Is there large weak mixing in heavy nuclei?

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Recent results in parity nonconserving neutron-nucleus scattering show a tendency for the parity nonconserving effects to predominantly have the same sign. Explanations of this phenomenon all require a weak matrix element between single-particle levels in heavy nuclei to be $\sim 100$ eV. This paper contains a discussion of this effect and how the same phenomenon will manifest itself in other systems. The gamma decay of $^{207}$Pb is given as an example of a system where the weak matrix element between single-particle levels in a heavy nucleus can be measured directly.

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I. INTRODUCTION

During the last few years a number of measurements of parity nonconserving (PNC) longitudinal asymmetries in the compound nucleus have been published [1-9]. Resonances were formed by scattering longitudinally polarized epithermal (1–1000 eV) neutrons from nuclear targets. The longitudinal asymmetry is the fractional difference of the resonance cross sections for positive and negative helicities, $\sigma_+$ and $\sigma_-$. The asymmetry for a given $p$-wave resonance $\mu$ may be expressed as a perturbation series in the weak interaction. The term in this series that is first order in the mixing matrix-element, $V_{\nu\mu}$ is [10]:

$$A_\mu = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = 2 \sum \frac{V_{\nu\mu}}{E_\nu - E_\mu} \frac{g_\nu}{g_\mu}. \quad (1)$$

In this expression for $A_\mu$, $V_{\nu\mu}$ is the matrix element of the weak interaction between the strong interaction eigenstates $\mu$ and $\nu$, $E_\nu$ and $E_\mu$ are their energies, and $g_\nu$ and $g_\mu$ are their neutron decay amplitudes. According to the statistical model of the compound nucleus $V_{\nu\mu}$, $g_\mu$, and $g_\nu$ behave as independent random variables and have random signs. The energy differences $E_\nu - E_\mu$ can be positive and negative. It thus came as a surprise when Frankle et al. [6,7] reported that all seven statistically-significant asymmetries measured for $^{232}$Th had positive signs. Of the statistically-significant asymmetries measured in other nuclei, four of six were positive. The fact that the signs of the measured asymmetries are predominantly positive is in contrast to expectations (based on compound nuclear models) that the average asymmetry be zero. The average asymmetry can be estimated as the average of the measured asymmetries multiplied by $\sqrt{E_\mu}$ to remove the threshold dependence of $A_\mu$. The result for $^{232}$Th is: $A_\mu \sqrt{E_\mu / 1 \text{ eV}} = 34 \pm 12\%$.

The phenomenon of the measured asymmetries having a common sign is the subject of a number of calculations [11–15]. These calculations all require a weak interaction strength two orders of magnitude larger than estimated, for example, by comparing the strength of the weak and strong coupling constants. The treatment of Bowman et al. [11] illustrates this point. The average asymmetry is written as:

$$A_\mu \sqrt{E_\mu / 1 \text{ eV}} = 4 \frac{g_\nu^0(1 \text{ eV})}{g_\mu^0(1 \text{ eV})} \frac{V_{\nu\mu}}{\hbar \omega}, \quad (2)$$

where $g_\nu^0(1 \text{ eV})$ and $g_\mu^0(1 \text{ eV})$ are the $p$- and $s$-wave scattering amplitudes for single-particle configurations evaluated at 1 eV neutron energy, $V_{\nu\mu}$ is the matrix element of the weak interaction between 5$s_1/2$ and 4$p_1/2$ neutron single-particle configurations of $^{233}$Th and $\hbar \omega \approx 7$ MeV is the spacing between single-particle configurations. Solving for $V_{\nu\mu}$ yields $V_{\nu\mu} \sim 100$ eV, because the ratio $g_\nu^0(1 \text{ eV})/g_\mu^0(1 \text{ eV})$ is typically 1000. Resonances measured with targets in the mass range of 230 are based on the 4$p_1/2$ single-particle configuration. Equation (2) is interpreted physically as the parity nonconserving longitudinal asymmetry arising from the admixture of the 4$p_1/2$ single-particle configuration with opposite-parity 4 and 5$s_1/2$ single-particle configurations. The 4 and 5$s_1/2$ configurations are located in energy 1$\hbar \omega$ below and above the 4$p_1/2$ configuration, respectively.

In this paper it is argued that a matrix element $V_{\nu\mu}$ produces measurable effects in gamma decay asymmetries between single-particle states in heavy nuclei. In particular, if the matrix elements are as large as 100 eV, measurements could be accomplished on short time scales. The feasibility of such experiments is analyzed.

II. STATES IN $^{207}$Pb

There are several necessary criteria for an experiment to measure neutron single-particle weak matrix elements of approximately 100 eV in heavy nuclei. The weak interaction induces mixing between states with the same angular momentum and opposite parity. The parity mixing results in a pseudoscalar observable in decay products of
the nucleus in question. The nucleus studied should have a mass approximately the same as for $^{232}$Th and $^{238}$U. The transition involved should be between good neutron single-particle states. The best single-particle neutron states occur in the nuclei $^{209}$Pb or $^{207}$Pb, which differ from the doubly-closed shell nucleus $^{208}$Pb by the addition or removal of one neutron. Because good shell model states are sought, the admixed opposite-parity state cannot be close in energy, but is $1\hbar \omega$ away from the normal parity state. If possible, the $\gamma$ transition to the opposite-parity component should have a larger transition amplitude than to the normal component, so that the size of the parity nonconserving asymmetries will be enhanced.

