

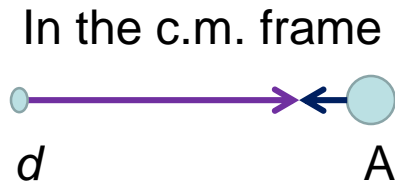
# Nuclear reactions

## Lecture 3

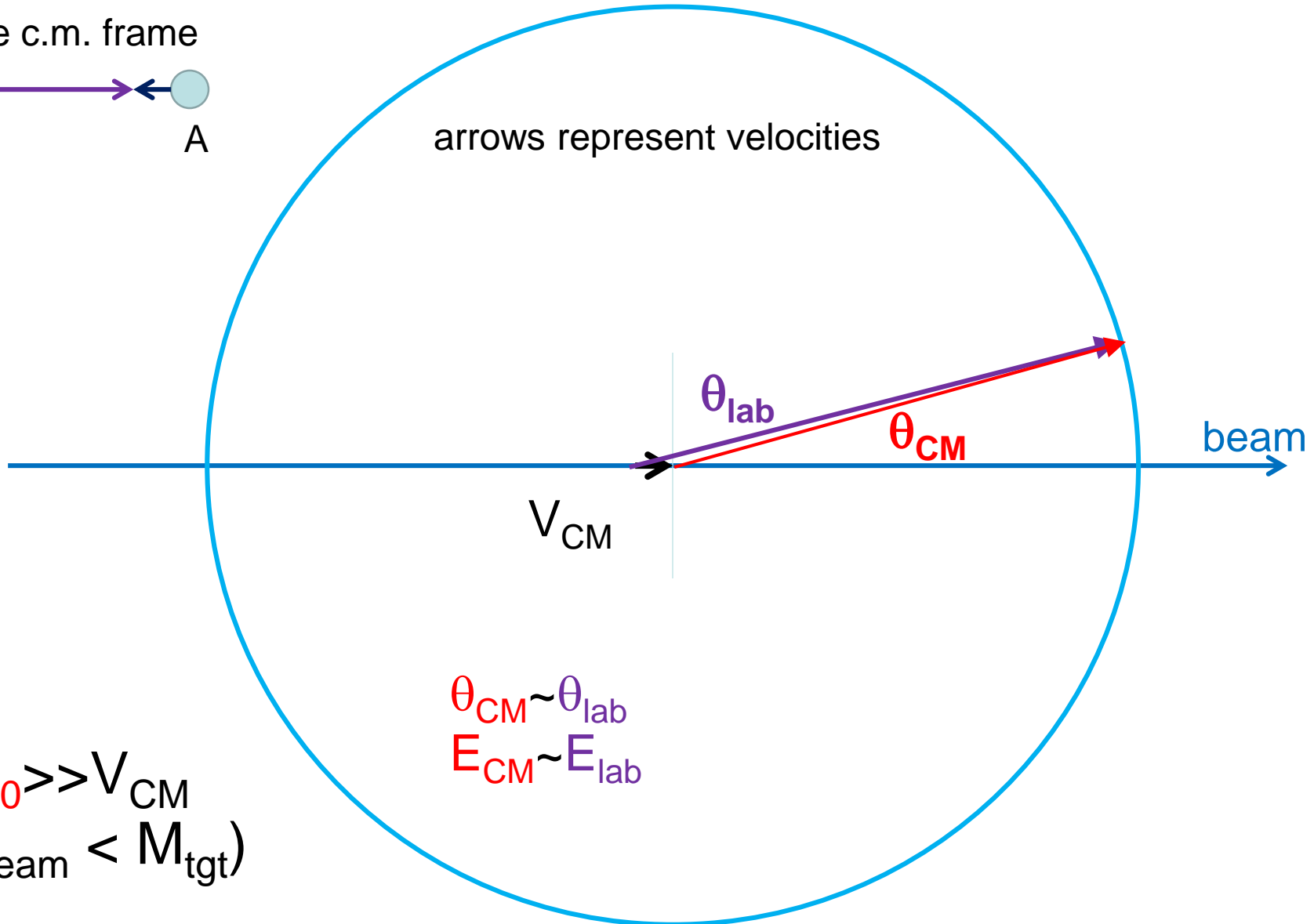
# Modern concrete examples, and some techniques

How we can actually learn things

# “Normal kinematics”



arrows represent velocities



$$\theta_{CM} \sim \theta_{lab}$$
$$E_{CM} \sim E_{lab}$$

$$V_0 \gg V_{CM}$$
$$(M_{beam} < M_{tgt})$$

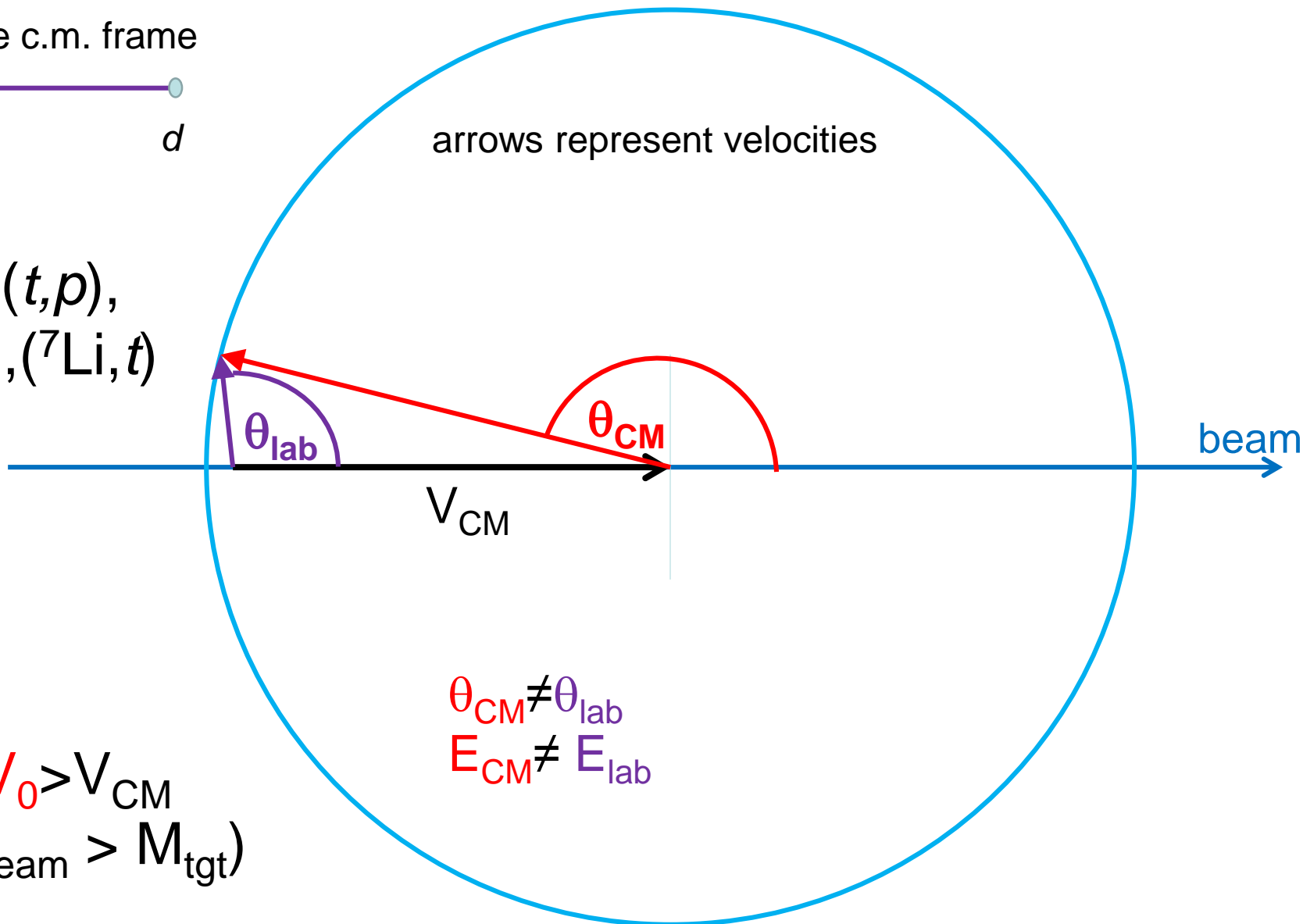
# “Inverse kinematics” I

In the c.m. frame



arrows represent velocities

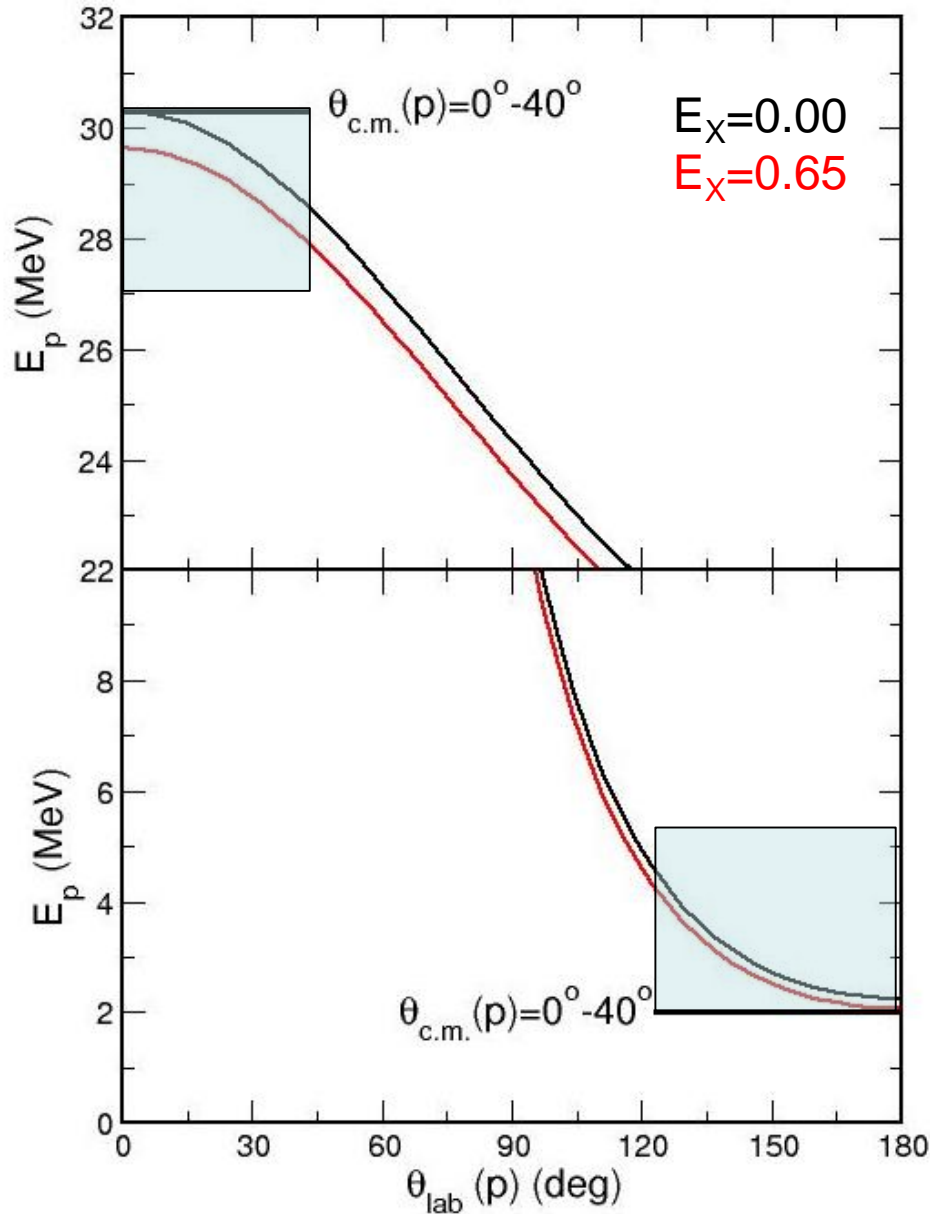
$(d,p)$ ,  $(t,p)$ ,  
 $({}^6\text{Li},d)$ ,  $({}^7\text{Li},t)$



$$\theta_{\text{CM}} \neq \theta_{\text{lab}}$$
$$E_{\text{CM}} \neq E_{\text{lab}}$$

$$V_0 > V_{\text{CM}}$$
$$(M_{\text{beam}} > M_{\text{tgt}})$$

# $^{13}\text{B}(d,p)^{14}\text{B}$ Kinematics



Normal Kinematics  
( $^2\text{H}$  beam +  $^{13}\text{B}$  target)

Inverse Kinematics  
( $^{13}\text{B}$  beam +  $^2\text{H}$  target)

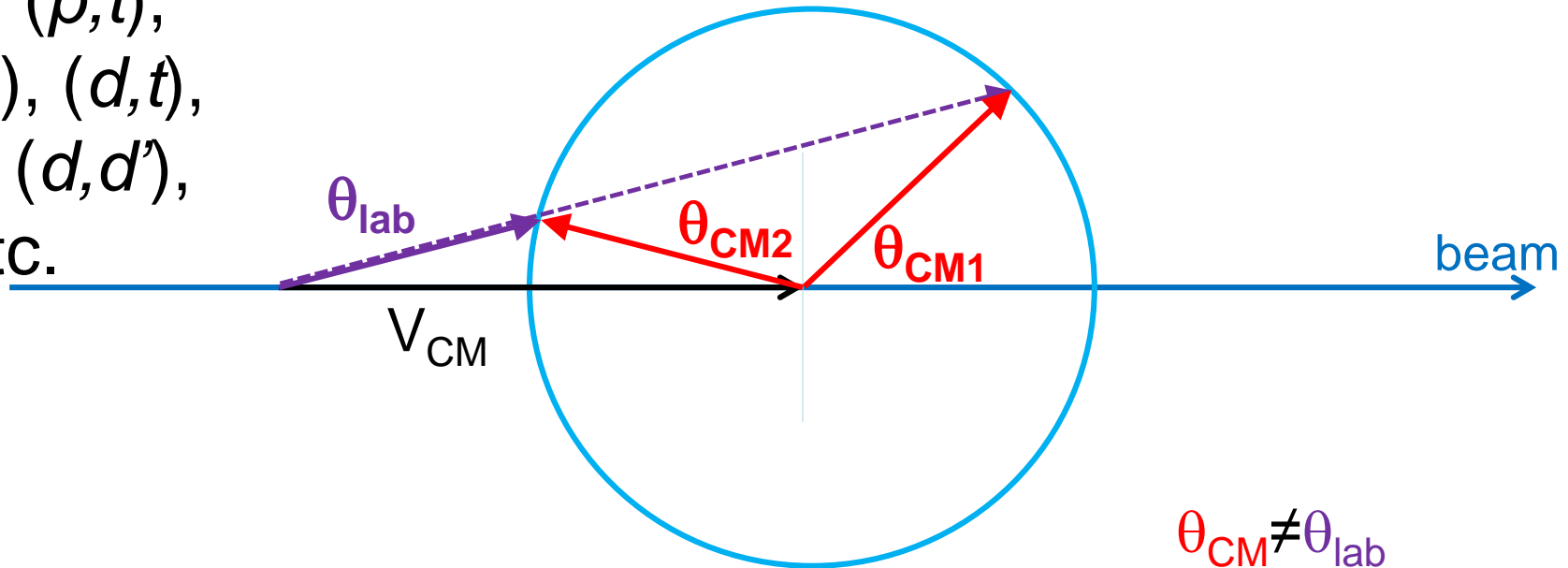
# “Inverse kinematics” II

In the c.m. frame



arrows represent velocities

$(p,d), (p,t),$   
 $(d,^3\text{He}), (d,t),$   
 $(p,p'), (d,d'),$   
etc.

