Absence of M3 Quenching in $^{26}$Mg

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In a high-resolution electron scattering experiment ten $3^+$ states in $^{26}$Mg have been identified, increasing the world data on M3 transitions severalfold. Differential cross sections have been measured and magnetization current densities and form factors have been deduced. It is found that while the level of agreement of the $B(M3)$ values with shell-model predictions varies from state to state, the summed $B(M3)$ strengths are in good agreement. No overall quenching of the M3 strength is found, and it is concluded that no renormalization of the free nucleon g factors is needed.

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The possible quenching of magnetic transitions in nuclei has been the subject of interest for a long time. Great interest was attached to the quenching of M1 and Gamow-Teller transitions because of the possible importance of delta-hole excitations. Interest in the M3 transitions naturally followed because “the M3 operator is crudely $E2 \times M1$” [1]. Indeed, Hynes et al. [2] reported large (a factor of approximately 3) quenching of the M3 multipole (relative to single particle model predictions) in a multipole analysis of magnetic elastic scattering from $^{17}$O. Subsequently, similar large M3 quenching was observed in several other nuclei of the 1s-0d shell [3]. Many possible sources for the effect were investigated, ranging from configuration mixing and meson exchange currents to delta-hole excitation. However, through all the excitement the experimental basis remained essentially confined to multipole analysis of magnetic elastic scattering of electrons, in which generally several multipoles are simultaneously involved (e.g., $M1$, $M3$, and $M5$ for $^{17}$O), and one must “peel off” the other multipoles in order to reveal the character of the M3 multipole. This unpeeling process is far from unique. As a result, while it is generally agreed that a major fraction of the observed M3 quenching arises from configuration mixing, it has not been possible to study it in a quantitative way by the analysis of magnetic elastic scattering. Clearly, the best way to study the possible quenching of the M3 operator is to measure “pure” M3 transitions and to compare them with the best available shell-model predictions. Unfortunately, in the entire 1s-0d shell only three M3 transitions have ever been measured, one each in $^{24}$Al, $^{24}$Na, and $^{34}$Cl. These $B(M3)$ values have been compared by Brown et al. [4] with the predictions of their multiconfiguration shell-model calculations. They conclude that the data require quenching of the free nucleon spin g factors by 30(5)% (isoscalar $g_0$) and 13(5)% (isovector $g_1$). Since this conclusion is based on extremely limited data from three different odd-odd nuclei and on several simplifying assumptions, an independent confirmation of the need for g-factor renormalization is sorely needed.

Ideally, one would like to investigate the possible M3 quenching for several transitions in one nucleus and to also study the radial structure of the M3 transition densities. Such data, which can be obtained only by inelastic scattering of electrons from even-even nuclei, would provide valuable insight into the possible contributions to M3 quenching which go beyond first-order configuration mixing. Unfortunately, in the entire 1s-0d shell, electron inelastic scattering for only two $3^+$ states has so far been reported. In both cases the data are meager and of poor quality, and lead to opposite conclusions; M3 quenching for $^{28}$Si [5], and M3 enhancement for $^{24}$Mg [6]. Thus a detailed examination of the possible quenching of the M3 operator has not been done so far. In this Letter we report the identification of the first ten $3^+$ states in $^{26}$Mg by inelastic scattering of electrons and present results of measurements of their M3 transition strengths. These data enable us to make the first exhaustive examination of M3 quenching. We compare our experimental results with well-known shell-model calculations of Brown, Radhi, and Wildenthal [7]. These calculations assume an inert $^{16}$O core and consider all allowed multiparticle excitations in the $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$ orbits. Two-body
FIG. 1. Results for the $3^+$ states at 7242, 8248, 3941, and 4350 keV. Top panels show form factors and bottom panels show magnetization current densities. In the $F_q^2(q)$ plots the solid points refer to $\theta = 154^\circ$ and the open points to $\theta \leq 90^\circ$. The solid curves and the hatched areas represent the results of the model-independent analysis described in the text. The dashed curves represent shell-model predictions.
matrix elements of the weakly atomic weight-dependent interaction are determined by fitting binding energies of 450 known states. Meson exchange current effects are not included. From these comparisons we draw the conclusion that there is no evidence for any overall quenching of the M3 operator.

The experiment was performed at the electron linear accelerator at NIKHEF-K at Amsterdam using the 600 MeV/c high-resolution QDD spectrometer [8]. Data were taken at energies ranging from 85 to 360 MeV and angles between 33° and 154°, covering momentum transfers from ~0.4 to ~3.0 fm⁻¹. A self-supporting target of 12.78 mg/cm² thickness and 99.4% ²⁶Mg enrichment was used. The energy resolution achieved ranged from FWHM = 17 to 35 keV at forward angles, and ≤ 48 keV at 154°. Level energies were determined with an uncertainty of ≤ 5 keV. More than seventy states in ²⁶Mg were identified below 10 MeV excitation [9]. In this Letter we only report the results for the 3⁺ states.

