EFFECTS OF TWO-PARTICLE–TWO-HOLE GROUND-STATE CORRELATIONS ON SPIN-DIPOLE TRANSITIONS IN $^{12}$N

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We have studied the effect of two-particle–two-hole ground-state correlations on the spin-dipole transitions in $^{12}$N. We found that the transition strengths in the energy range $E_x = 2-12$ MeV are quenched 25% by the ground state correlations. The tensor correlation is important for $1^-$ and $2^-$ states, while the spin-orbit force has an appreciable effect on $1^-$ states only.

The $(p, n)$ reaction at intermediate energy is an extremely useful probe for the study of $\sigma \cdot \tau$ correlations in nuclei [1]. The cross section at zero degrees has been studied in detail and the Gamow–Teller (GT) strength for $l = 0$ transition has been extracted systematically in many nuclei throughout the periodic table [1,2]. Moreover, the neutron spectra at larger angles ($\theta = 5-15^\circ$) show strong transitions characterized by $l = 1$ angular distributions with a large width of around 10 MeV for nuclei $A > 40$ [1]. These broad resonances are interpreted by an envelope of collective states with spin parities $2^-, 1^-$ and $0^-$ excited through the transition operators [3],

$$T_{\lambda \mu} = \sum_j [Y_{\lambda-1}(\hat{r}_j) \times \sigma_j]_{\lambda \mu} \tau_{-1} \quad \lambda = 2^- , 1^- , 0^-.$$  

We refer to them as spin-dipole transition operators. The particle–hole matrix elements for these operators are given by

$$\langle j_{\lambda}^\dagger l_{\mu} \rangle_{\lambda \mu} = \left[ (2 j_{\lambda} + 1)/4\pi \right]^{1/2} \langle j_{\lambda}^\dagger 0 | j_{\mu} | j_{\mu} \rangle \langle l_{\mu} | r_j | l_{\lambda} \rangle \times 1, \quad \lambda = 0^-,$$

$$\times \left\{ - \left( (2\lambda + 1)/[\lambda(\lambda + 1)] \right)^{1/2} \right\}$$

$$\times \left\{ (-)^{l_{\lambda} + 1/2 - l_{\mu}} \left( j_{\mu} + 1/2 \right) \right\}$$

$$- (-)^{l_{\lambda} + 1/2 - l_{\mu}} \left( j_{\mu} + 1/2 \right), \quad \lambda = 1^-,$$

$$\times \left[ \lambda \right]^{1/2} \left( 1 + (-)^{l_{\lambda} + 1/2 - l_{\mu}} \left( j_{\mu} + 1/2 \right) \right)$$

$$+ (-)^{l_{\lambda} + 1/2 - l_{\mu}} \left( j_{\mu} + 1/2 \right), \quad \lambda = 2^-.$$  

It has not been possible to resolve the broad peaks at angles ($\theta = 5-15^\circ$) in heavy nuclei into their components and confirm each transition strength partly due to the poor energy resolution of neutron spectra. Recently, in the light nucleus $^{12}$N, the spin-dipole states with $\lambda^\tau = 1^-, 2^-$ have been observed separately in the energy region $E_x = 2-12$ MeV [4]. From the experimental neutron spectrum for the $^{12}$C(p, n)$^{12}$N reaction at $E_p = 160$ MeV and $\theta = 8^\circ$, the extracted cross section obtained by adding the total yield (corrected for cosmic ray background) is given by

$$\sum_{E_x} \frac{d\sigma}{d\Omega} (E_x = 2-12 \text{ MeV})_{\text{exp}} = (12.0 \pm 1.8) \text{mb/sr},$$

while a shell-model calculation [4] gives the summed cross section,

$$\sum_{E_x} \frac{d\sigma}{d\Omega} (E_x = 2-12 \text{ MeV})_{\text{theory}} = 20 \text{mb/sr}.\quad (4)$$

The model prediction in ref. [4] took into account one-particle–one-hole (1p–1h) states built on the lowest six odd-parity states of the $A = 11$ system and the excitations from the $0s_{1/2}$-orbit were not allowed.
Table 1
Sum rule values $B(I=1, \lambda^\pi)$ for the spin-dipole transitions in $^{12}\text{N}$. The values in row 1 are obtained by the $1p$-$1h$ excitations from $0p_{3/2}$ and $0s_{1/2}$ states assuming a $0p_{3/2}$-closed shell. The shell-model calculation of ref. [4] in row 2 took into account only $1p$-$1h$ excitations from the lowest six odd-parity states of the $A=11$ system, while the present calculation in row [3] includes all configurations from $0p$- and $0s$-orbits. For details, see the text.

