${}^{36,38}\text{Ar}(d,\alpha){}^{34,36}\text{Cl}$ reactions induced by tensor polarized deuterons

C. M. Bhat, Y. Tagishi,* and E. J. Ludwig
University of North Carolina at Chapel Hill, Chapel Hill, North Carolina 27514
and Triangle Universities Nuclear Laboratory, Durham, North Carolina 27706

B. A. Brown
Michigan State University, East Lansing, Michigan 48824
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Exact finite-range distorted-wave Born approximation analyses of cross section and vector and tensor analyzing power angular distributions for some $^{36,38}\text{Ar}(d,\alpha)$ transitions induced by 16 MeV polarized deuterons have been performed using full sd-shell model wave functions and including a $D$-state amplitude for the $\alpha$ particle. The $L$-mixing ratios $R$ for several unnatural parity transitions have been determined from comparisons of the calculations with the tensor analyzing power data. Shell-model calculations are found to be in agreement with the experimentally obtained phases of $R$, while the predicted magnitudes agree well in some cases.

It has been shown$^1$ recently that the sensitivity of the tensor analyzing powers (TAP's) for the two-particle components of the wave functions of the target nucleus is a unique feature of the direct two-nucleon spin-1 transfer reactions. Hence, the TAP measurements can be used effectively in nuclear structure investigations. Several other studies of the TAP observables in $(d,\alpha)$ reactions were primarily directed to study the structure of the $\alpha$ particle.$^{2,3}$ A quantitative determination of the two-nucleon spectroscopic amplitudes with their respective phases, however, is interesting from the point of view of nuclear structure calculations. In this paper, we present a systematic nuclear structure investigation using the TAP observables to deduce properties of the strongly excited low-lying levels of $^{34}\text{Cl}$ and $^{36}\text{Cl}$, via $^{36,38}\text{Ar}(d,\alpha)$ reactions.$^4$ The measurements have been made using the vector- and tensor-polarized 16 MeV deuteron beam available at the Triangle Universities Nuclear Laboratory. A previous analysis$^5$ of the data using the zero-range DWBA code DWUCK2,$^5$ and assuming only a single prominent $L$ transfer, failed to reproduce the shapes of the TAP distributions. In the present analysis, the results of the full sd-shell model calculations which predict amplitudes and phases for allowed values of $L$ have been used. The $D$-state effect of the $\alpha$ particle has also been included in the exact finite-range calculations.

In direct one-step $(d,\alpha)$ transitions, the two-particle structure of the target nucleus ($A$) enters into the DWBA transition amplitude through a double parentage expansion$^6,7$ of the wave function for $A$.

$$\Phi_A(J_A,M_A) = \sum_{\eta,JT} \beta(\eta,JT) \Phi_B(J_B,M_B) \Phi(\eta,JT).$$

Here, the $\Phi_B$ and $\Phi$ are the residual nucleus and the two-particle (neutron-proton) wave functions, respectively, and the $\beta$ is a generalized two-nucleon spectroscopic amplitude for a two-particle configuration $\eta$ having spin $J$ and isospin $T$. In a cluster model of two-nucleon transfer reactions,$^8,9$ the amplitude $G(L,J)$ for the allowed $L$ and $J$ transfer will depend upon $\beta$ by

$$G(L,J) = \sum_\eta \beta(\eta,JT) G_{LJ}(\eta).$$

Here the $G_{LJ}(\eta)$ are the structure amplitudes defined by the product of symmetrized LS-JJ coupling coefficients and a Talmi-Moshinsky bracket. A significant advance in the shell-model calculations with a mass-dependent effective interaction which varies across the shell-model space$^8$ has enabled the prediction of improved two-nucleon spectroscopic amplitudes $\beta$ for sd-shell nuclei. Table I gives the predicted values of $\beta$ and the $L$-mixing ratios

$$R = G(L = J - 1, J)/G(L = J + 1, J)$$

for $(d,\alpha)$ transitions to some of the low-lying levels of $^{34}\text{Cl}$ and $^{36}\text{Cl}$ investigated here.

