1-Forbidden M1 Transition in $^{32}$S: A Test of Tensor Corrections to the Magnetic Dipole Operator


Institut für Kernphysik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany

D. I. Sober

Department of Physics, Catholic University of America, Washington, DC 20064

B. A. Brown

Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824

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The extremely weak 1-forbidden 1d$_{3/2} \rightarrow$ 2s$_{1/2}$ magnetic dipole (M1) transition from the ground state of $^{32}$S to a $J^\pi = 1^+$ state at $E_x = 7.003$ MeV was investigated by 165$^\circ$ and 180$^\circ$ electron scattering. The extracted strength of 0.0040(5)$\mu_5^2$ represents the smallest $B$(M1) value ever measured in electron scattering. A combined analysis with the isospin-analogous Gamow-Teller (GT) decays of the $J^\pi = 1^+$ ground states of $^{32}$Cl and $^{32}$P is performed. Empirical corrections of Brown and Wildenthal to the magnetic operators reasonably account for the data. Corrections derived from microscopic approaches are less successful pointing towards a fundamental, not yet understood problem in the description of the effective M1 and GT operators. [S0031-9007(98)08207-6]

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The phenomenon of quenching of the $M1$ and Gamow-Teller (GT) response in nuclei, i.e., the reduction of the experimentally observed strengths with respect to the best available model results, has attracted intense interest over the years (see, e.g., [1–6] and references therein). It has become clear that a large part of the reduction can be explained by tensor correlations induced by the renormalization of the transition strengths in (necessarily) truncated model spaces. However, there are also finite effects from non-nuclear degrees of freedom such as mesonic-exchange currents and excitations of nucleons to the $\Delta$ isobar. The different contributions can be incorporated by the introduction of effective $M1$ and GT operators [see Eqs. (1) and (2) below]. Spin, orbital, and tensor corrections to the operators for transitions in sd-shell nuclei have been derived from microscopic calculations of Arima et al. [2] and Towner and Khanna [7], as well as by Brown and Wildenthal [8] from empirical fits to a large body of data. Both methods agree quite well with each other except for the isovector $M1$ tensor corrections whose predicted magnitude is much smaller than the empirically found value.

Allowed $M1$ and GT transitions are usually dominated by the spin strength, and the tensor corrections are weak. In contrast, l-forbidden transitions are mainly governed by the tensor part, thus providing experimental insight into this otherwise hardly accessible contribution. The term “l-forbidden” refers to a selection rule for the one-body operator of $M1$ or GT transitions which does not allow a change of the radial quantum number.

The higher-order corrections to the l-forbidden transitions are theoretically expected to be dominated by $\Delta$ admixtures into the nuclear wave functions [2,7], and they are a unique observable in this respect. When scaled to the free-nucleon strength, the delta correction is expected to be essentially the same for the isovector $M1$ and GT operators. However, the analysis of the l-forbidden 1d$_{3/2} \rightarrow$ 2s$_{1/2}$ single-hole transitions in $A = 39$ nuclei gives an order of magnitude larger $M1$ strength relative to the GT strength [9–13]. While this result contradicts the calculations [2,7] it can be well explained by the empirical approach [8]. Indeed, this is one of the major problems remaining in our understanding of electromagnetic and $\beta$-decay observables in light nuclei which we address here.

One possibility of explaining this discrepancy is that the $A = 39$ transitions are not well described by pure 1d$_{3/2}$ and 2s$_{1/2}$ single-hole states, and that low-lying core excitations across from the sd shell to the fp shell, i.e., ground state correlations, could introduce some allowed strength in a way which is not well understood. Indeed, sizable $M1$ strength has been found [14] in the LS closed-shell nucleus $^{40}$Ca. Therefore it is important to examine data away from the end of the sd shell (where such low-lying core excitations should be less important) in order to see if the discrepancy persists. Unfortunately, it is very difficult to find very pure l-forbidden transitions [15], and small admixtures with allowed components make the extraction of the l-forbidden component uncertain [16].

