A comparison of Woods–Saxon and double-folding potentials for lithium scattering from light target nuclei

C L Woods†, B A Brown‡ and N A Jelley

Nuclear Physics Laboratory, Department of Nuclear Physics, University of Oxford, Keble Road, Oxford OX1 3RH, UK

Received 24 May 1982, in final form 6 August 1982

Abstract. Elastic scattering data are presented for 6Li at 36 MeV from 12C, 16O and 26Mg and for 7Li at 28.8 MeV from 15N and at 27 MeV from 25Mg over the range 15–95° in the centre-of-mass frame. Microscopic optical potentials are generated for these scattering systems using the double-folding model. The sensitivity of the potentials to the prescriptions used for the nuclear density calculations is studied. The elastic scattering data are analysed within the framework of the optical model using both Woods–Saxon potentials and double-folding model real potentials with Woods–Saxon imaginary wells. Equally good fits to the data are obtained with the phenomenological and microscopic potentials. The double-folding potentials must be normalised by factors of 0.5 or 0.6 to fit the data in all cases except for 6Li scattering from 16O, for which a normalisation of 0.8 is favoured. These results are supplemented by those from an inelastic scattering analysis using DWBA and the collective rotational model for the first 2 + states in 12C and 26Mg. The shapes of the phenomenological and microscopic potentials around the strong absorption radius are compared and the gradients are found to differ significantly for the 7Li + 15N and 6Li + 12C systems.

1. Introduction

The double-folding model which has been developed recently and is reviewed in an article by Satchler and Love (1979) derives the real part of the optical potential from a ‘realistic’ nucleon–nucleon interaction and the projectile and target nuclear densities. When used in optical-model analyses of elastic scattering it has been found to work well for all projectiles except lithium and beryllium, for which the potential must be normalised by a factor of approximately 0.6 to produce a good fit to the data (e.g., Satchler and Love 1979, Glover et al 1980, Cook et al 1981). Although the anomaly has been demonstrated for a wide range of medium-mass targets, only Stanley et al (1980) have published a detailed double-folding analysis for lithium scattering from a target with \( A < 24 \).

Several groups have attributed this anomaly to the effects of cluster break-up of the projectile in the field of the target (Glover et al 1980, Norbeck et al 1981, Thompson and Nagarajan 1981). The anomaly might therefore be expected to be present for lighter targets as well. The observed energy independence of the normalisation factor may also be understood in terms of this explanation of the anomaly. For energies well above the Coulomb barrier the break-up is a direct process, break-up following Coulomb excitation.
to excited states being negligible (Ollerhead et al. 1964, Pfeiffer et al. 1973). Thus at these
energies, where many channels are open, the cross section to one particular channel is
more-or-less constant.

Results obtained using several different prescriptions for the calculation of nuclear
densities in the p and sd shells for use in the double-folding model are presented here. The
sensitivity of the double-folding potentials to the parameters assumed for the density
calculation has been investigated paying particular attention to the dependence of the
potential on the prescription used for the addition of the valence nucleons. As examples of
this work, densities and potentials for the $^6\text{Li} + ^{26}\text{Mg}$, $^6\text{Li} + ^{16}\text{O}$, $^6\text{Li} + ^{12}\text{C}$, $^7\text{Li} + ^{25}\text{Mg}$ and
$^7\text{Li} + ^{15}\text{N}$ systems are discussed. These scattering systems were chosen because elastic
scattering data for $^6\text{Li}$ at 36 MeV from $^{12}\text{C}$, $^{16}\text{O}$ and $^{26}\text{Mg}$ and for $^7\text{Li}$ at 27 MeV from
$^{25}\text{Mg}$ and at 28.8 MeV from $^{15}\text{N}$ were available, having been obtained as part of a study of the
($^6\text{Li}$, $^7\text{Be}$) reaction from light nuclei which is reported elsewhere (Woods 1981). In this
paper the data are analysed within the framework of the optical model and the results
obtained using both Woods–Saxon potentials and the real double-folding potentials with
Woods–Saxon imaginary wells are compared.

A comparison of the shapes, as well as the depths, of the real Woods–Saxon and
double-folding model wells in the vicinity of the strong absorption radius is interesting in
the light of Kobos and Mackintosh’s (1981) argument that both the gradient and absolute
value of the potentials generated using the double-folding model may be incorrect in this
region. The results of such a comparison must be treated with caution if the shapes of the
potentials are determined from the elastic scattering analysis at one energy alone because
this scattering probes the potential over a limited region around the strong absorption
radius and therefore determines the depth at this point whilst leaving a large uncertainty in
the gradient. However, the gradient of the potential in this region is probed by inelastic
scattering and other direct reactions. The inelastic scattering data for $^6\text{Li}$ from the first 2$^-$
states in $^{12}\text{C}$ and $^{26}\text{Mg}$ were analysed using both the distorted-wave Born approximation
and the coupled-channels formalism. The interaction used in the form factor was taken as
the derivative of the optical-model potential, the excitation being treated in the collective
rotational model. The detailed calculations for this inelastic scattering are reported

2. Experimental method

Lithium beams with intensities of typically 100 nA were used to bombard self-supporting
targets of 120 $\mu$g cm$^{-2}$ thickness for the $^{25,26}\text{Mg}$ and $^{12}\text{C}$ experiments. Natural carbon
targets were used and the magnesium targets were isotopically enriched. For the $^6\text{Li} + ^{16}\text{O}$
scattering experiments both 120 $\mu$g cm$^{-2}$ $^6\text{Li}_2\text{O}$ and 100 $\mu$g cm$^{-2}$ Na$_2$O targets on a
20 $\mu$g cm$^{-2}$ carbon backing were used. The $^7\text{Li} + ^{15}\text{N}$ scattering experiments were
performed with 120 $\mu$g cm$^{-2}$ NaCN targets, isotopically enriched in $^{15}\text{N}$, on a 20 $\mu$g cm$^{-2}$
carbon backing.

The experimental arrangement, electronics for data collection and particle
identification technique are described elsewhere (Woods et al. 1980, Woods 1981). Angular
distributions were collected over the range 15–95° in the centre-of-mass frame in steps of
3° using three $\Delta E–E$ solid-state detector telescopes. The angle was known to an accuracy
of $\pm 0.15\degree$ and the energy resolution was approximately 200 keV FWHM at 20°. Standard
particle identification techniques were used to obtain energy spectra for the lithium ejectiles
without a large number of background events. Random errors on the data due to statistics.
dead-time uncertainties, non-uniformities in the target thickness, uncertainties in the FWHMs of the overlapping elastic scattering peaks at forward angles and unresolved impurities in the spectra taken at forward angles from composite targets are included in the error bars shown in the figures. The absolute normalisation (determined by Rutherford scattering) was accurate to ±8% for the $^{25,26}$Mg, $^{16}$O and $^{12}$C data and ±12% for the $^{15}$N data, this being somewhat poorer due to the presence of large quantities of $^{16}$O in these targets.