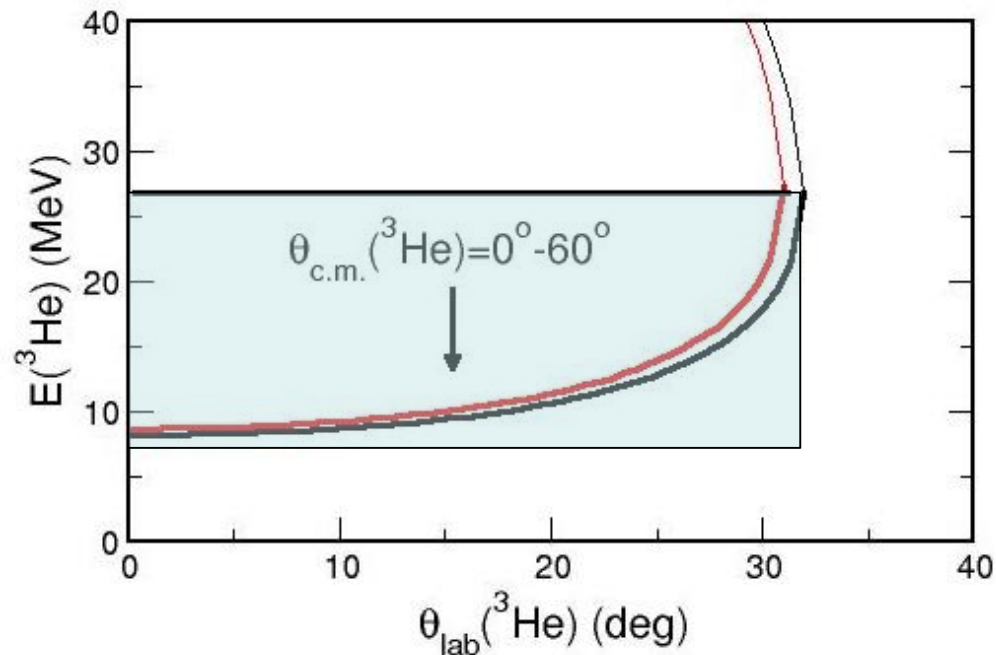
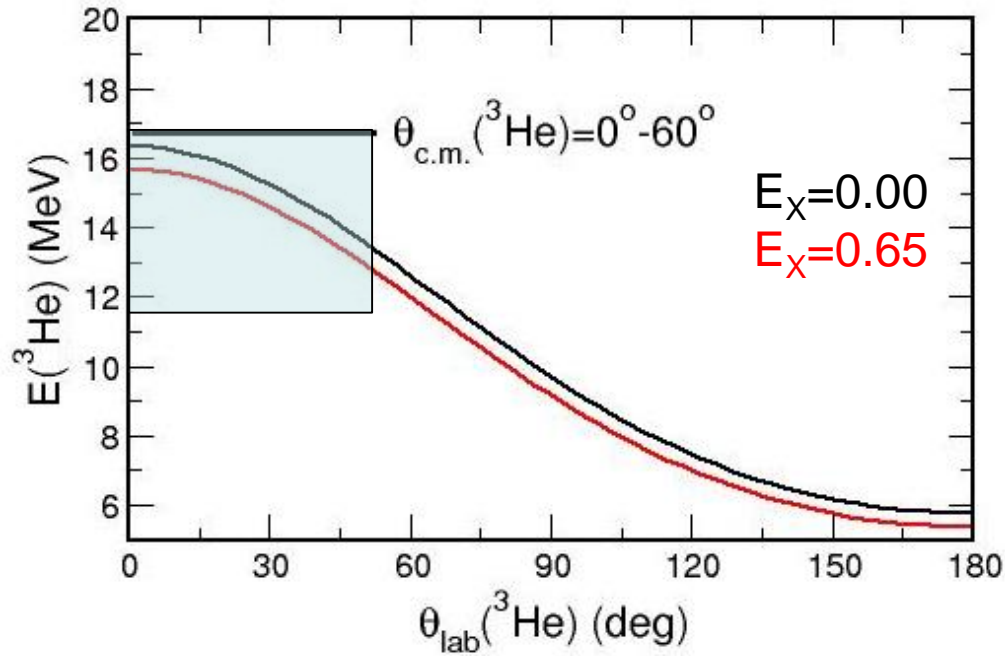


$\theta_{\text{CM}} \neq \theta_{\text{lab}}$   
 $E_{\text{CM}} \neq E_{\text{lab}}$   
Two solutions

$$V_0 < V_{\text{CM}}$$
$$(M_{\text{beam}} > M_{\text{tgt}})$$

# $^{15}\text{C}(d,^3\text{He})^{14}\text{B}$ Kinematics

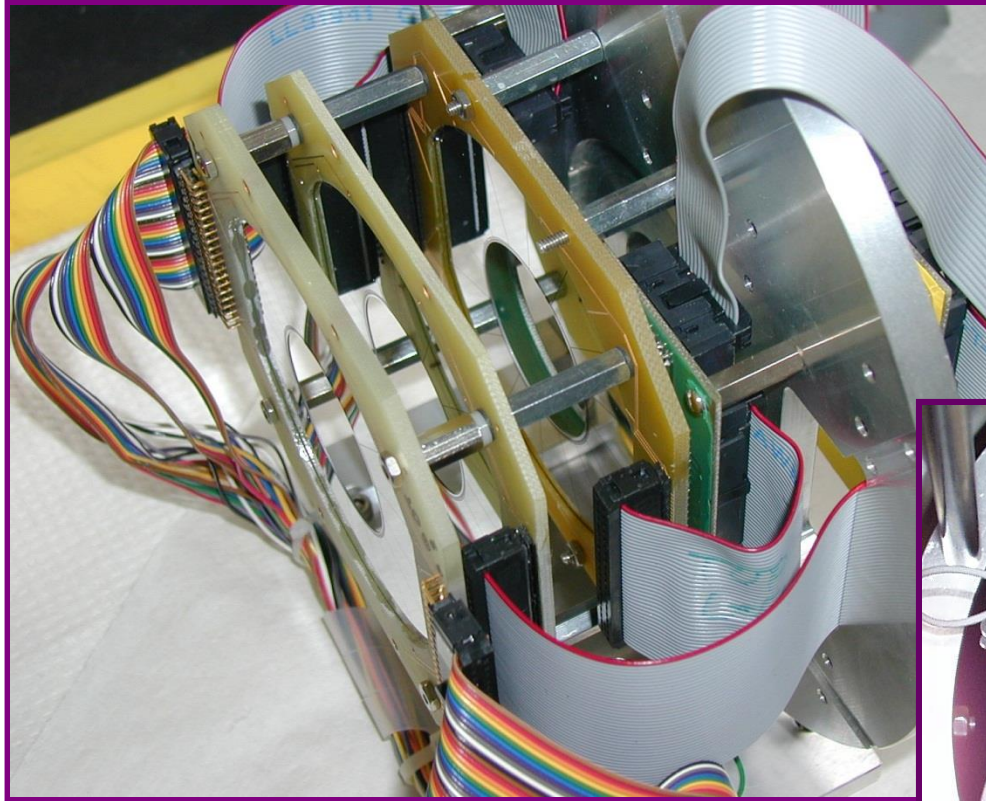
Normal Kinematics  
( $^2\text{H}$  beam +  $^{15}\text{C}$  target)



Inverse Kinematics  
( $^{15}\text{C}$  beam +  $^2\text{H}$  target)



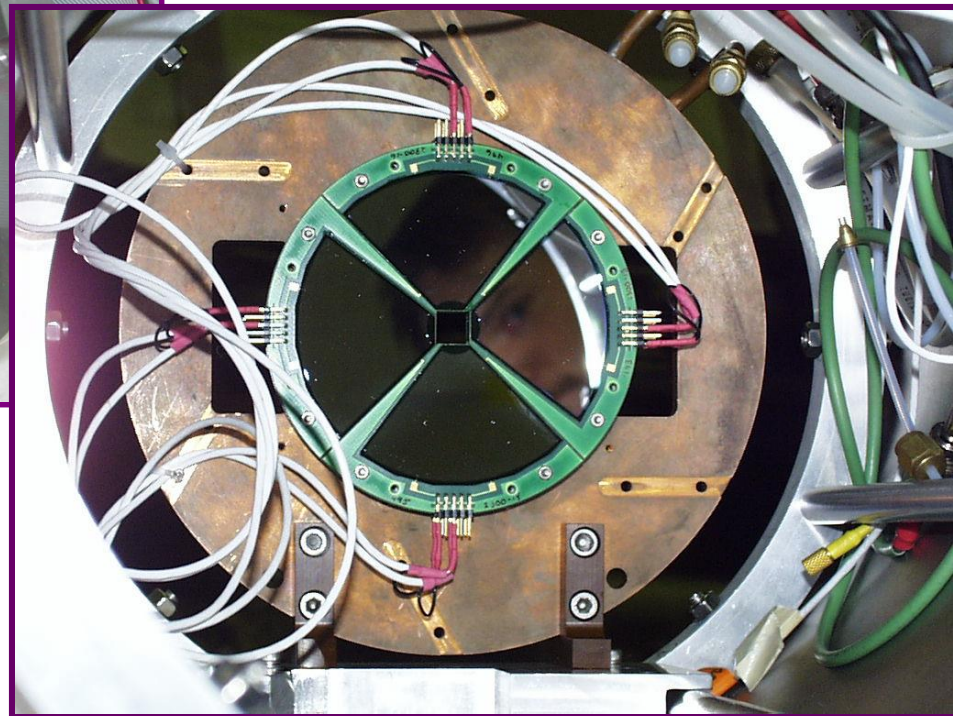
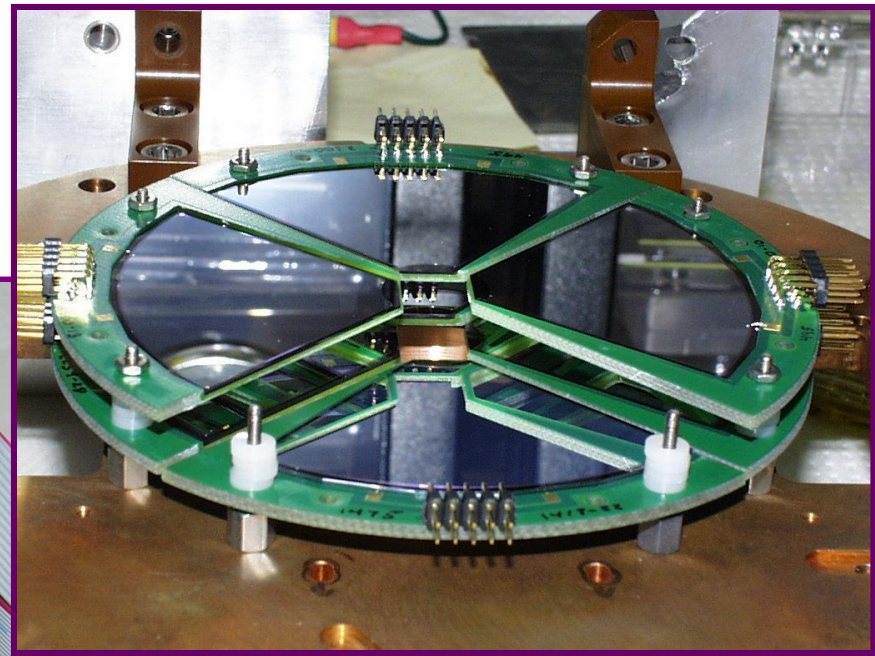
# Detectors



