E2 CORE-POLARIZATION CHARGE
FOR NUCLEI NEAR $^{16}$O AND $^{40}$Ca

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Abstract: Empirical electric-quadrupole core-polarization charges are extracted from the experimental $\gamma$-ray transitions probabilities and quadrupole moments of nuclei near $^{16}$O and $^{40}$Ca using Woods-Saxon wave functions which include two-hole excitations of the cores. The results are discussed in terms of microscopic and macroscopic models of the core-polarization charge which are modified to include binding-energy-dependent effects. The empirical results are $e_n = 0.34 \pm 0.02e$ and $e_p = 0.94 \pm 0.05e$ for $A = 18$ and $e_n = 0.57 \pm 0.03e$ and $e_p = 1.06 \pm 0.07e$ for $A = 42$. In both the macroscopic and microscopic models the isoscalar polarization charges imply that some of the isoscalar E2 strength in $^{16}$O and $^{40}$Ca lies above $\sqrt{2}\hbar\omega$. The isovector polarization charges are larger than predicted by the microscopic calculations using the Kuo-Brown interaction but are consistent with the macroscopic model where $E(T = 1) = 3.3 \hbar\omega$.

1. Introduction

There is considerable evidence that the conventional shell model can reproduce many observed properties of nuclei if appropriate effective operators within a limited model space are used 1). The effective operators are introduced to compensate for the severe space truncation which must be made to construct a tractable shell-model calculation. One of the oldest effective operators in nuclear physics is the effective charge which is used to account satisfactorily for electric quadrupole (E2) observables 2-5).

The experimental data for E2 observables in nuclei near the closed shells where the shell-model calculations are simplest are abundant, and it was realized early that empirical E2 effective charges for the proton and neutron of about $e_p = 1.5$ and $e_n = 0.5$ (in units of $e$) were needed 6) even in cases where the experimental level scheme was well described by the shell-model theory. This effect is now well established to arise predominantly from the virtual excitation of particles from the closed shells.

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to orbitals two units higher in the major quantum number $N \{N = 2(n-1)+l \text{ where } n = 1, 2, \ldots\}$. The amplitude of these $\Delta N = 2$ one particle-one hole states in the wave function is small but they have a large effect because the $\Delta N = 2$ transition strength enters into the E2 matrix elements linearly in the admixed amplitude. The $\Delta N = 2$ transition contains most of the nuclear E2 sum rule limit. The effective charge originating from these $\Delta N = 2$ contributions is often referred to as the core-polarization charge.

The simplest microscopic calculation for the core-polarization charge, carried out in first-order perturbation theory with a delta function residual interaction and $\Delta E = 2\hbar\omega$ for the energy denominator, yields qualitative agreement with the empirical values. However, one expects, in analogy with the well known 1p-1h $\Delta N = 1$ collective $3^-, T = 0$ excitations and $1^-, T = 1$ giant dipole excitations, that the $\Delta N = 2$, $2^+$ excitations have collective properties which would be important for a more quantitative understanding of the core-polarization charge.

There has been a renewed interest in the $\Delta N = 2$, $2^+$ excitations theoretically and experimentally. More sophisticated microscopic calculations for the core-polarization charge have been carried out which include the effects of higher order terms on the perturbation expansion. Many of these calculations have been made with effective interactions based on realistic nucleon-nucleon potentials. In addition, the $2^+$ collective properties have been described in the macroscopic model. Also there is now some direct experimental evidence for the location and transition strengths of the collective $\Delta N = 2$ states.

The most detailed calculations of the core-polarization charge have been carried out for light nuclei ($A \leq 40$). However it is known that for the light nuclei other effects exist which make it difficult to compare the calculated core-polarization charge directly with experimental quantities. The most important of these are changes in the radial wave functions for loosely bound valence orbitals (especially near $^{16}$O) and the large admixtures of core deformed wave functions made up of $\Delta N = 1$, 2p-2h and 4p-4h excitations.

The purpose of the present paper is twofold. First, we attempt to extract from the experimental observables for nuclei near $^{16}$O and $^{40}$Ca, effective charges which are the most closely related to the core-polarization charges. Secondly, these quantities are related in a consistent way to the existing theoretical calculations and subsequently to observable properties of the $\Delta N = 2$ collective $2^+$ states. The experimental effective charges are extracted from the measured quadrupole moments and E2 transition probabilities using Woods-Saxon single-particle wave functions. The Woods-Saxon parameters are chosen to reproduce the rms charge radii and single-particle binding energies. The experimental core-polarization charge is isolated by explicitly including microscopic 2p-2h components in the model-space wave functions and by emphasizing the states with highest spin which are believed to be the least influenced by these components.

The single-particle wave functions for the nuclei with $A = 15, 17, 39$ and 41 are
theoretically simplest, but the experimental data for these nuclei are not complete. For the experimental polarization charge we also consider the configuration mixed nuclei with \(N = 16, 18, 40\) and 42 for which the model-space wave functions are still relatively simple. In most cases the model-space wave functions are obtained using the Kuo-Brown \(^{31, 32}\) two-body effective interaction.

The following sections can be divided into two major parts. Sects. 3–6 give a complete discussion of how the empirical effective charges are obtained from the experimental data and are independent of sects. 2 and 7. Sect. 2 is meant to give an introduction and brief review of the theoretical models for the core-polarization charge with a discussion of how the theoretical models can be modified if Woods-Saxon wave functions are used. The comparison between the empirical and the theoretical results is discussed in sect. 7.

### 2. The E2 effective operator

For a particle or hole outside a closed shell nucleus the core-polarization charges \(\delta e_p\) and \(\delta e_n\) are defined by the ratio of the matrix elements of the \(Q_n^{(2)}\) single-particle operator between the exact wave functions \(\psi\) to its matrix elements between the single-particle wave functions \(\phi\):

\[
\langle \psi | \sum_{l=1}^{A} Q_n^{(2)}(l)[\frac{1}{2} - t_s(l)] e | \psi' \rangle = \langle \phi | Q_n^{(2)}(\frac{1}{2} - t_s) e + \delta Q_n^{(2)} | \phi' \rangle = \langle \phi | Q_n^{(2)}[(\frac{1}{2} - t_s)(e + \delta e_p) + (\frac{1}{2} + t_s) \delta e_n] | \phi' \rangle,
\]

where

\[
Q_n^{(2)} = r^2 Y_n^{(2)}(\theta, \phi),
\]

\[
\delta e_n = \frac{\langle \phi | \delta Q_n^{(2)} | \phi' \rangle}{\langle \phi | Q_n^{(2)} | \phi' \rangle},
\]

\(t_s = +\frac{1}{2}\) for a neutron and \(-\frac{1}{2}\) for a proton. For the \(LS\) closed cores \(^{16}\)O and \(^{40}\)Ca the most important difference for the E2 matrix elements between the wave functions \(\psi\) and \(\phi\) is that \(\psi\) contains \(\Delta N = 2\) \(1p-1h\) states whose coherent superpositions make up the \(J^p = 2^+\) \(T = 0, 1\) giant quadrupole states of the closed core.

The reduced matrix elements of eq. (2)

\[
\langle \phi || E2 || \phi' \rangle = (-1)^{j'-m} \left( \begin{array}{cc} j & j' \\ -m & \mu \\ m' & \nu \end{array} \right)^{-1} \langle \phi || E2 || \phi' \rangle,
\]

where

\(E2 = Q_n^{(2)}[(\frac{1}{2} - t_s)(e + \delta e_p) + (\frac{1}{2} + t_s) \delta e_n]\)

are directly related to the experimental quadrupole moments and \(B(E2)\) values:

\[
Q(j) = \sqrt{\frac{\hbar}{2\pi}} \left( \begin{array}{cc} j & 2 \\ -j & 0 \\ 0 & j \end{array} \right) \langle \phi || E2 || \phi' \rangle,
\]
Once the wave functions are specified these matrix elements can be evaluated and the effective charges can be extracted from the comparison with experimental data. This is done in sects. 3–6.

All detailed microscopic and macroscopic theoretical calculations for $\delta e$ have been previously carried out assuming harmonic oscillator single-particle wave functions; the result of those calculations will be denoted by $\delta e^{\text{mier}}$ and $\delta e^{\text{mier}}$, respectively. In this section the relationship of these calculated core-polarization charges to the properties of the valence-particle wave functions and the properties of the giant quadrupole states of $^{16}\text{O}$ and $^{40}\text{Ca}$ will be discussed. The special problems associated with multiparticle systems are discussed in subsect. 2.3.

2.1. STATE AND BINDING ENERGY DEPENDENCE

In the simplest microscopic calculation, $\delta e^{\text{mier}}$ can be obtained in first-order perturbation theory with a delta-function for the effective interaction and putting all $1p$-$1h$ states degenerate at an energy of $2\hbar\omega$. Assuming LS closed shells for $^{16}\text{O}$ and $^{40}\text{Ca}$ and using harmonic oscillator wave functions, the microscopic first-order expression for $\delta Q^{(2)}$, which involves a sum over all $\Delta N = 2$ $1p$-$1h$ states [see for example eq. (1) of ref. 18)], can be simplified to the exact form:

$$\delta Q^{(2)} = F(t_{2})^{\text{mier}(4)}k(r)^{\text{mier}(4)}Y_{\ell}^{(2)}(\theta, \phi),$$

where

$$F(t_{2})^{\text{mier}(4)} = [3V(1-\alpha) + V(1+3\alpha)(2t_{2})]/4\nu(\Delta E = 2\hbar\omega),$$

$$k(r)^{\text{mier}(4)} = r\int_0^r \frac{\rho(r')}{r'} dr,'$$

$$\rho(r) = \sum_{\text{core protons}} |\phi_{i}(r)|^2.$$

The delta-function interaction is parameterized as

$$V(r_{12}) = -V(1+\alpha\sigma_{1} \cdot \sigma_{2})\delta(r_{12}),$$

and $k(r)^{\text{mier}(4)}$ is given explicitly by

$$k(r)^{\text{mier}(4)} = -(v/\pi)^{3/2}vr(8vr^2 - 4)e^{-r^2}$$

for $^{16}\text{O}$ and

$$k(r)^{\text{mier}(4)} = -(v/\pi)^{3/2}vr(16v^2r^4 - 8vr^2 + 10)e^{-r^2}$$

for $^{40}\text{Ca}$.

For a single state which exhausts the energy-weighted sum rule for the multipolarity $\lambda$, the microscopic transition charge density $\rho(r)_{\ell}$ defined by

$$\langle 0||r^{4}Y^{(2)}||\lambda \rangle = \int \rho(r)_{\ell}r^{2\lambda+2}dr$$
CORE-POLARIZATION CHARGE

has been shown to be simply related to the ground-state density \( \rho(r) \) [refs. 33, 34)] by

\[
\rho(r)_w = N r^{4-1} \frac{d\rho(r)}{dr},
\]

where

\[
N = -\langle 0 | r^4 Y^{(4)} | \lambda \rangle / (2\lambda + 1) \langle 0 | r^{2\lambda+2} | 0 \rangle.
\]

Thus in eq. (5) \( k(r)^{\text{mier}(\theta)} \) is proportional to the transition charge density for the \( \Delta N = 2, 2^+ \rightarrow 0^+ \) transition in \( ^{16}O \) or \( ^{40}Ca \). In the \( LS \) closed shell model the degenerate \( \Delta N = 2, 1p-1h \) state contains all of the E2 strength. In the Tamm-Dancoff and random phase approximations the core-polarization charge depends mainly on the coherent isoscalar and isovector \( \Delta N = 2 \) states which are perturbed from \( 2\hbar \omega \). In this case the transition charge density is still proportional to \( r^{4-1} \frac{d\rho(r)}{dr} \) since the sum rule is nearly exhausted by these single coherent states.