<table>
<thead>
<tr>
<th></th>
<th>$\lambda^\pi$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1p$-$1h$</td>
<td>5.56</td>
<td>14.11</td>
<td>14.97</td>
<td></td>
</tr>
<tr>
<td>ref. [4]</td>
<td>3.79</td>
<td>10.76</td>
<td>16.03</td>
<td></td>
</tr>
<tr>
<td>present</td>
<td>4.56</td>
<td>12.61</td>
<td>17.48</td>
<td></td>
</tr>
<tr>
<td>ref. [4] ($E_x &lt; 12$ MeV)</td>
<td>3.34</td>
<td>9.70</td>
<td>15.00</td>
<td></td>
</tr>
<tr>
<td>present ($E_x &lt; 12$ MeV)</td>
<td>2.69</td>
<td>8.60</td>
<td>14.67</td>
<td></td>
</tr>
<tr>
<td>ref. [4] ($E_x &lt; 18$ MeV)</td>
<td>3.79</td>
<td>10.76</td>
<td>16.03</td>
<td></td>
</tr>
<tr>
<td>present ($E_x &lt; 18$ MeV)</td>
<td>2.69</td>
<td>8.60</td>
<td>14.67</td>
<td></td>
</tr>
</tbody>
</table>

We show in table 1 the results of a full $\hbar\omega$ $1p$-$1h$ configuration space calculation based on the Cohen-Kurath wave function for the $A=11$ system in comparison with the result of ref. [4]. A Yukawa-type potential with central, spin-orbit and tensor components is used for the present calculation;

\[
V(r) = \frac{V_c(r) + V_{LS}(r) + V_T(r)}{\mu_c/r} \\
V_c(r) = V_c(a_{11}p_{11} + a_{31}p_{31} + a_{13}p_{13} + a_{33}p_{33}) \\
\times (\mu_c/r) \exp(-r/\mu_c) \\
V_{LS}(r) = V_{LS}(a_{13}p_{13} + a_{33}p_{33}) \cdot S(\mu_{LS}/r) \\
\times \exp(-r/\mu_{LS}) \\
V_T(r) = V_T(a_{13}p_{13} + a_{33}p_{33})S_{12}(\mu_T/r) \\
\times \exp(-r/\mu_T),
\]

where $P_{2T+1,2S+1}$ is the projection operator of the $(T, S)$ channel and $S_{12}$ is the tensor operator. We adopted the parameter set of Millener and Kurath [5] which is listed in table 2. The summed cross section $d\sigma/d\Omega$ ($E_x = 2-12$ MeV) for the present calculation is $19$ mb/sr compared to $20$ mb/sr obtained in the restricted space calculation [4]. Both calculations show about 10% of the total transition strengths in the energy region $E_x = 12-18$ MeV. However, the full space calculation shows 15% of the total strength above $E_x = 18$ MeV, while there is no transition strength in this region in the calculation of ref. [4].

We study in this letter the effect of $2p$-$2h$ ground-state correlations on the spin-dipole transitions in $^{12}\text{N}$ using perturbation theory. The $2p$-$2h$ correlation is claimed as an important effect for the quenching of magnetic dipole and Gamow–Teller transition strengths in recent microscopic calculations [6,7]. The ground-state correlation is shown diagrammatically in fig. 1. The first-order perturbation theory gives the following wave function;

\[
\langle \phi \rangle = \langle \phi \rangle + \sum \frac{\langle (p'h^{-1})J, (p'h^{-1})J; 0^+|V|\phi \rangle}{E_0 - E_j(p'h)} \\
\times \langle (p'h^{-1})J, (p'h^{-1})J; 0^+ \rangle.
\]