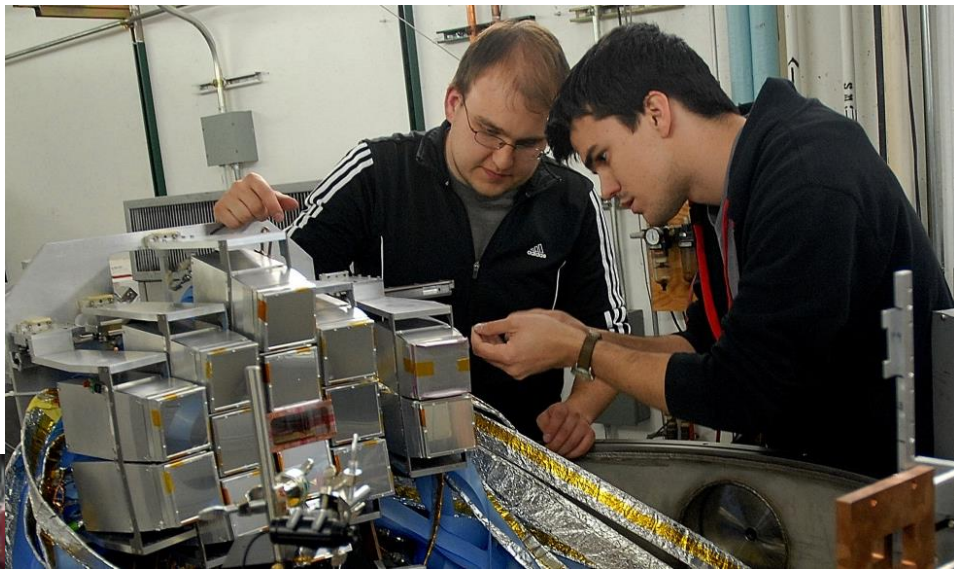
Segmented proton detectors

500 $\mu\text{m}$ /1000 $\mu\text{m}$  silicon  $E\Delta E$   
telescope

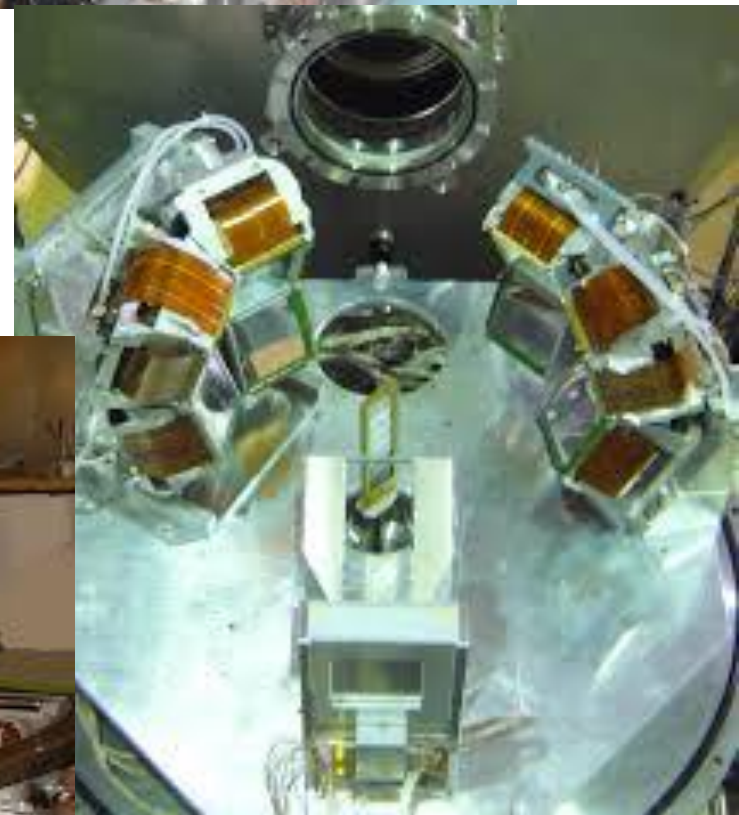
Can be far more complicated...







Many, many detectors  
and channels...

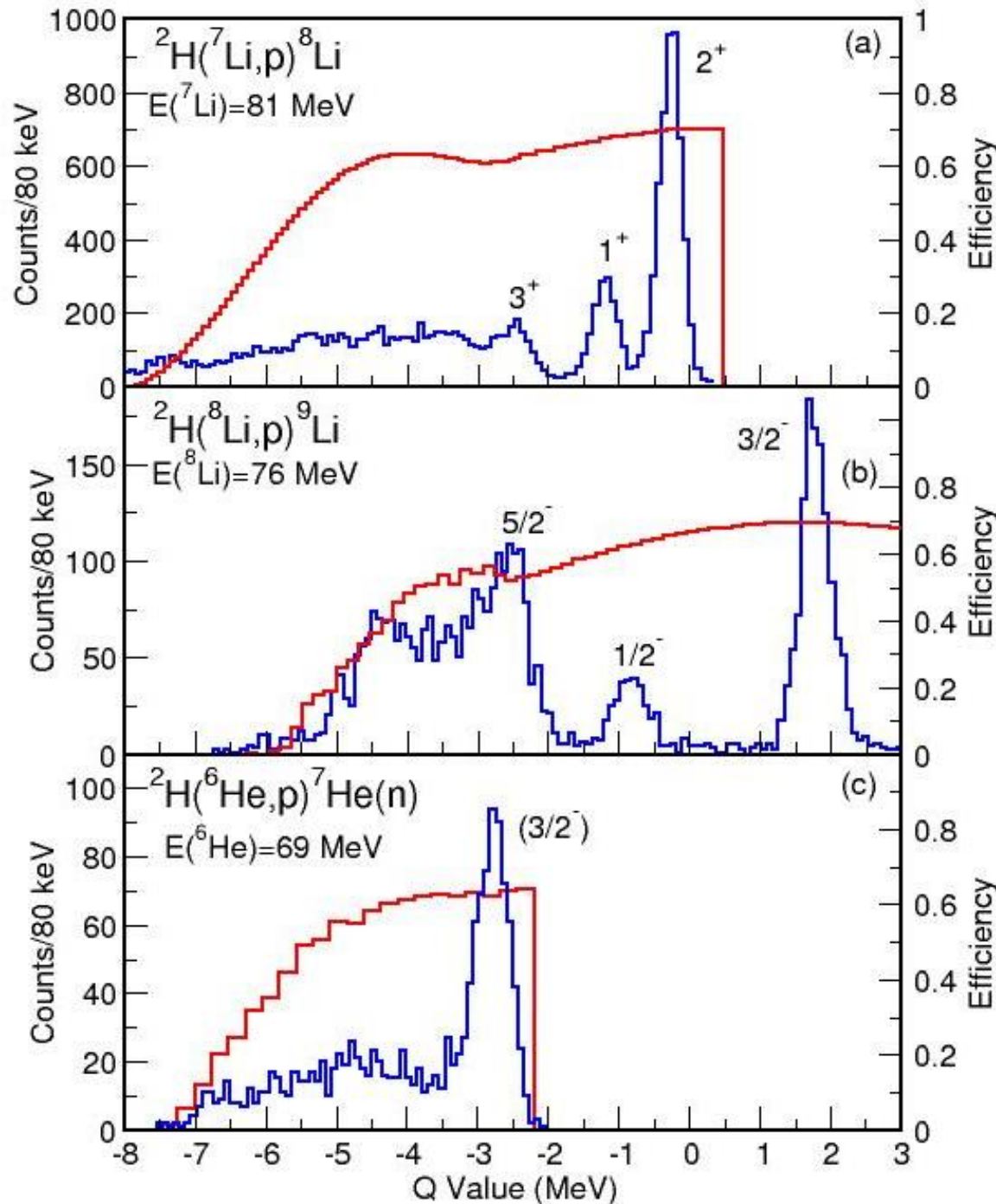


# Q-value spectra from ${}^7,8\text{Li}(d,p){}^8,9\text{Li}$ and ${}^6\text{He}(d,p){}^7\text{He}$

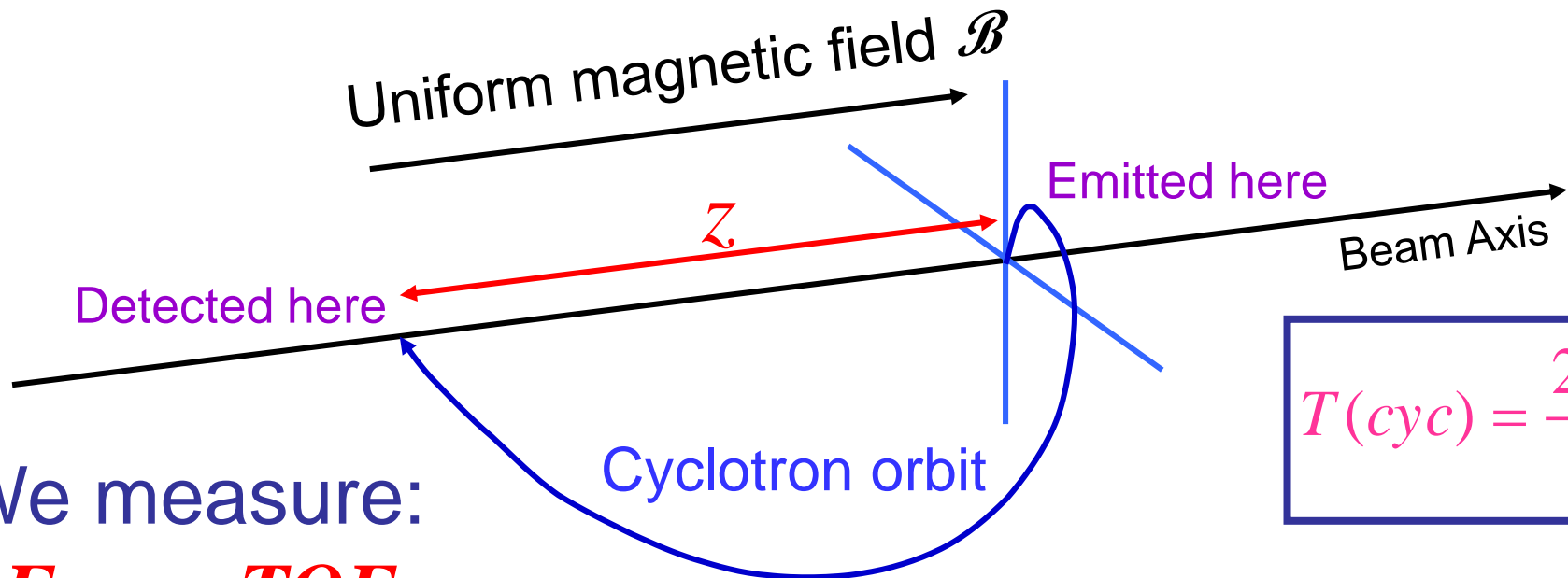
$p$ -Li or  $p$ - ${}^6\text{He}$  coincidences  
Efficiency from Monte Carlo simulations

Resolution is 400-600 keV FWHM

PRL **94**, 082502 (2005),  
PRC **72**, 061301(R) (2005)



# The solenoid approach to inverse kinematics



$$T(\text{cyc}) = \frac{2\pi m}{qB}$$

We measure:

$E_{lab}$ ,  $z$ ,  $TOF$

We deduce:

$E_{CM}$ ,  $\theta_{CM}$

$$z \propto \cos \theta_{CM}$$

$$E_{lab} = E_{CM} - A + Bz$$

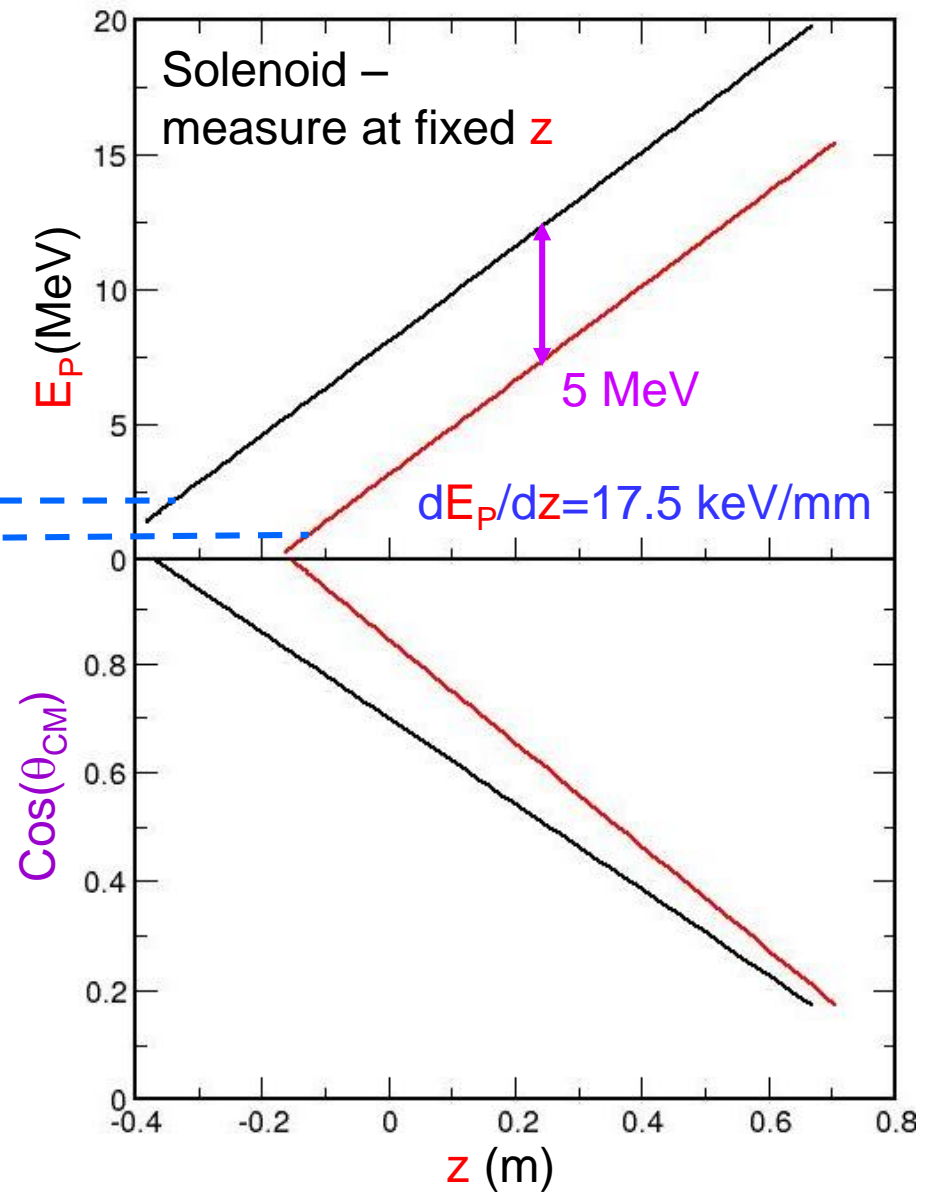
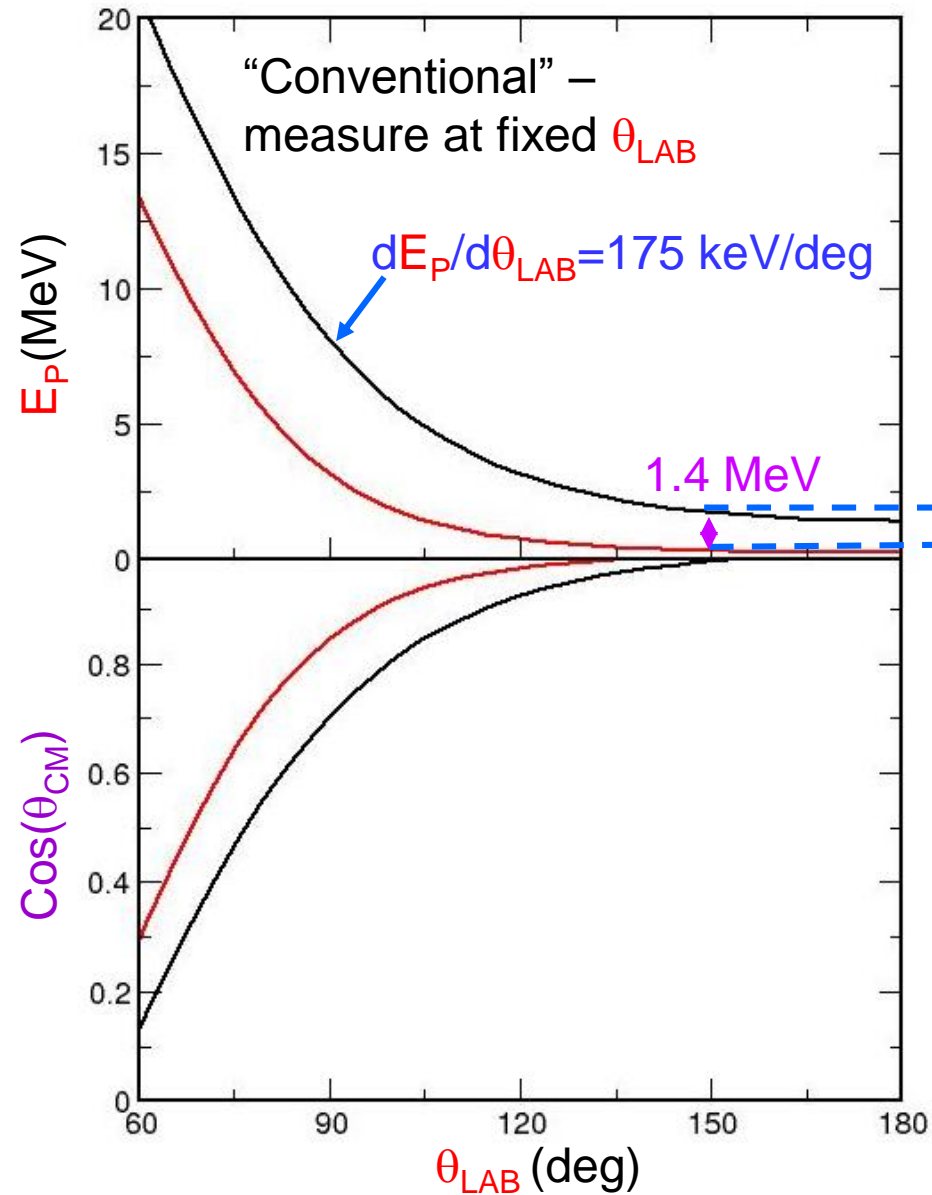
$$\Delta E_{lab} = \Delta E_{CM}$$

For a given state

For two states at fixed  $z$



# "Solenoid kinematics" for $(d,p)$





# HELICAL Orbit Spectrometer - HELIOS

$B_{MAX}=2.85\text{ T}$

2.35 m

0.9 m

Silicon Array

Target

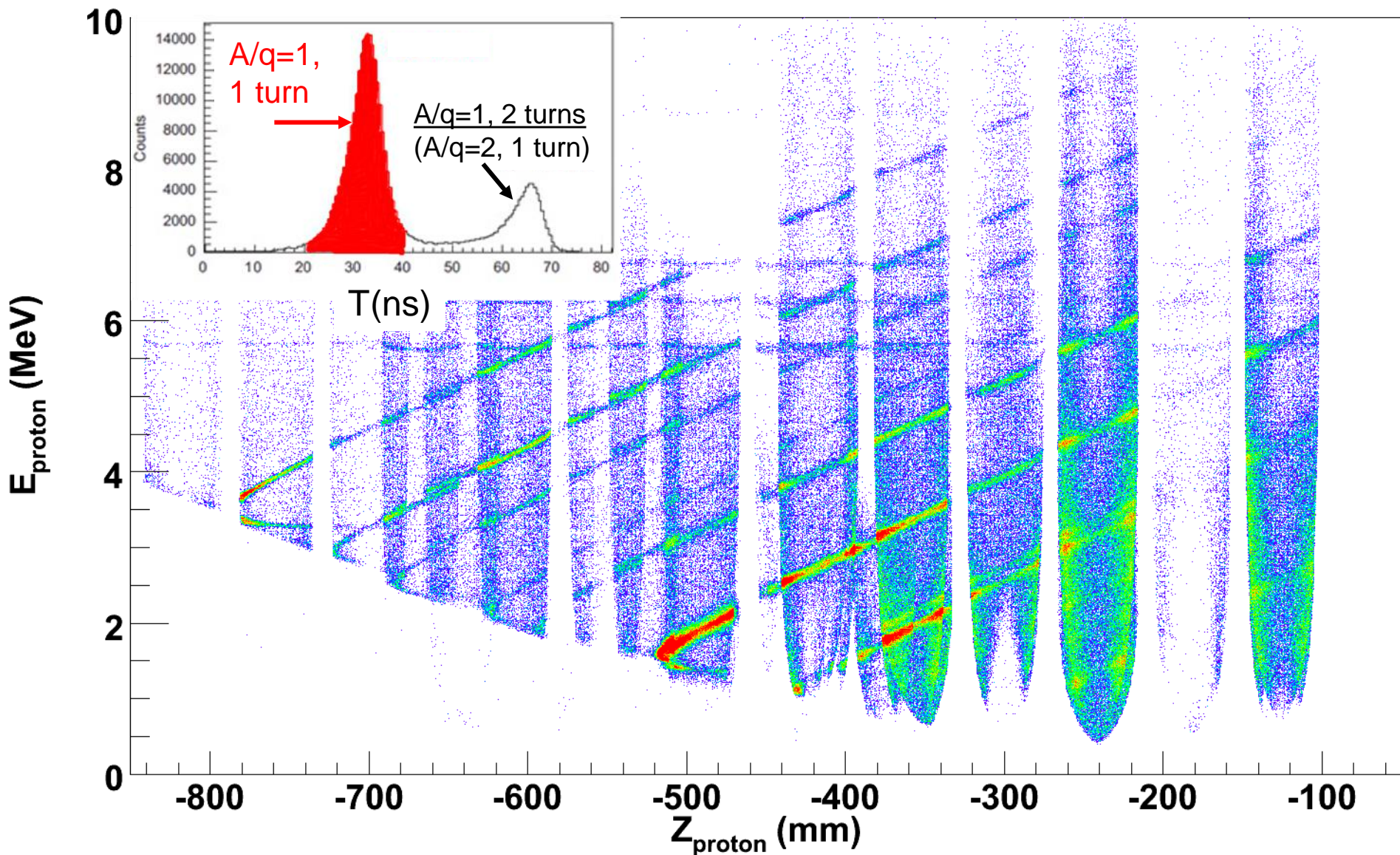
Laser  
rangerfinder

X-Y- $\theta$  positioning  
stage

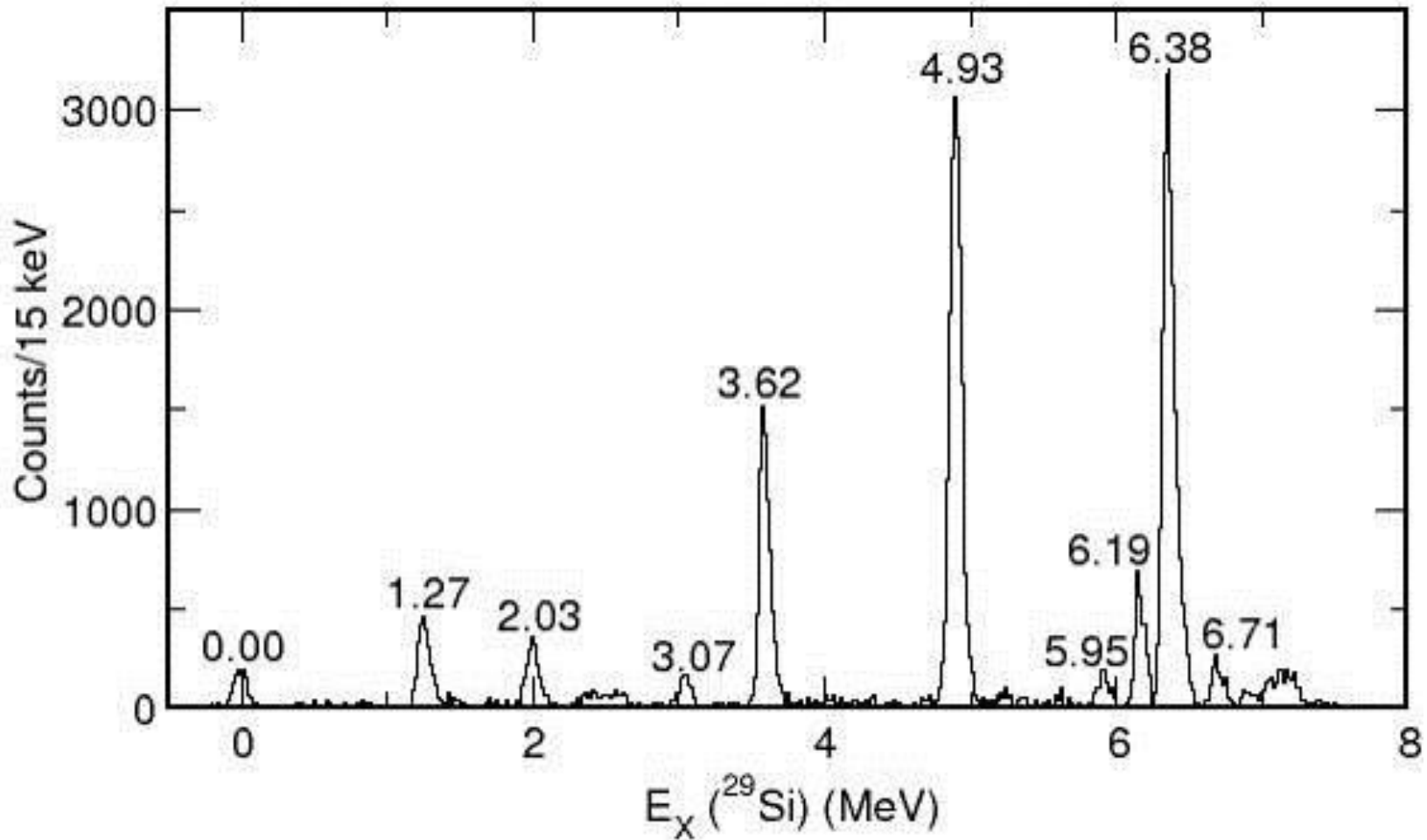
Beam

J.P. Schiffer, RIA equipment workshop 1999,  
NIMPRA **580**, 1290 (2007),  
J. C. Lighthall et al, NIMPRA **622**, 97 (2010)