3. Calculation of the nuclear densities and double-folding model potentials

The nuclear densities were calculated in a spherical single-particle potential model by summing the radial density probabilities of each nucleon weighted by the occupation probabilities as obtained from filling the lowest possible spherical levels or as obtained from the Cohen–Kurath (p)$^n$ and Chung–Wildenthal (sd)$^n$ configuration mixing calculations (Cohen and Kurath 1965, Chung 1976, Wildenthal 1977). A variety of Woods–Saxon and Skyrme Hartree–Fock potentials have been tried and from this experience the following procedure was found to be the most successful in reproducing the charge form factors. A Woods–Saxon shape was assumed with a different well depth for each orbit but with the same potential radius and diffuseness. The results were insensitive to small variations in the spin–orbit potential parameters and these were fixed at $r_o = 1.1$ fm, $a_o = 0.65$ fm and $V_o = 6.0$ MeV. The $N + 2$ parameters for $N$ orbits were adjusted to reproduce exactly the $N$ separation energies and the $\langle r^2 \rangle$ and $\langle r^4 \rangle$ moments of the charge distribution. The separation energy was taken as the binding energy difference between the ground state of the ‘target’ nucleus with mass $A$ and the centroid energy of levels in the nucleus with mass $A - 1$ corresponding to pick-up of a particular $(nlj)$ value. For states which have a small occupation probability in the target nucleus the separation energy is in general much larger than the centroid energy obtained from levels in nuclei with both mass $A - 1$ and $A + 1$. (The densities are not very sensitive to the assumptions made about the separation energy for deep hole states as long as the potential shape is readjusted to reproduce the $\langle r^2 \rangle$ and $\langle r^4 \rangle$ radii.) The oscillations in the interior of the charge density are generally not as well reproduced because they are sensitive to oscillations in the interior of the potential which are excluded ab initio in the Woods–Saxon potential. However, these details of the interior density have little effect on the calculated heavy-ion angular distributions. It is chiefly the tails of the density distributions that determine the double-folding (DF) potential in the peripheral region which is probed in elastic scattering and direct reaction experiments.

The $\langle r^2 \rangle$ and $\langle r^4 \rangle$ charge moments for $^{12}$C, $^{16}$O and $^{40}$Ca have been extracted from a model-independent analysis of the electron scattering form factors (Sick and McCarthy 1970, Sick 1974, Li et al 1974). The plane-wave form factors for these nuclei, calculated with the densities obtained using the above prescription, are in good agreement with experiment (plotted against $q_{min}$) up to the second minimum. (Above the second minimum the form factors are more sensitive to the interior oscillations mentioned above.) For $^6$Li the potential parameters were fitted to the RMS radius, 2.57 ± 0.10 fm, and the position of the first minimum in the form factor.

These potentials were interpolated to calculate the densities for $^{15}$N, $^{25}$Mg and $^{26}$Mg by introducing a smooth mass dependence in the parameters as well as an isovector potential for the deep hole states where the $A$ to $A - 1$ separation energy is not well known. The
Table 1. Parameters for the density calculations. (The potentials are in MeV, radii and diffuseness parameters in fm.)

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$V_r$</th>
<th>$r_r$</th>
<th>$a_r$</th>
<th>$V_s$</th>
<th>$r_s$</th>
<th>$a_s$</th>
<th>$r_c$</th>
<th>Occupation number†</th>
<th>SE†</th>
<th>$r_{eh}$ norm‡</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$O</td>
<td>varied</td>
<td>1.364</td>
<td>0.58</td>
<td>6.0</td>
<td>1.10</td>
<td>0.65</td>
<td>1.414</td>
<td>ssm</td>
<td>§</td>
<td>no</td>
</tr>
<tr>
<td>$^{15}$N</td>
<td>p: 51.686</td>
<td>1.315</td>
<td>0.519</td>
<td>6.0</td>
<td>1.315</td>
<td>0.519</td>
<td>1.422</td>
<td>ssm varied</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>varied</td>
<td>1.316</td>
<td>0.55</td>
<td>6.0</td>
<td>1.10</td>
<td>0.65</td>
<td>1.350</td>
<td>mixed†</td>
<td>†</td>
<td>no</td>
</tr>
<tr>
<td>$^{25}$Mg</td>
<td>n: 51.692</td>
<td>1.281</td>
<td>0.773</td>
<td>6.0</td>
<td>1.10</td>
<td>0.65</td>
<td>1.358</td>
<td>mixed†</td>
<td>†</td>
<td>yes</td>
</tr>
<tr>
<td>$^{25}$Mg</td>
<td>p: 55.839</td>
<td>1.279</td>
<td>0.765</td>
<td>6.0</td>
<td>1.10</td>
<td>0.65</td>
<td>1.362</td>
<td>ssm varied</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>$^6$Li</td>
<td>p: 55.906</td>
<td>1.304</td>
<td>0.748</td>
<td>6.0</td>
<td>1.10</td>
<td>0.65</td>
<td>1.559</td>
<td>ssm varied</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>$^{7}$Li</td>
<td>n: 48.672</td>
<td>1.330</td>
<td>0.65</td>
<td>6.0</td>
<td>1.10</td>
<td>0.65</td>
<td>1.533</td>
<td>ssm varied</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>$^5$Li</td>
<td>p: 58.728</td>
<td>1.330</td>
<td>0.65</td>
<td>6.0</td>
<td>1.10</td>
<td>0.65</td>
<td>1.533</td>
<td>ssm</td>
<td>§§</td>
<td></td>
</tr>
</tbody>
</table>

† See table 2 for occupation numbers and separation energies (SE).
‡ $r_{eh}$ norm indicates whether a 'fine adjustment' was applied as explained in the text.
|| See text for explanation of binding energy procedure.
§ Taken from Nucl. Phys. A 227 (1977) 77.