Using this perturbed wave function (6), we obtain the modified transition strength,

\[
\langle (p'h^{-1})\lambda ||T_\lambda||\phi \rangle = \langle (p'h^{-1})\lambda ||T_\lambda||\phi \rangle(1 - \alpha).
\]
where

$$\alpha = \sum_{p'h'} \frac{\langle \langle ph^{-1} \rangle \lambda | V'(p'h'^{-1} \rangle \lambda \rangle}{E_\lambda (ph, p'h') - E_0} \langle \langle p'h'^{-1} \rangle \lambda | T_{\lambda} || 0 \rangle \rangle. \tag{8}$$

We calculate the particle–hole (p–h) matrix elements with the Yukawa-type potential with central, spin–orbit and tensor components as given in eq. (5). The p–h energy difference is taken to be 15.4 MeV. The oscillator length is determined as \( b = 1.64 \text{ fm} \) in order to reproduce the mean charge radius of \(^{12}\text{C} \). The calculated values of \( \alpha \) are given in table 3.

The central interaction gives \( \alpha = 0.13 \) which decreases the spin-dipole transition strength by about 25%. This positive value of \( \alpha \) is due to the fact that the spin–isospin p–h interaction is repulsive. We also calculated \( \alpha \) by using the parameter set for the central interaction given by Ferrell and Visscher \[8\]. This gave a slightly larger value of \( \alpha = 0.15 \).

The tensor correlation increases the values of \( \alpha \) for the state with \( \lambda' = 1 \) by 10–20%, while decreasing the value of \( \alpha \) for \( \lambda' = 0 \) by about the same amount. The transition strength of the \( 2^- \) state is not changed much by the tensor correlation. This result can be understood intuitively by the following argument given by Mottelson \[9\].

The tensor interaction can be given in the form

$$V_T(r) = F(r) \sum_\lambda \left( \begin{array}{c} \lambda' \\ \lambda' \end{array} \right) \frac{\sqrt{4\pi}}{4} \left( \frac{2\sqrt{5}}{\sqrt{15}} \right) \left\{ \begin{array}{c} 2^- \\ 1^- \end{array} \right\} \left( \begin{array}{c} \lambda \\ \lambda' \end{array} \right), \tag{11}$$

where we choose \( \lambda' = 1 \) only in the expansion (10). Since the coupling strength \( F(r) \) is repulsive for the tensor interaction, the tensor correlation for the \( 1^- \) state is additive to that of the central interaction. On the other hand, the two contributions tend to cancel each other in the case of the \( 0^- \) state. A smaller tensor correlation for the \( 2^- \)