The structure of the $\alpha$ particle in the $(d,\alpha)$ reaction appears in the transition amplitude through the overlap$^2,9$

$$\langle \phi_d, \phi_x \mid \phi_\alpha \rangle = \frac{1}{2} \sum_{L' = 0,2} (L'M'|1\sigma_x \mid 1 - \sigma_d) \times u_{L'}(r) Y_L^M(\hat{r}),$$

where $\phi_d$, $\phi_x$, and $\phi_\alpha$ are some normalized internal wave functions for the incident deuteron, the transferred cluster, and the $\alpha$ particle, respectively, while $u_{L'}(r)$ is the radial wave function of the deuteron in the $\alpha$ particle. A parameter $D_2$, which has been extracted from previous analyses of the $(d,\alpha)$ reactions,$^2$ is a measure of the asymptotic $L' = 2$ component in the $\alpha$ particle.$^9$

Exact finite-range DWBA calculations have been carried out using the DWBA code TWFNR.$^{10}$ A deuteron cluster pickup is assumed throughout the analyses. The radial wave functions of deuterons in the $\alpha$ particle and in the target nucleus have been generated using a Woods-Saxon central potential consistent with their separation energies. The interaction that induces the transfer is taken to be a function of the bound state variables of the cluster $x$ in the $\alpha$ particle. The deuterons and $\alpha$ particle optical model parameters have been taken from Ref. 11 and Ref. 12, respectively.

Recently, a range of radius ($r_0$) and diffuseness ($a_0$)
parameters which provides a matching of two-nucleon microscopic and cluster form factors have been suggested.\textsuperscript{13} Zero-range DWBA calculations of differential cross sections were made using cluster form factors with $r_0$ and $d_0$ values within this range and shown to compare favorably to analogous calculations using microscopic form factors. We have extended this study by examining the sensitivity of the vector analyzing power (VAP) and TAP as well as the differential cross sections to the bound-cluster geometry parameters. It is found that only the magnitude of the differential cross section is highly sensitive to small variations in $r_0$ and $a_0$, while the predicted VAP and TAP are less sensitive to this variation. In the present analysis we use $r_0=1.25$ fm and $a_0=0.75$ fm.

The sensitivity of the TAP observables on $D_2$ has been studied previously\textsuperscript{12,13} using unnatural parity transitions in ($\bar{d},\alpha$) reactions. In some cases, ambiguities in the best fit values of $D_2$ were observed\textsuperscript{13} to be rather serious due to the uncertainties in the $L$ mixing. In the present work the $D_2$ parameter has been varied in the range $-0.3 \leq D_2 \leq 0.0$ fm\textsuperscript{2}. Figure 1(a) shows the results of DWBA calculations made using the predicted $G(L,J)$ for the 0.46 MeV (1\textsuperscript{+}) level in $^{34}$Cl. This level is known to show characteristic $L=0+2$ transfer with a dominant $L=2$ component (see Table I). We find that the calculated TAP's, in particular $A_{yy}$, are not sensitive to the $D_2$ parameter in the $\alpha$ particle. This is in agreement with estimates based on the peripheral model\textsuperscript{9} which predicts that the magnitudes of $A_{yy}$ for unnatural parity ($\bar{d},\alpha$) transitions having a dominant $L=J+1$ transfer will be relatively large without including $D$-state effects of the $\alpha$ particle and hence less sensitive to the $D$-state contributions. For natural parity ($\bar{d},\alpha$) transitions from a target with $J^\pi=0^+$, there is a unique $L$ transfer; hence the TAP measurements on such a transition are interesting from the point of view of