# Stable beam, no recoil: $^{28}\text{Si}(d,p)^{29}\text{Si}$



# $^{28}\text{Si}(d,p)^{29}\text{Si}$ Excitation-energy spectrum

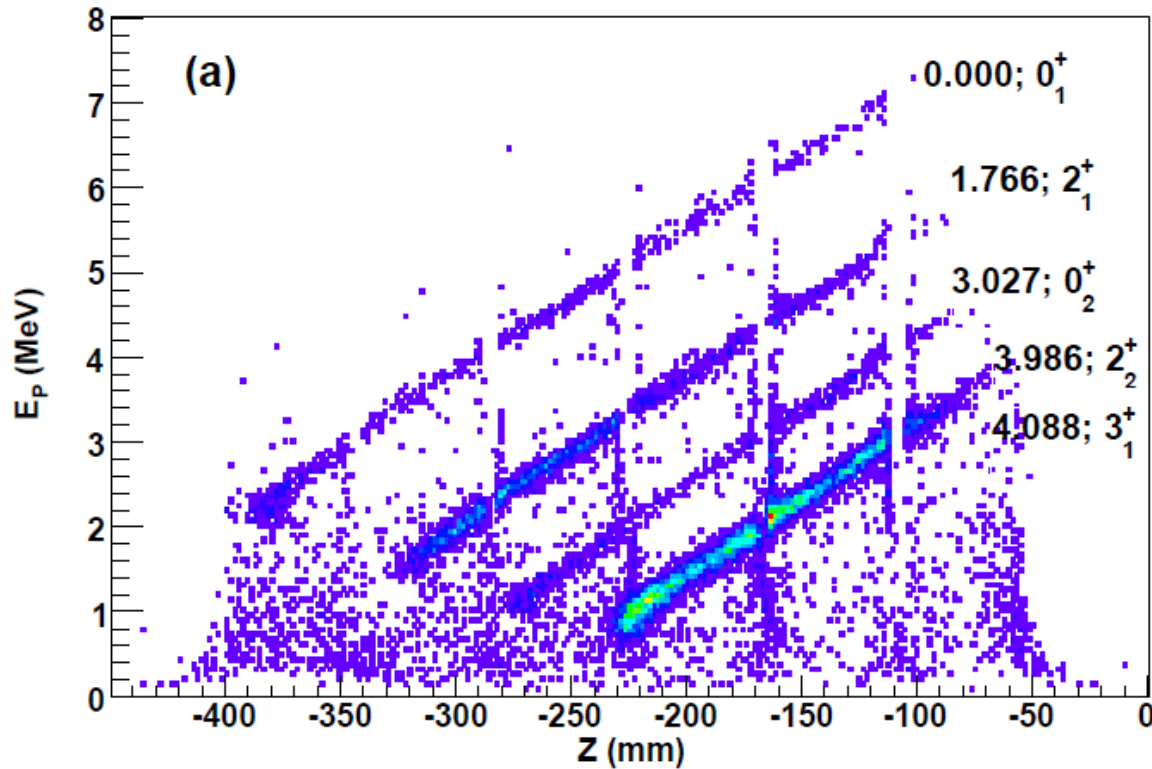


Typical resolution ~ 120 keV FWHM  
Best resolution ~ 80 keV FWHM

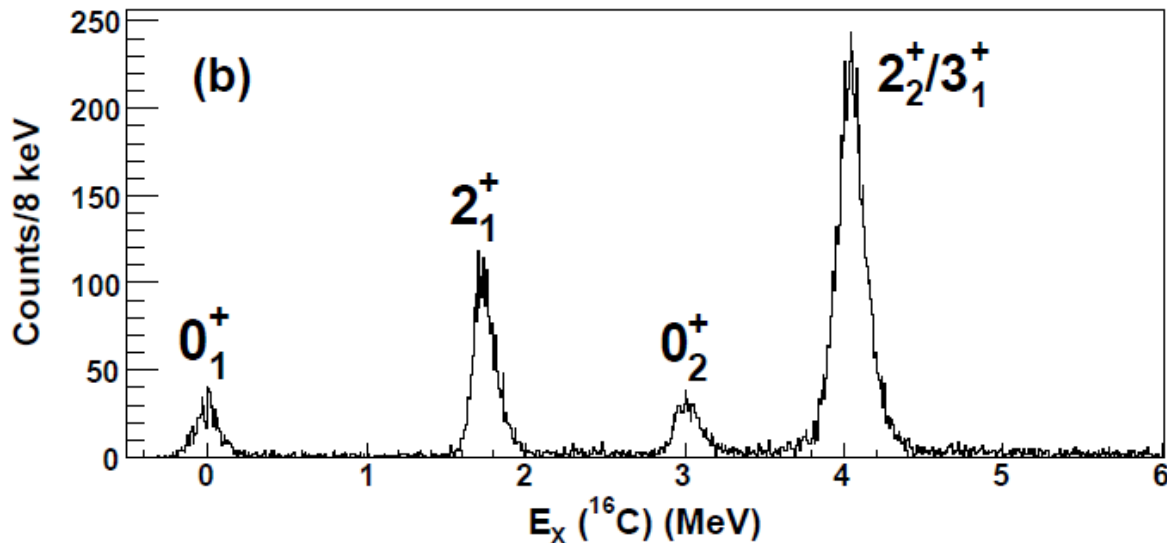


# Unstable beam $^{15}\text{C}(d,p)^{16}\text{C}$

Proton energy-position  
correlation with recoil



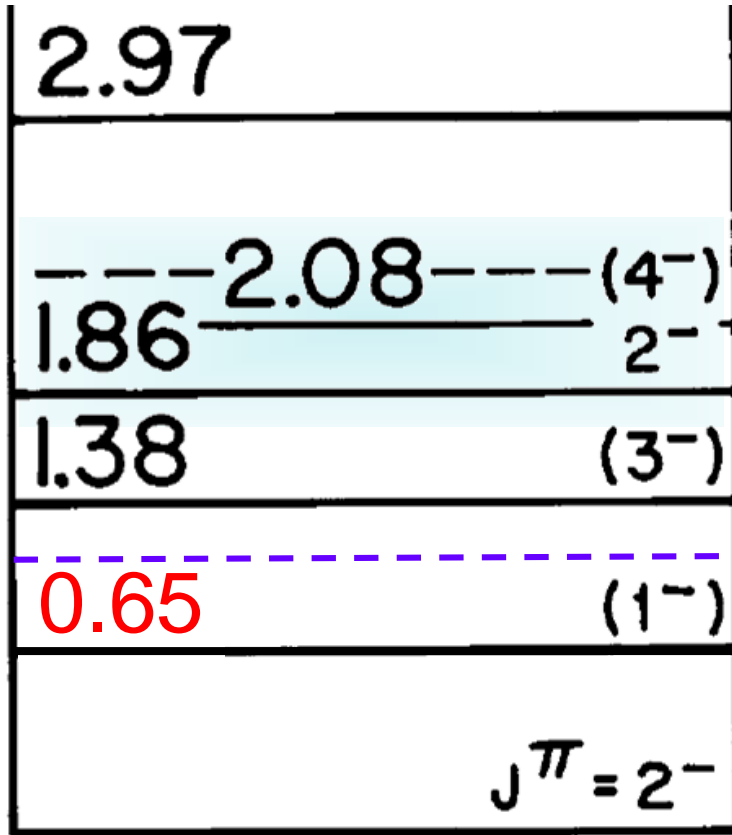
$^{16}\text{C}$  Excitation-energy  
spectrum  
Resolution is  
 $\sim 150$  keV FWHM  
 $I(^{15}\text{C}) \sim 10^6$  pps



# $^{14}\text{B}$ – as far as you can go with $N=9$ (and still be stable)

- We expect  $^{14}\text{B}$  to have a neutron structure similar to  $^{15}\text{C}$  with one neutron in  $1s_{1/2}$  orbital
- But: Complications due to  $p_{3/2}$  proton hole lead to multiplets of states, configuration mixing.
- Identification of the  $L=0$  and  $2$  strength can pin down the single-particle energies

# States in $^{14}\text{B}$



$^{14}\text{B}$

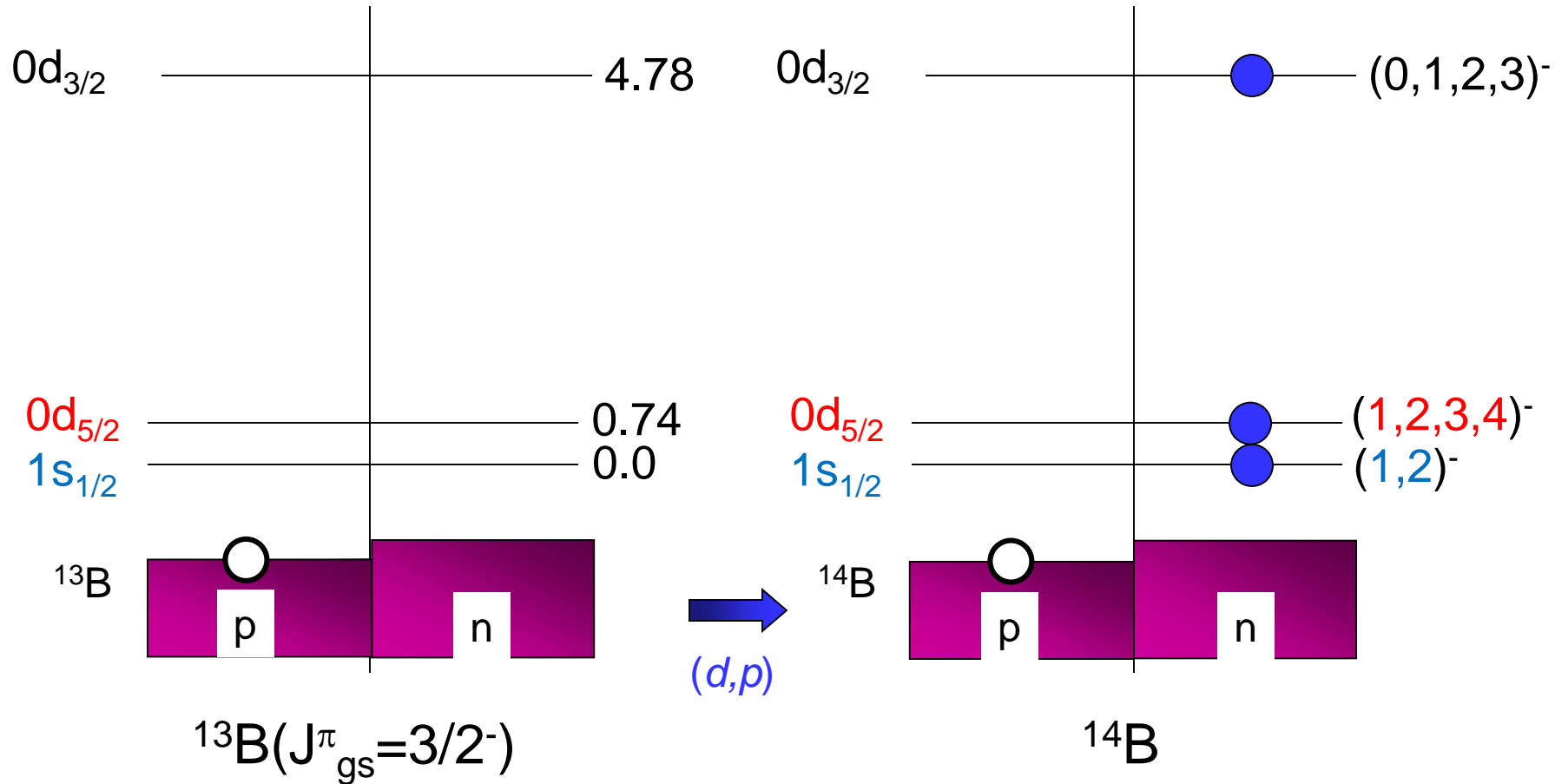
From most recent TUNL A=14 compilation (1991)

$2^-_2$  state is broad ( $\Gamma \sim 1$  MeV)

$S_n = 0.969$  MeV

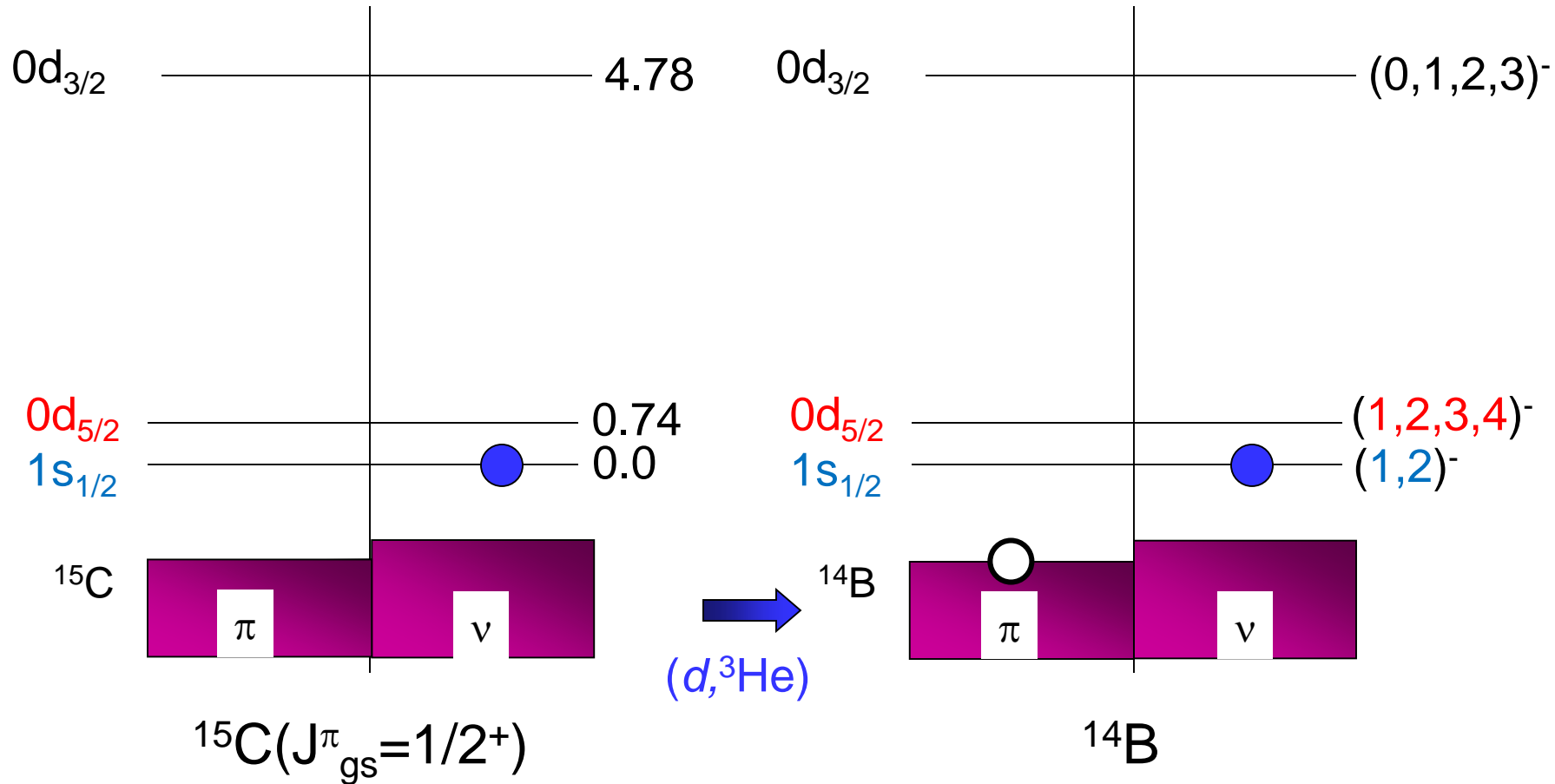
Most information is from  $^{14}\text{C}(^7\text{Li}, ^7\text{Be})^{14}\text{B}$  and analogies to the  $^{12}\text{B}$  spectrum. More recent  $^{14}\text{Be}$   $\beta$ -decay work suggests positive-parity levels not shown here