Two of the most hindered allowed beta decays in sd-shell nuclei are those for the mirror decays of $J^\pi = 1^+$ ground states in $^{32}$P and $^{32}$Cl with isospin $T = 1$ to the $J^\pi; T = 0^+$; 0 ground state (g.s.) of $^{32}$S [17]. Their structure in the simplest shell model as well as in the full sd-shell configuration mixing model [17] is dominated by a 1d$_{3/2} \rightarrow$ 2s$_{1/2}$ transition. Thus, the transitions in $A = 32$ nuclei are perhaps the best case available for a study of l-forbidden strength towards the middle of the sd
shell. In this work, we have measured for the first time a $B(M1)$ value for the extremely weak $I$-forbidden transition to the isospin-analog $1^+$ state in $^{32}\text{S}$ by inelastic electron scattering. This information is of particular interest for a combined analysis with the GT strengths to test the $sd$-shell wave functions and the effects of higher-order corrections.

The $^{32}\text{S}(e,e')$ reaction has been investigated at the superconducting Darmstadt electron linear accelerator (SDLALINC) with a new $\Theta = 180^\circ$ scattering system [18] coupled to a large solid-angle Q-CLAM magnetic spectrometer [19]. Incident electron energies were $E_0 = 42, 66,$ and $82$ MeV. Additional data were available from experiments at a high-resolution energy-loss spectrometer positioned at $\Theta = 165^\circ$ for $E_0 = 30, 35,$ and $49$ MeV. In total, a momentum transfer range $q = 0.3-0.8$ fm$^{-1}$ was covered. A self-supporting Li$_2$S target containing natural sulfur (95.6% abundance of $^{32}\text{S}$) of 28 mg/cm$^2$ areal density was used. Average beam currents were several $\mu$A. Figure 1 presents typical spectra taken at both spectrometers. Energy resolutions of about 40 keV (FWHM) at 165$^\circ$ and 100 keV at 180$^\circ$ (limited by the target thickness) were achieved. The line contents of the weak transition to the 7.003 MeV state were obtained from a simultaneous fit of all spectra including the additional close-by levels indicated in Fig. 1. For an extraction of the form factor the data measured at 165$^\circ$ were normalized to elastic scattering taking into account the Coulomb distortion of the wave functions. Because of the rapid variation of the elastic scattering cross section as a function of the effective detection angle, the 180$^\circ$ measurements were normalized to the prominent $M1$ transition in $^6\text{Li}$ at 3.562 MeV where detailed form factor measurements are available [20].

The extracted form factor of the $1^+$ transition is displayed in Fig. 2. It rises monotonously in the investigated $q$ range indicating a first maximum at $q_{\text{eff}} = 1$ fm$^{-1}$. Such a $q$ dependence deviates strongly from the typical behavior observed for spin-dominated $M1$ transitions. This is demonstrated by a calculation for a pure $1d_{5/2} \rightarrow 1d_{3/2}$ spin-flip transition (dashed line) which shows the first maximum at an effective momentum transfer of about 0.5 fm$^{-1}$ and cannot reproduce the data at all. However, when using the shell-model wave functions obtained with the unified $sd$-shell (USD) interaction [21,22] a very good description of the $q$ dependence is achieved (solid line). The transition strength is extracted from a normalization of the form factor to the data and an extrapolation to the photon point $k = E_\gamma/\hbar c$. One finds $B(M1) = 0.0040(5)\,\mu\text{J}$ which constitutes, to the best of our knowledge, the smallest $M1$ strength ever measured in electron scattering experiments.