![FIG. 1. Angular distributions of $A_{xx}$ and $A_{yy}$ for (a) the 0.46-MeV 1\textsuperscript{+} transition and (b) the 1.89-MeV 2\textsuperscript{+} transition in $^{36}$Ar ($\bar{d},\alpha$)$^{34}$Cl reaction at 16 MeV. The curves are results of exact finite range DWBA calculations for (i) the pure $S$ state of the $\alpha$ particle (dotted curve), (ii) the $S+D$ states of the $\alpha$ particle with $D_2=-0.3$ fm\textsuperscript{2} (dashed curve), and (iii) $S+D$ states of $\alpha$ particle with $D_2=-0.2$ fm\textsuperscript{2} (solid curve). All these curves have been obtained by using $G(L,J)$ derived from full $sd$-shell model calculations.](image-url)
studying the $D$-state effect in the $\alpha$ particle. In the present case, the 1.89 MeV ($2^+$) level in $^{34}$Cl is found to be excited fairly strongly and has a characteristic $L=2$ shape for the angular distribution of differential cross section. The calculations for this state show a sensitivity to the effect of the $D$ state of the $\alpha$ particle, especially for $A_{xx}$, and suggests a value of $D_2 = -0.20 \pm 0.05$ fm$^2$ to provide an improved comparison to the observed $A_{xx}$. We find the $A_{yy}$ angular distribution is less sensitive to the $D_2$ parameter for this transition [see Fig. 1(b)]. Previously, a number of attempts had been made to estimate $D_2$ values both by theoretical$^9$ and by semiempirical methods.$^{2,3}$ A recent theoretical$^{14}$ estimate of $D_2$ based on a realistic Hamiltonian that includes three-nucleon interaction is $-0.24$ fm$^2$, which is in good agreement with the value suggested from our study.

Figure 2 shows the results of DWBA analyses for lying unnatural parity transitions in the $^{36,38}$Ar($d,\alpha$) reaction. The dashed curves are obtained with the amplitudes $G(L,J)$ derived from the full sd-shell model calculations, and $D_2 = -0.2$ fm$^2$ as indicated by the analysis of the data for the 1.89 MeV level in $^{36}$Cl and the theoretical estimates.$^{14}$ The shapes of the angular distributions of differential cross sections are well reproduced in all cases. The angular distributions of VAP, $A_y$ (not shown in the figure), are also well described. However, the comparisons of the calculations to the measured TAP's, $A_{xx}$ and $A_{yy}$, differ significantly in some cases. Although the shape of the predicted angular distribution of the differential cross sections and the VAP are not sensitive to the small variations in the magnitudes and the signs of $R$, the $A_{xx}$ and $A_{yy}$ are found to be sensitive to $R$. In this work we have treated $R$ as a free parameter and searched for the best fit values for each transition. For $^{36}$Ar($d,\alpha$) transitions, the best fit could be obtained by varying $R$ slightly around the shell-model predictions. Larger variations in $R$ were necessary for transitions in the $^{38}$Ar($d,\alpha$) reaction. The solid curves in Fig. 2 are the results of DWBA calculations made with the best fit $L$-mixing ratios given in Table I. In the present study, we find that the $L=J+1$ component is dominant in all unnatural parity transitions and the relative phases between $L=J+1$ and $L=J-1$ components are very well reproduced by the full sd-shell model calculations. However, the overall fit to the $A_{xx}$ and $A_{yy}$ for the 1.16 MeV transition in the $^{38}$Ar($d,\alpha$) reaction remains poor. The ($d,\alpha$) transition strength in this case is smaller than the strength observed for the other unnatural parity transitions studied here and hence it may be necessary to consider higher-ordered processes$^{15}$ in this case.

In summary, we have performed an analysis of the relatively strong $^{36,38}$Ar($d,\alpha$) transitions assuming only a one-step process and demonstrated that the TAP observables can be used to test the shell-model predictions of two-nucleon cluster spectroscopic amplitudes. We find that all unnatural parity transitions show a dominant $L=J+1$ component and hence the $A_{xx}$ and $A_{yy}$ are found to be less sensitive to the $D_2$ parameter of the $\alpha$ particle than for previously studied $L=J-1$ transitions.$^{1}$ The predicted TAP's for the natural parity transitions, however, show some sensitivity to the $D_2$ parameter, and suggests their usefulness in determining a $D_2$ value without the previously observed ambiguities since they occur with a unique $L$. The $L$-mixing ratios for several unnatural parity transitions have been deduced and compared with the shell-model calculations. A good agreement is obtained for the $L$-mixing ratios, $R$, in the case of $^{36}$Ar($d,\alpha$) data. The extracted values of $R$ for the $^{38}$Ar($d,\alpha$) reaction, however, differ considerably. In all cases the predicted phases for $L$-mixing ratios in unnatur-
al parity transitions are in agreement with the present determinations.

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*Present address: Tandem Accelerator Center, University of Tsukuba, Ibaraki 305, Japan.

5P. D. Kunz, DWBA code DWUCK2 (unpublished).
10M. Toyama and M. Igarashi, TWOFNR (unpublished).