The complete potential had the form

$$V_T(r) = U_r(r) + U_s(r) + U_C(r)$$

where

$$U_r(r) = V_r(1 + e^{a_r})^{-1},$$

$$U_s(r) = 2V_s (I \cdot \sigma) r^{-1} d(1 + e^{a_s})^{-1}/dr,$$

$U_C(r)$ is the potential for a uniformly charged sphere with charge $Z - 1$ and radius $r_C A^{1/3}$, $A$ is the mass number for the complete nucleus and

$$\rho_i = (r - r_i A^{1/3})/a_i.$$ 

The values used for these parameters for each nucleus are shown in table 1 and explained below. The RMS radii based on these potential parameters are in agreement with experiment to within 1%. Thus we have some confidence that the neutron distribution can be

Table 2. Occupation numbers and separation energies for the density calculations.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Orbit</th>
<th>1s(_{1/2})</th>
<th>1p(_{3/2})</th>
<th>1p(_{1/2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}$C</td>
<td>Occupation number</td>
<td>n: 2.000</td>
<td>p: 2.000</td>
<td>n: 3.241</td>
</tr>
<tr>
<td>SE (MeV)</td>
<td>-44.66</td>
<td>-41.60</td>
<td>-18.72</td>
<td>-15.96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Orbit</th>
<th>1d(_{5/2})</th>
<th>2s(_{1/2})</th>
<th>1d(_{3/2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{25}$Mg</td>
<td>Occupation number</td>
<td>n: 4.75</td>
<td>p: 3.16</td>
<td>n: 0.60</td>
</tr>
<tr>
<td>SE (MeV)</td>
<td>-14.0</td>
<td>-14.0</td>
<td>-12.0</td>
<td>-12.0</td>
</tr>
</tbody>
</table>
Lithium scattering from light target nuclei

calculated to about this level of accuracy for nuclei with \( N \neq Z \). (For nuclei with \( N = Z \)
\( \rho_5(r) \approx \rho_5^*(r) \).) A final 'fine adjustment' (which was always less than 1\% in the RMS radius)
was sometimes applied by making a renormalisation based on the monopole Tassie model
\( \Delta \rho(r) \approx 3 \rho_5^*(r) + r \frac{\partial \rho_5(r)}{\partial r} \) to reproduce the experimental charge RMS radius exactly.

For \(^7\text{Li}\) several prescriptions were used to generate the densities and the sensitivity of
the double-folding potentials to the parameters used in the density calculations was studied. These tests were necessary to confirm that the 'anomaly' found in fitting the scattering data
was not caused by the use of an incorrect density distribution to describe the projectile.

The nucleus \(^7\text{Li}\) has a large static quadrupole moment \((-4.5 \pm 0.5 \text{ e fm}^2)\) producing a
significant contribution to the experimental charge form factor for momentum transfers
above \(2 \text{ fm}^{-1}\), and the monopole charge form factor can therefore only be fitted to the
experimental form factor for smaller momentum transfers. In the region \( q^2 < 2 \text{ fm}^{-2}\) the
charge form factors of \(^6\text{Li}\) and \(^7\text{Li}\) differ very little (Suelzle \textit{et al} 1967) and the RMS radii
are very similar, their values being \(2.41 \pm 0.10 \text{ fm}\) for \(^7\text{Li}\) (deJager \textit{et al} 1974) and
\(2.57 \pm 0.10 \text{ fm}\) for \(^6\text{Li}\) (Bumiller \textit{et al} 1972). Thus, to evaluate the \(^7\text{Li}\) charge density the optimised calculation for \(^6\text{Li}\) was first performed and a single neutron added to the valence
shell in \(^7\text{Li}\) using the same well parameters but with a separation energy of 9.41 MeV, this
being the separation energy of the \(^7\text{Li} - ^6\text{Li}\) system plus the centroid energy of the final
states in \(^6\text{Li}\). As explained above, the density distribution in the important tail region is
expected to be sensitive to the choice of prescription for the addition of the last neutron. To
determine how sensitive it was, these \(^7\text{Li}\) results were compared with a calculation
performed for the complete \(^7\text{Li}\) nucleus using the optimised \(^6\text{Li}\) parameters. The resulting
RMS charge radius of 2.39 fm agreed well with the average experimental value of 2.41 fm
obtained from deJager \textit{et al} (1974). The RMS neutron radius was 2.542 fm when the density
was calculated using a separation energy of 9.41 MeV for the last neutron and 2.619 fm
when calculated using the same separation energy for all the valence neutrons. The two
density distributions differed by 6\% at 4 fm and by 20\% at 6 fm. However, DF potentials
generated with these two \(^7\text{Li}\) densities and a \(^{25}\text{Mg}\) density were very similar, differing by
only 2\% at 6 fm and by approximately 8\% in the region of the strong absorption radius,
\(R_{ss}\).

The double-folding potentials were calculated using the method described by Satchler
and Love (1979) with the effective interaction

\[
v(r_{12}) = 6315 \frac{e^{-4r}}{4r} - 1961 \frac{e^{-2.5r}}{2.5r} - 81 \delta(r_{12}) \quad (\text{MeV})
\]

where

\[
r = |r_{12}| = |r_1 - r_2|
\]

and \(r_{12}\) (fm) is the vector joining the point at which the density is taken in one nucleus to
that at which it is taken in the other.

All the \(^6\text{Li} + ^{26}\text{Mg}\) double-folding potentials generated using the density prescriptions
described above had large central depths (typically 280 MeV) and a shape which could be
represented roughly as a Woods–Saxon squared. Densities calculated using different
prescriptions for the valence nucleons in \(^{26}\text{Mg}\) yielded potentials which varied by typically
5\% at the strong absorption radius.

Two \(^7\text{Li} + ^{25}\text{Mg}\) double-folding potentials were generated, using the two different
prescriptions for the addition of the last neutron to the \(^7\text{Li}\) nucleus as described above and
using simple shell-model occupation numbers for the \(^{25}\text{Mg}\) density and varying the binding
energies. These potentials differed by only 2% at 6 fm and approximately 8% in the region of $R_{sa}$. Their shapes and depths were very similar to those of the $^6 Li + ^{26} Mg$ potentials described above.

Three $^6 Li + ^{16} O$ potentials were calculated using different prescriptions for both the $^6 Li$ and $^{16} O$ densities to test the sensitivity of the shape as well as the normalisation of the potential around the strong absorption radius to the nuclear densities. All three potentials differed by less than 1% for radii up to 9 fm, the strong absorption radius being approximately 7.3 fm. Double-folding potentials were also generated for the $^7 Li + ^{15} N$ and $^6 Li + ^{12} C$ systems. These potentials had central depths of between 200 and 250 MeV and shapes which were approximately Woods–Saxon raised to the power 1.5.

4. Optical-model analysis of the elastic scattering

The optical-model potential searches were performed using the search routine DWAVF which contains the elastic scattering subroutines from MARS (Tamura and Low 1974) together with the NAG library search routine E04DF. Phenomenological Woods–Saxon imaginary potentials were used with the real double-folding potentials because Satchler and Love (1979) have shown that generally this gives more accurate predictions for the elastic scattering cross sections than those obtained using imaginary wells of the same shape as the real double-folding potentials. In the double-folding analysis the real potential was supplied point by point and its overall normalisation was searched together with the parameters of the imaginary well. The optical potential used in the Woods–Saxon analysis had the form

$$U(r) = \frac{-V}{1 + e^{-r}} - \frac{i W_{V}}{1 + e^{-ai}} + V_C$$

if a volume-imaginary well shape was assumed and

$$U(r) = \frac{-V}{1 + e^{-r}} - 4i W_{S} \frac{d}{d x_i} (1 + e^{a_i})^{-1} + V_C$$

if a surface-imaginary well shape was assumed where

$$x_k = (r - r_k A^{1/3})/a_k$$

$$V_C = (Z_1 Z_2 e^2 / 2 R_C) [3 - (r/R_C)^2]$$

$$= Z_1 Z_2 e^2 / r$$

for $r \leq R_C$ and

$$r > R_C$$

with $R_C = r_C A^{1/3}$.

