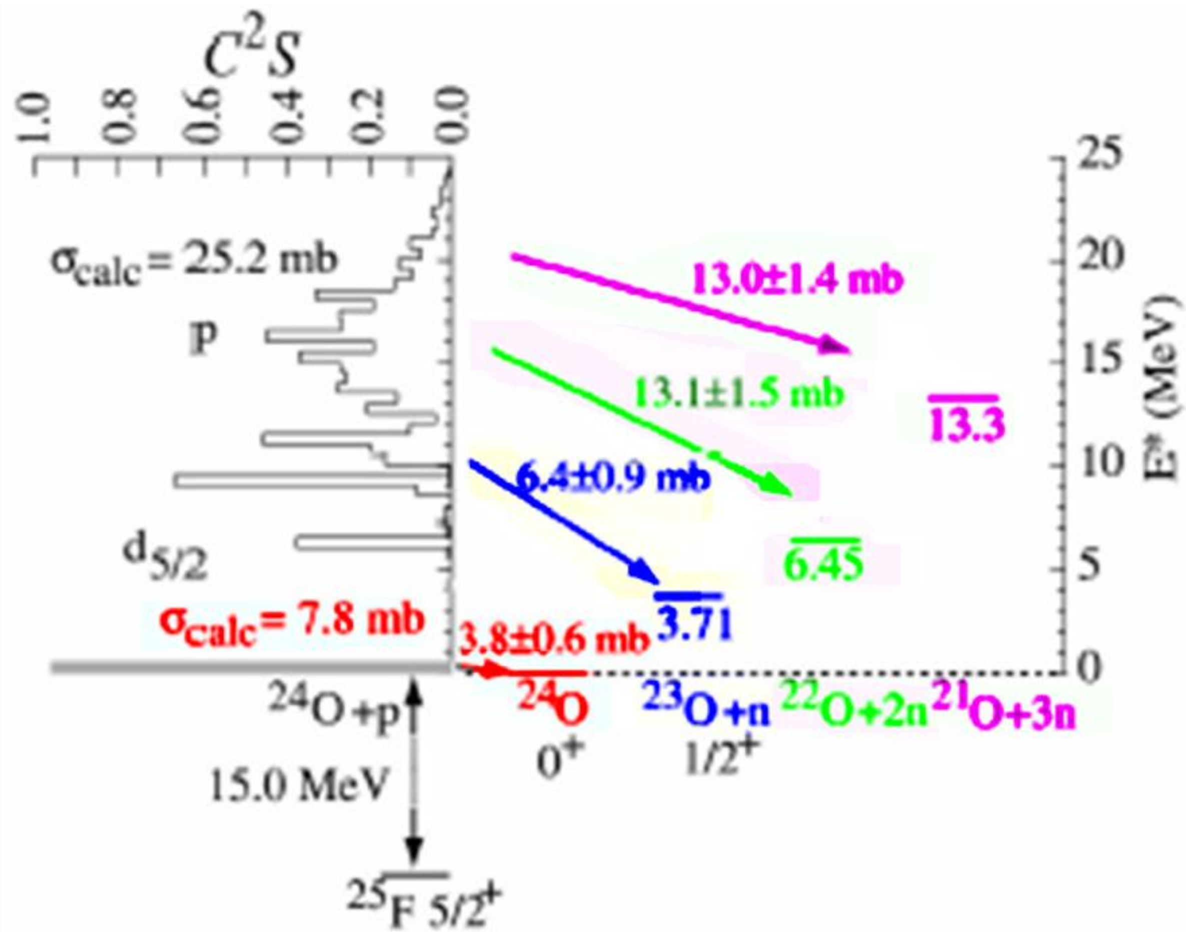
$\theta_a+\theta_b\neq 90^o$





p Knock-out from ^{25}F





$^{25}F+^{12}C \rightarrow {}^{24}O+X$: probes the structure of the ^{25}F ground state. $\sigma_{MEAS} \sim .5\sigma_{CALC}$

M. Thoennessen et al, PRC **68**, 044318 (2003)

A summary

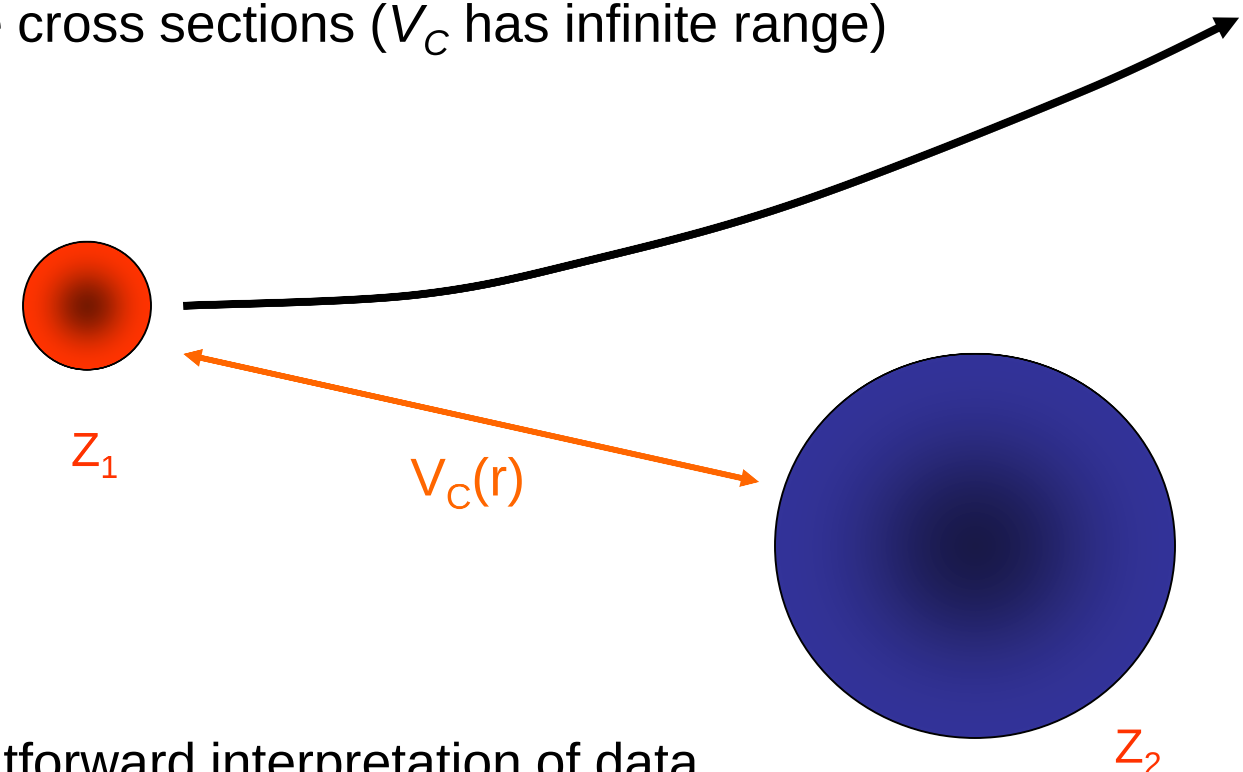
- Direct reactions are essential tools for the understanding of the structure of nuclei, and they are also not new. There are *many* well-understood tools at our disposal.
- They may not be new, but they are sure going to tell us a lot about exotic nuclei, as they have already!
- Care must be undertaken when doing detailed comparisons between theory and experiment.
- The trend is towards more predictability and less model dependence – this is important in the era where we are exploring new and uncharted territory.
- We've said nothing about how hard it is to study such reactions with exotic beams – it is! Very! Tomorrow Kate Jones will tell you.