As it was first pointed out by Fallieros and Ferrell 5), the state dependence of the core-polarization charge given by eqs. (3) and (5):

\[
\delta e_n^{\text{mier}(\theta)} = F(t) \frac{\langle \phi | k(r)^{\text{mier}(\theta)} | \phi' \rangle_{\text{HO}}}{\langle \phi | r^2 | \phi' \rangle_{\text{HO}}}
\]

can easily be understood. It depends only on radial integrals and hence only on the \( n, l, n' \) and \( l' \) quantum numbers of the valence wave functions. That is, \( \delta e_n^{\text{mier}(\theta)} \) is independent of any combination of the \( \pm \) signs for the orbitals with \( j = j' \pm \frac{1}{2} \) and \( j' = j' \pm \frac{1}{2} \). The functions \( k(r)^{\text{mier}(\theta)} \) and \( r^2 \) are similar inside the radius \( r \approx 1.1A^{1/3} \), but \( k(r)^{\text{mier}(\theta)} \) decrease while \( r^2 \) increases outside this radius (see fig. 1). Thus, if the radial wave functions of the valence particles lie mostly inside the nuclear surface, the effective charges are state independent, while if a substantial part of the radial wave function lies outside the nuclear surface, the effective charges are relatively reduced.

| Table 1 |

Radial integrals of \( r^2 \) and \( k(r)^{\text{mier}(\theta)} \) for harmonic oscillator wave functions and the E2 core-polarization charges for a delta-function interaction

<table>
<thead>
<tr>
<th>( A )</th>
<th>( nl )</th>
<th>( n'l' )</th>
<th>( \langle r^2 \rangle_{\text{HO}} \times 10^{-4} ) ( (\text{fm}^2) )</th>
<th>( \langle k(r)^{\text{mier}(\theta)} \rangle_{\text{HO}} \times 10^{-3} ) ( (\text{fm}^{-3}) )</th>
<th>( \delta e_n^{\text{mier}(\theta)} ) ( e )</th>
<th>( \delta e_p^{\text{mier}(\theta)} ) ( e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>1p</td>
<td>1p</td>
<td>7.44</td>
<td>77.8</td>
<td>0.533</td>
<td>0.177</td>
</tr>
<tr>
<td>17</td>
<td>1d</td>
<td>1d</td>
<td>10.62</td>
<td>75.1</td>
<td>0.374</td>
<td>0.120</td>
</tr>
<tr>
<td></td>
<td>1d</td>
<td>2s</td>
<td>9.60</td>
<td>45.1</td>
<td>0.248</td>
<td>0.080</td>
</tr>
<tr>
<td>40</td>
<td>1d</td>
<td>1d</td>
<td>13.53</td>
<td>85.6</td>
<td>0.573</td>
<td>0.183</td>
</tr>
<tr>
<td></td>
<td>1d</td>
<td>2s</td>
<td>12.22</td>
<td>68.7</td>
<td>0.509</td>
<td>0.163</td>
</tr>
<tr>
<td>41</td>
<td>1f</td>
<td>1f</td>
<td>17.53</td>
<td>87.6</td>
<td>0.452</td>
<td>0.145</td>
</tr>
<tr>
<td></td>
<td>1f</td>
<td>2p</td>
<td>14.58</td>
<td>49.4</td>
<td>0.307</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td>2p</td>
<td>2p</td>
<td>17.53</td>
<td>54.5</td>
<td>0.281</td>
<td>0.090</td>
</tr>
</tbody>
</table>

\( ^{a}) \langle f(r) \rangle = \langle n|f(r)|n'\rangle \).

\( ^{b}) b^2 = 1.18A^{1/3} \) for \( A = 16 \) and 17 and \( b^2 = 1.13A^{1/3} \) for \( A = 40 \) and 41.

\( ^{c}) k(r)^{\text{mier}(\theta)} \) is given by eq. (6).

\( ^{d}) \) First order core-polarization charge with the interaction \( V(r_{12}) = -500(1+0.12\sigma_1 \cdot \sigma_2) \delta(r_{12}) \) MeV \( \cdot \text{fm}^2 \) and \( \Delta E = 2\hbar \omega \).
The integrals \(\langle r^2 \rangle_{\text{HO}}\) and \(\langle k(r)^{\text{mier}(d)} \rangle_{\text{HO}}\) are given in table 1. The \(\delta e_{\text{M}}^{\text{mier}(d)}\) given in table 1 were calculated with the interaction strengths \(V = 500 \text{ MeV} \cdot \text{fm}^3\) and \(\alpha = 0.12\) which are typical values needed to fit empirical matrix elements within the (\(sd\))^7 model space. The ratio of the proton and neutron core-polarization charges is a constant \(\frac{1}{3}(1 - 3\alpha) = 0.32\).

Eq. (8) for \(\delta e_{\text{M}}^{\text{mier}(d)}\) is useful in understanding the core-polarization charges obtained from more realistic interactions. The results of previous calculations of \(\delta e_{\text{M}}^{\text{mier}}\) using the central finite-range Kallio-Koltviet interaction \(^9,^{10,17}\) show almost exactly the same orbital dependence as \(\delta e_{\text{M}}^{\text{mier}(d)}\) and the \(j\)-independence is still preserved. The effect of a central finite-range interaction is to introduce a more general radial dependence \(k(r)^{\text{mier}}\) obtained from a folding integral of \(k(r)^{\text{mier}(d)}\) with the radial dependence of the interaction. The width of the surfaced peaked function \(k(r)^{\text{mier}}\) is broadened relative to \(k(r)^{\text{mier}(d)}\) but the position of the maxima are nearly the same [see fig. 3 of ref. 5]). For a non-central interaction the situation is complicated even further by the introduction of a complete \(nij\) and \(n'lj'\) dependence for \(\delta Q_{\text{m}}^{(2)}\). However, the fact that the calculated values of \(\delta e_{\text{M}}^{\text{mier}}\) given by Kuo and Osnes \(^{14}\) using the Kuo-Brown effective interaction qualitatively follow the orbital dependence of eq. (8) indicates that the most important aspect of the state dependence is contained in the zero-range form \(k(r)^{\text{mier}(d)}\).

An important question is how the values of \(\delta e_{\text{M}}^{\text{mier}}\) are modified if, for example, Woods-Saxon (WS) wave functions are used instead of harmonic oscillator wave functions. A reasonable assumption is that the \(\Delta N = 2\) transition charge density is still proportional to \(r d\rho(r)/dr\) and that \(\rho(r)\) itself can be described by harmonic oscillator wave functions. The most important region for the integrals of \(k(r)^{\text{mier}(d)}\) is near the nuclear surface where \(\rho(r)_{\text{exp}}\) obtained from model independent analysis \(^{35}\) of electron scattering data is well described by \(\rho(r)_{\text{HO}}\).

The modification to \(\delta e_{\text{M}}^{\text{mier}}\) is then simply given by the ratios of radial integrals:

\[
\delta e_{\text{M}} = \frac{\langle \phi | r^2 | \phi' \rangle_{\text{HO}}}{\langle \phi | r^2 | \phi' \rangle_{\text{WS}}} \frac{\langle \phi | k(r)^{\text{mier}, l} | \phi' \rangle_{\text{WS}}}{\langle \phi | k(r)^{\text{mier}, l} | \phi' \rangle_{\text{HO}}} \delta e_{\text{M}}^{\text{mier}},
\]

where the label WS means that the valence particle wave functions have been calculated in a self-consistent way which can, for example, be approximated by using a Woods-Saxon potential. The quantity \(\delta e_{\text{M}}\) will be referred to as the binding-energy dependent core-polarization charge. This correction will be estimated by assuming \(k(r)^{\text{mier}} = k(r)^{\text{mier}(d)}\).

The parameters given in tables 2 and 3 for the Woods-Saxon potential are discussed in sect. 3. The ratios \(\langle r^2 \rangle_{\text{WS}}/\langle r^2 \rangle_{\text{HO}}\) and \(\langle k(r)^{\text{mier}(d)} \rangle_{\text{WS}}/\langle k(r)^{\text{mier}(d)} \rangle_{\text{HO}}\) are given in tables 4 and 5, respectively. For loosely bound valence particles the ratios \(\langle r^2 \rangle_{\text{WS}}/\langle r^2 \rangle_{\text{HO}}\) increase, however, the ratios \(\langle k(r)^{\text{mier}(d)} \rangle_{\text{WS}}/\langle k(r)^{\text{mier}(d)} \rangle_{\text{HO}}\) decrease which means that the value of the effective charge given by eq. (8) can be significantly reduced from \(\delta e_{\text{M}}\).

The form of the microscopic effective charge given by eq. (8) is similar to the result
Table 2
Harmonic oscillator and Woods-Saxon parameters

<table>
<thead>
<tr>
<th></th>
<th>(^{16}\text{O})</th>
<th>(^{40}\text{Ca})</th>
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<tbody>
<tr>
<td>(r_0) (fm)</td>
<td>1.324</td>
<td>1.329</td>
</tr>
<tr>
<td>(a) (fm)</td>
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<td>0.65</td>
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<tr>
<td>(\hbar\omega) (MeV)</td>
<td>14.0</td>
<td>10.7</td>
</tr>
<tr>
<td>(\langle r^3 \rangle^{1/3}) (fm)</td>
<td>2.72 (^{a}))</td>
<td>3.48 (^{a}))</td>
</tr>
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</table>

\(^{a}\) Ref. \(^{41}\).

Table 3
Proton and neutron single-particle energies for holes \(A = 16\) and \(40\), and particles, \(A = 17\) and \(41\) \(^{a}\))

<table>
<thead>
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<th>(nl)</th>
<th>(A = 16, 17)</th>
<th>(A = 40, 41)</th>
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<td></td>
<td>(e_n)</td>
<td>(e_p)</td>
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<tr>
<td>(1s_{\frac{1}{2}})</td>
<td>-44 (^{b}))</td>
<td>-21.81</td>
</tr>
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<td>(1p_{\frac{1}{2}})</td>
<td>-18.44</td>
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</tr>
<tr>
<td>(2s_{\frac{1}{2}})</td>
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<td>-18.11</td>
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<td>(1d_{\frac{3}{2}})</td>
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</table>

\(^{a}\) Experimental values from ref. \(^{42}\).

\(^{b}\) Theoretical values from Elton and Swift \(^{76}\).

Table 4
Radial integral of \(r^3\) for Woods-Saxon wave functions

<table>
<thead>
<tr>
<th>(A)</th>
<th>(nl)</th>
<th>(n'l'f')</th>
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<th>Neutron</th>
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<td></td>
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<td>(\langle r^3 \rangle_{\text{ws}}) (fm(^3))</td>
<td>(\langle r^3 \rangle_{\text{wo}})</td>
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<td>1.15</td>
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<tr>
<td></td>
<td>(1f_{\frac{3}{2}})</td>
<td>(2p_{\frac{1}{2}})</td>
<td>17.2</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>(1f_{\frac{3}{2}})</td>
<td>(1f_{\frac{3}{2}})</td>
<td>18.0</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>(2p_{\frac{3}{2}})</td>
<td>(2p_{\frac{3}{2}})</td>
<td>23.7</td>
<td>1.35</td>
</tr>
</tbody>
</table>
TABLE 5
Radial integrals of $k(r)_{\text{micr}(\delta)}$ for Woods-Saxon wave functions

<table>
<thead>
<tr>
<th>$A$</th>
<th>$nlJ$</th>
<th>$n'J'$</th>
<th>Proton $\langle k(r)<em>{\text{micr}(\delta)} \rangle</em>{ws}$ (fm$^{-3} \times 10^{-5}$)</th>
<th>$\langle k(r)<em>{\text{micr}(\delta)} \rangle</em>{\text{BO}}$</th>
<th>Neutron $\langle k(r)<em>{\text{micr}(\delta)} \rangle</em>{ws}$ (fm$^{-3} \times 10^{-5}$)</th>
<th>$\langle k(r)<em>{\text{micr}(\delta)} \rangle</em>{\text{BO}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>$1p_{\uparrow}$</td>
<td>$1p_{\uparrow}$</td>
<td>75.5</td>
<td>0.970</td>
<td>75.8</td>
<td>0.975</td>
</tr>
<tr>
<td>17</td>
<td>$1d_{\uparrow}$</td>
<td>$1d_{\uparrow}$</td>
<td>60.0</td>
<td>0.814</td>
<td>64.2</td>
<td>0.873</td>
</tr>
<tr>
<td></td>
<td>$2s_{\uparrow}$</td>
<td>$2s_{\uparrow}$</td>
<td>24.9</td>
<td>0.554</td>
<td>31.0</td>
<td>0.688</td>
</tr>
<tr>
<td>40</td>
<td>$1d_{\uparrow}$</td>
<td>$2s_{\uparrow}$</td>
<td>62.6</td>
<td>0.911</td>
<td>64.0</td>
<td>0.932</td>
</tr>
<tr>
<td></td>
<td>$1d_{\uparrow}$</td>
<td>$1d_{\uparrow}$</td>
<td>85.1</td>
<td>0.994</td>
<td>84.6</td>
<td>0.989</td>
</tr>
<tr>
<td>41</td>
<td>$1f_{\uparrow}$</td>
<td>$1f_{\uparrow}$</td>
<td>85.1</td>
<td>0.972</td>
<td>88.5</td>
<td>1.010</td>
</tr>
<tr>
<td></td>
<td>$2p_{\uparrow}$</td>
<td>$2p_{\uparrow}$</td>
<td>39.9</td>
<td>0.808</td>
<td>44.9</td>
<td>0.909</td>
</tr>
<tr>
<td></td>
<td>$2p_{\uparrow}$</td>
<td>$2p_{\uparrow}$</td>
<td>43.8</td>
<td>0.803</td>
<td>49.6</td>
<td>0.910</td>
</tr>
</tbody>
</table>

Fig. 1. Plots for $^{40}$Ca of (a) the harmonic oscillator ground-state charge distribution $\rho(r)$ and the Woods-Saxon radial dependence $f(r)$, (b) the microscopic and macroscopic transition densities, and (c) products of the valence particle or hole single-particle wave functions.
of the macroscopic particle-vibration coupling model where the radial dependence of $k(r)$, which originates from the coupling interaction is approximated by \(^{19}\)

$$k(r)^{\text{macro}} = r dV(r)/dr,$$

where $V(r)$ is the average shell-model potential.