The six parameters $V$, $r$, $a$, $W$, $r_i$ and $a_i$ were allowed to vary independently. The shapes of the elastic scattering curves were insensitive to the value of the Coulomb radius parameter, $r_C$, within the range $1.2 \text{ fm} \leq r_C \leq 1.9 \text{ fm}$ and its value was fixed at 1.3 fm. The fits to the data were optimised by minimising the value of $\chi^2$ as defined by the equation

$$\chi^2 = \frac{1}{N} \sum_{i} \frac{(\sigma_i(\theta_i) - \sigma_C(\theta_i))^2}{(\Delta \sigma_i(\theta_i))^2}$$

where $N$ is the number of data points less the number of free parameters and $\Delta \sigma_i(\theta_i)$ is the standard deviation on the $i$th data point.

The $\Delta \sigma_i(\theta_i)$ included random errors but not the overall normalisation uncertainty. For each scattering system searches were performed from several initial optical potentials,
these being taken from the literature as referenced in the optical potential parameter tables. A comprehensive search of each parameter in turn was not performed and therefore the different families of potentials which are related by discrete or continuous ambiguities were not explored in detail.

5. Elastic scattering analysis results

In this section the detailed results of the optical-model analysis are presented for each scattering system. In general, equally good fits to the elastic scattering data were obtained using phenomenological Woods–Saxon potentials and double-folding (DF) potentials. Searches using a real DF potential and starting from imaginary potentials obtained in the Woods–Saxon analysis which had different shapes yielded final imaginary wells which were identical in the region \( r > 6 \) fm. Thus the use of the DF model removed some of the ambiguity in the shape of the imaginary well although it made no direct predictions about the imaginary potential. The double-folding potential for \( ^6\text{Li} \) scattering from \( ^{16}\text{O} \) required normalisation by a factor of 0.8 to fit the data while those for the other systems studied needed to be normalised by factors of 0.5 or 0.6. These normalisation factors were checked to be insensitive to changes by one standard deviation in the absolute normalisation of the experimental data. The shapes of the real Woods–Saxon and double-folding potentials are similar around the strong absorption radius for scattering from \( ^{26}\text{Mg}, ^{25}\text{Mg} \) and \( ^{16}\text{O} \) but the slopes of the double-folding potentials are steeper than those for the Woods–Saxon potentials for scattering from \( ^{15}\text{N} \) and \( ^{12}\text{C} \).

5.1. \( ^6\text{Li} + ^{26}\text{Mg} \) optical-model analysis results

The initial optical potential parameters for the Woods–Saxon analysis are shown in table 3 above their corresponding final, searched parameters and the resulting values of \( \chi^2 \). All the optical potentials gave equally good fits to the elastic scattering data, the angular distributions being shown in figure 1. The potentials are plotted in figure 2. The real wells are identical in the region \( r > 6 \) fm, showing that in this case the data are sufficient to probe the potential over an extended region around the strong absorption radius (8 fm) and thus determine the slope, as well as the depth, of the real well at this point.

The double-folding analysis was performed using the real \( ^6\text{Li} + ^{26}\text{Mg} \) potential which is described in § 3. Each of the final imaginary potentials of table 3 was used as an initial potential and the imaginary parameters were searched together with the overall normalisation of the real well. A single final potential was obtained. The nearest scattering

![Table 3. \( ^6\text{Li} + ^{26}\text{Mg} \) initial and final OM parameters.](image)

<table>
<thead>
<tr>
<th>Potential</th>
<th>( V ) (MeV)</th>
<th>( r_r ) (fm)</th>
<th>( a_r ) (fm)</th>
<th>( W ) (MeV)</th>
<th>( r_i ) (fm)</th>
<th>( a_i ) (fm)</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moore et al (1975)</td>
<td>37.20</td>
<td>1.750</td>
<td>0.650</td>
<td>29.20</td>
<td>1.500</td>
<td>1.140</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>32.59</td>
<td>1.725</td>
<td>0.724</td>
<td>24.99</td>
<td>1.682</td>
<td>0.886</td>
<td>6.0</td>
</tr>
<tr>
<td>Schumacher et al (1973)</td>
<td>208.6</td>
<td>1.210</td>
<td>0.750</td>
<td>19.80</td>
<td>1.790</td>
<td>0.890</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>222.6</td>
<td>1.209</td>
<td>0.743</td>
<td>22.39</td>
<td>1.702</td>
<td>0.897</td>
<td>4.9</td>
</tr>
<tr>
<td>Schumacher et al (1973)</td>
<td>161.9</td>
<td>1.210</td>
<td>0.800</td>
<td>17.30</td>
<td>1.850</td>
<td>0.890</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>152.16</td>
<td>1.335</td>
<td>0.724</td>
<td>18.72</td>
<td>1.770</td>
<td>0.918</td>
<td>7.4</td>
</tr>
</tbody>
</table>
system for which imaginary parameters were available from a DF analysis was $^6\text{Li} + ^{40}\text{Ca}$ at 34 MeV (Satchler and Love 1979). These parameters were also used as initial values in the searches. The two final potentials are shown in table 4. The double-folding potentials produced fits to the data similar in quality to those from the Woods—Saxon analysis.

The DF potentials needed to be normalised by a factor of approximately 0.6 to fit the data, as found by other groups for $^6\text{Li}$ scattering. The normalised double-folding potentials are shown in figure 2 together with the phenomenological potentials. The gradients of all the real potentials are very similar in the tail region and the values of the Woods—Saxon and normalised double-folding wells differ negligibly within the range $6 < r < 9$ fm around the strong absorption radius. All the imaginary wells, whether they are associated with real Woods—Saxon or double-folding potentials, have the same depth at $r = 7$ fm. Thus they pass through a point of constant depth at a ‘critical radius’ near the strong absorption radius, the potential at this point determining the elastic scattering predictions (Satchler 1974). This type of relationship between the shapes of a family of potentials will be referred to as an ‘invariant point’ ambiguity.