Tomorrow – short discussion of two more kinds of reactions, and then some experimental techniques.

Special case: Coulomb Excitation

Distance is larger than range of nuclear force, so
1 and 2 are excited by the Coulomb force only

Large cross sections (V_C has infinite range)



Typically Z_2 is large. Au or Pb targets are common

Straightforward interpretation of data
because we understand the Coulomb force:
Direct measurements of nuclear matrix elements
lead to “measurements” of deformation, very useful spectroscopic tool

Another modification to that optical model potential...

$$U_\ell(r) = -i\frac{2m}{k\hbar^2}\frac{\pi}{50}(Ze)^2 B(E2\uparrow)$$
$$\times\left[\left(\frac{\eta^2 k^2(3\bar{\ell}^2+\eta^2)}{\bar{\ell}^2(\bar{\ell}^2+\eta^2)^2} - \frac{\eta k^2}{\bar{\ell}^3}\arctan\frac{\bar{\ell}}{\eta}\right)\frac{1}{r^3}\right.$$
$$\left. + \frac{4\eta k\bar{\ell}^2}{(\bar{\ell}^2+\eta^2)^2}\frac{1}{r^4} + \frac{2\bar{\ell}^4}{(\bar{\ell}^2+\eta^2)^2}\frac{1}{r^5}\right].$$

$U_l(r)$ is imaginary (it takes flux away from the elastic channel)
It depends on *Z*, B(E2), and *l*
It has a long range, and
You can see the effects in elastic scattering...

Glendenning, pp 123

Effects of long-range absorption due to Coulomb Excitation

Radial dependence of $U_l(r)$ for various values of l

Influence on elastic $^{16}O+^{184}W$ scattering

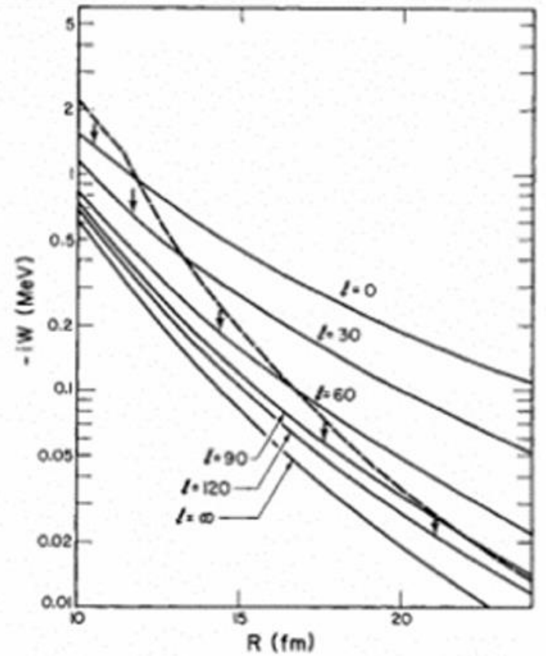


Fig. 14.1. The ℓ-dependent potential of $^{16}O + ^{184}W$, at 90 MeV plotted for several values of ℓ (——). The Love-Terasawa-Satchler ℓ-independent potential (---) tracks it near the classical turning points indicated by the arrows (From Baltz *et al.* 1979.)

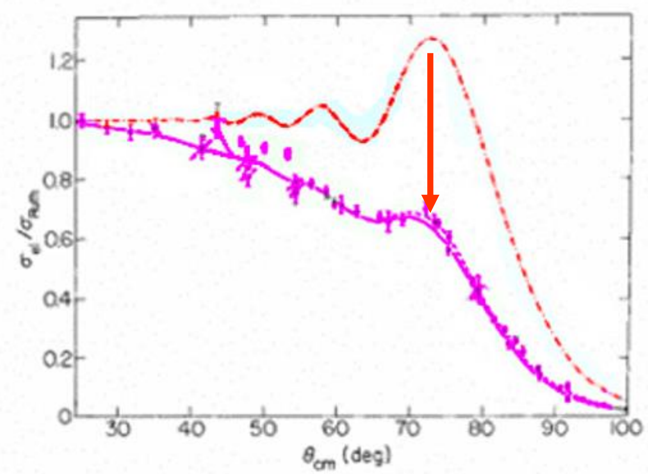
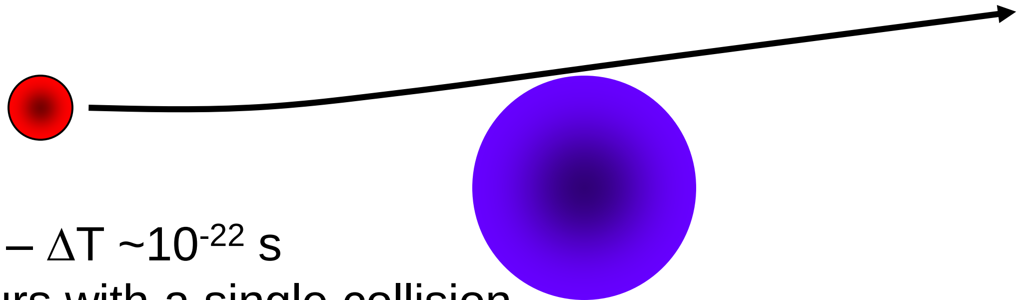


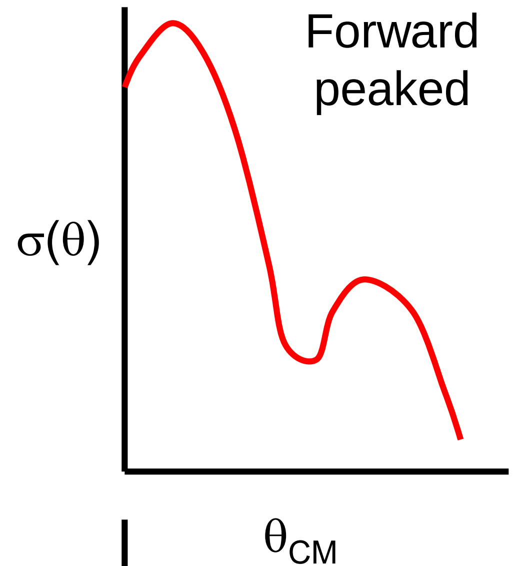
Fig. 14.2. The elastic cross section for $^{16}O + ^{184}W$ at 90 MeV, where ℓ-dependent potential (——), LTS potential (- - -), and no long-range adsorption (-·-), compared with the theory. It is the long-range absorption that causes the fall off prior to the Fresnel peak, which is usually a fluctuation above 1. (Data is from Brookhaven, Calculations by Baltz *et al.*, 1979).