Because of the extreme weakness of the transition, one may ask how sensitive the results are to details of the model wave functions. As pointed out above, the free-nucleon transition strength for $I$-forbidden transitions is zero. According to our present understanding, nonzero strength might be induced by (i) configuration mixing with allowed shell-model components within the $sd$ shell, (ii) configuration mixing with allowed shell-model components outside the $sd$ shell, (iii) $\Delta$ admixtures into the nuclear wave functions, and (iv) mesonic exchange currents [1]. The full $sd$-shell model calculations contain

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**FIG. 1.** Spectra of the $^{32}\text{S}(e,e')$ reaction at $E_0 = 49.5$ MeV, $\Theta = 165^\circ$, and $E_0 = 82.2$ MeV, $\Theta = 180^\circ$ in the energy region of the $I$-forbidden $M1$ transition. The states marked by arrows were consistently included in the data analysis.

**FIG. 2.** Form factor of the $I$-forbidden transition to the $J^\pi = 1^+$, $E_x = 7.003$ MeV state in $^{32}\text{S}$. Full circles: Data measured at 180$^\circ$. Full triangles: Data measured at 165$^\circ$. Dashed line: Calculation for a pure $1d_{5/2} \rightarrow 1d_{3/2}$ spin-flip excitation. Solid line: Calculation with wave functions from the USD interaction 21 and free nucleon $g$ factors.
the effect of (i) to all orders. Effects [(ii)–(iv)] are usually treated in perturbation theory in terms of an effective GT operator

\[ \text{GT}_{\text{eff}} = \frac{g_A}{g_V} \{ S + \delta_S(\text{GT}) S + \delta_L(\text{GT}) L \]

\[ + \delta_P(\text{GT}) \sqrt{8\pi} [ Y^{(2)} \otimes S ]^{\Lambda J-1} \}, \]

and an effective isovector \( M^1 \) operator

\[ M_{1\text{eff}} = g_S S + g_L L \]

\[ + \frac{g_S \delta_S(M^1)}{g_L} \frac{S + \delta_L(M^1) L \]

\[ + \frac{\delta_P(M^1)}{g_L} \sqrt{8\pi} [ Y^{(2)} \otimes S ]^{\Lambda J-1} \}, \]

where \( g_V \) and \( g_A \) are the weak Fermi and GT coupling strengths, \( g_A/g_V = -1.2599(25) \). \( g_S = -4.706 \) is the isovector spin g factor, and \( g_L = 0.5 \) is the isovector orbital g factor. The difference between \( \delta_S(\text{GT}) \) and \( \delta_S(M^1) \) is dominated by mesonic exchange currents (iv). The empirical values for \( \delta_S \) obtained from a global fit of many sd-shell data [17] as well as specific transitions in \( ^{32}\text{S} \) give a significant enhancement of \( \delta_S(M^1) \) over \( \delta_S(\text{GT}) \) consistent with the microscopic calculations [2,7].

The predictive power of calculations using the USD interaction [21,22] and effective operators is demonstrated by an investigation of the \( B(M^1) \) strength distribution in \( ^{32}\text{S} \) also obtained from the experiments described above. This reaction has been studied previously in electron scattering, but with conflicting results on the strengths of two of the most prominent \( M^1 \) transitions [27,28]. The upper part of Fig. 3 presents the experimental \( B(M^1) \) distribution derived from the present data up to an excitation energy of 12 MeV (details will be presented elsewhere [29]). In the middle part, a calculation using empirical effective g factors [8] is shown. Clearly, one finds excellent agreement even on a level-to-level basis. The two prominent transitions at \( E_x = 11.5 \text{ MeV} \) exhibit a strong mixing in the calculation, and the sharing of the strength is very sensitive to the assumed energy difference. Thus, one should rather compare the sum of both with experiment which again agrees satisfactorily. The bottom part of Fig. 3 shows the squared \( 1d_{3/2} \rightarrow 2s_{1/2} \) one-body transition density associated with \( J^T;T = 0^+;0 \rightarrow J^T;T = 1^+;1 \) transitions in \( ^{32}\text{S} \). It is concentrated in the lowest state predicted at \( E_x = 7.058 \text{ MeV} \) confirming the \( l \)-forbidden character of the transition to the level observed experimentally at 7.003 MeV.