5.2. $^7\text{Li} + ^{25}\text{Mg}$ optical-model analysis results

The initial and final optical potential parameters for the Woods—Saxon analysis are shown in table 5. The grazing angular momentum for potential 1 is $l \approx 18$ while that for potentials 2 and 3 is $l \approx 16$. These values correspond to strong absorption radii, $R_a$, of 9.2 and 8.3 fm.
Lithium scattering from light target nuclei

Figure 2. Illustrating the similarity between the Woods–Saxon and double-folding (A) real and (B) imaginary optical potentials for $^6\text{Li} + ^{26}\text{Mg}$ scattering at 36 MeV: full curve, potential 1 of table 3; broken curve, potential 2 of table 3; chain curve, potential 3 of table 3; dotted curve, potentials 1 and 2 of table 4.

respectively. The potentials, which are plotted in figure 3, give equally good fits to the elastic scattering data (figure 4), although they differ markedly in shape.

Double-folding potential searches were performed using the final imaginary parameters from the Woods–Saxon analysis and using 34 MeV $^7\text{Li} + ^{40}\text{Ca}$ imaginary parameters from Satchler and Love (1979) with both the $^7\text{Li} + ^{25}\text{Mg}$ double-folding potentials described in § 3. These DF potentials were generated with different prescriptions for the addition of the last neutron in the $^7\text{Li}$ density calculation and differed by approximately 8% at $R_{\text{sc}}$.

Two final potentials were obtained and their parameters are given in table 6. The final

<table>
<thead>
<tr>
<th>DF potential label</th>
<th>Overall normalisation</th>
<th>$W$ (MeV)</th>
<th>$r_i$ (fm)</th>
<th>$a_i$ (fm)</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.576</td>
<td>15.26</td>
<td>1.881</td>
<td>0.837</td>
<td>6.1</td>
</tr>
<tr>
<td>2</td>
<td>0.573</td>
<td>15.27</td>
<td>1.888</td>
<td>0.825</td>
<td>6.0</td>
</tr>
</tbody>
</table>
Table 5. $^7\text{Li} + ^{25}\text{Mg}$ initial and final OM parameters.

<table>
<thead>
<tr>
<th>Potential</th>
<th>$V$ (MeV)</th>
<th>$r_i$ (fm)</th>
<th>$a_i$ (fm)</th>
<th>$W$ (MeV)</th>
<th>$r_i$ (fm)</th>
<th>$a_i$ (fm)</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moore et al (1974)</td>
<td>54.1</td>
<td>1.78</td>
<td>0.58</td>
<td>10.95</td>
<td>2.15</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>55.58</td>
<td>1.951</td>
<td>0.367</td>
<td>7.946</td>
<td>2.338</td>
<td>0.990</td>
<td>4.9</td>
</tr>
<tr>
<td>White and Kemper (1974)</td>
<td>49.7</td>
<td>1.78</td>
<td>0.58</td>
<td>8.52</td>
<td>1.78</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>46.21</td>
<td>1.708</td>
<td>0.665</td>
<td>22.34</td>
<td>1.806</td>
<td>0.861</td>
<td>3.4</td>
</tr>
<tr>
<td>Moore et al (1975)</td>
<td>195.3</td>
<td>1.21</td>
<td>0.78</td>
<td>31.2</td>
<td>1.67</td>
<td>0.89</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>116.0</td>
<td>1.389</td>
<td>0.735</td>
<td>57.31</td>
<td>1.552</td>
<td>0.800</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Values for the overall normalisation of the DF potentials were 0.48 and 0.50. These normalisation factors agree with that found by Glover et al (1980) in their analysis of $^7\text{Li}$ scattering at 34 MeV and indicate that the anomaly is present at least down to energies of 27 MeV for $^7\text{Li}$ scattering on $^{25}\text{Mg}$.

Figure 3. Illustrating the similarity between the (A) double-folding and (B) Woods–Saxon real potentials and between the corresponding imaginary potentials (C and D respectively) for $^7\text{Li} + ^{25}\text{Mg}$ scattering at 27 MeV: dotted curve, potential 1 of table 6; plus curve, potential 2 of table 6 (also shown with the Woods–Saxon curves for comparison); full curve, potential 1 of table 5; broken curve, potential 2 of table 5; chain curve, potential 3 of table 5.
Lithium scattering from light target nuclei

Figure 4. Elastic scattering data and optical-model angular distributions calculated using both Woods–Saxon and double-folding model potentials for $^7\text{Li} + ^{25}\text{Mg}$ at 27 MeV: full curve, potential 1 of table 5; broken curve, potentials 2 and 3 of table 5 and potentials 1 and 2 of table 6.

The final DF potentials are plotted with the Woods–Saxon potentials in figure 3. The slopes of all except one of the Woods–Saxon potentials are very similar in the tail region and the values of these potentials differ by $\leq 10\%$ in the region $6 < r < 10$ fm. However, the success of a different shaped Woods–Saxon potential in predicting these data equally well indicates that the data for this scattering system are insufficient to test the correctness of the shape of the DF potentials.

5.3. $^6\text{Li} + ^{16}\text{O}$ optical-model analysis results

In common with other groups which have performed optical-model analyses of $^6\text{Li}$ scattering from $^{16}\text{O}$ (e.g., Schumacher et al 1973), difficulty was experienced in finding a potential which gave a good fit to the data. In addition to the standard volume-imaginary Woods–Saxon potential shape, searches were performed using surface-imaginary wells, Woods–Saxon squared real wells and both surface- and volume-imaginary wells. However,

<table>
<thead>
<tr>
<th>DF potential label</th>
<th>Overall normalisation</th>
<th>$W$ (MeV)</th>
<th>$r_1$ (fm)</th>
<th>$a_1$ (fm)</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.477</td>
<td>250.9</td>
<td>1.198</td>
<td>0.769</td>
<td>3.6</td>
</tr>
<tr>
<td>2</td>
<td>0.497</td>
<td>93.32</td>
<td>1.449</td>
<td>0.778</td>
<td>3.7</td>
</tr>
</tbody>
</table>
no potential was found which produced a better fit than the conventional volume-
imaginary shaped potentials. The Woods–Saxon parameter sets are shown in table 7.
Potential 3 of the table gives a better fit to the forward-angle data than that of potentials 1
and 2 although it fails to fit the data at angles larger than $57^\circ$ in the centre-of-mass frame
(figure 5) and therefore has a $\chi^2$ which is a factor of two larger than those for potentials 1
and 2.