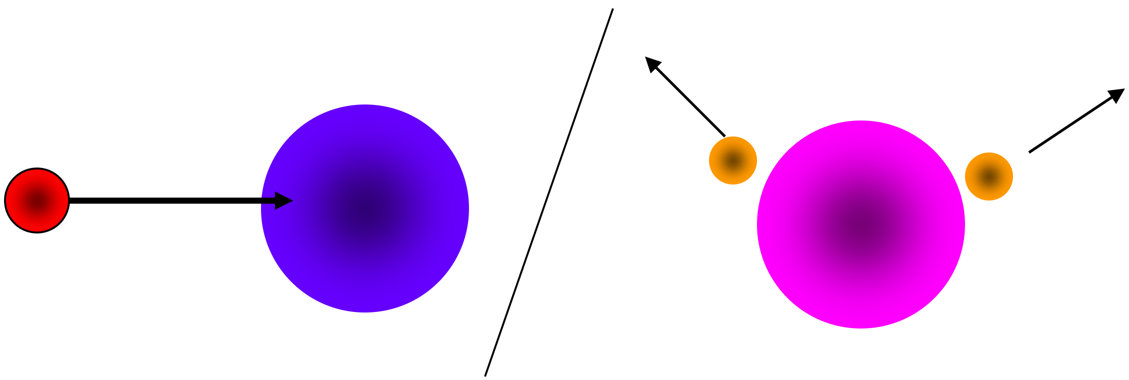
A. J. Baltz et al, NPA **327**, 221 (1979)

Direct vs. Compound reactions

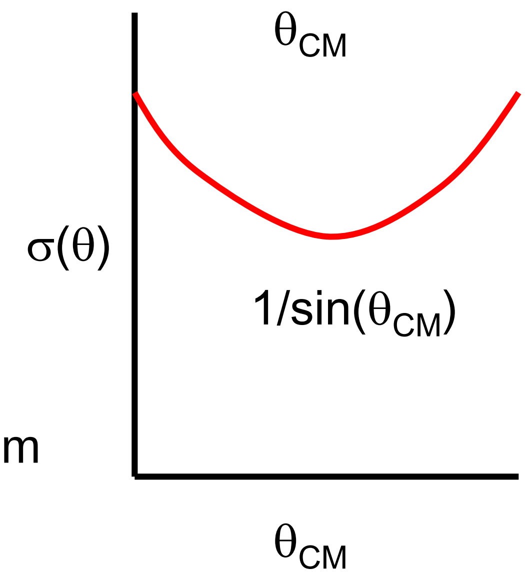


Fast – $\Delta T \sim 10^{-22}$ s
Occurs with a single collision
Smooth energy dependence
Examples: (*d,p*), (^{3}He,*d*),(*p,d*), (*d*,^{3}He)





Slow(er) – $\Delta T \sim 10^{-20}$ s or more
Proceeds through many complex states in compound system
Memory of beam, target is lost.
Particles are emitted isotropically in the CM
Examples: (α,p), HI(HI,*p,n*,α)

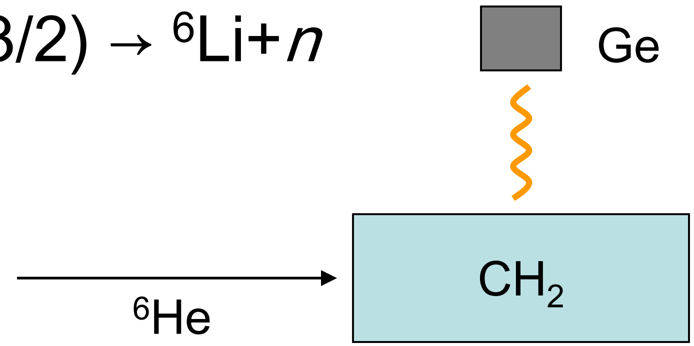


Two other direct processes: Charge exchange and knockout

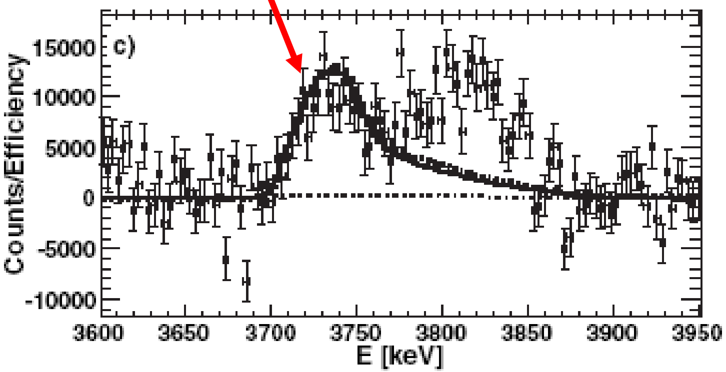
- Charge exchange – change a *p* to an *n* or vice-versa:
  - examples: (*p*,*n*), (^{3}He,*t*),(*d*,^{2}He)
  - Populates "Isobaric analog states"
  - Samples Gamow-Teller strength at small angles/low momentum transfer – like β decay.
- Knock-out: The projectile "knocks out" a particle from the target nucleus
  - examples: (*e*,*e'p*),(*p*,2*p*), (*p*,*np*) etc.
  - can be used to complement other direct transfer reactions, sensitive to nuclear structure

Charge exchange – an example

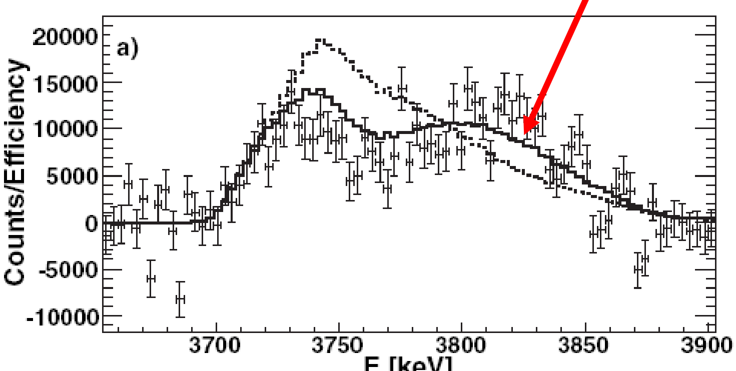
$^{6}He + p \rightarrow {}^{7}Li^{*}(T=3/2) \rightarrow {}^{6}Li + n$



Analog of ^{7}He g.s.



Tail of analog of ^{7}He $1/2^{-}$?