# Simple considerations for $^{13}\text{B}(d,p)^{14}\text{B}$



$(d,p)$  populates single-neutron states in  $^{14}\text{B}$

# Simple considerations for $^{15}\text{C}(d,^3\text{He})^{14}\text{B}$



$(d,^3\text{He})$  populates proton-hole states in  $^{14}\text{B}$

# $\nu(sd)$ states in $^{14}\text{B}$ with $(d,p)$

$$\psi(2_1^-) = \nu(1s_{1/2})\pi(0p_{3/2})^{-1}$$

$$\psi(2_2^-) = \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

2-

$$\psi(1_1^-) = \nu(1s_{1/2})\pi(0p_{3/2})^{-1}$$

$$\psi(1_2^-) = \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

1-

$$\psi(3_1^-) = \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

3-

$$\psi(4_1^-) = \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

4-

Assuming that there is no mixing, states are pure

# $\nu(sd)$ states in $^{14}\text{B}$ with $(d,p)$

$$\psi(2_1^-) = \alpha_2 \nu(1s_{1/2}) \pi(0p_{3/2})^{-1} + \beta_2 \nu(0d_{5/2}) \pi(0p_{3/2})^{-1}$$

$$\psi(2_2^-) = -\beta_2 \nu(1s_{1/2}) \pi(0p_{3/2})^{-1} + \alpha_2 \nu(0d_{5/2}) \pi(0p_{3/2})^{-1}$$

2-

$$\psi(1_1^-) = \alpha_1 \nu(1s_{1/2}) \pi(0p_{3/2})^{-1} + \beta_1 \nu(0d_{5/2}) \pi(0p_{3/2})^{-1}$$

$$\psi(1_2^-) = -\beta_1 \nu(1s_{1/2}) \pi(0p_{3/2})^{-1} + \alpha_1 \nu(0d_{5/2}) \pi(0p_{3/2})^{-1}$$

1-

$$\psi(3_1^-) = \alpha_3 \nu(0d_{5/2}) \pi(0p_{3/2})^{-1}$$

3-

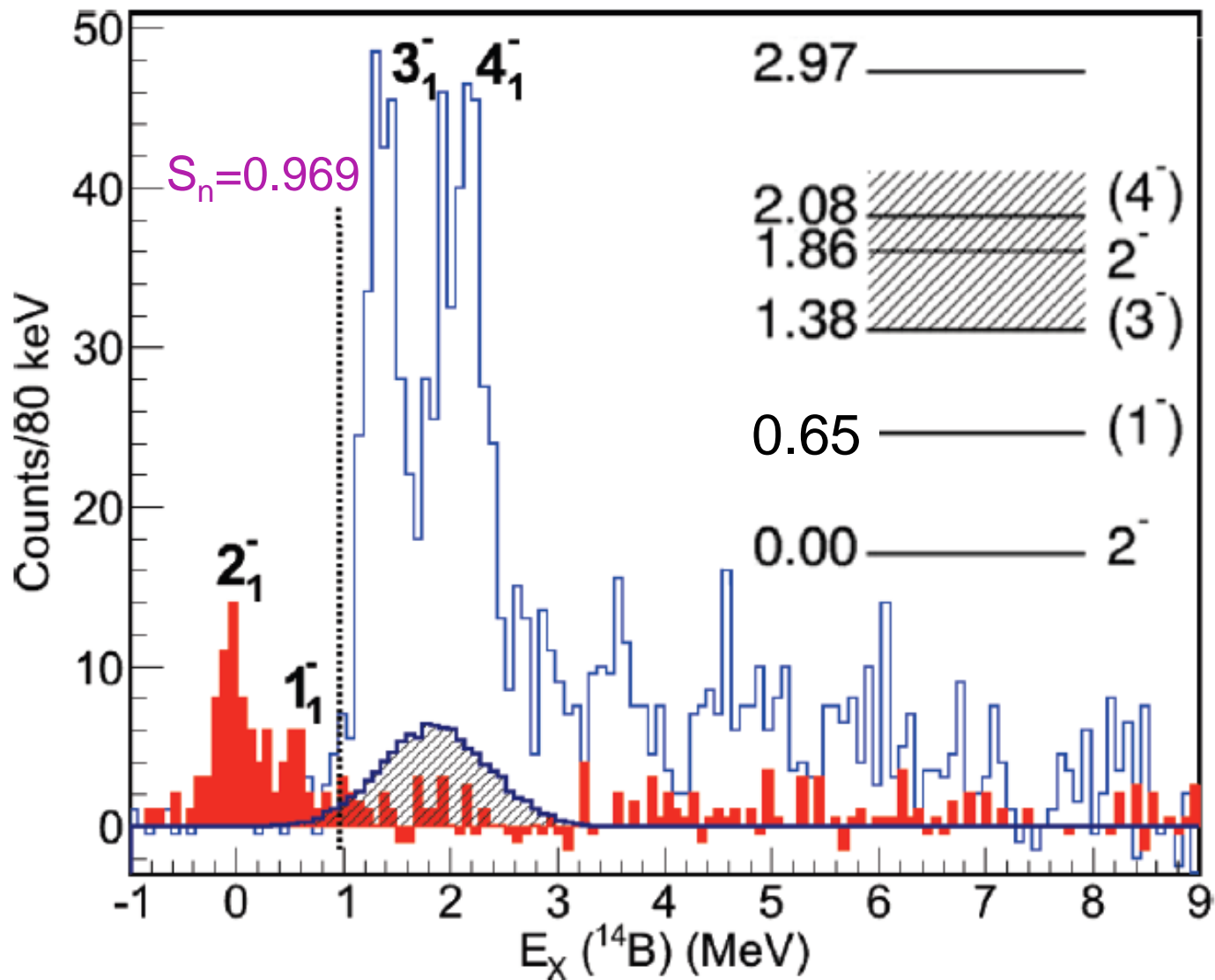
$$\psi(4_1^-) = \alpha_4 \nu(0d_{5/2}) \pi(0p_{3/2})^{-1}$$

4-

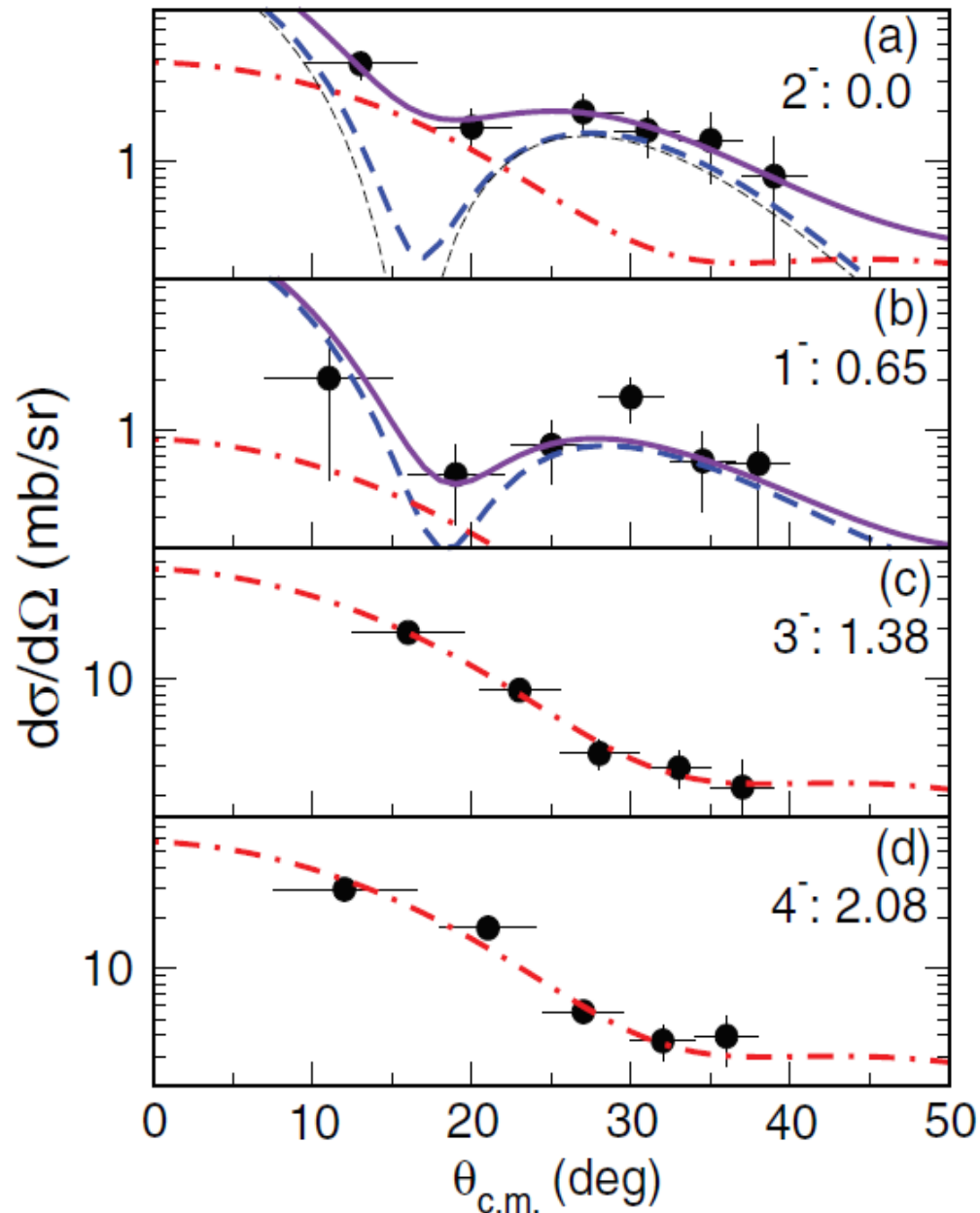
$(d,p)$  spectroscopic factors give us the  $\alpha$ 's and the  $\beta$ 's



# $^{14}\text{B}$ Excitation-energy spectrum



# $^{13}\text{B}(d,p)^{14}\text{B}$ angular distributions



Blue:  $L=0$

Red:  $L=2$

Violet:  $L=0 + L=2$

$2^-(0.00): S_0=.71 \quad S_2=.17$

$1^-(0.65): S_0=0.96 \quad S_2=.06$

$3^-(1.38): S_2=1.00$  (fixed)

$4^-(2.08): S_2=1.00$

OMP fit 30 MeV  $d+^{12}\text{C}$ ,  $p+^{12,13}\text{C}$   
elastic scattering

# Sum rules with simple, pure states:

All  $S = 1.0$  and  $(2J_I + 1 = 4)$

$$\#holes = 1 \times \frac{5}{4} + 1 \times \frac{3}{4} = 2 (s_{1/2})$$

$$\#holes = 1 \times \frac{5}{4} + 1 \times \frac{3}{4} + 1 \times \frac{7}{4} + 1 \times \frac{9}{4} = 6 (d_{5/2})$$

$$J_F = \quad 2 \quad 1 \quad 3 \quad 4$$

# Sum rules with observed states:

Measured  $S$  ( $2J_I+1=4$ )

$$\#holes = 0.71 \times \frac{5}{4} + 0.96 \times \frac{3}{4} = 1.6 \quad (s_{1/2})$$

$$\#holes = 0.17 \times \frac{5}{4} + 0.06 \times \frac{3}{4} + 1 \times \frac{7}{4} + 1 \times \frac{9}{4} = 4.3 \quad (d_{5/2})$$

$$J_F = 2$$

$$1$$

$$3$$

$$4$$

We're missing two states!

# A game with sum rules...

$$\psi(2_1^-) = \alpha_2 \nu(1s_{1/2}) \pi(0p_{3/2})^{-1} + \beta_2 \nu(0d_{5/2}) \pi(0p_{3/2})^{-1}$$

$$\psi(2_2^-) = -\beta_2 \nu(1s_{1/2}) \pi(0p_{3/2})^{-1} + \alpha_2 \nu(0d_{5/2}) \pi(0p_{3/2})^{-1}$$

2-

$$\psi(1_1^-) = \alpha_1 \nu(1s_{1/2}) \pi(0p_{3/2})^{-1} + \beta_1 \nu(0d_{5/2}) \pi(0p_{3/2})^{-1}$$

$$\psi(1_2^-) = -\beta_1 \nu(1s_{1/2}) \pi(0p_{3/2})^{-1} + \alpha_1 \nu(0d_{5/2}) \pi(0p_{3/2})^{-1}$$

1-

Assume  $\psi(2_2^-)$  and  $\psi(1_2^-)$  are the orthogonal partners of  $\psi(2_1^-)$  and  $\psi(1_1^-)$ . We already know  $\alpha_2, \beta_2$  and  $\alpha_1, \beta_1$ , so we can then guess the spectroscopic factors for  $\psi(2_2^-)$  and  $\psi(1_2^-)$ .

Then:

Experimentally,  $\Sigma S[J_f]/4 = 5.9$  ( $L=2$ ) and  $1.9$  ( $L=0$ ) compared to the sum-rule values of  $6.0$  and  $2.0$ .