### Table 3
Normalization factors \( \alpha \) for spin dipole transitions.

<table>
<thead>
<tr>
<th>( \lambda' )</th>
<th>(p–h)</th>
<th>( \langle h^{-1} p \lambda | T | 0 \rangle )</th>
<th>( \alpha(V_C) )</th>
<th>( \alpha(V_C + V_T) )</th>
<th>( \alpha(V_C + V_T + V_{LS}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0^- )</td>
<td>((2s_{1/2}, 1p_{1/2}^{-1}))</td>
<td>-0.654</td>
<td>0.093</td>
<td>0.077</td>
<td>0.077</td>
</tr>
<tr>
<td></td>
<td>((1d_{3/2}, 1p_{3/2}^{-1}))</td>
<td>1.463</td>
<td>0.135</td>
<td>0.101</td>
<td>0.116</td>
</tr>
<tr>
<td>( 1^- )</td>
<td>((1d_{3/2}, 1p_{1/2}^{-1}))</td>
<td>1.035</td>
<td>0.135</td>
<td>0.154</td>
<td>0.147</td>
</tr>
<tr>
<td></td>
<td>((2s_{1/2}, 1p_{1/2}^{-1}))</td>
<td>0.925</td>
<td>0.135</td>
<td>0.154</td>
<td>0.147</td>
</tr>
<tr>
<td></td>
<td>((1d_{3/2}, 1p_{3/2}^{-1}))</td>
<td>1.851</td>
<td>0.135</td>
<td>0.156</td>
<td>0.180</td>
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<tr>
<td></td>
<td>((1d_{5/2}, 1p_{3/2}^{-1}))</td>
<td>-1.388</td>
<td>0.135</td>
<td>0.156</td>
<td>0.180</td>
</tr>
<tr>
<td></td>
<td>((2s_{1/2}, 1p_{3/2}^{-1}))</td>
<td>0.654</td>
<td>0.135</td>
<td>0.156</td>
<td>0.180</td>
</tr>
<tr>
<td>( 2^- )</td>
<td>((1d_{3/2}, 1p_{1/2}^{-1}))</td>
<td>-0.463</td>
<td>0.135</td>
<td>0.085</td>
<td>0.089</td>
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<tr>
<td></td>
<td>((1d_{5/2}, 1p_{1/2}^{-1}))</td>
<td>1.035</td>
<td>0.135</td>
<td>0.133</td>
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<tr>
<td></td>
<td>((1d_{3/2}, 1p_{3/2}^{-1}))</td>
<td>2.266</td>
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<td>0.133</td>
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<td></td>
<td>((1d_{5/2}, 1p_{3/2}^{-1}))</td>
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<td>0.145</td>
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<td>((2s_{1/2}, 1p_{3/2}^{-1}))</td>
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<td>0.129</td>
<td>0.127</td>
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<td></td>
<td>((2s_{1/2}, 1p_{3/2}^{-1}))</td>
<td>1.463</td>
<td>0.135</td>
<td>0.127</td>
<td>0.127</td>
</tr>
</tbody>
</table>
The state is also quite reasonable since the coefficient for $2^-$ in eq. (11) is several times smaller than those for $0^-$ and $1^-$. The results given in table 3 are slightly different from that expected from eq. (11) because of the exchange term contribution.

As can be seen in the last column of table 3, the spin–orbit two-body force has an appreciable effect only on $1^-$ states. The spin–orbit two-body operator is given by

$$L \cdot S = \frac{1}{2}(r_1 - r_2) \times (p_1 - p_2) \cdot (s_1 + s_2).$$

(12)

When eq. (12) is expanded in terms of the spin-dipole operators, the important terms for the direct two-body matrix elements in eq. (8) are

$$L \cdot S \propto r_1 \left\{ \left[ s_1 \times Y_1(\hat{r}_1) \right]^{(1)} \times P_2 \right\}^{(0)} + r_2 \left\{ \left[ s_2 \times Y_1(\hat{r}_2) \right] \times P_1 \right\}^{(0)},$$

(13)

where the spin-dipole operator can couple to $\lambda^\pi = 1^-$ only. This is the reason why the spin–orbit force contributes appreciably to the value of $\alpha$ for the states with $\lambda^\pi = 1^-$, but not for the states with $\lambda^\pi = 0^-$ and $2^-$. In summary, we have studied the effect of the $2p-2h$ ground state correlation on the spin-dipole transition in $^{12}$N. We found that the net effect of the central, tensor and spin–orbit forces, averaging over the transition matrix elements, gives the values $\alpha = 0.089, 0.139$ and $0.122$ for $\lambda^\pi = 0^-$, $1^-$ and $2^-$, respectively. Because the experimental data in the region $E_x = 2–12$ MeV is dominated by $1^-$ and $2^-$ resonances, the value $\alpha$ substantially decreases the spin-dipole transition strength by about 25%. The tensor correlation contributes 10–20% to the renormalization factors $\alpha$ for $0^-$ and $1^-$ states, while the $2^-$ state is insensitive to the tensor force, the effect of the two-body spin–orbit correlation is appreciable only in the transition strength of the $1^-$ state. The experimental cross section in the energy region $E_x = 2–12$ MeV is 40% less than the $1p-1h$ shell-model prediction in the full configuration space and the present study suggests that the $2p-2h$ ground-state correlations explain the major part (60%) of this missing strength. The transition strengths might be decreased further by meson-exchange currents [7] and $\Delta$-hole couplings [10] together with higher $2p-2h$ excitations above $1h\omega$ configuration space [5].

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References