<table>
<thead>
<tr>
<th>Potential</th>
<th>$V$ (MeV)</th>
<th>$r_e$ (fm)</th>
<th>$a_e$ (fm)</th>
<th>$W$ (MeV)</th>
<th>$r_i$ (fm)</th>
<th>$a_i$ (fm)</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schumacher et al (1973)</td>
<td>222.3</td>
<td>1.21</td>
<td>0.80</td>
<td>11.8</td>
<td>2.017</td>
<td>1.035</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>228.8</td>
<td>1.179</td>
<td>0.782</td>
<td>9.420</td>
<td>2.161</td>
<td>0.766</td>
<td>22</td>
</tr>
<tr>
<td>Schumacher et al (1973)</td>
<td>164.3</td>
<td>1.21</td>
<td>0.826</td>
<td>10.6</td>
<td>2.017</td>
<td>1.064</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>172.8</td>
<td>1.167</td>
<td>0.838</td>
<td>7.630</td>
<td>2.297</td>
<td>0.697</td>
<td>21</td>
</tr>
<tr>
<td>Orloff and Daehnick (1973)</td>
<td>10.1</td>
<td>2.296</td>
<td>0.649</td>
<td>3.32</td>
<td>2.296</td>
<td>0.649</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>37.34</td>
<td>1.732</td>
<td>0.804</td>
<td>8.352</td>
<td>2.211</td>
<td>0.812</td>
<td>44</td>
</tr>
</tbody>
</table>

Figure 5. Elastic scattering data and optical-model angular distributions calculated using (A)
Woods–Saxon and (B) double-folding model potentials for $^6\text{Li} + ^{16}\text{O}$ at 36 MeV: full curve
A, potentials 1 and 2 of table 7; broken curve A, potential 3 of table 7; broken curve B, potential 1 of table 8; full curve B, potential 2 of table 8; plus curve B, potential 3 of table 8.
Lithium scattering from light target nuclei

Figure 6. Illustrating the similarity between the (A) double-folding model and (B) Woods–Saxon real potentials and between the corresponding imaginary potentials (C and D respectively) for $^6$Li + $^{16}$O scattering at 36 MeV: triangle curve, potential 1 of table 8; chain curve, potential 2 of table 8; plus curve, potential 3 of table 8; full curve, potential 1 of table 7 (also shown with double-folding curves for comparison); broken curve, potential 2 of table 7; cross curve, potential 3 of table 7.

The potentials are shown in figure 6, which illustrates that the real potentials are related by the invariant point ambiguity, having the same depth at $r \approx 7$ fm close to the strong absorption radius (7.3 fm). The imaginary well number 3 is slightly deeper than the others at the critical radius, accounting, perhaps, for its success in fitting the elastic scattering data in a different region to the other potentials.

In the double-folding analysis both the final imaginary well parameters and the overall normalisation of the real well were sensitive to the initial value of this normalisation. As it was varied between 0.4 and 1.2, three local minima were found in $\chi^2$ (figure 11) and the corresponding three potentials are shown in table 8 and plotted in figure 6 with the Woods–Saxon potentials. The best fit is comparable with that produced in the Woods–Saxon analysis and is obtained with an overall normalisation of 0.83 for the DF potential. Both the real and imaginary parts of this potential are similar in the tail region to the best Woods–Saxon potentials.

In double-folding model analyses of $^6$Li scattering from heavier targets at similar
energies, other groups have found that the real potential must be normalised by a factor of approximately 0.6 to fit the data (as found for the $^6\text{Li} + ^{26}\text{Mg}$ analysis presented in § 5.1). The normalisation factor of 0.83 obtained in this analysis of $^6\text{Li}$ scattering from $^{16}\text{O}$ was therefore surprising. However, if the normalisation is constrained to the local minimum value close to 0.6 in this analysis then the fit to the elastic scattering data deteriorates by a factor of two. A comparison of the shapes of the potentials obtained in the Woods–Saxon and double-folding analyses is illuminating. At $r_{sc}$ the depths of the real Woods–Saxon potentials are approximately 0.92 MeV while that of the unnormalised DF potential is 1.12 MeV. This suggests that the DF potential may need to be renormalised by a factor of approximately 0.82 to fit the elastic scattering data, in agreement with the best normalisation obtained in the analysis.

To test whether this result was due to an incorrect choice of density for the $^{16}\text{O}$ nucleus, two $^6\text{Li} + ^{16}\text{O}$ DF potentials were generated using different well parameters for the $^{16}\text{O}$ density calculation but choosing parameters which reproduced the separation energies and $\langle r^2 \rangle$ and $\langle r^4 \rangle$ moments of the charge distribution in both cases. Searches were also performed with these potentials and in both cases the best fits to the data were obtained with a normalisation of 0.83 and an imaginary potential which was almost identical to that of potential 2 in table 8. The same $^6\text{Li}$ density was used in this analysis as for the $^{26}\text{Mg}$ analysis described above. The change in the overall normalisation from 0.6 for $^{26}\text{Mg}$ to 0.82 for $^{16}\text{O}$ therefore seems to be due to the different target. No target dependence of the "$^6\text{Li}$ anomaly" has been reported previously.

### 5.4. $^7\text{Li} + ^{15}\text{N}$ optical-model analysis results

The Woods–Saxon potentials obtained in the $^6\text{Li} + ^{16}\text{O}$ analysis were used as initial parameter sets for the $^7\text{Li} + ^{15}\text{N}$ scattering and the final potential parameters are shown in table 9. The strong absorption radius for the elastic scattering was 7.5 fm. Both the real and imaginary wells pass through invariant points in this region. Angular distributions from the Woods–Saxon and double-folding analyses are shown in figure 7.

In the double-folding analysis the final value of the normalisation of the real well and the final imaginary well shape were found to be sensitive to the initial value of the normalisation, there being a local minimum in the value of $\chi^2$ for a normalisation of 0.78

<p>| Table 8. $^6\text{Li} + ^{16}\text{O}$ double-folding model potential parameters. |
|----------------|----------------|---------------|---------------|---------------|---------------|---------------|</p>
<table>
<thead>
<tr>
<th>DF potential label</th>
<th>Overall normalisation</th>
<th>$W$ (MeV)</th>
<th>$r_i$ (fm)</th>
<th>$a_i$ (fm)</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.09</td>
<td>14.04</td>
<td>2.110</td>
<td>0.577</td>
<td>36</td>
</tr>
<tr>
<td>2</td>
<td>0.825</td>
<td>8.446</td>
<td>2.216</td>
<td>0.740</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>0.560</td>
<td>6.618</td>
<td>2.324</td>
<td>0.919</td>
<td>41</td>
</tr>
</tbody>
</table>

| Table 9. Final $^7\text{Li} + ^{15}\text{N}$ OM parameters. |
|----------------|---------------|---------------|---------------|---------------|---------------|---------------|
| Potential     | $V$ (MeV)    | $r_i$ (fm)   | $a_i$ (fm)   | $W$ (MeV)    | $r_i$ (fm)   | $a_i$ (fm)   | $\chi^2$ |
|----------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 1               | 284.7         | 0.840         | 0.907         | 8.982         | 2.416         | 0.670         | 6.6         |
| 2               | 219.5         | 1.065         | 0.839         | 9.008         | 2.378         | 0.707         | 6.6         |
Lithium scattering from light target nuclei

(*Figure 7.*) Elastic scattering data and optical-model angular distributions calculated using both Woods-Saxon and double-folding model potentials for $^7\text{Li} + ^{15}\text{N}$ at 28.8 MeV: full curve, potentials 1 and 2 of table 9; broken curve, potential 1 of table 10; dotted curve, potential 2 of table 10.