In $^{16}\text{O}$ and $^{40}\text{Ca}$, when $V(r)$ and $\rho(r)$ are parameterized by the Fermi distribution shape [eq. (25)], about a 20\% larger value of $r_o$ is needed for $V(r)$ than for $\rho(r)$. Thus, since $k(r)^{\text{macro}}$ has a maximum at about 0.7 fm larger radius than the maximum of $k(r)^{\text{micro(3)}}$ derived from the density distribution (see fig. 1) the state dependence of the effective charge for the $1d_\downarrow$-$2s_\uparrow$ and $1d_\uparrow$-$1d_\downarrow$ matrix elements using $k(r)^{\text{macro}}$ is about 50\% less than the microscopic result. It is interesting that the macroscopic state dependence is similar to the results of microscopic calculations with density dependent interactions \(^{17}\).

The difference between the microscopic and macroscopic transition density has also recently been discussed for $^{208}\text{Pb}$ [ref. 36]), where the difference between $V(r)$ and $\rho(r)$ is less. For nuclei around $^{208}\text{Pb}$ Astner et al. \(^{37}\) found that the experimental state dependence of the effective charges were approximately accounted for by assuming $\delta e_n \propto \langle \phi | k(r)^{\text{macro}} | \phi' \rangle_{\text{ws}} \langle \phi | r^2 | \phi' \rangle^{-1}_{\text{ws}}$.

In the harmonic oscillator macroscopic model a state independent core-polarization charge for nuclei with $N \approx Z$ has been estimated \(^{19-21}\) as

$$\delta e_n^0 = 0.50 + 0.32(2t_s).$$

Since a state independent effective charge can be obtained by using the harmonic oscillator potential $V(r)_{\text{HO}} = \frac{1}{2}m\omega^2r^2$ for the macroscopic transition density:

$$\delta e_n^0 = F(t_s) \frac{\langle \phi | r dV(r)_{\text{HO}} | \phi' \rangle}{\langle \phi | r^2 | \phi' \rangle} = F(t_s)m\omega^2,$$

\(^{11}\)

### Table 6

State dependence of the core-polarization charges in the macroscopic model with $k(r)^{\text{macro}} = r dV(r)/dr$

<table>
<thead>
<tr>
<th>$A$</th>
<th>$nlj$</th>
<th>$n'l'j'$</th>
<th>$\langle k(r)^{\text{macro}} \rangle_{\text{HO}}$</th>
<th>$\langle \langle k(r)^{\text{macro}} \rangle_{\text{HO}} \rangle_{\text{HO}}$</th>
<th>Proton $\langle k(r)^{\text{macro}} \rangle_{\text{ws}}$</th>
<th>Neutron $\langle k(r)^{\text{macro}} \rangle_{\text{ws}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>1p_\uparrow</td>
<td>1p_\uparrow</td>
<td>0.988</td>
<td>0.920</td>
<td>0.938</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>1d_\uparrow</td>
<td>1d_\uparrow</td>
<td>0.897</td>
<td>0.597</td>
<td>0.693</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1d_\uparrow</td>
<td>2s_\uparrow</td>
<td>0.843</td>
<td>0.356</td>
<td>0.499</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>1d_\uparrow</td>
<td>2s_\uparrow</td>
<td>1.207</td>
<td>1.188</td>
<td>1.207</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1d_\uparrow</td>
<td>1d_\uparrow</td>
<td>1.112</td>
<td>1.069</td>
<td>1.070</td>
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</tr>
<tr>
<td>41</td>
<td>1f_\uparrow</td>
<td>1f_\uparrow</td>
<td>1.115</td>
<td>1.034</td>
<td>1.098</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1f_\uparrow</td>
<td>2p_\uparrow</td>
<td>1.131</td>
<td>0.891</td>
<td>1.034</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2p_\uparrow</td>
<td>2p_\uparrow</td>
<td>0.970</td>
<td>0.635</td>
<td>0.792</td>
<td></td>
</tr>
</tbody>
</table>
which defines $F(t_s)$, the state dependent macroscopic effective charge is given by

$$
\delta e_{\text{macr}} = \frac{1}{m\omega^2} \left\langle \phi | k(r) \delta e_{\text{macr}} | \phi' \right\rangle_{\text{HO}} \delta e_{\text{macr}}^0,
$$

where $k(r) \delta e_{\text{macr}} = V_0 r d f(r)/dr$, and $f(r)$ is given by eq. (25). The binding energy dependence is obtained by replacing $\delta e_{\text{macr}}$ and $k(r) \delta e_{\text{macr}}$ in eq. (9) with $\delta e_{\text{macr}}$ and $k(r) \delta e_{\text{macr}}$, respectively. The result is

$$
\delta e_{\text{macr}} = \frac{1}{m\omega^2} \left\langle \phi | k(r) \delta e_{\text{macr}} | \phi' \right\rangle_{\text{WS}} \delta e_{\text{macr}}^0.
$$

For the present calculation, $V_0 = 50$ MeV was used for $k(r) \delta e_{\text{macr}}$. The calculated values of the ratios used in eqs. (13) and (14) are given in table 6.

2.2. RELATION WITH GIANT QUADRUPOLE STATES

Due to the residual interaction, the energies of the coherent $1p-1h$ $2^+$ states are lowered for the $T = 0$ combination and raised for the $T = 1$ combination. Since these particular states contain most of the sum rule limit the radial dependence of the transition density can still be approximated by eq. (7). However, the shift in energy can change the core-polarization charges.

For the $N \approx Z$ nuclei, the isoscalar effective charge $\delta e_0 = \frac{1}{2}(\delta e_n + \delta e_p)$ and isovector effective charge $\delta e_1 = \frac{1}{2}(\delta e_n - \delta e_p)$ are directly related to the $T = 0$ and $T = 1$ states, respectively. In the macroscopic model in which the particles couple to vibrations of the core, the effective charge $\delta e_T$ is proportional to the transition strength to the ground state divided by the excitation energy of the collective $1p-1h$ $2^+$ with isospin $T$. In general if the $2^+$ state is fragmented over several energies then

$$
\delta e_T \propto W_T \equiv \sum \frac{B(E2; 2^+_i \rightarrow 0^+ \text{g.s.})}{E(2^+_i)}
$$

[The dependence on the energy of the low lying $\gamma$-transitions has been neglected since in most cases $E_\gamma \lesssim 1 \text{ MeV} \ll E(2^+_i) \approx 2 \hbar \omega$.]

The macroscopic state independent core-polarization charge given by eq. (11) has been obtained by calculating the energy of the $T = 0$ and $1$ states in the RPA approximation and assuming that the single collective states each contain half of the classical energy weighted sum rule for $N = Z$ nuclei. This sum rule is given as

$$
S_T = \frac{1}{2} S_{\text{class}} \equiv \frac{1}{2} \sum_i E(2^+_i) B(E2; 2^+_i \rightarrow \text{g.s.}) = \frac{5Z}{8\pi m} \frac{\hbar^2}{e^2} \langle r^2 \rangle,
$$

where $\langle r^2 \rangle$ is the mean-square charge radius and where $S_T = 187 \text{ MeV \cdot W.u.}$ for $^{16}\text{O}$ and $S_T = 232 \text{ MeV \cdot W.u.}$ for $^{40}\text{Ca}$ (W.u. is the Weisskopf unit; 1 W.u. = 2.40 $e^2 \cdot \text{fm}^4$ for $^{16}\text{O}$ and 1 W.u. = 8.13 $e^2 \cdot \text{fm}^4$ for $^{40}\text{Ca}$). With the RPA solutions of
$E_{T=0} = \sqrt{2} \hbar \omega$ and $E_{T=1} = 3.3 \hbar \omega$ given by Hamamoto\(^{19}\) the state independent core-polarization charge in terms of $W_T = S_T/E_T^2$ is

\[
\begin{align*}
\delta e_0^0 &= 1.05 W_0^\text{macr} \\
\delta e_1^0 &= 3.6 W_1^\text{macr}
\end{align*}
\]

for $^{16}$O,

\[
\begin{align*}
\delta e_0^0 &= 0.49 W_0^\text{macr} \\
\delta e_1^0 &= 1.7 W_1^\text{macr}
\end{align*}
\]

for $^{40}$Ca,

(17)

where $W_T$ is in units of W.u./MeV. Substituting these relations into eq. (14) the state and binding energy dependent core-polarization charges for some cases of interest are obtained:

\[
\begin{align*}
\delta e_n &= 0.73 W_0^\text{macr} + 2.49 W_1^\text{macr} \\
\delta e_p &= 0.63 W_0^\text{macr} - 2.15 W_1^\text{macr} \\
\delta e_n &= 0.52 W_0^\text{macr} + 1.80 W_1^\text{macr} \\
\delta e_p &= 0.37 W_0^\text{macr} - 1.28 W_1^\text{macr} \\
\delta e_n &= 0.54 W_0^\text{macr} + 1.87 W_1^\text{macr} \\
\delta e_p &= 0.51 W_0^\text{macr} - 1.76 W_1^\text{macr}
\end{align*}
\]

(18)

In order to obtain analogous relations between the core-polarization charge and $W_T$ in the microscopic theory, we consider the five sets of quantities $\delta e$, $E(2^+_f)$ and $B(E2; 2^+_r \rightarrow g.s.)$ given by the five approximations used by Kuo and Osnes\(^{14}\). Treating $\delta e$ and $W_T^\text{macr}$ as numerical parameters the five approximations can be contained within the modified linear relation:

\[
\delta e_T^\text{macr} = G_T W_T^\text{macr} + H_T,
\]

(19)

where $G_T$ and $H_T$ are orbital dependent constants. For some cases of interest:

\[
\begin{align*}
\delta e_0^\text{macr} &= 1.09 W_0^\text{macr} - 0.03 \\
\delta e_1^\text{macr} &= 0.50 W_1^\text{macr} - 0.01 \\
\delta e_n &= 0.84 W_0^\text{macr} \\
\delta e_p &= 0.30 W_1^\text{macr} \\
\delta e_0^\text{macr} &= 0.73 W_0^\text{macr} - 0.07 \\
\delta e_1^\text{macr} &= 0.33 W_1^\text{macr} - 0.01
\end{align*}
\]

(20)

Including the binding energy corrections given by eq. (9):

\[
\begin{align*}
\delta e_n &= 0.77 W_0^\text{macr} + 0.35 W_1^\text{macr} - 0.03 \\
\delta e_p &= 0.63 W_0^\text{macr} - 0.29 W_1^\text{macr} - 0.01 \\
\delta e_n &= 0.40 W_0^\text{macr} + 0.14 W_1^\text{macr} \\
\delta e_p &= 0.26 W_0^\text{macr} - 0.09 W_1^\text{macr} \\
\delta e_n &= 0.72 W_0^\text{macr} + 0.32 W_1^\text{macr} - 0.08 \\
\delta e_p &= 0.66 W_0^\text{macr} - 0.30 W_1^\text{macr} - 0.05
\end{align*}
\]

(21)
The results given by eqs. (18) and (21) are useful in relating the observed core-polarization charges directly with the properties of the giant quadrupole states. The values of the isoscalar coupling constant $G_0$ are very similar in the microscopic and macroscopic theory, but the isovector coupling constant is much larger in the macroscopic theory than in the microscopic theory. This difference is closely related to the strength of the isovector exchange in the two-body interactions. In the macroscopic model the isovector core-polarization charge is obtained from the charge dependent part of the static nuclear potential. It is interesting but perhaps not surprising that the microscopic core-polarization charges obtained with the Skyrme interaction $\{VBIII' of ref. 17\}$ are nearly identical to the macroscopic result, since the parameters of the Skyrme interaction were obtained by fitting the ground-state properties of nuclei. The microscopic results of Kuo and Osnes which are adopted for comparison here are based in contrast on a basic nucleon-nucleon interaction which apparently has a relatively smaller isovector component.