In the standard notation the plane wave Born approximation (PWBA) cross section for electron scattering for the excitation of an unnatural parity state with J = λ and parity = (−1)¹+λ, in an even-even nucleus, is given in terms of the magnetic form factor F₇₄₄(q) by

$$\frac{d\sigma(q)}{d\Omega} = Z^2\sigma_{\text{Mott}}(q)f_{\text{rec}}\left(\frac{1}{2} + \tan^2\theta/2\right)\left|F_{\lambda\lambda}(q)\right|^2.$$  \hfill (1)

The magnetization current transition density Jₐₖ(r) is related to Fₗₖ(q) as

$$\left|F_{\lambda\lambda}(q)\right|^2 = \frac{4\pi}{Z}\sqrt{2J + 1}\int_0^\infty J_{\lambda\lambda}(r)j_\lambda(qr)r^2\,dr.$$  \hfill (2)

Model-independent analysis of the measured cross sections was done by using the distorted wave Born approximation code FOUBES1 [10] to determine the best fit magnetization current density Jₐₖ(r). The reduced transition probabilities were determined from these fits as

$$B(M\lambda \lambda) = \left[\frac{4J_f + 1}{4J_i + 1}\right]\int_0^\infty r^{4+2}J_{\lambda\lambda}(r)\,dr.$$  \hfill (3)

Comparison of the theoretical PWBA form factors with the experimental form factors was made by plotting the latter as a function of $q(\text{effective}) = q[1 + 6.8/E_{\text{inc}}(\text{MeV})]$.

In Fig. 1 we show four examples of our results for $\left|F_{\lambda\lambda}(q)\right|^2$ and Jₐₖ(r). In these figures the solid line curves and hatched areas show the results of the model-independent analysis of the data, and the dashed curves show predictions from the shell-model calculations [7].

The first three excited 3⁺ states of ²⁶Mg are well known. In the present experiment seven additional 3⁺ states were identified (Table I). This was made possible by several features of our data, and the limits on J⁺ assignments imposed by existing γ-decay and particle transfer experiments. First, the candidate states had to have no identifiable longitudinal excitation. Pure transverse transitions could be identified unambiguously because the Rosenbluth factor $(\frac{1}{2} + \tan^2\theta/2)$ in Eq. (1) is equal to 19.8 for $\theta = 154°$, while it is ≤ 1 at $\theta ≤ 90°$. Second, it was assumed that the observed γ decays from the candidate states are limited to E0, E1, M1, or E2. Finally, the measured form factor had to have a shape similar to one for the excitation of a well-established 3⁺ state. This last criterion was used only in conjunction with other evidence supporting the assignment because of the recognition that 3⁺ states can be formed within the s-d shell via a number of different transitions, $s_{1/2} \rightarrow d_{5/2}$, $d_{3/2} \rightarrow d_{5/2}$, $d_{5/2} \rightarrow d_{3/2}$, and $d_{3/2} \rightarrow d_{3/2}$, and the interplay between them can give rise to a variety of shapes to the form factor (see Fig. 1). We present two examples of our identification procedure.

The form factor for the 3⁺ state at 7242 keV is shown in Fig. 1. This state is known to decay to 2⁺, 3⁺, or 4⁺. Its pure transverse character in our experiment, illustrated by the excellent agreement between the filled symbols (θ = 154°) and open symbols (θ ≤ 90°), immediately leads to $J^\pi = 3^\pi$. As shown in the figure, the experimental form factor and the transition current density for this state are in excellent agreement with the shell-model predictions.

The state at 8248 keV is known to γ decay 100% to the 2⁺ state, which limits its possible $J^\pi$ to 0⁺, 1⁺, 2⁺, 3⁺, and 4⁺. As shown in Fig. 1, its pure transverse character is firmly established by the excellent agreement between the forward angle and 154° values of the form factors for $q_{\text{eff}} > 1.5$ fm⁻¹. This leaves $J^\pi = 1^+$ and 3⁺ as the only possibilities. We assign $J^\pi = 3^+$ because the form factor has the characteristic shape predicted by the shell-model calculations for the well-established 3⁺ states, and is completely unlike that for 1⁺ transitions which show a sharp peak at $q = 0.5$ fm⁻¹. Once again the dashed lines show the excellent agreement of our data with the shell-model predictions.