$(\chi^2 = 9.4)$, the true minimum corresponding to a value of 0.61 for the normalisation and having $\chi^2 = 6.7$ (figure 11).

These two potentials are shown in table 10 and are illustrated together with the Woods-Saxon potentials in figure 8. The imaginary potentials possess the same invariant point ambiguity as those obtained in the Woods-Saxon analysis. The gradient of the real DF potential is considerably steeper than those of both Woods-Saxon potentials in the vicinity of the strong absorption radius. A visual comparison of the DF and Woods-Saxon real potentials suggests that the DF potential must be normalised by a factor of approximately 0.78 in order to pass through the same invariant point at $R_{\text{abs}}$ (7.5 fm) as the Woods-Saxon potentials and thus fit the data. However, a normalisation of 0.61 gives a better fit to the data at large angles than that produced with a normalisation of 0.78. Semiclassically, the large-angle scattering is produced by low partial waves as these have small impact parameters. This suggests that normalisation by a factor of 0.61 produces a

Table 10. $^7\text{Li} + ^{15}\text{N}$ double-folding model potential parameters.

<table>
<thead>
<tr>
<th>DF potential label</th>
<th>Overall normalisation</th>
<th>$W$ (MeV)</th>
<th>$r_i$ (fm)</th>
<th>$a_i$ (fm)</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.611</td>
<td>7.325</td>
<td>2.373</td>
<td>0.825</td>
<td>6.7</td>
</tr>
<tr>
<td>2</td>
<td>0.785</td>
<td>110.4</td>
<td>1.538</td>
<td>0.748</td>
<td>9.4</td>
</tr>
</tbody>
</table>
potential with the correct magnitude over a radial region just inside $R_{sa}$, where the partial waves $I=12$ and $I=13$ produce a significant contribution to the scattering. The DF potential multiplied by 0.61 is indeed closer to the depths of the Woods–Saxon potentials within the region $6 < r < 7$ fm. These results suggest that the gradient of the DF potential for $^7$Li $+ ^{15}$N may be too steep in the vicinity of $R_{sa}$ whereas it is correct for the scattering systems described previously. However, as explained above, such conclusions must be treated with caution if they are based on an analysis of the elastic scattering only because this probes the potential over a limited region around $R_{sa}$.

5.5. $^6$Li $+ ^{12}$C optical-model analysis results

In the Woods–Saxon analysis searches were performed using both volume-imaginary and surface-imaginary potentials and the initial and final parameters are shown in table 11. All four final potentials produced equally good fits to the elastic scattering data and the angular distributions are shown in figure 9. The real and imaginary wells are plotted in figure 10 and it is seen that the potentials are related by the invariant point ambiguity,
Lithium scattering from light target nuclei

Table 11. $^6$Li + $^{12}$C initial and final OM parameters.

<table>
<thead>
<tr>
<th>Potential</th>
<th>$V$ (MeV)</th>
<th>$r_r$ (fm)</th>
<th>$a_r$ (fm)</th>
<th>$W_V$ (MeV)</th>
<th>$W_S$ (MeV)</th>
<th>$r_i$ (fm)</th>
<th>$a_i$ (fm)</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schumacher et al (1973)</td>
<td>173.2</td>
<td>1.208</td>
<td>0.802</td>
<td>8.9</td>
<td>2.17</td>
<td>0.945</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>177.4</td>
<td>1.189</td>
<td>0.788</td>
<td>9.349</td>
<td>2.159</td>
<td>0.815</td>
<td>13.3</td>
<td></td>
</tr>
<tr>
<td>Bassani et al (1972)</td>
<td>281.0</td>
<td>1.550</td>
<td>0.640</td>
<td>27.60</td>
<td>1.970</td>
<td>0.360</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>475.2</td>
<td>1.051</td>
<td>0.708</td>
<td>16.16</td>
<td>1.076</td>
<td>0.988</td>
<td>13.9</td>
<td></td>
</tr>
<tr>
<td>Cunsolo et al (1978)</td>
<td>250.0</td>
<td>1.354</td>
<td>0.65</td>
<td>30.0</td>
<td>1.354</td>
<td>0.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>220.2</td>
<td>1.291</td>
<td>0.696</td>
<td>12.93</td>
<td>1.100</td>
<td>1.057</td>
<td>15.4</td>
<td></td>
</tr>
<tr>
<td>Bindal et al (1974)</td>
<td>69.84</td>
<td>1.74</td>
<td>0.735</td>
<td>16.64</td>
<td>2.242</td>
<td>0.386</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>129.4</td>
<td>1.103</td>
<td>0.888</td>
<td>6.91</td>
<td>2.382</td>
<td>0.739</td>
<td>14.9</td>
<td></td>
</tr>
</tbody>
</table>

having different slopes in the tail region but all passing through a point of constant depth near the strong absorption radius, $R_{sa} = 7$ fm. An analysis of the inelastic scattering to the $2^+$ (4.44 MeV) state in $^{12}$C was performed (Woods 1981). The four potentials predicted

![Figure 9. Elastic and inelastic scattering data and optical-model angular distributions calculated using both Woods–Saxon and double-folding model potentials for $^6$Li + $^{12}$C at 36 MeV: full curve, potential 1 of table 11; broken curve, potential 2 of table 11; plus curve, potential 3 of table 11; dotted curve, potential 4 of table 11; chain curve, potential 1 of table 12.](image-url)
differing amounts of structure in the inelastic angular distribution, indicating that the predictions are sensitive to the diffuseness of the real or imaginary potential in this reaction. The best fit to the data was obtained with potential 4, suggesting that this potential had the best combination of real and imaginary diffusenesses to the extent that these can be studied within the collective model.

The double-folding model analysis was performed with only volume Woods–Saxon imaginary potentials, the two searched potentials of table 11 being used as starting parameters. As before, it was found that the final potential depended on the initial value of the normalisation of the real well but not on the initial parameters of the imaginary well. Only one potential was found which gave as good a fit to the data as the purely Woods–Saxon potentials and its parameters are shown in table 12. The normalisation of the DF potential was 0.62. The DF potential is less deep at $R$ than the Woods–Saxon real potentials (figure 10) and the corresponding imaginary potential is deeper at $R$ than those obtained in the Woods–Saxon analysis.