2.3. SYSTEMS WITH MORE THAN ONE VALENCE NUCLEON

The discussions in subsects. 2.1 and 2.2 which follow from eqs. (1)–(3) hold exactly only for one particle or hole outside the closed core. In some cases it is necessary to take the empirical effective charges from nuclei with more than one valence nucleon. For these cases eq. (3) must be modified by including sums over state-dependent one-body and two-body operators:

$$\delta e_{is} = \frac{\langle \phi | \sum r \delta Q_r^{(2)}(i) + \sum \delta Q_r^{(2)}(i, j) | \phi' \rangle}{\langle \phi | Q_r^{(2)} | \phi' \rangle}. \quad (22)$$

The two-body operator, which is necessary in order to take into account the Pauli principle for more than one valence nucleon, is small compared to the one-body operator because in perturbation theory it involves several terms which tend to cancel. The two-body effective charge has been discussed by Harvey and Khanna for $A = 18$ and by Dieperink and Brussaard for $^{40}K$.

The experimental core-polarization charges are extracted assuming only a one-body state-independent operator, and they are examined for differences which could be attributed to the two-body effective operator. Since a state-independent operator is assumed in some cases where there are several orbital components involved due to configuration mixing, the extracted polarization charge reflects only the dominant component in these cases.

One of the most difficult problems for several valence nucleons is how to treat the "single-particle" binding energy. In this paper we make the approximation that the effective single-particle binding energy for the component of a specific orbital in a multiparticle state is

$$e'_j = e_j + \langle \phi_j | V | \phi_j \rangle \! / \! n, \quad (23)$$

where $e_j$ is the single-particle energy in the one particle system and $\langle \phi_j | V | \phi_j \rangle$ is the
where 

effective interaction energy for the \( n \)-particle system in the state \( J \). The behavior of the ratios \( \langle r^2 \rangle_{WS}/\langle r^2 \rangle_{BO} \) and \( \langle k(r)_{\text{mier}}(\theta) \rangle_{WS}/\langle k(r)_{\text{mier}}(\theta) \rangle_{BO} \) for the \( 1d_{\frac{3}{2}} \)-\( 1d_{\frac{3}{2}} \) and \( 1d_{\frac{3}{2}} \)-\( 2s_{\frac{1}{2}} \) matrix elements as a function of \( \Delta s = e_i - e_j \) is shown in table 7. Values of \( \Delta s \) are estimated empirically from \( \Delta s = (B.E.(8+Z, 8+N) - B.E.(16\text{O}) - N_{2n}(1d_{\frac{3}{2}}) - Z_{p}(1d_{\frac{3}{2}}))/\text{(N+Z)} \). Typical values of \( \Delta s \) are \( \Delta s(1\text{8O} 0^+) = 1.9 \text{ MeV}, \Delta s(1\text{8O} 2^+) = 0.9 \text{ MeV}, \Delta s(1\text{8O} 4^+) = 0.1 \text{ MeV}, \Delta s(1\text{9F} 4^+) = 3.7 \text{ MeV}, \) and \( \Delta s(2\text{0Ne} 2^+) = 6.0 \text{ MeV}. \)

### 3. Radial matrix elements

The Woods-Saxon \( ^{40} \) wave functions for the valence particles were calculated for an average field which has the form

\[
V(r)_{\text{WS}} = V_0 f(r) + V_u \frac{1}{r} \frac{df(r)}{dr} - \sigma + V(r)_{\text{Coul}},
\]

where

\[
f(r) = 1 \left[ 1 + \exp \left( \frac{r - R}{a} \right) \right], \quad R = r_0(A - 1)^{\frac{1}{3}},
\]

\[
V(r)_{\text{Coul}} = \begin{cases} \frac{Ze^2}{2R_0} \left[ 3 - \left( \frac{r}{R_0} \right)^2 \right], & r \leq R_0 \\ \frac{Ze^2}{r}, & r \geq R_0, \end{cases}
\]

with \( R_0 = 3.48 \text{ fm for } 16\text{O} \) and \( R_0 = 4.48 \text{ fm for } 40\text{Ca} \).

A conventional value \( a = 0.65 \text{ fm} \) of the diffuseness parameter was assumed and \( r_0 \) was then chosen to reproduce the rms charge radii obtained for \( 16\text{O} \) and \( 40\text{Ca} \) in...
electron scattering measurements $^{41}$)

$$\langle r^2 \rangle_{\exp}^{\pm} = \left[ \sum_{\text{core protons}} \langle j| r^2 |j\rangle_{\text{WS}}/Z + \langle r^2 \rangle_{\text{proton}} \right]^{\pm},$$

where the finite size of the proton, $\langle r^2 \rangle_{\text{proton}} = 0.8$ fm [ref. $^{41}$]. A summary of the WS parameters are given in table 2. The radial wave functions for the valence particles were then calculated numerically; $V_0$ was chosen to reproduce the experimental single-particle binding energy for each orbital considered ($V_0 \approx 50$ MeV and $V_\text{as} \approx 15$ MeV). The value of $A$ was taken to be 16 (40) for hole orbitals and 17 (41) for particle orbitals near $^{16}$O ($^{40}$Ca). The adopted single-particle binding energies taken from ref. $^{42}$] are given in table 3. The radial wave functions for unbound orbitals were approximated by assuming a 0.1 MeV binding energy. The harmonic oscillator parameters needed to reproduce the rms radii are $\hbar \omega = 14.0$ MeV for $^{16}$O and $\hbar \omega = 10.7$ MeV for $^{40}$Ca.

There are at least two small effects which have not been included in the above determination of the Woods-Saxon and harmonic oscillator parameters. The c.m. correction should be included for the matrix elements of $E_0 = r^2$ which determine the rms charge radii $^2$, but, as shown by Fallieros and Ferrell $^3$, there are no equivalent corrections for the $Q^{(2)}$ matrix elements. The harmonic oscillator rms radii should be modified to

$$\left[ \sum_{\text{core protons}} \langle j| r^2 |j\rangle_{\text{HO}}/Z + \langle r^2 \rangle_{\text{proton}} - \frac{3}{2\hbar A} \right]^{\pm}. $$

This leads to a 3.8% decrease in $\hbar \omega$ for $^{16}$O and a 1.2% decrease in $\hbar \omega$ for $^{40}$Ca. Also the ground states of $^{16}$O and $^{40}$Ca have been assumed to be closed shells. Calculations for the core excitations indicate that the ground state of $^{16}$O includes about 27% 2p-2h and 4% 4p-4h. Using harmonic oscillator wave functions this leads to a 1.9% increase in the sum over core protons and hence a 1.9% increase in $\hbar \omega$ for $^{16}$O. Using the same wave functions for $^{40}$Ca leads to a 0.5% increase in $\hbar \omega$. In total these corrections lead to a decrease of 1.9% and 0.7% in $\hbar \omega$ for $^{16}$O and $^{40}$Ca, respectively. These changes are small compared to the other theoretical and experimental errors in the present paper [see below and eqs. (30)] and hence they have been neglected. Their major effect would be to reduce the empirical proton core-polarization charges in table 10 by 0.019e and 0.007e for $^{16}$O and $^{40}$Ca, respectively.

Clearly, the values chosen for the parameters $a$ and $r_0$ are not unique. Thus it is important to determine the sensitivity of various matrix elements to changes in these parameters. In the extreme case of tightly bound particles when the harmonic oscillator and Woods-Saxon matrix elements are the same, the requirement of reproducing the rms radius is enough to fix the individual valence particle matrix elements. However, the loosely bound 2$s_\pm$ and 1$d_\pm$ single-particle wave functions near $^{16}$O are more sensitive to changes in $a$ and $r_0$. For small changes from our parameters,
$\Delta r_0 = r_0 - 1.324$ and $\Delta a = a - 0.65$, the requirement to reproduce rms radius implies $\Delta r_0 = -0.32\Delta a$. The dependence of the individual $\langle r^2 \rangle_w$ matrix elements on $\Delta r_0$ and $\Delta a$ can thus be reduced to a dependence on $\Delta a$:

**neutron:**
\[
\langle 1d_{\frac{5}{2}} | r^2 | 1d_{\frac{3}{2}} \rangle_w = 13.2 + 4.1\Delta a,
\]
\[
\langle 1d_{\frac{3}{2}} | r^2 | 2s_{\frac{1}{2}} \rangle_w = 14.0 + 6.8\Delta a,
\]

**proton:**
\[
\langle 1d_{\frac{5}{2}} | r^2 | 1d_{\frac{3}{2}} \rangle_w = 15.0 + 6.3\Delta a,
\]
\[
\langle 1d_{\frac{3}{2}} | r^2 | 2s_{\frac{1}{2}} \rangle_w = 17.5 + 10.1\Delta a.
\]

These relations are used to relate the uncertainty in the diffuseness parameter to uncertainties in the extracted effective charges.

4. Model-space wave functions

The $B(E2)$ values and quadrupole moments for the states of interest near $^{16}O$ and $^{40}Ca$ are obtained using wave functions derived by diagonalizing an effective one-plus-two-body Hamiltonian. These wave functions, or in some cases the $E2$ matrix elements themselves, are taken from previous calculations. In the simplest model space considered it is assumed that $^{16}O$ and $^{40}Ca$ form inert cores. These will be referred to as zero-hole (0h) wave functions. More extended wave functions, which include some part of what is often referred to as the deformed states, involve the excitation of two particles from the closed core (2h wave functions). The $E2$ matrix elements for the wave functions which include 2h components are given by

\[
\langle 0h+2h|E2|0h'+2h \rangle = \sqrt{1-\alpha^2} \sqrt{1-\alpha'^2} \langle 0h|E2|0h' \rangle + \alpha\alpha' \langle 2h|E2|2h' \rangle. \quad (27)
\]

For the diagonal case it is clear that the $E2$ matrix elements will be enhanced by the addition of the 2h components only if $\langle 2h|E2|2h \rangle > \langle 0h|E2|0h \rangle$. This inequality depends upon the mass number and $T'$, that is, whether or not $\langle 0h|E2|0h \rangle$ is dominated by protons or neutrons.

The 0h wave functions for $A = 15, 17, 39$ and 41 are simply the single-particle wave functions. The 0h wave functions for the $A = 18$ and 42 nuclei were obtained by diagonalizing the Kuo-Brown effective interaction $^{31, 32}$ in the $(2s-1d)^2$ and $(1f-2p)^2$ model spaces, respectively. These wave functions are given by Kuo and Brown $^{31, 32}$. The $(2s-1d)^{-1}(1f-2p)^1(T = 1)$ wave functions for $^{40}K$ were calculated by Jackson et al. $^{43}$ again using the Kuo-Brown interaction. The $(1p)^{-1}(2s-1d)^1(T = 1)$ wave functions for $^{16}N$ were calculated by Gillet et al. $^{44}$ using a different effective interaction, however, in the case of interest ($J' \to J = 0^- \to 2^-$) the $E2$ matrix element is dominated by one component.