An example of a transition that fits these criteria is the $M4$ 1064 keV gamma transition in $^{207}$Pb. The energy level diagram of $^{207}$Pb is shown in Fig. 1. The 1064 keV gamma ray is emitted in the transition from the $I_{13/2}^+$ second excited state to the $2f_{5/2}^-$ first excited state. Both of these states have a strong single-particle structure, as evidenced by one-neutron pickup data from $^{208}$Pb (see, for example, Table I in Ref. [16]). One major-shell spacing higher in energy, there is a $3d_{5/2}^+$ single-particle configuration (off the top of the scale in the figure). The weak interaction will induce an opposite-parity admixture into the $2f_{5/2}^-$ state from the $3d_{5/2}^+$ strength, which is located $1\hbar \omega$ higher in energy. The admixed $5/2^+$ strength opens up an $E4$ component to the 1064 line. Note that the $d$-$f$ mixing considered in this paper can be related in a simple way to the $s$-$p$ mixing observed in the neutron-nucleus experiments by using a shell model with harmonic oscillator wave functions.

The only PNC matrix element considered in this paper, $\langle 3d_{5/2}^+ | V_{\text{weak}} | 2f_{5/2}^- \rangle$, is between single-hole states. There are other $5/2^+$ states in $^{207}$Pb, starting at 2.2 MeV excitation energy. The lowest of these have the structure $| (j^{-1} \otimes 3^-) \frac{5}{2}^+ \rangle$, where $j = 3p_{1/2}$, $2f_{5/2}$, or $3p_{3/2}$ and $3^-$ is the low-lying octupole vibration of $^{208}$Pb. The parity nonconserving mixture of this 2-particle–1-hole state with the $3d_{5/2}^+$ single-hole state is small. In addition, the $E4$ matrix element $| (1\frac{13}{2}^+ | E4 | (j^{-1} \otimes 3^-) \frac{5}{2}^+ \rangle$ is zero. Likewise, the higher-lying $5/2^+$, 2-particle–1-hole states with strong $E4$ matrix elements will also have a small mixing with the $3d_{5/2}^+$ single-hole state.

We have carried out estimates of the PNC matrix element within the framework of the weak meson-exchange potential. Matrix elements of the meson-exchange potential are dominated by the one-body part of the two-body meson-exchange potential. The one-body part originates from the two-body potential coherently summed over semidiagonal diagrams. We have calculated the direct and exchange terms in this summation for the $\pi$ and $\rho$ exchange parts of the "best" PNC meson-nucleon couplings, $f_\pi$ and $h_\rho^o$, respectively, of Desplanque, Donoghue, and Holstein (DDH) [16]. The result for the PNC matrix element is $| \langle 3d_{5/2}^+ | V_{\text{weak}} | 2f_{5/2}^- \rangle | = | 1.9 - 2.9 | eV = 1.0 eV$ for the neutron-hole single-particle PNC matrix element, where the first term is due to $\rho$ exchange and the second term is due to $\pi$ exchange. Oscillator radial wave functions were used and the summation was carried out over all orbitals up to the $3p_{1/2}$. The value of the matrix element obtained for other values of the coupling constants $f_\pi$ and $h_\rho^o$ can be obtained by scaling the results above for the "best" DDH values. For example, if we take the value of $f_\pi$ as 1/4 of the DDH value, as suggested by the $^{18}$F experiment [17], then $| \langle 3d_{5/2}^+ | V_{\text{weak}} | 2f_{5/2}^- \rangle | = | 1.9 - 0.7 \rangle eV = 1.2 eV$.

Another calculation of the matrix element $| \langle 3d_{5/2}^+ | V_{\text{weak}} | 2f_{5/2}^- \rangle |$ has been done by Horowitz and Yilmaz [18] in a relativistic Hartree-Fock model. Using the "best" DDH values for $f_\pi$ and $h_\rho^o$, they find $| \langle 3d_{5/2}^+ | V_{\text{weak}} | 2f_{5/2}^- \rangle | = | 1.591 - 1.748 + 0.109 | = 0.049 eV$ in the relativistic model, where the first term is for $\rho$ exchange, the second for $\pi$ exchange and the third for $\omega$ exchange. Their accompanying nonrelativistic calculation gives the result $| \langle 3d_{5/2}^+ | V_{\text{weak}} | 2f_{5/2}^- \rangle | = | 1.657 - 2.126 + 0.073 | = 0.396 eV$, for $\rho$, $\pi$, and $\omega$ exchange, respectively.