A comparison of the depths of the real wells at $R_\text{as}$ shows that a normalisation of 0.88 is required if the double-folding potential is to pass through the same invariant point as the
Woods–Saxon potentials. However, the normalised DF potential is similar to Woods–Saxon potential number 4 in the region around 6 fm just inside the strong absorption radius. This suggests that the gradient of the double-folding model prediction is too steep in the vicinity of $R_{1s}$, being significantly larger than that of potential number 4 which gave the correct predictions for the inelastic scattering angular distribution. Thus for $^7\text{Li} + ^{15}\text{N}$ and $^6\text{Li} + ^{12}\text{C}$ the double-folding model appears to give incorrect predictions for the slope, as well as the absolute normalisation of the optical potential around the strong absorption radius. However, the slope of the $^6\text{Li} + ^{26}\text{Mg}$ double-folding potential agrees with that of the corresponding Woods–Saxon potentials, all of which fit the elastic scattering data and the inelastic scattering data (Woods 1981) populating the $2^+$ (1.81 MeV) state in $^{26}\text{Mg}$.

6. Conclusions

Generally, several potentials, related by the invariant point ambiguity, were found to give equally good fits to the data for each scattering system. The real potentials for $^7\text{Li}$ scattering from $^{25}\text{Mg}$ belonged to different discrete-ambiguity families, the fact that a different type of ambiguity was found for this system probably being due to the choice of initial parameters. In all cases equally good fits to the data were obtained with microscopic real potentials, generated using the double-folding model, and with phenomenological Woods–Saxon real potentials.

Several prescriptions were used to generate the nuclear densities for the double-folding model potential calculations and the sensitivity of the potentials to the tails of the nuclear densities was studied. For all three targets the $^6\text{Li}$ scattering potentials generated by these different methods differed by $\leq 4\%$ at the strong absorption radius. The $^7\text{Li} + ^{25}\text{Mg}$ potential varied by approximately 8\% at the strong absorption radius when different prescriptions were used for the addition of the last neutron to the $^7\text{Li}$ nucleus.

For $^6\text{Li}$ scattering from $^{26}\text{Mg}$ and $^{12}\text{C}$ at 36 MeV, overall normalisations of 0.60 and 0.62 were required respectively. This value agrees with results found by other groups for $^6\text{Li}$ scattering from a large number of targets at various energies and shows that the anomaly is also present for $^6\text{Li}$ scattering from $^{12}\text{C}$. However, for $^6\text{Li}$ scattering from $^{16}\text{O}$ the DF potential required normalisation by 0.83 to give the best fit to the data. The difference between the $^{16}\text{O}$ and $^{26}\text{Mg}$ and $^{12}\text{C}$ results cannot be due to the choice of density for $^6\text{Li}$ and does not appear to depend strongly on the method used to construct the $^{16}\text{O}$ density. It is unlikely to be caused by coupled-channels effects for the deformed $^{12}\text{C}$ and $^{26}\text{Mg}$ nuclei because Clarke et al (1981) have shown that inclusion of these effects explicitly does not change the normalisation factor.

For $^7\text{Li}$ scattering from $^{25}\text{Mg}$ at 27 MeV a normalisation of the double-folding potential by 0.49 was necessary to fit the data and for $^7\text{Li}$ scattering from $^{15}\text{N}$ at 28.8 MeV a normalisation of 0.61 was needed. These values agree with those obtained by other
groups at higher energies, indicating that the anomaly is present down at least to an incident energy of 27 MeV for $^{25}\text{Mg}$ and that it is also present for the lighter target, $^{15}\text{N}$.

Several local minima in $\chi^2$ were found when searching for the best normalisation factor of the double-folding potential to fit the $^{12}\text{C}$, $^{15}\text{N}$ and $^{16}\text{O}$ data. How well the normalisation factor $N$ was determined is illustrated in figure 11. These curves were obtained by evaluating $\chi^2$ allowing only the diffuseness and depth of the imaginary well to vary for values of $N$ between 0.2 and 1.2. As can be seen, such an analysis shows that great care must be taken when assessing the uncertainty in the normalisation factor of a double-folding potential in comparisons with other experimental values or theoretical predictions.

The variety of shapes of Woods–Saxon potentials which produce equally good fits to the elastic scattering data indicates that these data alone are often insufficient to test the correctness of the shapes of the double-folding model optical potentials. The $^6\text{Li} + ^{26}\text{Mg}$ elastic and inelastic scattering analyses together suggest that the double-folding model predicts the correct shape for this optical potential although the overall normalisation is wrong. However, the $^6\text{Li} + ^{12}\text{C}$ analyses suggest that the gradient as well as the normalisation of the double-folding potential is incorrect for this system, being too steep around the strong absorption radius. The analysis shows that double-folding potentials which are ‘anomalous’ may or may not have an incorrect shape in the tail region. In the light of these results a detailed study of the shapes of double-folding potentials which are not ‘anomalous’ when used in elastic scattering analyses would be interesting.

A direct comparison of the potentials found in this work with model-independent potentials such as those calculated by Kobos and Mackintosh would clearly be instructive. For example, the analysis presented above shows that the DF potential for $^7\text{Li}$ scattering from $^{15}\text{N}$ at 28.8 MeV must be normalised by approximately 0.6 to fit the elastic scattering data, its gradient being steeper in the vicinity of $R_{ss}$ than that of the best Woods–Saxon potentials, and its value at $R_{ss}$ being 0.78 times that of the Woods–Saxon potentials. These results are similar to the finding of Kobos and Mackintosh (1981) that the ratio of the DF and model-independent potentials for $^6\text{Li} + ^{24}\text{Mg}$ scattering at 88 MeV is 0.8 although the

![Figure 11](image.png)

Figure 11. Illustrating how well the normalisation factor $N$ of the double-folding potential is determined by the elastic scattering data. The curves were obtained by varying only the diffuseness and depth of the imaginary well for values of $N$ between 0.2 and 1.2.
Lithium scattering from light target nuclei

DF potential must be normalised by 0.6 to fit the data, the gradient of the DF potential being too large around $R_{sa}$. The two results suggest that the Woods–Saxon potential for $^7\text{Li} + ^{15}\text{N}$ scattering at 28.8 MeV is similar in both magnitude and slope at $R_{sa}$ to the model-independent potential, while both the magnitude and slope of the DF potential are incorrect.

It would also be instructive to derive values for the normalisations of the DF model potentials for $^6\text{Li}$ and $^7\text{Li}$ scattering from these nuclei theoretically. The values found in this work could be tested by an analysis such as that of Thompson and Nagarajan (1981) which includes the effects of break-up of the $^6\text{Li}$ nucleus into $\alpha + d$ explicitly.

Acknowledgments

The financial support of the Science and Engineering Research Council is gratefully acknowledged. We thank R Lindsay for useful discussions concerning the double-folding model calculations.

References

Chung W 1976 Thesis Michigan State University
Cohen S and Kurath D 1965 Nucl. Phys. 73 1
deJager C W, deVries H and deVries C 1974 Atomic Data and Nuclear Data Tables 14 479
Kobos A M and Mackintosh R S 1981 unpublished
Oilerhead R W, Chasman C and Bromley D A 1964 Phys. Rev. 134 B74
Sick I 1974 Nucl. Phys. A 218 509
Woods C L 1981 Thesis Oxford University