The coefficients $A$ and $B$ for the theoretical $E2$ matrix elements written in the form $Ae_p + Be_n$ are given in tables 8 and 9 for cases of interest in nuclei near $^{16}O$ and $^{40}Ca$, respectively.
<table>
<thead>
<tr>
<th>Nucleus</th>
<th>No.</th>
<th>Q-moment</th>
<th>( Q_{\exp} ) ( (e \cdot fm^2) )</th>
<th>Ref.</th>
<th>( Q_{\exp} ) ( (e \cdot fm^2) )</th>
<th>( \sqrt{B(E2)}_{\exp} ) ( (e^2 \cdot fm^4) )</th>
<th>( Q_{\exp}, \sqrt{B(E2)}<em>{\exp} = A</em>{\exp} + B_{\exp} )</th>
<th>0h wave functions *</th>
<th>0h+2h wave functions *</th>
<th>% 0h component</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{18}\text{N})</td>
<td>1</td>
<td>( ^{3}<em>{-} \rightarrow ^{1}</em>{-} )</td>
<td>7.1±0.9</td>
<td></td>
<td>2.7±0.2</td>
<td></td>
<td>2.10</td>
<td>2.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^{16}\text{N})</td>
<td>2</td>
<td>( 0^{-} \rightarrow 2^{-} )</td>
<td>4.22±0.10</td>
<td>(*)</td>
<td>2.05±0.024</td>
<td></td>
<td>0.08</td>
<td>4.45</td>
<td>0.08</td>
<td>6.50</td>
</tr>
<tr>
<td>(^{17}\text{O})</td>
<td>3</td>
<td>( Q(4^{+}) )</td>
<td>-2.56±0.05</td>
<td>(*)</td>
<td>-2.56±0.05</td>
<td></td>
<td>-6.20</td>
<td>-7.69</td>
<td>-0.17</td>
<td>-5.84</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>( \frac{1}{2}^{+} \rightarrow \frac{1}{2}^{+} )</td>
<td>6.30±0.06</td>
<td>(*)</td>
<td>2.51±0.012</td>
<td></td>
<td>4.69</td>
<td>6.85</td>
<td>0.40</td>
<td>4.45</td>
</tr>
<tr>
<td>(^{17}\text{F})</td>
<td>5</td>
<td>( Q(4^{+}) )</td>
<td>10±2</td>
<td>(*)</td>
<td>10±2</td>
<td></td>
<td>-6.20</td>
<td>-8.74</td>
<td>-5.84</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>( \frac{1}{2}^{+} \rightarrow \frac{1}{2}^{+} )</td>
<td>66.4±1.4</td>
<td>(*)</td>
<td>8.15±0.09</td>
<td></td>
<td>4.69</td>
<td>8.49</td>
<td>4.45</td>
<td>0.40</td>
</tr>
<tr>
<td>(^{18}\text{O})</td>
<td>7</td>
<td>( 4^{+} \rightarrow 2^{+} )</td>
<td>3.42±0.18</td>
<td>(*)</td>
<td>1.85±0.05</td>
<td></td>
<td>3.66</td>
<td>5.05</td>
<td>0.17</td>
<td>3.62</td>
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<tr>
<td></td>
<td>8</td>
<td>( 2^{+} \rightarrow 0^{+} )</td>
<td>9.6±0.4</td>
<td>(*)</td>
<td>3.10±0.07</td>
<td></td>
<td>3.98</td>
<td>5.12</td>
<td>0.29</td>
<td>3.97</td>
</tr>
<tr>
<td>(^{18}\text{F})</td>
<td>9</td>
<td>( Q(5^{+}) )</td>
<td>13±2</td>
<td>(*)</td>
<td>13±2</td>
<td></td>
<td>-6.20</td>
<td>-6.20</td>
<td>-7.58</td>
<td>-7.08</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>( 5^{+} \rightarrow 3^{+} )</td>
<td>17.4±0.8</td>
<td>(*)</td>
<td>4.17±0.09</td>
<td></td>
<td>2.18</td>
<td>2.18</td>
<td>2.96</td>
<td>2.67</td>
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<tr>
<td></td>
<td>11</td>
<td>( 3^{+} \rightarrow 1^{+} )</td>
<td>16.7±0.6</td>
<td>(*)</td>
<td>4.90±0.07</td>
<td></td>
<td>2.27</td>
<td>2.27</td>
<td>3.19</td>
<td>3.00</td>
</tr>
<tr>
<td>(^{18}\text{Ne})</td>
<td>12</td>
<td>( 4^{+} \rightarrow 2^{+} )</td>
<td>25±4</td>
<td>(*)</td>
<td>5.0±0.4</td>
<td></td>
<td>3.66</td>
<td>5.71</td>
<td>3.62</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>( 2^{+} \rightarrow 0^{+} )</td>
<td>70±17</td>
<td>(*)</td>
<td>8.4±1.0</td>
<td></td>
<td>3.98</td>
<td>5.64</td>
<td>3.97</td>
<td>0.29</td>
</tr>
</tbody>
</table>

\* The 0h \( B(E2) \) values for \( A = 15 \) and 17 were calculated from single-particle wave functions, for \( A = 16 \), from the wave functions of Gillet and Jenkins \(^{44}\) (case \( f_b \)), and for \( A = 18 \) from the wave functions of Kuo and Brown \(^{31}\). The harmonic oscillator \( 0h+2h \) \( B(E2) \) values are given by Ellis and Engeland \(^{39}\) for the pairs of mirror nuclei from which the coefficients \( A \) and \( B \) can be deduced. The \( E2 \) matrix elements of Ellis and Engeland have been increased by the factor \( b^2 \) (present)/\( b^2 \) (Ellis) = 1.073 where \( b^2 = \hbar/mc^2 \).

\* Ref. \(^{77}\).

\* Ref. \(^{45}\).

\* Ref. \(^{63,50}\), see text sect. 5.

\* Ref. \(^{55,56}\), see text sect. 5.

\* Ref. \(^{77}\), see text sect. 5.

\* Ref. \(^{79}\).

\* Ref. \(^{80}\).

\* Ref. \(^{81}\).


**Table 9**

Experimental and theoretical B(E2) values and quadrupole moments for nuclei near $^{40}$Ca

<table>
<thead>
<tr>
<th>Nucleus No.</th>
<th>Q-moment</th>
<th>Ref.</th>
<th>$Q_{exp}$ (e$^2$ fm$^2$)</th>
<th>$Q_{exp}$ (e$^2$ fm$^2$)</th>
<th>$Q_{exp}$, $\sqrt{B(E2)_{exp}}$ (e$^2$ fm$^2$)</th>
<th>$Q_{lab}, \sqrt{B(E2)<em>{lab}} = A</em>{52} - B_{52}$</th>
<th>0h wave functions *)</th>
<th>0h+2h wave functions *)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E2 transition</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>14</td>
<td>$\frac{1}{2}^+$ $\rightarrow$ $\frac{3}{2}^+$</td>
<td>29 ± 6</td>
<td>5.4 ± 0.6</td>
<td>4.87</td>
<td>4.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{40}$K</td>
<td>15</td>
<td>Q($\frac{1}{2}^+$)</td>
<td>5.2 ± 0.2</td>
<td>5.2 ± 0.2</td>
<td>5.41</td>
<td>5.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{40}$K</td>
<td>16</td>
<td>$\frac{1}{2}^+$ $\rightarrow$ $\frac{3}{2}^+$</td>
<td>90 ± 60</td>
<td>9.5 ± 4.0</td>
<td>4.87</td>
<td>4.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{41}$Ca</td>
<td>17</td>
<td>Q(4+)</td>
<td>−6.7 ± 0.8</td>
<td>−6.7 ± 0.8</td>
<td>−3.75</td>
<td>−8.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{41}$Ca</td>
<td>18</td>
<td>$\frac{1}{2}^-$ $\rightarrow$ $\frac{3}{2}^-$</td>
<td>50 ± 25</td>
<td>7 ± 2</td>
<td>6.60</td>
<td>6.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{42}$Sc</td>
<td>19</td>
<td>$\frac{1}{2}^-$ $\rightarrow$ $\frac{3}{2}^-$</td>
<td>≥110</td>
<td>≥10.5</td>
<td>6.60</td>
<td>7.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{42}$Ca</td>
<td>20</td>
<td>6$^+$ $\rightarrow$ 4$^+$</td>
<td>6.44 ± 0.19</td>
<td>2.53 ± 0.04</td>
<td>4.11</td>
<td>4.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{42}$Ca</td>
<td>21</td>
<td>4$^+$ $\rightarrow$ 2$^+$</td>
<td>65 ± 7</td>
<td>8.1 ± 0.4</td>
<td>5.78</td>
<td>5.78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{42}$Ca</td>
<td>22</td>
<td>2$^+$ $\rightarrow$ 0$^+$</td>
<td>81.5 ± 3.0</td>
<td>9.03 ± 0.17</td>
<td>5.58</td>
<td>5.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{43}$Sc</td>
<td>23</td>
<td>5$^+$ $\rightarrow$ 7$^+$</td>
<td>25.1 ± 2.3</td>
<td>3.34</td>
<td>3.34</td>
<td>3.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{43}$Sc</td>
<td>24</td>
<td>3$^+$ $\rightarrow$ 1$^+$</td>
<td>34.6 ± 6.3</td>
<td>3.18</td>
<td>3.18</td>
<td>3.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{43}$Sc</td>
<td>25</td>
<td>2$^+$ $\rightarrow$ 0$^+$</td>
<td>6.3 ± 4.3</td>
<td>3.22</td>
<td>3.22</td>
<td>3.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{43}$Ti</td>
<td>26</td>
<td>6$^+$ $\rightarrow$ 4$^+$</td>
<td>76.3 ± 2.3</td>
<td>3.39</td>
<td>3.39</td>
<td>3.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{43}$Ti</td>
<td>27</td>
<td>4$^+$ $\rightarrow$ 2$^+$</td>
<td>&lt;230</td>
<td>&lt;15</td>
<td>6.35</td>
<td>6.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{43}$Ti</td>
<td>28</td>
<td>2$^+$ $\rightarrow$ 0$^+$</td>
<td>134 ± 3.9</td>
<td>5.58</td>
<td>5.58</td>
<td>5.58</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*) The 0h $B(E2)$ values for $A = 39$ and 41 were calculated from single-particle wave functions, for $A = 40$ from the wave functions of Jackson et al. 43) and for $A = 42$ from the wave functions of Kuo and Brown 44).

b) The harmonic oscillator 0h+2h $B(E2)$ values for $A = 42$ are given by Flowers and Skouras 39) (method B) for several values of $\delta$ from which the coefficients $A$ and $B$ can be deduced. The E2 matrix elements of Flowers and Skouras have been increased by the factor $b^2$(present)/$b^2$(Flowers) = 1.049 where $b^2 = \hbar^2/m$. The 0h+2h $Q$-moment of $^{39}$K was calculated only with (1f$^2$)(2d$^2$)−2 2h components.

a) Refs. 61, 62, see text sect. 5.

b) Ref. 58, see text sect. 5.

a) Refs. 48, 59, 60, see text sect. 5.

b) Ref. 53, see text sect. 5.

b) Refs. 62, 64, see text sect. 5.
respectively, using the 0h wave functions. The results are given using the harmonic oscillator and Woods-Saxon radial matrix elements discussed in the last section. With harmonic oscillator wave functions the coefficients $A$ and $B$ are symmetric for pairs of mirror nuclei and $A = (-1)^{A-Z}$ for nuclei with $N = Z$.

Extended 0h+2h wave functions have been calculated for $A = 17$ and 18 by Ellis and Engeland (29) and for $A = 42$ by Flowers and Skouras (30). These wave functions are complicated. However, the E2 matrix elements for the states of interest using harmonic oscillator radial matrix elements can be obtained directly from the respective papers. The 0h parts of the wave functions are consistent since again the Kuo-Brown interaction was used. The coefficients $A$ and $B$ using the present harmonic oscillator parameters are given in tables 8 and 9. The product $\sqrt{1-x^2}\sqrt{1-y^2}$ which indicates the amount of 0h component in the E2 matrix element of eq. (27) is also given in these tables. The coefficients $A$ and $B$ using Woods-Saxon wave functions can easily be estimated since the form of the 0h part of the wave function is not significantly changed by the addition of 2h components. Thus

$$\langle 0h+2h||E2||0h'+2h'\rangle_{WS} \\
\approx \langle 0h+2h||E2||0h'+2h'\rangle_{HO} + \sqrt{1-x^2}\sqrt{1-y^2}[\langle 0h||E2||0h'\rangle_{WS} - \langle 0h||E2||0h'\rangle_{HO}] + ax'[\langle 2h||E2||2h'\rangle_{WS} - \langle 2h||E2||2h'\rangle_{HO}].$$

The last term proportional to the small factor $ax'$ is neglected and the 0h+2h WS matrix elements are obtained from the terms which contain previously calculated quantities.

5. Experimental data

Some of the experimental data concerning the quadrupole moments and $B(E2)$ values for nuclei near $^{16}O$ and $^{40}Ca$ are well established in the compilations (45-48) but in the last few years much new and more accurate data have become available. In this section the data will be briefly discussed with an emphasis on the uncertainties which are still present and the discrepancies which exist in the literature.

The quadrupole moments of $^{17}O$, $^{39}K$ and $^{40}K$ are deduced from hyperfine splitting measurements and thus depend on an accurate knowledge of the atomic wave functions which must include the atomic polarization effect (49, 50). Using the experimental results of Harvey (51), Schaefer et al. (49) deduce $Q(17O) = -2.562 \, e \cdot fm^2$ using atomic wave functions which include a shielding effect due to the atomic polarization. The uncertainty in the shielding correction is believed to be 2% or less (49). This more accurate value of $^{17}O$ $Q$-moment is consistent with the value $Q = -2.65 \pm 0.3 \, e \cdot fm^2$ quoted previously (46). The hyperfine splitting for $^{39}K$ and $^{40}K$ has been measured by Ney et al. (52, 53). Including the antishielding corrections of
Sternheimer et al.\textsuperscript{50}) the deduced quadrupole moments are $Q(^{39}\text{K}) = 5.2 \pm 0.2 e \cdot \text{fm}^2$ and $Q(^{40}\text{K}) = -6.7 \pm 0.8 e \cdot \text{fm}^2$. These errors include an estimated 3\% uncertainty\textsuperscript{50} in the Sternheimer antishielding correction. This value of the $^{39}\text{K}$ moment is different from the value in the 1965 compilation of Lindgren\textsuperscript{54}), $Q(^{39}\text{K}) = 9 \pm 2 e \cdot \text{fm}^2$, which is commonly quoted in the literature.