# $^{13}\text{B}(d,p)^{14}\text{B}$

## Spectroscopic factors

Excitation energies and relative spectroscopic factors from the shell model

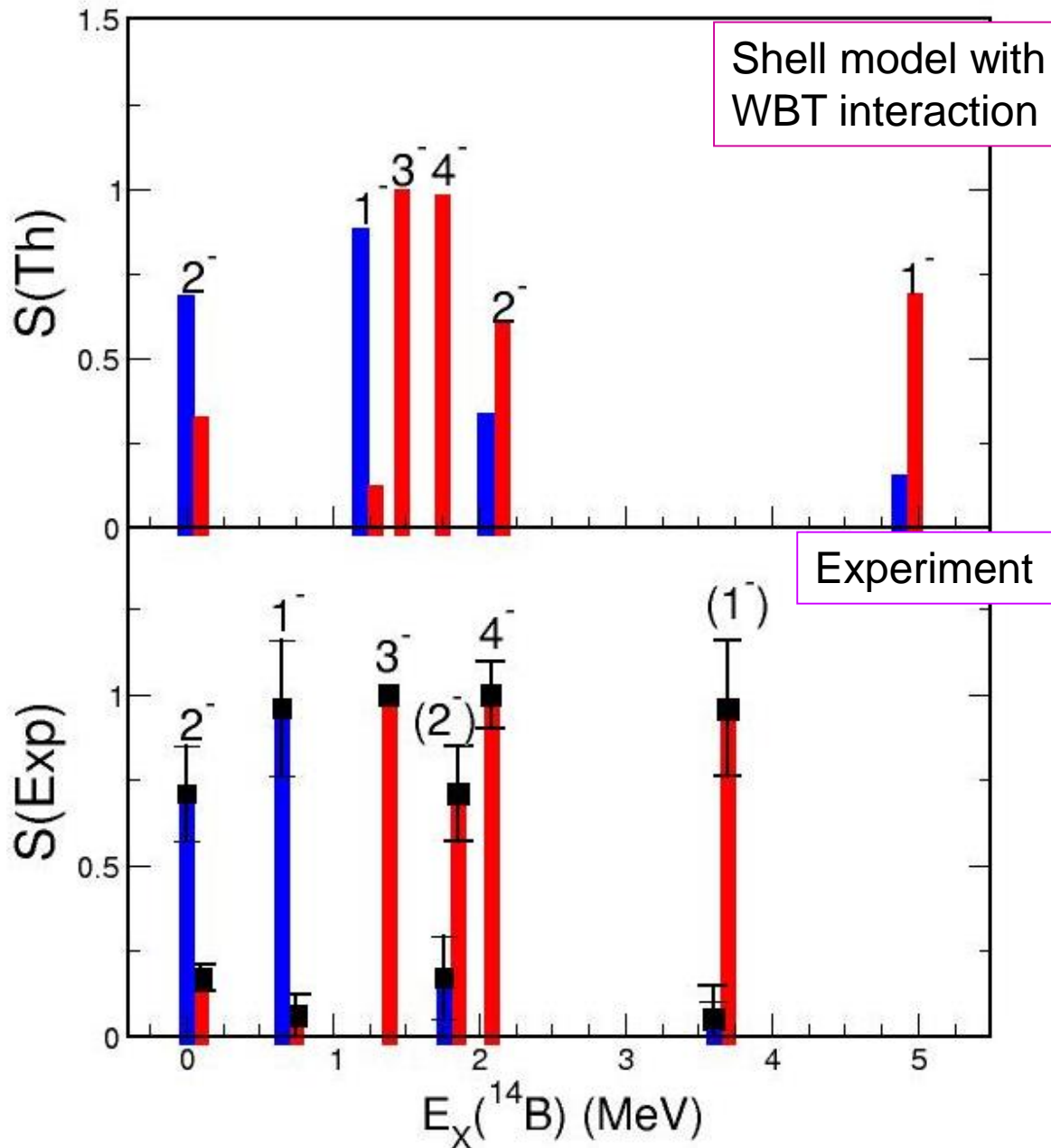
Blue:  $L=0, 1s_{1/2}$

Red:  $L=2, 0d_{5/2}$

2<sup>-</sup> mixed  $L=0+2$ ,

1<sup>-</sup> mostly  $L=0$

Reasonable agreement  
But *caveat emptor!*



## Another game...

Can we *estimate* the  $1s_{1/2}$  and  $0d_{5/2}$  single-particle energies in  $^{14}\text{B}$ ?

No narrow states left.

We assume  $^{14}\text{B}(2^-_2)$  is the broad state at 1.86 MeV, and we assume  $^{14}\text{B}(1^-_2)$  is a broad state at 3.6 MeV (there seems to be something there in  $(d,p)$  with  $L=2$ ).

$$\langle E_0 \rangle_j = \frac{\sum_i (2J_i + 1) S_i E_{xi}}{\sum_i (2J_i + 1) S_i}$$

Then:

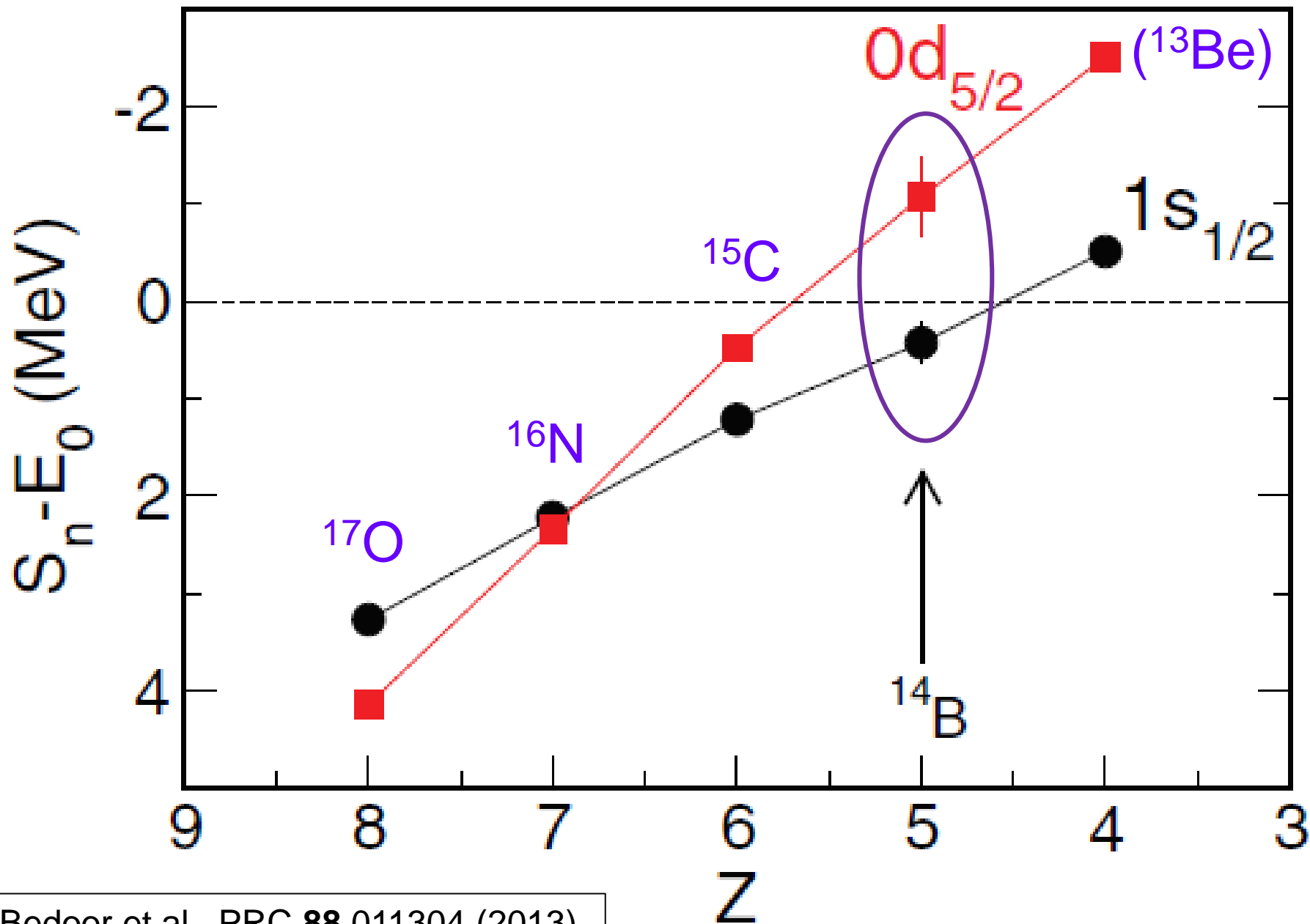
$$\langle E_0 \rangle(1s_{1/2}) = 0.55 \text{ MeV}$$

$$\langle E_0 \rangle(0d_{5/2}) = 1.94 \text{ MeV}$$

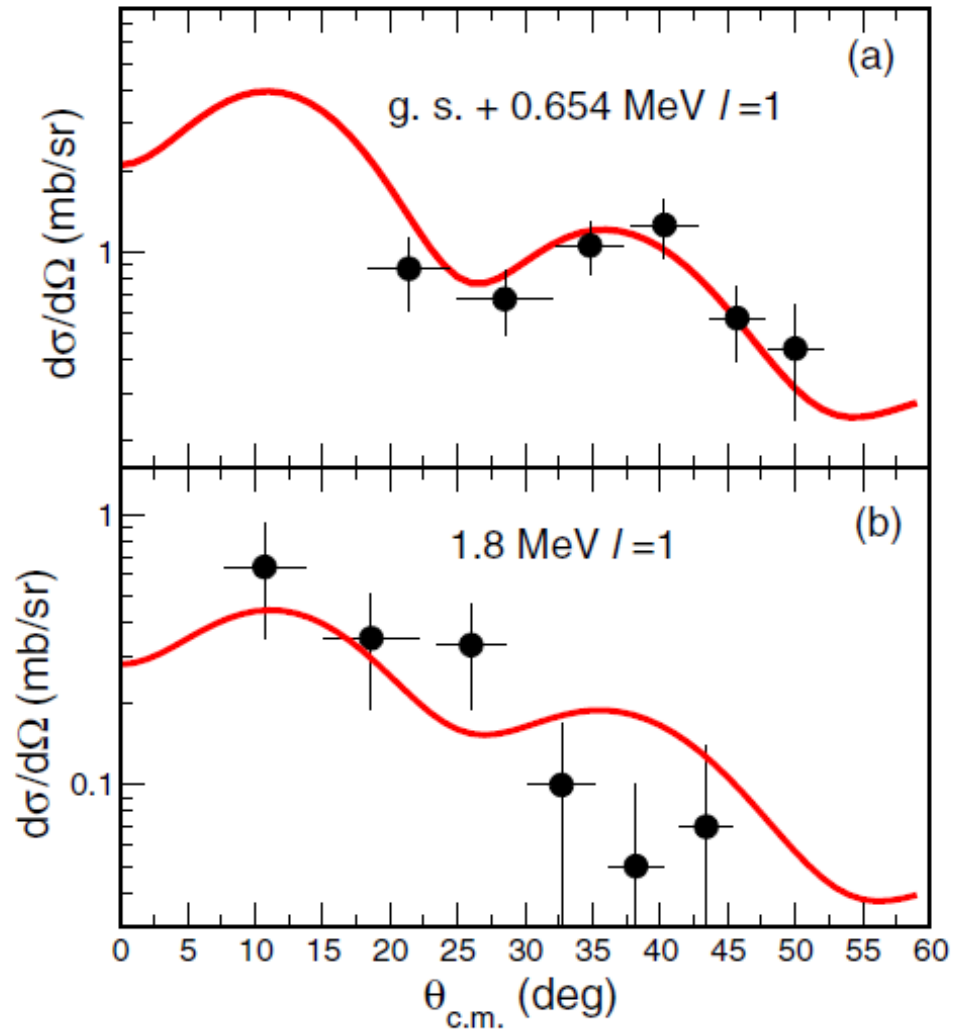
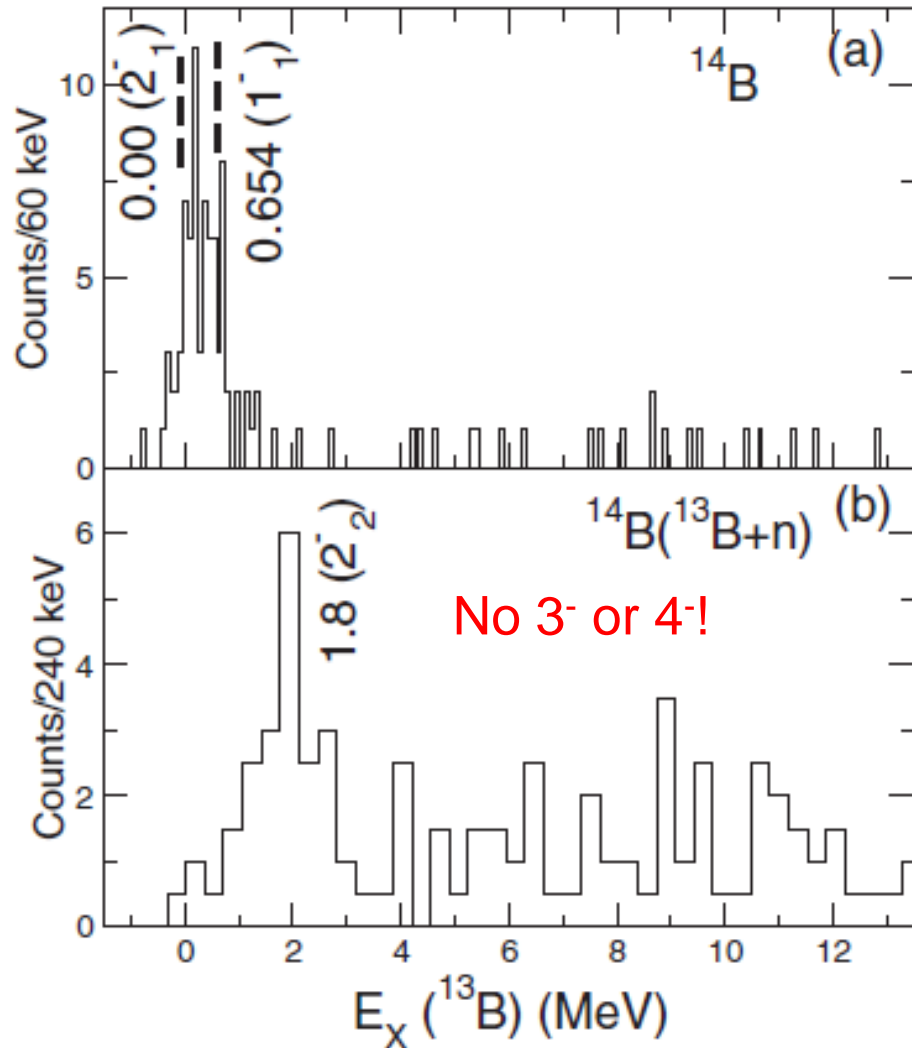
$$\langle E_0 \rangle(0d_{5/2}) - \langle E_0 \rangle(1s_{1/2}) = 1.39 \text{ MeV}$$