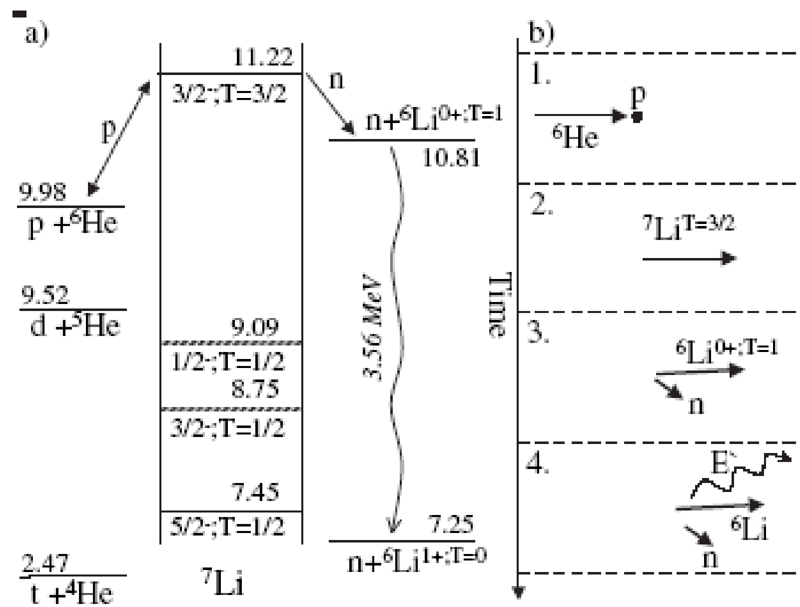
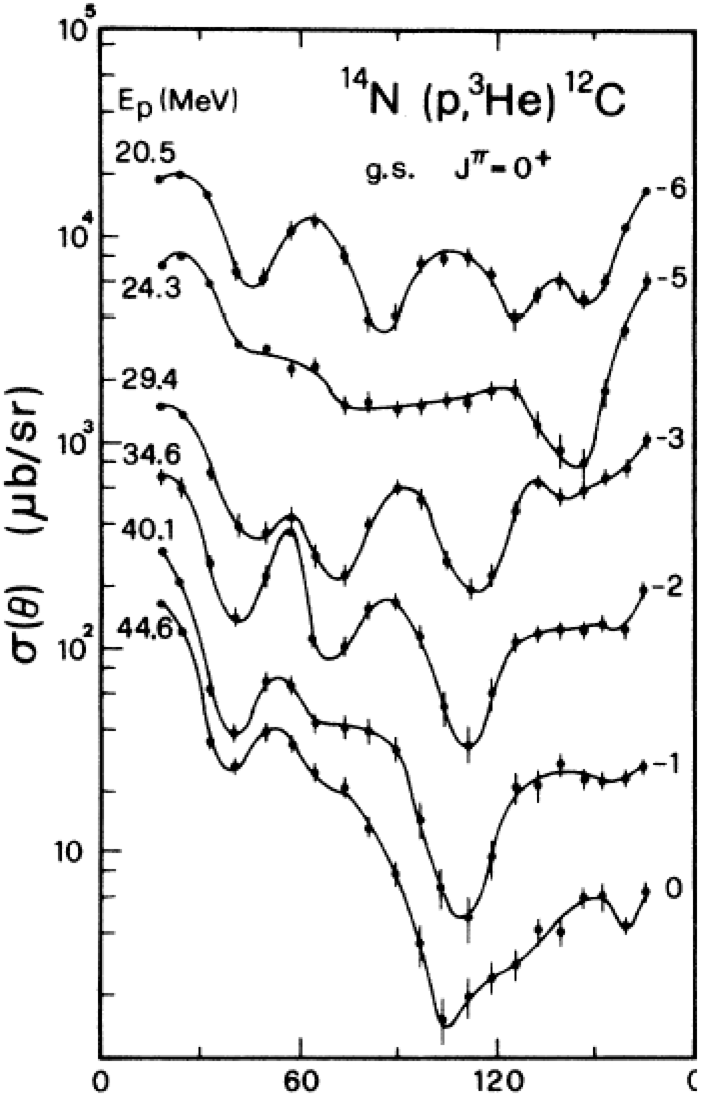
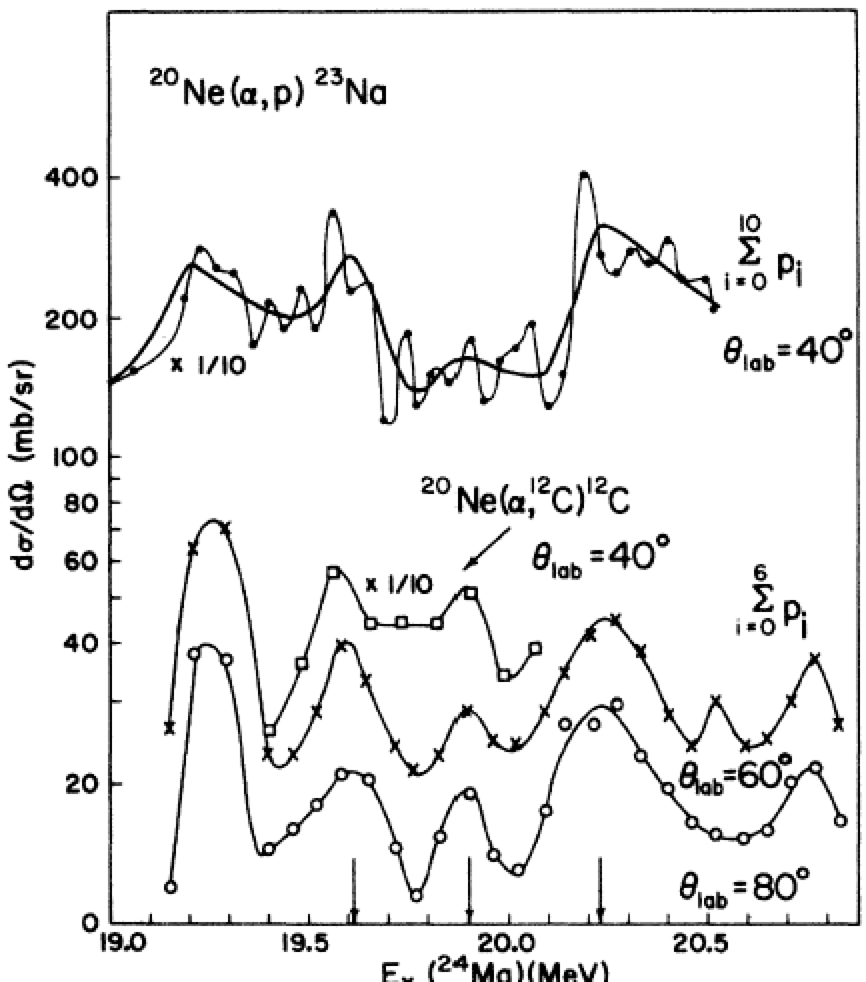


FIG. 1. (a) Decay pathways for the $T = 3/2$ resonance in ^{7}Li, and (b) the successive kinematics stages of the studied reaction.

P. Boutachkov et al, PRL **95**, 132502 (2005).



$^{14}N\ (p,^3He)\ ^{12}C$
E_p (MeV)
g.s. $J^\pi = 0^+$
20.5
24.3
29.4
34.6
40.1
44.6
-6
-5
-3
-2
-1
0
$\sigma(\theta)$ (μb/sr)
10^5
10^4
10^3
10^2
10
0
60
120