The ratios of the quadrupole moments of $^{17}\text{F}(\frac{1}{2}^+)$, $^{18}\text{F}(5^+)$, $^{19}\text{F}(\frac{3}{2}^+)$ and $^{20}\text{F}(2^+)$ have recently been reported\textsuperscript{33}). The magnitudes $|Q(^{17}\text{F} \frac{1}{2}^+)| = 10 \pm 2 e \cdot \text{fm}^2$ and $|Q(^{18}\text{F} 5^+)| = 13 \pm 3 e \cdot \text{fm}^2$ are deduced using a previous measurement\textsuperscript{56}) of $|Q(^{19}\text{F} \frac{3}{2}^+)| = 12 \pm 2 e \cdot \text{fm}^2$.

Two different values for the mean lifetime, $\tau = 24.8 \pm 1.3$ ps [ref.\textsuperscript{57})] and $\tau = 46 \pm 3$ ps [ref.\textsuperscript{58})], have been reported for the $^{18}\text{O}$ 4\textsuperscript{+} state. Both lifetimes were measured with the recoil-distance method but using different reactions. The former value, $\tau = 24.8 \pm 1.3$ ps, leads to a $B(E2)$ value [$B(E2) = 816/E_\gamma^2(\text{MeV} \cdot \text{ps})$] which is consistent with theoretical expectations (see discussion below). The latter value, $\tau = 46 \pm 3$ ps, leads to a $B(E2)$ value which implies quite different neutron effective charges in $^{17}\text{O}$ and $^{18}\text{O}$ which could not easily be explained theoretically. For the discussion below the larger value of the lifetime will be assumed to be in error; a definitive experimental answer on this point would be important.

For the $^{39}\text{K} \frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ transition a $B(E2) = 115 \pm 40 e^2 \cdot \text{fm}^4$ is deduced from the average mean lifetime of $\tau = 70 \pm 25$ fs [ref.\textsuperscript{48})] for the $\frac{1}{2}^+$ state together with the fact that the M1 component of this transition has been measured to be small in (e, e') experiments\textsuperscript{59, 60}). However, a different value of $B(E2) = 44 \pm 10 e^2 \cdot \text{fm}^4$ was deduced directly from an (e, e') experiment which was also consistent with a resonance fluorescence measurement of the total width [$\Gamma(\frac{1}{2}^+) = 4.6 \pm 1.3$ meV]\textsuperscript{59}). For the present comparison we adopt a value of $B(E2; ^{39}\text{K} \frac{1}{2}^+ \rightarrow \frac{3}{2}^+) = 90 \pm 60 e^2 \cdot \text{fm}^4$. If the M1 component of the $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ transition in $^{39}\text{K}$ is small it is reasonable, to assume that this is also the case in $^{39}\text{Ca}$. Assuming a pure E2 transition a value of $B(E2) = 29 \pm 6 e^2 \cdot \text{fm}^4$ is obtained from the average of two mean lifetime measurements\textsuperscript{61, 62}) for the $^{39}\text{Ca} \frac{1}{2}^+$ level, $\tau = 310 \pm 60$ fs.

The most recent measurement\textsuperscript{63}) of the $^{41}\text{Ca} \frac{3}{2}^-$ mean lifetime of $\tau = 800 \pm 70$ fs is in disagreement with previous measurements which averaged\textsuperscript{64}) $\tau = 450 \pm 120$ fs. We adopt an average value of $\tau = 600 \pm 300$ fs which gives $B(E2; ^{41}\text{Ca} \frac{3}{2}^- \rightarrow \frac{1}{2}^-) = 50 \pm 25 e^2 \cdot \text{fm}^4$. Only an upper limit has been obtained for the lifetime of the $^{41}\text{Sc} \frac{3}{2}^-$ level\textsuperscript{48}).

Finally, two different values have been reported for the mean lifetime at the $^{42}\text{Ti} 6^+$ state, $\tau = 26 \pm 5$ ns [ref.\textsuperscript{65}] and $\tau = 5 \pm 2$ ns [ref.\textsuperscript{66}). As discussed in ref.\textsuperscript{66}) the longer lifetime leads to a $B(E2)$ value which implies a much smaller proton effective charge than can be expected theoretically. A remeasurement\textsuperscript{67}) of the $^{42}\text{Ti} 6^+$ lifetime with the same reaction as used in ref.\textsuperscript{66}) yielded a more accurate result of $\tau = 4.5 \pm 0.5$ ns which supports the short lifetime, however, an independent measurement is needed. This result yields $B(E2; ^{42}\text{Ti} 6^+ \rightarrow 4^+) = 28 \pm 3 e^2 \cdot \text{fm}^4$. The $B(E2)$ values for other $A = 42$ nuclei have been summarized previously\textsuperscript{66, 68}).
6. Experimental effective charges

The experimental values of the quadrupole moments, \( Q_{\text{exp}} \), and the reduced transition probabilities, \( B(E2)_{\exp} \), for the cases of interest are given in tables 8 and 9 for nuclei near \(^{16}\text{O}\) and \(^{40}\text{Ca}\), respectively.

The theoretical values of \( Q \) and \( B(E2) \) for the 0\( h \) and 0\( h+2h \) model-space wave functions discussed in sect. 4 are also given in tables 8 and 9. The theoretical values are parameterized by the neutron and proton effective charges in the form \( A e_p + B e_n \).

By equating these theoretical values with the measured value, \( C \pm 4C \), one obtains the relation:

\[
\frac{B}{C} = -\frac{e_p}{e_n} \frac{A}{C} + \frac{1}{e_n}
\]

If \( e_p \) and \( e_n \) are state-independent quantities, then the points \((A/C, B/C)\) plotted on an \((x, y)\) graph should form a straight line which crosses the \( x \)-axis at \( 1/e_p \) and crosses the \( y \)-axis at \( 1/e_n \).

The points \((A/C, B/C)\) are plotted for nuclei near \(^{16}\text{O}\) in fig. 2 using \( A \) and \( B \) calculated with harmonic oscillator 0\( h \) wave functions, Woods-Saxon 0\( h \) wave functions,

![Fig. 2. Plots for nuclei near \(^{16}\text{O}\) of the quantities \( A/C \) versus \( B/C \) with \( A \) and \( B \) determined from: (a) 0\( h \) harmonic oscillator wave functions (b) 0\( h \) Woods-Saxon wave functions and (c) 0\( h+2h \) Woods-Saxon wave functions. The quantities \( A, B \) and \( C \) are given in table 8. The numbers labeling \((A/C, B/C)\) refer to the labels in table 8 and the line represents the effective charges needed for the \( A = 17 \frac{1}{2}^+ \rightarrow \frac{1}{2}^+ \) transitions (4 and 6). The neutron and proton effective charges are the values of \( 1/y \) and \( 1/x \), respectively, where the line crosses the axis.](image-url)
### Table 10
Experimental and theoretical proton effective charges

<table>
<thead>
<tr>
<th>A</th>
<th>No. *)</th>
<th>nU</th>
<th>n'U'</th>
<th>$\delta e_{p\text{\thinspace eff}}^{\text{exp}}$</th>
<th>$\delta e_{p\text{\thinspace eff}}^{\text{th}}$ without binding energy correction</th>
<th>$\delta e_{p\text{\thinspace eff}}^{\text{th}}$ with binding energy correction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0h WS wave functions</td>
<td>0h+2h WS wave functions</td>
<td>1st order RPA (bare)</td>
</tr>
<tr>
<td>15</td>
<td>(1)</td>
<td>1p₂</td>
<td>1p₂</td>
<td>0.25±0.10</td>
<td>0.188 b) 0.731 b) ≈0.37 i) ≈0.21 i) 0.18</td>
<td>0.177 0.690 ≈0.35 ≈0.20 0.17</td>
</tr>
<tr>
<td>17</td>
<td>(4, 6)</td>
<td>1d₂</td>
<td>2s₂</td>
<td>−0.040±0.011 (±0.06)</td>
<td>0.070±0.014 (±0.06)</td>
<td>0.119 0.394 0.222 0.130 0.15</td>
</tr>
<tr>
<td>17</td>
<td>(3, 5)</td>
<td>1d₂</td>
<td>1d₂</td>
<td>0.14±0.23 (±0.04)</td>
<td>0.24±0.27 (±0.04)</td>
<td>0.103 0.477 0.245 0.128 0.16</td>
</tr>
<tr>
<td>18</td>
<td>(7, 11)</td>
<td>1d₂</td>
<td>1d₂</td>
<td>−0.07±0.03 (±0.04)</td>
<td>−0.06±0.03 s) (±0.04)</td>
<td>0.253 b) 3.784 b) ≈0.49 i) ≈0.39 i) 0.22</td>
</tr>
<tr>
<td>39</td>
<td>(16)</td>
<td>2s₂</td>
<td>1d₂</td>
<td>1.1±0.9</td>
<td>−0.03±0.04</td>
<td>−0.02±0.05</td>
</tr>
<tr>
<td>41</td>
<td>(19)</td>
<td>2p₂</td>
<td>1f₂</td>
<td>≥0.4</td>
<td>0.147 1.269 0.265 0.223 0.20</td>
<td>0.102 0.884 0.185 0.156 0.16</td>
</tr>
<tr>
<td>42</td>
<td>(20, 23)</td>
<td>1f₂</td>
<td>1f₂</td>
<td>0.08±0.05</td>
<td>0.06±0.07 s)</td>
<td>0.155 2.171 0.366 0.278 0.20</td>
</tr>
</tbody>
</table>

*) This number refers to the specific transitions considered from tables 8 and 9.

b) Except where indicated these values are taken from Kuo and Osnes 14).

i) Eq. (13).

s) Eq. (9).

*) Eq. (14).

i) Uncertainty due to the radial integrals, assuming a variation in the diffuseness parameter of $\Delta u = ±0.10$ fm, see text, sect. 6.

s) These underlined values indicate the experimental effective charges which are believed to be closest to the $\Delta N = 2$ core-polarization charge.

Ref. 9).

i) Estimated from the preceding two columns and eq. (19).
<table>
<thead>
<tr>
<th>A No.</th>
<th>n/</th>
<th>n'/?</th>
<th>$\delta_{e_{\text{exp}}}$</th>
<th>$\delta_{e_{\text{th}}}$ without binding energy correction</th>
<th>$\delta_{e_{\text{th}}}$ with binding energy correction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>0h WS wave functions</td>
<td>0h+2h WS wave functions</td>
<td>1st order RPA (bare)</td>
</tr>
<tr>
<td>15</td>
<td>1p½</td>
<td>1p½</td>
<td></td>
<td></td>
<td>0.483 0.954 0.63 0.40 0.81 0.472 0.933 0.62 0.39 0.78</td>
</tr>
<tr>
<td>16 (2)</td>
<td>1d 1a</td>
<td>2a</td>
<td>0.301±0.004 (±0.02)</td>
<td></td>
<td>0.269 0.517 0.357 0.230 0.69 0.127 0.244 0.168 0.108 0.41</td>
</tr>
<tr>
<td>17 (4, 6)</td>
<td>1d</td>
<td>2a</td>
<td>0.366±0.002 (±0.02) 0.337±0.002 (±0.02)</td>
<td></td>
<td>0.329 0.655 0.443 0.272 0.74 0.232 0.461 0.312 0.192 0.57</td>
</tr>
<tr>
<td>17 (3, 5)</td>
<td>1d</td>
<td>1d</td>
<td>0.333±0.007 (±0.01) 0.330±0.009 (±0.01)</td>
<td></td>
<td>0.367±0.010 0.342±0.010</td>
</tr>
<tr>
<td>18 (7, 11, 12)</td>
<td>1d</td>
<td>1d</td>
<td></td>
<td></td>
<td>0.697 4.109 0.87 0.73 0.99 0.730 3.256 0.91 0.76 0.99</td>
</tr>
<tr>
<td>39 (14)</td>
<td>1d</td>
<td>2a</td>
<td>1.25±0.14</td>
<td></td>
<td>0.782 5.398 0.98 0.81 0.91 0.823 5.679 1.03 0.98 0.88</td>
</tr>
<tr>
<td>41 (18)</td>
<td>1f</td>
<td>2p½</td>
<td>1.0±0.3</td>
<td></td>
<td>0.381 1.446 0.468 0.407 0.93 0.337 1.280 0.414 0.360 0.85</td>
</tr>
<tr>
<td>42 (20, 23, 24, 26)</td>
<td>1f</td>
<td>1f</td>
<td>0.612±0.010 0.396±0.012</td>
<td></td>
<td>0.499 2.416 0.656 0.533 0.91 0.509 2.465 0.669 0.544 0.90</td>
</tr>
</tbody>
</table>

*See footnotes to table 10.

a) Ref. [18].
and Woods-Saxon 0h +2h wave functions in parts a, b and c, respectively. The error bars on the points (A/C, B/C) represent the experimental error ± ΔC.