# Evolution of $1s_{1/2}$ and $0d_{5/2}$ energies



# $2^-_2$ confirmation from $^{15}\text{C}(d,^3\text{He})^{14}\text{B}$

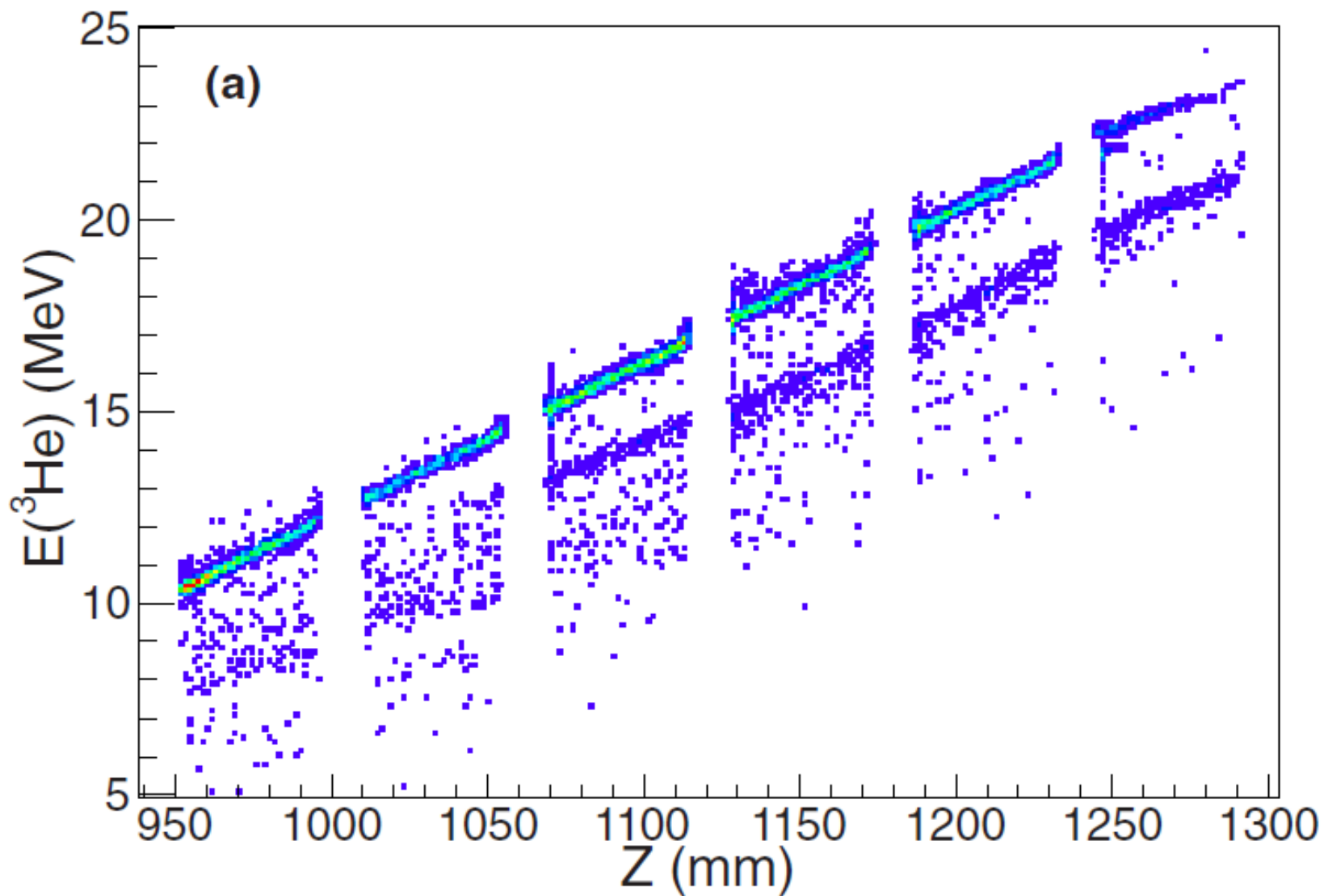


Removal of a  $0p_{3/2}$  proton

# Preaching and Conclusion

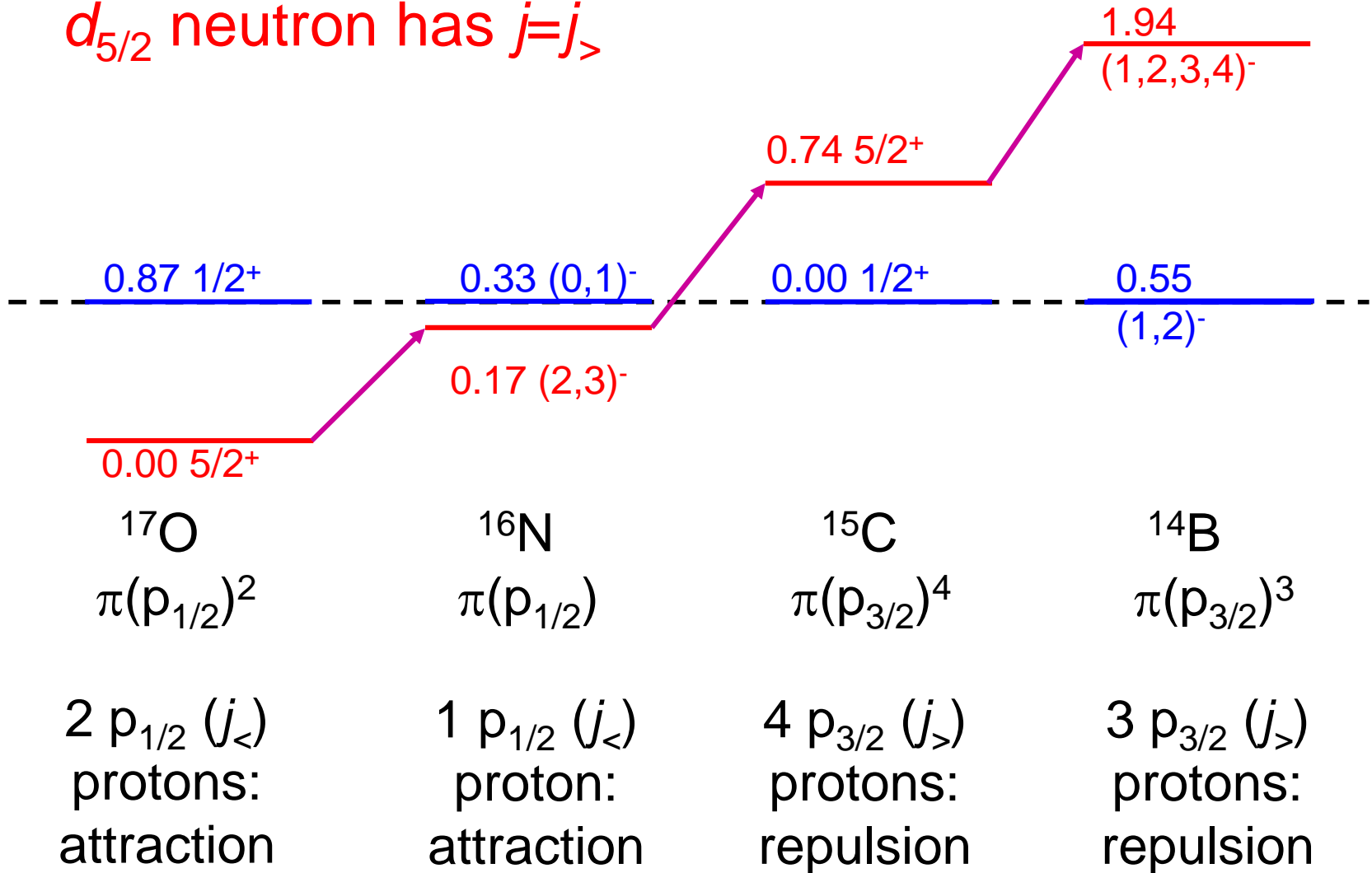
- Remember history – basic understanding embedded in early work, often obscured by nuance and details accumulated over the years
- Put results in context – nuclear physics progresses by assembly of a puzzle with many parts, individual measurements are pieces but **don't lose sight of the Big Picture**
- Technical advances can help provide better data, but equally important are imagination and insight in the design of experiments and the interpretation of data.
- **There is a lot that I did not cover. Take this and run with it!**

# $^{14}\text{C}(d, ^3\text{He})^{13}\text{B}$ in HELIOS



# Evolution of $1s_{1/2}$ - $0d_{5/2}$ splitting outside N=8

$d_{5/2}$  neutron has  $j=j_>$



# Heavy-Ion transfer and ANC's

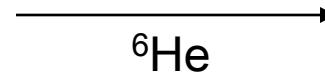
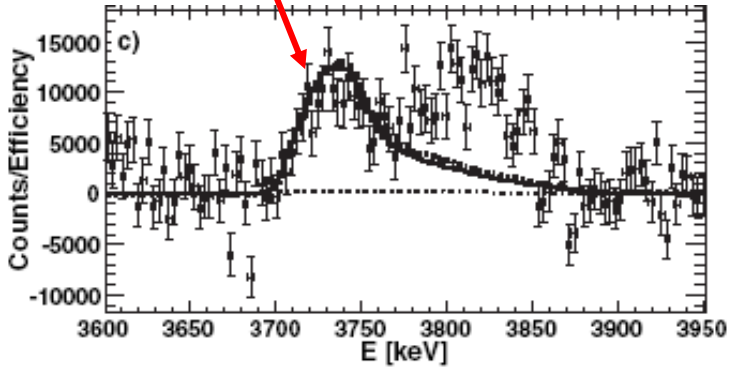
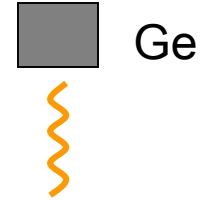
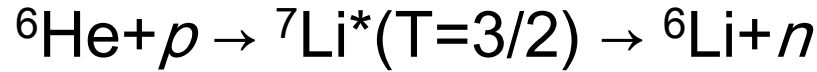
- Peripheral collisions – close or head-on collisions lead to more complex processes
- Samples the tail of the wave functions-  
“Asymptotic Normalization Coefficients” or  
“ANCs”
- **Why do it this way?** Many astrophysical processes occur at very low energies and are extremely peripheral.
- Analyze in a very similar way.
- Are the approaches consistent?...

# Two other direct processes: Charge exchange and knockout

- **Charge exchange** – change a  $p$  to an  $n$  or vice-versa:
  - examples:  $(p,n)$ ,  $({}^3\text{He},t)$ ,  $(d,{}^2\text{He})$
  - Populates “**Isobaric analog states**”
  - Samples Gamow-Teller strength at small angles/low momentum transfer – like  $\beta$  decay.
- **Knock-out**: The projectile “knocks out” a particle from the target nucleus
  - examples:  $(e,e'p)$ ,  $(p,2p)$ ,  $(p,np)$  etc.
  - can be used to complement other direct transfer reactions, sensitive to nuclear structure

# Charge exchange – an example

Analog of  ${}^7\text{He}$  g.s.



Tail of analog of  ${}^7\text{He}$  1/2-

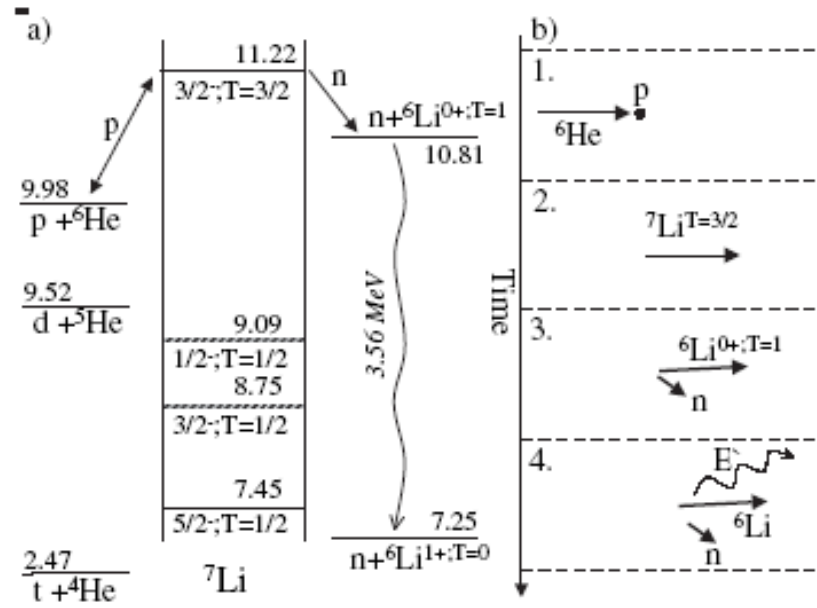
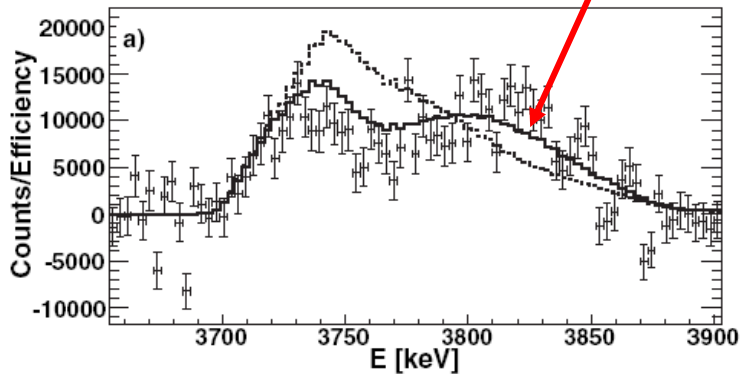


FIG. 1. (a) Decay pathways for the  $T = 3/2$  resonance in  ${}^7\text{Li}$ , and (b) the successive kinematics stages of the studied reaction.



# Typical CN angular distributions



Angular distributions are forward-backward symmetric