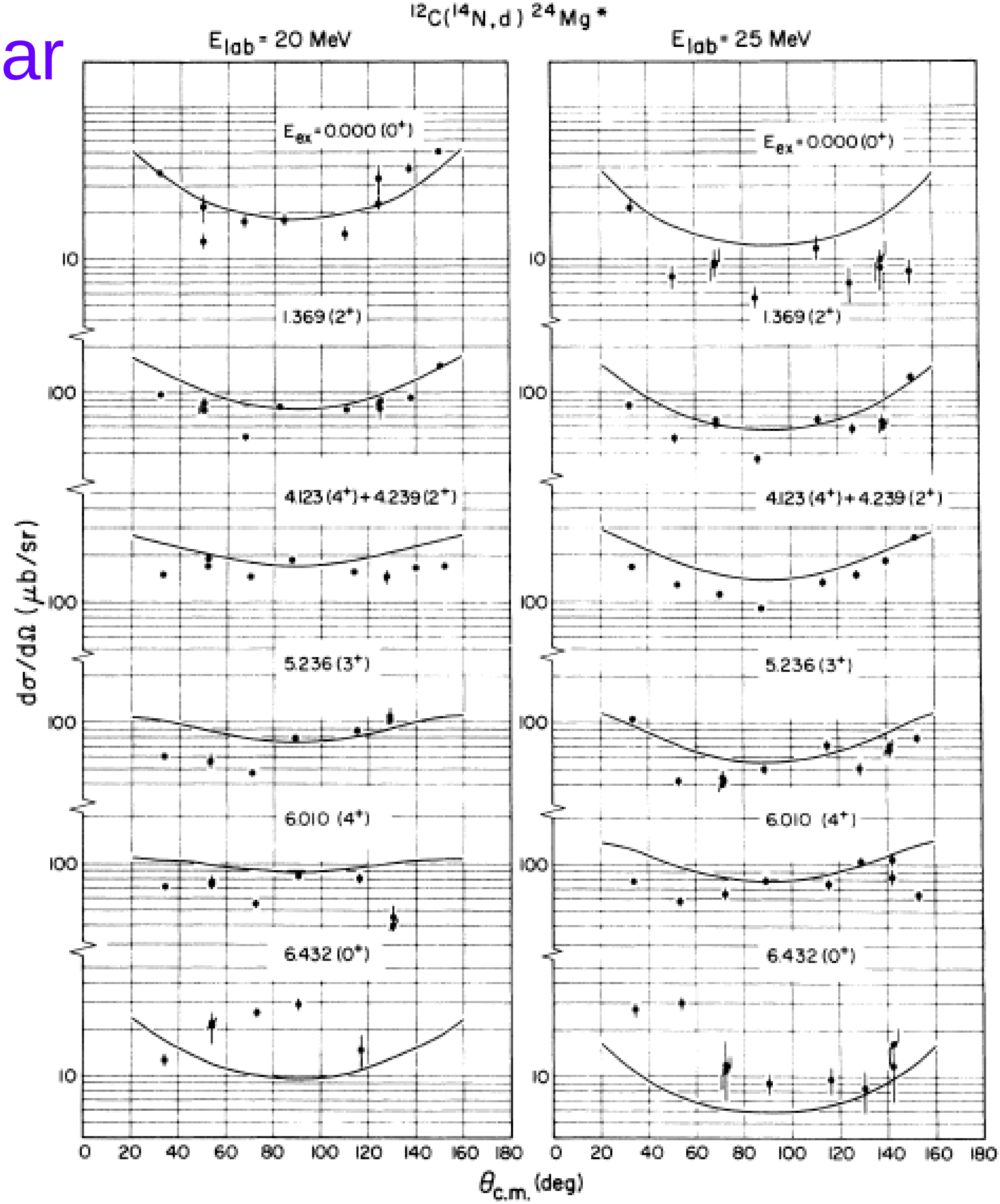


$^{20}Ne(\alpha,p)\ ^{23}Na$
$\sum_{i=0}^{10} p_i$
$\theta_{lab} = 40°$
x 1/10
$^{20}Ne(\alpha,^{12}C)^{12}C$
$\theta_{lab} = 40°$
x 1/10
$\sum_{i=0}^{6} p_i$
$\theta_{lab} = 60°$
$\theta_{lab} = 80°$
$d\sigma/d\Omega$ (mb/sr)
400
200
100
80
60
40
20
0
19.0
19.5
20.0
20.5
E_x (^{24}Mg)(MeV)

Typical CN angular distributions

$^{12}C(^{14}N,d)^{24}Mg$

Angular distributions are forward-backward symmetric



$\nu(sd)^2$ states in ^{16}C

$$\psi(0^+{}_1) = \alpha_0(1s_{1/2})^2 + \beta_0(0d_{5/2})^2 + \delta(0d_{3/2})^2$$

$$\psi(0^+{}_2) = -\beta_0(1s_{1/2})^2 + \alpha_0(0d_{5/2})^2 + \delta'(0d_{3/2})^2$$

0^+

$$\psi(2^+{}_1) = \alpha_2(1s_{1/2})(0d_{5/2}) + \beta_2(0d_{5/2})^2 + \gamma_2(1s_{1/2})(0d_{3/2}) + \delta_2(0d_{3/2})(0d_{5/2})$$

$$\psi(2^+{}_1) = -\beta_2(1s_{1/2})(0d_{5/2}) + \alpha_2(0d_{5/2})^2 + \gamma_2'(1s_{1/2})(0d_{3/2}) + \delta_2'(1d_{5/2})(0d_{3/2})$$

2^+

$$\psi(3^+{}_1) = \alpha_3(1s_{1/2})(0d_{5/2}) + \beta_3(0d_{3/2})(0d_{5/2})$$

3^+

$$\psi(4^+{}_1) = \alpha_4(0d_{5/2})^2 + \beta_4(0d_{3/2})(0d_{5/2})$$

4^+

$\nu(sd)^2$ states in ^{16}C – no $(0d_{3/2})$

$$\psi(0^+{}_1) = \alpha_0(1s_{1/2})^2 + \beta_0(0d_{5/2})^2 \cancel{+ \delta(0d_{3/2})^2}$$

$$\psi(0^+{}_2) = -\beta_0(1s_{1/2})^2 + \alpha_0(0d_{5/2})^2 \cancel{+ \delta'(0d_{3/2})^2}$$

0^+

$$\psi(2^+{}_1) = \alpha_2(1s_{1/2})(0d_{5/2}) + \beta_2(0d_{5/2})^2$$
$$\cancel{+\gamma_2(1s_{1/2})(0d_{3/2})+\delta_2(0d_{3/2})(0d_{5/2})}$$

$$\psi(2^+{}_1) = -\beta_2(1s_{1/2})(0d_{5/2}) + \alpha_2(0d_{5/2})^2$$
$$\cancel{+\gamma_2'(1s_{1/2})(0d_{3/2})+\delta_2'(1d_{5/2})(0d_{3/2})}$$

2^+

$$\psi(3^+{}_1) = \alpha_3(1s_{1/2})(0d_{5/2}) + \cancel{\beta_3(0d_{3/2})(0d_{5/2})}$$

3^+

$$\psi(4^+{}_1) = \alpha_4(0d_{5/2})^2 \cancel{+\beta_4(0d_{3/2})(0d_{5/2})}$$

4^+

$\nu(sd)^2$ states in ^{16}C with (d,p)

$$\psi(0^+{}_1) = \alpha_0(1s_{1/2})^2 + \beta_0(0d_{5/2})^2$$

$$\psi(0^+{}_2) = -\beta_0(1s_{1/2})^2 + \alpha_0(0d_{5/2})^2$$

0$^+$

$$\psi(2^+{}_1) = \alpha_2(1s_{1/2})(0d_{5/2}) + \beta_2(0d_{5/2})^2$$

$$\psi(2^+{}_1) = -\beta_2(1s_{1/2})(0d_{5/2}) + \alpha_2(0d_{5/2})^2$$

2$^+$

$$\psi(3^+{}_1) = \alpha_3(1s_{1/2})(0d_{5/2})$$

3$^+$

$$\psi(4^+{}_1) = \alpha_4(0d_{5/2})^2$$

4$^+$

(d,p) spectroscopic factors tell us the values of the α's and the β

^{16}C – Previous work

PRL 40, 1236 (1978)