There are two ways that the weak single-particle mixing in the $5/2^-$ state could be measured. The interference of the $E4$ and $M4$ strengths either induces a circular polarization of the 1064 keV gamma line from an unpolarized nucleus, or induces a forward-backward asymmetry of the gamma-ray angular distribution from polarized $^{207}$Pb nuclei. The circular polarization is

$$P_\gamma = \frac{2}{E_d - E_f} \times \left| \langle 3d_{5/2}^+ | V_{\text{weak}} | 2f_{5/2}^- \rangle \right|^2 \times \left| \langle 1\frac{13}{2}^+ | E4 | 3d_{5/2}^+ \rangle \right|^2 \times \left| \langle 1\frac{13}{2}^+ | M4 | 2f_{5/2}^- \rangle \right|^2.$$  

(3)

The expression for the forward-backward asymmetry from polarized $^{207}$Pb decay is similar to the expression for $P_\gamma$.

The normal parity $M4$ transition rate is known from the measured lifetime of the metastable $1i_{13/2}^-$ level. The ratio $| E4 | / | M4 |$, where $| E4 | = | 1\frac{13}{2}^+ | E4 | 3d_{5/2}^+ \rangle$ and $| M4 | = | 1\frac{13}{2}^+ | M4 | 2f_{5/2}^- \rangle$, can be found in two ways. The Weisskopf estimate for the $E4$ gamma decay
amplitude is
\[ \langle E4 \rangle \propto \sqrt{\Gamma(E4)} = \sqrt{1.1 \times 10^{-5} A^{6/3} E_0^9} = 28.83/s, \]
and the ratio of decay amplitudes is:
\[ \frac{\langle E4 \rangle}{\langle M4 \rangle} = \sqrt{\frac{\Gamma(E4)_{\text{eiss}}}{\Gamma(M4)_{\text{Exp}}}} = \sqrt{0.775} = 6. \]

Another estimate for \( E4 \) can be obtained from a shell-model calculation [19]. The results are
\[ \frac{\langle E4 \rangle}{\langle M4 \rangle} = \frac{\Gamma(E4)_{\text{sm}}}{\Gamma(M4)_{\text{Exp}}} = \frac{56.6s^{-1}}{0.775s^{-1}} = 8.5, \]
where \( \Gamma(M4) \) is from experiment and \( \Gamma(E4) \) is from the shell-model calculation. The shell-model calculation is based on Woods-Saxon radial wave functions and used a value of 1e for the effective neutron charge. The neutron effective charge was obtained by comparing the calculated \( E4 \) strength of the \( 0^+ \) to \( 4^+ \) transition in \( ^{206}\text{Pb} \) to the experimental value of \( 1.7 \times 10^8 \text{e}^2\text{fm}^6 \) [20]. The shell-model estimate of \( \sim 10 \) is used in the rest of this paper.

III. EXPERIMENTAL SENSITIVITY

This section contains an estimate of the sensitivity for two techniques for measuring the weak single-particle mixing. First, consider a measurement of the circular polarization of the 1064 keV line. The required components are a \( ^{207}\text{Bi} \) source, a gamma polarimeter, fast gamma detectors, and a fast data-acquisition system. The circular polarization sensitivity of a polarimeter made of the iron-cobalt alloy Permedur is 1.6% at 1 MeV, for a 7.2 cm length [17]. For a two interaction length gamma polarimeter, a 1 mCi source produces a count rate of \( \sim 500 \) kHz in the 1064 keV line (including the effect of the 84% branch to the 13/2\(^+ \) state in the \( ^{207}\text{Bi} \) decay). The circu-
lar polarization sensitivity for such a polarimeter would be \( \sim 1\% \). At such high count rates the gamma flux must be measured with very fast detectors, such as intrinsic CsI crystals, and recorded with a very fast multichannel analyzer system. For a \( 3 \times 10^6 \) s run (\( \sim 35 \) days), the potential sensitivity of a circular polarization measurement would be \( \sim 13 \text{ eV} \).

A measurement of the forward-backward asymmetry requires the following components: a dilution refrigerator to cool the source below \( \sim 10 \) mK, two Ge detectors to be placed at 0° and 180° with respect to the source polarization direction, and a \( ^{207}\text{Bi} \) source implanted in an Fe or Ni crystal. The \( ^{207}\text{Bi} \) nuclei are polarized by the large hyperfine field within the Fe or Ni crystal. The experimental observable is the forward-backward asymmetry of emitted gamma rays. Assuming a 10 \( \mu\text{Ci} \) \( ^{207}\text{Bi} \) source and \( \sim 50\% \) efficient Ge crystals, a sensitivity of \( \sim 5 \text{ eV} \) could be achieved with a 30 day run.

Both of these experiments share some sources of systematic error (e.g., field effects on the source and detectors). There are also unique systematic effects for each of the two experiments. All of these systematic effects require in-depth study before an experiment can be performed.

IV. CONCLUSIONS

We conclude that there is a need for a better measurement for single-particle weak mixing matrix elements in heavy nuclei. The transition considered in \( ^{207}\text{Pb} \) is sensitive to the weak mixing between single-particle states. The interpretation of experimental results could be done with a shell-model calculation in \( ^{207}\text{Pb} \). Even if the weak mixing matrix element is not as large as 100 eV, the levels considered in \( ^{207}\text{Pb} \) give a unique opportunity to measure a PNC single-particle weak matrix element in a heavy nucleus.

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