<table>
<thead>
<tr>
<th>State</th>
<th>Expt. E (keV)</th>
<th>Theory E (keV)</th>
<th>$B(M3)$ (10²μ₂₂² fm⁴) Expt.</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>3⁺</td>
<td>3941</td>
<td>3921</td>
<td>3.4(11)</td>
<td>1.4</td>
</tr>
<tr>
<td>3⁺</td>
<td>4350</td>
<td>4510</td>
<td>7.0(30)</td>
<td>4.5</td>
</tr>
<tr>
<td>3⁺</td>
<td>6125</td>
<td>6268</td>
<td>1.9(4)</td>
<td>6.0</td>
</tr>
<tr>
<td>3⁺</td>
<td>7242</td>
<td>7281</td>
<td>11.7(20)</td>
<td>9.8</td>
</tr>
<tr>
<td>3⁺</td>
<td>7724</td>
<td>7602</td>
<td>2.3(7)</td>
<td>0.6</td>
</tr>
<tr>
<td>3⁺</td>
<td>8248</td>
<td>8004</td>
<td>11.8(21)</td>
<td>9.7</td>
</tr>
<tr>
<td>3⁺</td>
<td>8456</td>
<td>8404</td>
<td>4.0(13)</td>
<td>16.3</td>
</tr>
<tr>
<td>3⁺</td>
<td>9042</td>
<td>9115</td>
<td>9.6(43)</td>
<td>0.07</td>
</tr>
<tr>
<td>3⁺</td>
<td>9423</td>
<td>9304</td>
<td>4.0(15)</td>
<td>0.03</td>
</tr>
<tr>
<td>3⁺</td>
<td>9902</td>
<td>9576</td>
<td>1.6(15)</td>
<td>0.01</td>
</tr>
<tr>
<td>Σ B(M3)</td>
<td></td>
<td></td>
<td>57.3(66)</td>
<td>48.4</td>
</tr>
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</table>

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Lest one gets the idea that we have excellent state-by-state agreement with the shell-model predictions, we display in Fig. 1 the results for the 3$^+_1$ and 3$^+_2$ states. The 3$^+_2$ form factor is in excellent agreement with theory, but the 3$^+_1$ form factor is not.

We have measured $F^M_N(q)$ for all 3$^+$ states over an extended range of $q$. However, it is not possible to make a global comparison of theory and experiment in terms of $F^M_N(q)$ because form factor shapes vary from state to state. Even for a specific state, comparison at the different maxima leads to different conclusions (e.g., 3$^+_1$ state in Fig. 1). The only consistent way to compare theory and experiment appears to be in terms of $B(M3)$, which, in our case, is determined as an integral over the transition density [Eq. (4)] and uses all the measured data. Our results for individual $B(M3)$ are listed in Table I, and presented as a ladder diagram for $\sum B(M3)$ in Fig. 2. We note that while the energies of the 3$^+$ states are predicted remarkably well by the theory, the $B(M3)$ predictions are widely scattered about the experimental values, making a state-by-state comparison rather meaningless. For this reason, comparison between theory and experiment for a specific state should not be used as a basis for any conclusion about the global $M3$ operator, as was done in Refs. [4–6].

It appears that the only reasonable thing to do is to compare the sum of $B(M3)$ over all the 3$^+$ states. In Fig. 2 we present such a comparison. It is seen that for the first ten states

$$\frac{\sum B(M3)_{\text{expt}}}{\sum B(M3)_{\text{theo}}} = \frac{57.3(66)}{48.3} = 1.19(14). \quad (4)$$

This result indicates that while there is plenty of room to improve the theoretical predictions for individual states, there is no evidence for any global quenching of the $M3$ operator. Since the reduced matrix element $\sqrt{B(M3)}$ can be written down in terms of the four $g$ factors, the orbital $g$ factors $g^o_x$ and $g^o_y$, and the spin $g$ factors $g^s_x$ and $g^s_y$, in the shell-model calculations used here these $g$ factors were assumed to have their free nucleon values, the result in Eq. (4) above can be interpreted to mean that no overall renormalization of the $g$ factors from their free nucleon values is suggested by the collective data for the $M3$ transitions presented in this paper.

We believe that the above conclusion is unlikely to be affected by the consideration of meson exchange current (MEC) effects. While MEC effects on $M3$ transitions have not been as extensively studied as those for $M1$ transitions, it is the consensus of all the studies made so far [11] that MEC contributions to $M3$ form factors are less than those for the $M1$ transitions and are no more than 5% for momentum transfers, $q \leq 3 \text{ fm}^{-1}$. In any case, we hope that the extensive data on $M3$ transitions provided by our measurements will encourage detailed theoretical investigations of all second-order effects which have not been included in the present shell-model calculations.

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