$(sd)^2$ States in $^{14,16}C$

H. T. Fortune,[a] M. E. Cobern,[b] S. Mordechai,[c] G. E. Moore,[d] S. Lafrance, and R. Middleton
Physics Department, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 20 December 1977)

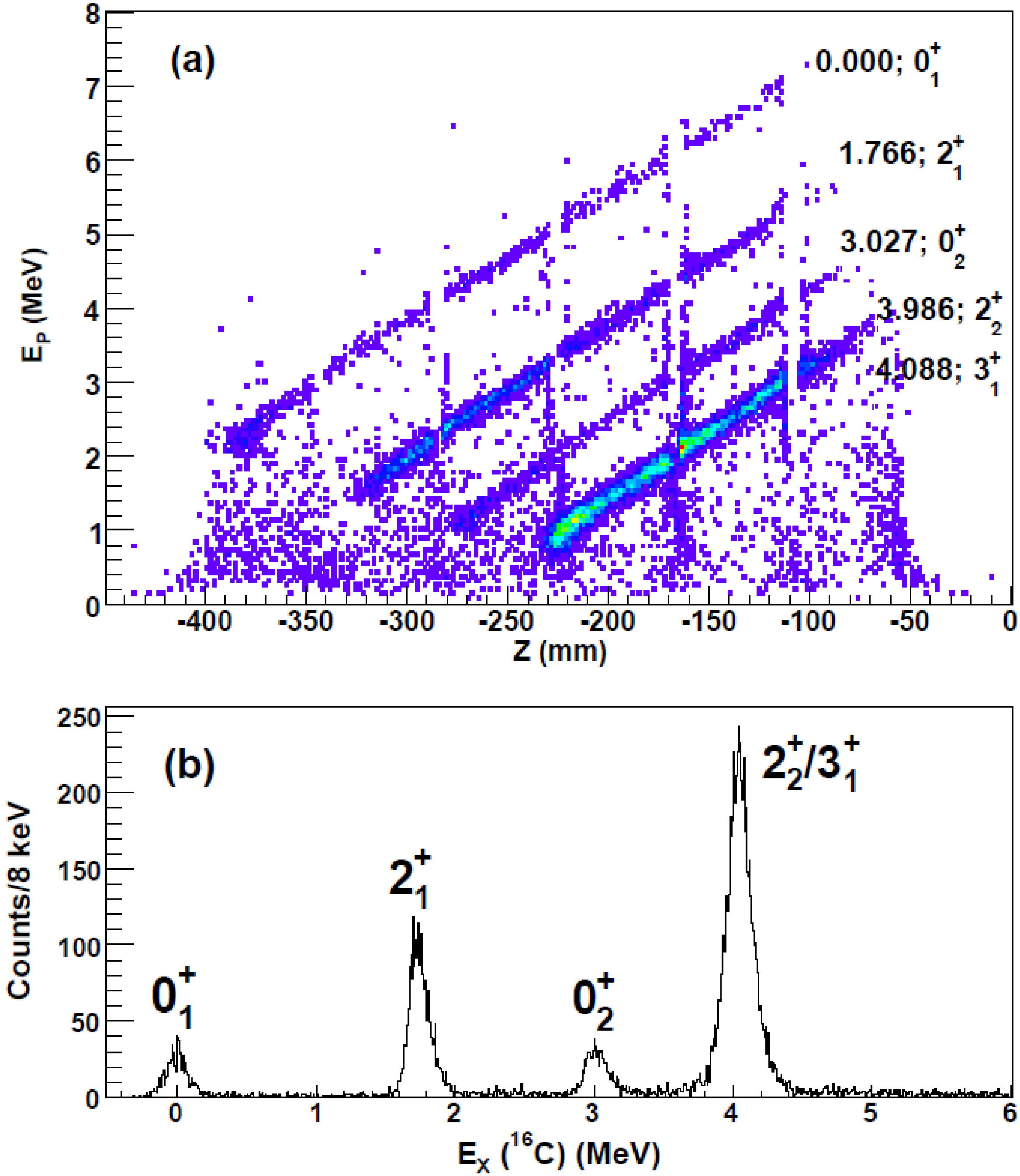
Wave functions from empirical interactions derived from ^{18}O – test with $^{15}C(d,p)^{16}C$

TABLE I. Wave functions for predominantly $(sd)^2$ states in $^{14,16}C$.

Wave functions for ^{14}C		E_x (MeV) in ^{14}C				E_x (MeV) in ^{16}C		Wave functions for ^{16}C	
$(1d\frac{5}{2})^2$	$(2s\frac{1}{2})^2$	Calc	Exp	J^π	M.E.	Exp	Calc	$(1d\frac{5}{2})^2$	$(2s\frac{1}{2})^2$
0.6765	0.7364	6.286	6.577	0^+	LSF	0.0	−0.110	0.6821	0.7313
0.5132	0.8582	6.876			Kuo		0.490	0.5222	0.8528
0.7364	−0.6765	9.734	9.746	0^+	LSF	3.020	3.338	0.7313	−0.6821
0.8582	−0.5132	9.079			Kuo		2.668	0.8528	−0.5222
[illegible]	[illegible]	[illegible]	10.736	4^+	LSF	4.136	[illegible]	[illegible]	[illegible]
0.9245	0.3813	10.327			Kuo		3.995	0.9454	0.3259

Strongly configuration-mixed wave functions
lowest 0^+,2^+ states mostly $1s_{1/2}$ and $0d_{5/2}$
B(E2) calculated with these wave functions using "standard" effective charges matches LBNL result exactly!