The lines in fig. 2 indicate the effective charges for the A = 17 ½ → ½ transitions. The results using the 0h and 0h +2h wave functions are qualitatively the same. For the transitions ¹⁷O ½ → ½, ¹⁸F 5 → 3 and ¹⁷F ½ → ½ which primarily involve the 2s₁⁻¹d₁ matrix element consistent effective charges are obtained especially with the 0h +2h wave functions: eₙ = 0.34 and eₚ = 1.07.

For transitions which primarily involve only the 1d₁ orbital, consistent effective charges are obtained in some cases; i.e. for ¹⁷O 3)), ¹⁸O 4 → 2, ¹⁸F 3 → 1 and ¹⁸Ne 4 → 2. However, the enhancement of the A = 18 2 → 0 transitions cannot be explained using the same effective charges even with the 0h +2h wave functions considered here. Even though it is very difficult to provide a microscopic description of these 2 → 0 transitions, the other 2d₁ E2 transitions and quadrupole moments seem to be adequately described. The ¹⁸F 5+ quadrupole moment is not in as good agreement but this may be due to the large experimental error. Using the 0h +2h wave functions, for the A = 17 1d₁ matrix elements we obtain, eₙ = 0.33 and eₚ = 1.24 (±0.27); and for A = 18, eₙ = 0.34 and eₚ = 0.94.

The results for the empirical effective charges for nuclei near ¹⁶O are summarized terms of the polarization charges δeₙ = eₙ - 1 and δeₚ = eₚ in tables 10 and 11, respectively. It is interesting that with the Woods-Saxon 0h +2h wave functions the neutron polarization charge, δeₙ = 0.34, to within 5% is both state and number independent.

The proton polarization charge is zero to within about ±0.1e. This result is quite different from the analysis with harmonic oscillator wave functions which gives δeₚ ≈ 0.5 (see fig. 2a). In a similar analysis with Woods-Saxon wave functions, Harvey and Khanna obtained a large value for the proton polarization charge. However, their ratios <1d₁ | r² | nlj>ₚ <1d₁ | r² | nlj>ₚ were assumed to be the same for protons and neutrons and are much different for the proton than the present value of the ratios given in table 4.

The errors indicated in tables 10 and 11 for the experimental polarization charges are due only to the experimental errors in C which are in some cases very small. Errors in the theoretical matrix elements should also be considered due to uncertainties in the radial wave functions, the effective interaction, and the extent to which the core deformed components have been adequately described. Errors due to the radial wave functions can be estimated from eqs. (26). In the worst case, an uncertainty for the diffuseness parameter of Δa = ±0.10 fm for the <1d₁ | r² | 2s₁> proton matrix element leads to an uncertainty in the effective proton charge of ±0.06e. Overall this leads to uncertainties in δeₚ of ±(0.04e - 0.06e) and in δeₙ of ±(0.01e - 0.02e); these are in most cases larger than the experimental errors.

The points (A/C, B/C) are plotted for nuclei near ⁴⁰Ca in fig. 3, and the extracted polarization charges are summarized in tables 10 and 11. In this case the experimental information is not as accurate, especially for the A = 41 nuclei. The best experi-
Fig. 3. Plots for nuclei near \( ^{40}\text{Ca} \) of the quantities \( A/C \) versus \( B/C \) with \( A \) and \( B \) determined from (a) Oh Woods-Saxon wave functions and (b) Oh+2h Woods-Saxon wave functions. The quantities \( A, B \) and \( C \) are given in table 9. The numbers labeling \( (A/C, B/C) \) refer to the labels in table 9 and the line represents the effective charges needed for the best fit to the \( A = 42 \) \( 6^+ \to 4^+, 5^+ \to 7^+ \) and \( 3^+ \to 1^+ \) transitions (20, 23, 24 and 26). The neutron and proton effective charges are given by the values of \( 1/y \) and \( 1/x \), respectively, where the line crosses the axes.

The neutron and proton effective charges needed for the \( ^{39}\text{K} \) quadrupole moment using Oh wave functions is \( e_p = 0.97 \); this value is changed insignificantly when Oh+2h wave functions are used.

Due to the larger angular momentum and relatively higher Coulomb barrier for the valence protons, the difference between Woods-Saxon and harmonic oscillator wave functions is not as large for nuclei near \( ^{40}\text{Ca} \) as it was for \( ^{16}\text{O} \). However the extracted values of the effective charges for the \( A = 42 \) system are very sensitive to the amplitude of the 2h or deformed component. This is especially clear in the larger enhancement of the \( 2 \to 0 \) transitions for \( A = 42 \) compared to \( A = 18 \). For the
polarization charge analysis the most important question is the amount of 2h component in the $A = 42$ $6 \rightarrow 4$ transitions. For these transitions the 2h component of the 0h+2h wave functions may have been overestimated by Flowers and Skouras [30] in order to achieve better overall agreement for all transitions.

The E2 transitions in $^{42}$Ca for both the ground-state band and excited deformed band have been analyzed in terms of 0h and deformed components by Towsley et al. [28]. With their coexistence model, there is enough experimental information for the $J = 0^+, 2^+$ and $4^+$ states to extract a constant effective charge, $\alpha$ and $\langle 2h|E2|2h' \rangle$ for each spin (see eq. (27)). They obtain $\alpha = 0.646 \pm 0.039$ using harmonic oscillator wave functions with $b^2 = 3.46$ fm$^2$. Using the present Woods-Saxon wave functions this becomes $\alpha = 0.57 \pm 0.03$ which is closer to the value of $\alpha$ extracted from the $6^+ \rightarrow 4^+$ transition in $^{42}$Ca using the 0h wave functions ($\alpha = 0.62$) than to the value extracted using the 0h+2h wave functions ($\alpha = 0.40$). Thus the $6^+ \rightarrow 4^+$ transition in $^{42}$Ca appears to have a smaller 2h component than is given by the Flowers and Skouras 0h+2h wave functions.

Until the microscopic wave functions in the $^{40}$Ca region are better understood theoretically the most reliable neutron polarization charge is then $\delta e_n = 0.57 \pm 0.03$ for the 1f$^+_4$ orbital. Using the 0h+2h wave functions small proton polarization charges are obtained; $\delta e_p = -0.02 \pm 0.05$ for 1d$^+_1$ and $\delta e_p = 0.06 \pm 0.07$ for 1f$^+_4$.

7. Discussion

The empirical E2 core-polarization charges for various valence orbitals near $^{16}$O and $^{40}$Ca are summarized in tables 10 and 11. The most striking features are the very small proton polarization charge, $\delta e_p^{exp} \approx 0$, for both $^{16}$O and $^{40}$Ca and the orbital and number independent neutron polarization charge, $\delta e_n^{exp} \approx 0.34$, for $^{16}$O. The neutron polarization charge is larger for $^{40}$Ca, $\delta e_n^{exp} \approx 0.57$, than for $^{16}$O.

In this section we discuss the comparison of these results with the present theoretical understanding of the nuclear polarization phenomenon. In the first section, the experimental results for the core-polarization charges are compared with the results of microscopic and macroscopic calculations including the binding-energy corrections discussed in subsect. 2.1. In subsect. 7.2 a model dependent analysis of the core-polarization charges is made based on the discussion in subsect. 2.2 to extract information concerning the isoscalar and isovector quadrupole vibrations of $^{16}$O and $^{40}$Ca. Finally in subsect. 7.3 the effective charge for the entire region $16 < A < 46$ is discussed.

Unfortunately it is very difficult to obtain unambiguous experimental information on the $\Delta N = 2$ two-body effective operator defined by eq. (22). The largest two-body effect would be from the net contribution of blocking of the $N-2 \rightarrow N$ excitations and enhancement of the $N \rightarrow N+2$ excitations as a function of the number of protons in the model space $N$. The experimental data for the proton rich nuclei is especially inadequate for this comparison. Due to the contribution of both blocking and en-
TABLE 12
Properties of the $\Delta N = 2$ giant quadrupole states in $^{16}$O and $^{40}$Ca

<table>
<thead>
<tr>
<th></th>
<th>Microscopic theory *)</th>
<th></th>
<th>Macroscopic theory *)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st order RPA (bare)</td>
<td>RPA (screened)</td>
<td>RPA (screened, emp. s.p.)</td>
<td>Experiment microscopic analysis of $\delta E^{exp}$ *)</td>
<td>Experiment macroscopic analysis of $\delta E^{exp}$ *)</td>
</tr>
<tr>
<td>$^{16}$O $T = 0$</td>
<td>$E_0$ *)</td>
<td>28.0</td>
<td>$\approx 14.5$</td>
<td>$\approx 20.0$</td>
<td>$\approx 31$</td>
</tr>
<tr>
<td></td>
<td>$S_0$ *)</td>
<td>184</td>
<td>130</td>
<td>148</td>
<td>225</td>
</tr>
<tr>
<td></td>
<td>$W_0$ *)</td>
<td>0.234</td>
<td>0.538</td>
<td>0.350</td>
<td>0.215</td>
</tr>
<tr>
<td>$^{16}$O $T = 1$</td>
<td>$E_1$</td>
<td>28.0</td>
<td>$\approx 34$</td>
<td>$\approx 33$</td>
<td>39-49</td>
</tr>
<tr>
<td></td>
<td>$S_1$</td>
<td>184</td>
<td>211</td>
<td>223</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>$W_1$</td>
<td>0.234</td>
<td>0.189</td>
<td>0.208</td>
<td>0.153</td>
</tr>
<tr>
<td>$^{40}$Ca $T = 0$</td>
<td>$E_0$</td>
<td>21.0</td>
<td>$\approx 6.8$</td>
<td>$\approx 15.5$</td>
<td>$\approx 18.7$</td>
</tr>
<tr>
<td></td>
<td>$S_0$</td>
<td>240</td>
<td>176</td>
<td>207</td>
<td>234</td>
</tr>
<tr>
<td></td>
<td>$W_0$</td>
<td>0.545</td>
<td>2.911</td>
<td>0.783</td>
<td>0.640</td>
</tr>
<tr>
<td>$^{40}$Ca $T = 1$</td>
<td>$E_1$</td>
<td>21.0</td>
<td>$\approx 25$</td>
<td>$\approx 26$</td>
<td>$\approx 29$</td>
</tr>
<tr>
<td></td>
<td>$S_1$</td>
<td>240</td>
<td>277</td>
<td>314</td>
<td>345</td>
</tr>
<tr>
<td></td>
<td>$W_1$</td>
<td>0.545</td>
<td>0.403</td>
<td>0.465</td>
<td>0.412</td>
</tr>
</tbody>
</table>

*) Taken from Kuo and Omes 14).

b) From ref. 19) where $E_0 = \sqrt[3]{2k_0}$, $E_1 = 3.3k_0$, $S_0 = S_1 = \frac{1}{2}S_{\text{class}}$ (eq. (16)) and $W_T = S_T/E_T^2$.

c) $W_T$ is obtained from eqs. (21) and (30) and $E_T$ is obtained assuming $E_T = (S_T/W_T)^4$.

d) $W_T$ is obtained from eqs. (18) and (30) and $E_T$ is obtained assuming $E_T = (S_T/W_T)^4$.

e) Energy in MeV. The notation $\approx$ indicates the approximate centroid energy of a multiplet.

f) $S_T = \Sigma_i[B(E2; 2^+T \rightarrow 0^+) g.s.]/E_i (W.u./MeV)$.

$W_T = \Sigma_i[B(E2; 2^+T \rightarrow 0 g.s.)]/E_i (W.u./MeV)$. 
hancement effects the two-body effective charge is relatively small and will be neglected for the discussion below.