$^{15}C(d,p)^{16}C$ with HELIOS



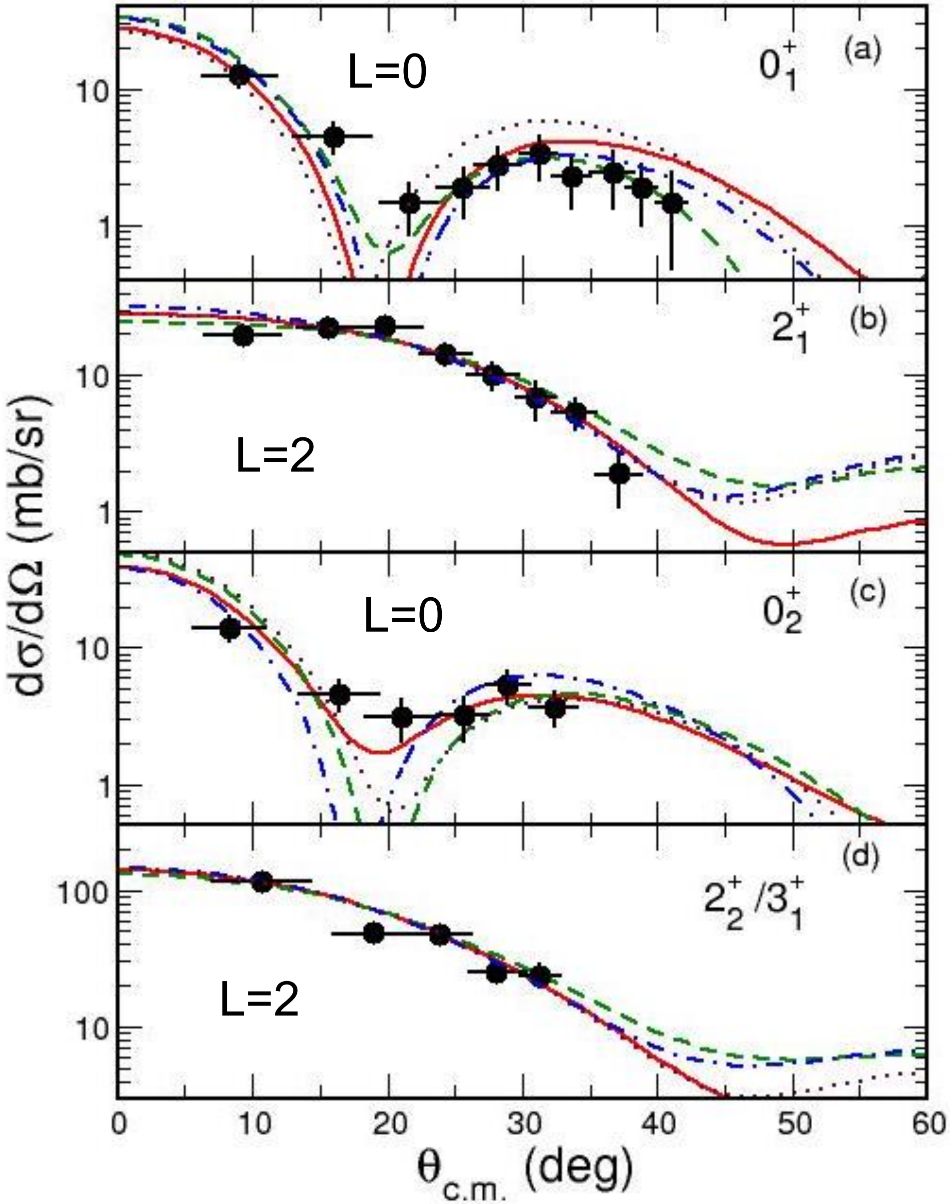
Proton energy-position correlation

(d,p) samples the $\nu(1s_{1/2})$ content of the wave functions for positive-parity states

^{16}C Excitation-energy spectrum

PRL **105**, 132501 (2010)

$^{15}C(d,p)^{16}C$ angular distributions



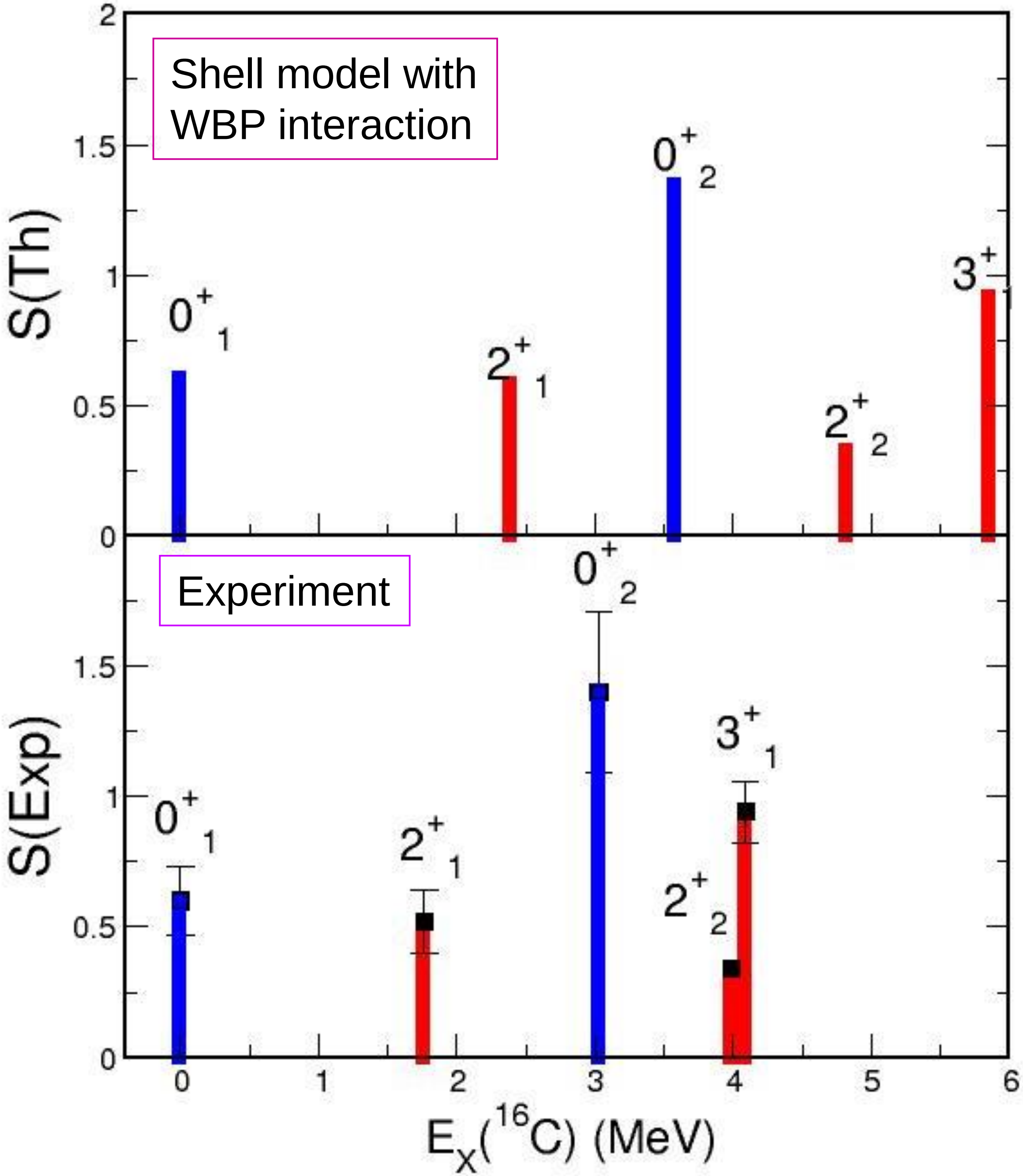
Curves are DWBA calculations with various optical-model potentials.

Spectroscopic factors obtained from the average over four sets of OMP.

Relative uncertainties in SF dominated by OMP variations
Absolute uncertainty (~30%) from beam-integration uncertainty

PRL **105**, 132501 (2010)

$^{15}C(d,p)^{16}C$ Spectroscopic factors



Excitation energies and relative spectroscopic factors from the shell model

Blue: L=0

Red: L=2

Agreement for SF is excellent!
No need for exotica

PRL **105**, 132501 (2010)

Sum Rules and ^{15}C(d,p)^{16}C

- ^{15}C(d,p)^{16}C: J^{π}_{i}=1/2^{+}, J^{π}_{f}= 0^{+} (1$s_{1/2}$),
  or (2,3)$^{+}$ (0$d_{5/2}$)
  - #holes = 6($d_{5/2}$) or 1($s_{1/2}$)
  - McF & F say: 6=ΣS x [J_f]/2 ($d_{5/2}$) or
    1=ΣS x [J_f]/2 ($s_{1/2}$)
  - This implies ΣS[J_f]/6=6.0 or 1.0 (maximum) for 0$d_{5/2}$ or 1$s_{1/2}$ single-particle strength
  - Experimentally, ΣS[J_f]/2=5.0 (L=2) and 1.0 (L=0)
  - We miss L=2 strength at high excitation energies (the shell model also tells us this).

Empirical $\nu(sd)^2$ residual interaction for 0^+

$$|0_1^+> = \alpha\,|(1s_{1/2})^2> + \beta\,|(0d_{5/2})^2>$$
$$|0_2^+> = -\beta\,|(1s_{1/2})^2> + \alpha\,|(0d_{5/2})^2>$$

$$\alpha = \sqrt{S(0_1^+)\times[J_f]/[J_i]} = 0.55$$
$$\beta = \sqrt{S(0_2^+)\times[J_f]/[J_i]} = 0.84$$

$[J]=2J+1$

$$\begin{pmatrix} E^0_{1/2}+\delta_{1/2;1/2} & \delta_{1/2;5/2} \\ \delta_{1/2;5/2} & E^0_{5/2}+\delta_{5/2;5/2} \end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = E_X\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

Single-particle energies E^0 from ^{15}C.

$< j_1 j_2 \vert v \vert j_1' j_2' >$	$(j_1 j_2, j_1' j_2')$ $(1/2\ 1/2, 1/2\ 1/2)$	$(5/2\ 5/2, 5/2\ 5/2)$	$(1/2\ 1/2, 5/2\ 5/2)$
Exp	-0.92(28)	-3.60(28)	-1.39(12)
LSF	-1.54	-2.78	-1.72
WBP	-2.12	-2.82	-1.32

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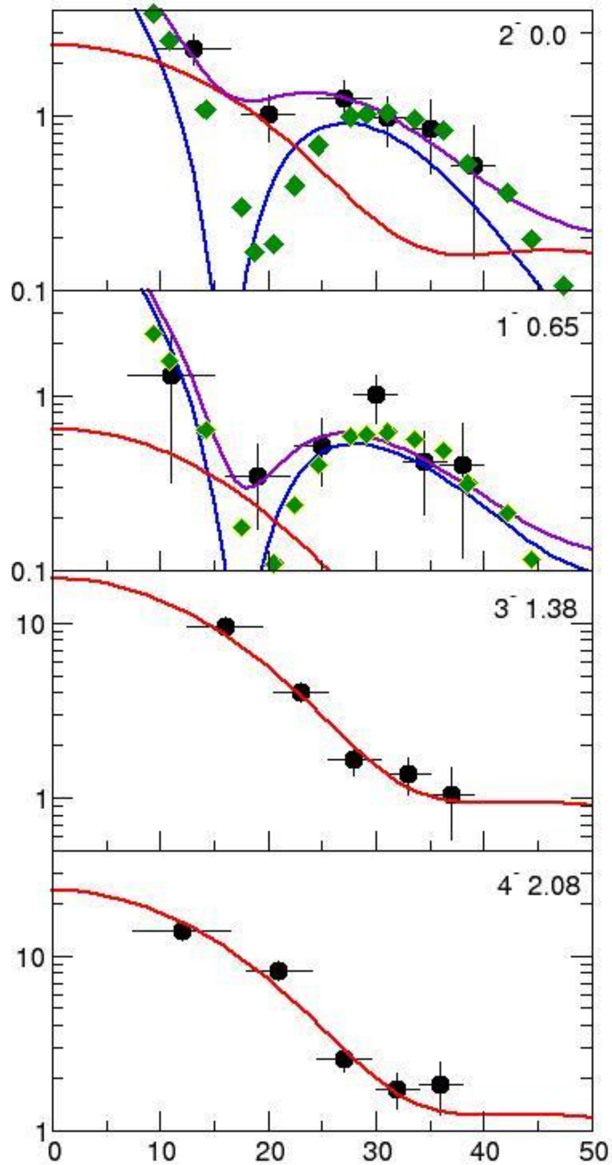
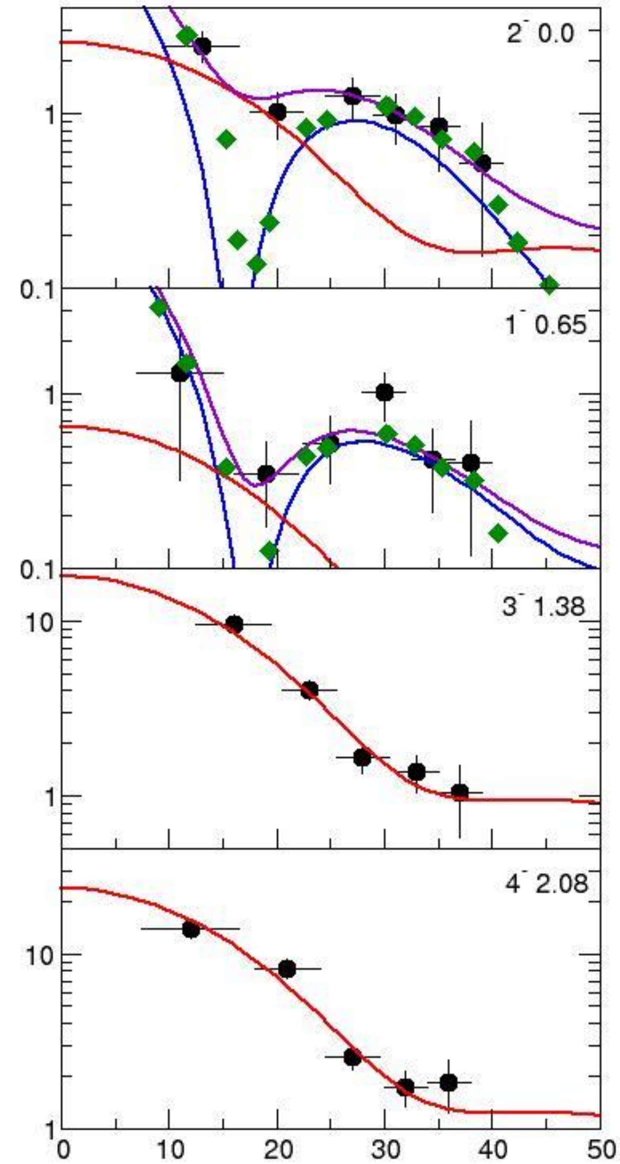


$^{16}O(d,p)^{17}O(1/2^+)$ 26 MeV



$^{16}O(d,p)^{17}O(1/2^+)$ 36 MeV

Channel coupling and inelastic scattering

Optical Potential *U(r)*

$$(E_\alpha - T_{\alpha l} - U_\alpha) u_\alpha^0 = 0$$

Elastic channel

$$(E_{\alpha'} - T_{\alpha l} - U_\alpha) u_{\alpha'} = V_{\alpha\alpha'} u_\alpha^0$$

Inelastic channels

Coupling matrix elements explicitly treat flux going to inelastic channels

Coupled differential equations for *u(r)*