7.1. COMPARISON OF EXPERIMENTAL AND THEORETICAL CORE-POLARIZATION CHARGES

The results of various theoretical calculations of the core-polarization charges are given on the right side of tables 10 and 11. The first four columns are based on the microscopic calculations of Kuo and Osnes, assuming harmonic oscillator radial wave functions. The results in the first three columns were obtained using the Kuo-Brown effective interaction and an unperturbed single-particle energy difference of $2\hbar\omega$ for: (i) the first order approximation ($1p-1h$ states unperturbed at $2\hbar\omega$), (ii) the RPA approximation, and (iii) the RPA approximation including the self-screening of the $1p-1h$ interaction. The fourth column gives the screened RPA result with "empirical" single-particle energies. The values in the first four columns are taken from ref. 14) for the particle orbitals. The first two columns for the hole orbitals are taken from ref. 9) and the third and fourth columns are estimated from these results using eq. (19) and $W^\text{mle}$ from table 12. The fifth column gives the macroscopic estimate of the polarization charge based on eqs. (11) and (13).

The next five columns give the binding energy dependent core-polarization charge which is obtained by replacing the harmonic oscillator valence particle wave functions with Woods-Saxon wave functions, eqs. (9) and (14).

The empirical state independent neutron charge for the $1d_\frac{5}{2}-1d_\frac{1}{2}$ and $1d_\frac{5}{2}-2s_\frac{1}{2}$ matrix elements cannot be explained by the theoretical core-polarization charges. With and without the binding energy correction a difference between these matrix elements of about 50\% and 20\%, respectively, is expected theoretically. This problem most likely indicates an inadequacy in the $0h+2h$ model space wave functions for the $A = 17$ nuclei. In particular the $4h$ components probably contribute due to the strong four-particle correlation which is known to be important for $^{16}O$. On the other hand, the $4h$ components may not be so important for the $A = 18$ high-spin wave functions since much of the four-particle correlation is already included in the $2h$ ($2h-4p$) wave functions. Thus, the empirical state independence of the neutron effective charge is probably fortuitous. If the deformed components can be better accounted for in future calculations it would be interesting to reanalyze the state dependence of the effective charge.

The best quantitative comparison with the calculated core-polarization charges should thus be obtained with the $A = 18$ and $A = 42$ high-spin results:

\begin{align}
\delta\epsilon_a^{\text{exp}} &= 0.34 \pm 0.02 \quad A = 18, \\
\delta\epsilon_p^{\text{exp}} &= -0.06 \pm 0.05 \quad 1d_\frac{5}{2}-1d_\frac{1}{2}, \\
\delta\epsilon_a^{\text{exp}} &= 0.57 \pm 0.03 \quad A = 42, \\
\delta\epsilon_p^{\text{exp}} &= 0.06 \pm 0.07 \quad 1f_{\frac{3}{2}}-1f_{\frac{5}{2}},
\end{align}
where the error for $A = 18$ includes an uncertainty of 0.1 fm for the diffuseness parameter. The effective single-particle binding energy in both cases is nearly the same as in the one particle systems ($\Delta s \approx 0$) since the interaction energy is small for the ($T = 1$) higher spin states.

The most detailed theoretical results are given by the last three columns in tables 10 and 11. These all give about equally good qualitative agreement with the empirical values, however, quantitatively none give perfect agreement. The quantitative disagreements are best discussed in terms of the isoscalar and isovector core-polarization charges.

7.2. PROPERTIES OF THE GQR IN $^{16}$O AND $^{40}$Ca

A fruitful way to proceed is to extract the empirical inverse energy weighted sum, $W_{T}^{pol}$, given by eqs. (18) and (21) [note that eq. (18) is valid for all of the binding-energy-corrected microscopic approximations given in tables 10 and 11]. The sum $W_{T}^{pol}$ can then be compared directly with the theoretical calculations of this quantity as well as direct experimental information concerning the GQR in $^{16}$O and $^{40}$Ca. For example if one assumes that the GQR are described by single states which exhaust the classical energy weighted sum rule, $W_{T}^{pol}$ immediately gives the excitation energy $E_T = (S_T/W_{T}^{pol})^2$. In relationship to states in $^{16}$O and $^{40}$Ca it should be emphasized that $W_0^{pol}$ does not include contributions from the low-lying 2p-2h and 4p-4h 2$^+$ states since these are considered part of the model space.

The results of this analysis are given in table 12 along with the theoretically calculated values for $E_s, S$ and $W$. First for the $T = 0$ GQR in $^{16}$O, the microscopic and macroscopic analysis for $W_{T}^{pol}$ give the same results; $W_{0}^{pol} \approx 0.20$ W.u./MeV and $E_0 \approx 31$ MeV assuming $E_T = (\frac{1}{2}S_{class}/W_{T}^{pol})^2$. This energy is in agreement with the results of the microscopic screened-RPA calculation with empirical single-particle energies, but is 10 MeV higher than the macroscopic estimate of $\sqrt{2\hbar\omega} = 19.8$ MeV. The $T = 0$ E2 strength which has been observed in $(\alpha, \alpha')$ [ref. 24]) and (e, e') [ref. 26]) experiments is all below 30 MeV; this discrepancy would be accounted for if a large fraction of the E2 strength lies unobserved above 30 MeV. The E2 strength in $^{16}$O is observed to be more spread out (below 30 MeV) than is observed for the $T = 0$ GQR in heavier nuclei.

The results for $W_{T=1}^{pol}$ for $^{16}$O are very different in the microscopic and macroscopic analysis due to the difference in the isovector coupling strength discussed in subsect. 2.2. The best consistency is found between the macroscopic analysis for $W_{1}^{pol}$ and the macroscopic calculation. These give an excitation energy of approximately 46 MeV for the $T = 1$ GQR and support the need for a large isovector component in the effective interaction. Unfortunately no direct experimental evidence has been reported for the $T = 1$ GQR.

Very similar conclusions are obtained for the analysis of the GQR in $^{40}$Ca. The sum $W_0^{pol}$ is in agreement with the microscopic screened-RPA calculation with empirical single-particle energies but the energy is 5 MeV higher than the macroscopic
estimate of $\sqrt{2}\hbar \omega = 15.1$ MeV. The $(\alpha, \alpha')$ experimental excitation energy is $E_T = 0 = 17.9 \pm 3.5$ MeV [ref. 23]. From the macroscopic analysis for $W_{\alpha}^{\alpha}$, the isovector GQR is predicted to lie at about 41 MeV in $^{40}$Ca.

It has been suggested 69) that the macroscopic estimate of the isoscalar GQR can be generalized to include finite range and non-local effects in the single-particle potential by introducing an effective mass $m^*/m$ which modifies the energy to $E(T = 0) = \sqrt{2}\hbar \omega (m/m^*)^4$. The discrepancy between the energies obtained from the core-polarization charges and the simple macroscopic estimate of $E(T = 0) = \sqrt{2}\hbar \omega$ corresponds to $m^*/m \approx 0.41$ for $^{16}$O and $m^*/m \approx 0.57$ for $^{40}$Ca. These values are much less than the value $m^*/m \approx 1$ suggested for heavy nuclei from the fact that the energy of the GQR are consistent with the simple macroscopic estimate of $\sqrt{2}\hbar \omega$ and that they exhaust the energy-weighted sum rule 70). However, the $T = 0$ GQR, at least in $^{16}$O, does not seem to be concentrated in one state. A recent analysis of $\alpha$ inelastic scattering on $^{16}$O [ref. 71)] shows there are at least five states below 14 MeV which exhaust about 13% of the sum rule and about eleven states in the GQR region from 18–25 MeV which exhaust only about 23% of the sum rule. The states from 18–25 MeV give $W_0 \approx 0.096$ and the remaining inverse-energy weighted strength could be accounted for by a state at $\approx$ 34 MeV which exhausts about 64% of the sum rule. With this situation it is difficult to interpret the effective mass.

7.3. EFFECTIVE CHARGES IN THE REGION 16 < A < 56

The core-polarization charge has been discussed for the systems with only one or two nucleons in the model space not only because these are the simplest to describe but also because these are most sensitive to the isovector effective charge. In nuclei with three or more nucleons in the model space which have ground states with isospin $T < T_{\text{max}} = \frac{1}{2}I$, the low-lying E2 transitions are dominated by the isoscalar component. The adequacy of $\Delta N = 0$ and 1 wave functions for the sd shell nuclei can be tested by comparing the needed isoscalar effective charge with the isoscalar core-polarization charge.

Since most shell-model calculations are carried out with harmonic oscillator wave functions, it is convenient to define a new quantity $e^\text{eff}_{\alpha}$:

$$\langle r^2 \rangle_{\text{HO}} e^\text{eff}_{\alpha} = \langle r^2 \rangle_{\text{WS}} [(\frac{1}{2} - t_s)e + \delta e_{\alpha}].$$

Using the macroscopic model to relate $\delta e_{\alpha}$ with the experimental quantities for the $A = 18$ and 42 systems (eqs. (30)) we obtain

$$e^\text{eff}_{\alpha} = \frac{\langle r^2 \rangle_{\text{WS}}}{\langle r^2 \rangle_{\text{HO}}} \left[ (\frac{1}{2} - t_s)e + \frac{k(r)_{\text{WS}}}{m_\omega^2 \langle r^2 \rangle_{\text{WS}}} \delta e_{\alpha}^0 \right],$$

where

$$\delta e_{\alpha}^0 \approx \begin{cases} 0.20 + 0.62 t_s & \text{for } ^{16}\text{O} \\ 0.29 + 0.48 t_s & \text{for } ^{40}\text{Ca}. \end{cases}$$

(32)
In the case when \( \langle r^2 \rangle_{\text{WB}} \approx \langle r^2 \rangle_{\text{HO}} \), which may be realized when there are about four or more particles in the sd shell, for the \( \langle 1d||E2||1d \rangle \) matrix element, \( \delta e_{\text{eff}}^0 = 0.18 \) and \( \delta e_{\text{eff}}^1 = 0.28 \) for particles near \(^{16}\text{O}\), and \( \delta e_{\text{eff}}^0 = 0.32 \) and \( \delta e_{\text{eff}}^1 = 0.27 \) for holes near \(^{40}\text{Ca}\). Thus the isoscalar effective charge is predicted to increase as \( A \) increases in the sd shell whereas the isovector effective charge is constant.

Many (sd)^n calculations have previously been carried out for 20 < \( A < 38 \); the comparison with experiment requires typically \( \delta e_{\text{eff}} = 0.5-0.6 \) [refs. 72-74], which is about twice as large as the core-polarization estimate. This discrepancy may indicate the inadequacy of the model-space wave functions within the sd shell due to the effective interaction or truncation effects, or it may indicate that \( \Delta N = 1 \) contributions, \( (p)^{n-1}(sd)^{n+1} \) and \( (sd)^{n-1}(fp)^n \), are more important than is generally considered.

Finally we briefly discuss the effective charge for the fp shell nuclei. Because of the large number of configurations in the (fp)^n model space with \( n > 5 \) it is prohibitive to carry out exact calculations. However, for the 1f_2 shell nuclei the energy levels especially for the yrast bands can be well described by simple (1f_2)^n wave functions with an effective interacting taken from the two-particle spectra of \(^{42}\text{Sc}\), \(^{48}\text{Sc}\) and \(^{54}\text{Co}\) [ref. 75]). With these wave functions the effective charges for the transitions between states with \( T < T_{\text{max}} = \frac{1}{2}n \) are in general very much larger than the \( A = 42 \) polarization charges, for example, for the \( N = 28 \) nuclei \( e_p \approx 1.9 \) [ref. 68]). Most of this large effective charge must then be due to a strong \( \Delta N = 0 \), \( T = 0 \) correlation which involves mainly the 2p_1 orbital. Similar effects are probably present in the upper fp shell nuclei where wave functions of the form \( (2p_1, 1f_2, 2p_4)^n \) can be used to describe the energy levels with some effective interaction, but the effective charges (as well as the effective interactions) are strongly influenced by \( \Delta N = 0 \), \( T = 0 \) correlations involving the 1f_2 orbital.

7.4. COMMENTS ON FURTHER INVESTIGATION

More accurate and more reliable values of the empirical core-polarization charge will depend on: (i) A better microscopic treatment of the core deformed states. The \( A = 18 \) and \( 42 \) \( 2^+ \rightarrow 0^+ \) enhancements relative to the higher spin transitions are still not microscopically understood. (ii) More accurate experimental measurements of lifetimes and moments are needed. As discussed in sect. 5, the present experimental results in some cases are conflicting. The experimental information for \( A = 15, 39 \) and 41 is especially lacking.

The present analysis of \( W_T^{\text{pol}} \) indicates that some \( T = 0 \) GQR strength may lie as yet unobserved above 30 MeV in \(^{16}\text{O}\). The \( T = 1 \) GQR are predicted at about 48 and 41 MeV in \(^{16}\text{O}\) and \(^{40}\text{Ca}\), respectively. It would be interesting to locate these states directly.

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