We now proceed to a combined analysis of the \( l \)-forbidden \( M^1 \) and GT transitions from the set of isospin-analog \( 1^+ \) states in \( ^{32}\text{S}, ^{32}\text{Cl}, \) and \( ^{32}\text{P} \), respectively. The importance of using the combined information on both quantities, i.e., \( M^1 \) and GT strengths, was demonstrated in Ref. [24]. The complete set of data allows a particularly stringent test. Furthermore, as was pointed out above, only the comparison reveals the theoretical problems in full depth [1,10–12]. The asymmetry in the \( \beta \)-decay probabilities of \( ^{32}\text{Cl} \) and \( ^{32}\text{P} \) indicates that isospin mixing effects must be considered, and we include this by carrying out an isospin-mixed calculation in the full proton-neutron basis with the Ormand-Brown [30] isospin nonconserving Hamiltonian added onto the USD. The \( M^1 \) and GT matrix elements are given in Table I as calculated with the empirical Brown-Wildenthal (BW) \( \delta \) parameters. In each case, the breakdown of the matrix element in terms of its spin, orbital, and tensor contributions is given.

Inspection of Table I shows that the tensor pieces of the \( l \)-forbidden matrix elements are largest, but the spin and orbital contributions cannot be neglected. The experimental asymmetry in the GT matrix elements 0.046:0.012 is close to 4 while the BW calculation yields 0.024:0.012, a ratio of about 2. For \( ^{32}\text{P} \) decay, where the effects due to

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<th>( ^{32}\text{P} )</th>
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<tr>
<td>( B(M^1) )</td>
<td>( 0.052 )</td>
<td>( -0.009 )</td>
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<tr>
<td>( M(S) )</td>
<td>0.076</td>
<td>-0.009</td>
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<td>( M(L) )</td>
<td>0.119</td>
<td>0.003</td>
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<tr>
<td>( M(P) )</td>
<td>-0.136</td>
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isospin mixing in the calculation are smallest, experiment and theory are close (0.013:0.012). The predicted matrix element $|M| = 0.058$ for the $M1$ transition in $^{32}$S is in good agreement with the experimental value of 0.063(3). Considering the weakness of all investigated transitions, we conclude that the BW results provide a reasonably consistent description of the data.

On the other hand, a calculation with analytically determined correction factors from Towner and Khanna (TK) including isospin, while resulting in comparable spin and orbital $M1$ matrix elements, exhibits the aforementioned difference of a much smaller tensor correction to the isovector $M1$ strength $[M(P) = -0.070$ for TK compared to $-0.136$ for BW]. Because of the sign difference between $M(P)$ and the spin and orbital pieces, this leads to a larger total matrix element $|M| = 0.087$. The transition strength $B(M1) = |M|^2$ from the TK approach amounts to 0.0076$\mu^2$, exceeding the experimental result by a factor of 2. Furthermore, nearly equal GT strengths are predicted for the $^{32}$Cl (0.039) and $^{32}$P (0.030) g.s. decays in contrast to the pronounced experimental asymmetry (although the individual description of the $^{32}$Cl result agrees better with the data than the BW calculation). These discrepancies await solution which at present is not in sight.

It was pointed out in [11–13] that there is no simple way to improve the microscopic calculations of the $M1$ transitions, e.g., by increasing the interaction strength, because core polarization and $\Delta$ isobar contributions to the $l$-forbidden $M1$ and GT matrix elements scale strictly within the models. Thus, explanations should focus on orbital or meson-exchange contributions. The $M1$ orbital matrix elements obtained from BW and TK are very close (0.119 and 0.105, respectively, see Table I) pointing to meson-exchange currents as the most likely source for the discrepancies.

In summary, we have measured the extremely weak $l$-forbidden $M1$ transition in $^{32}$S in high-resolution electron scattering under backward angles including 180°. The anomalous momentum transfer dependence of the form factor can be reproduced by shell-model calculations with the USD interaction. The derived $B(M1)$ value allows, for the first time, a combined analysis of the analog $M1$ and GT strengths of an $l$-forbidden transition away from the shell closure. The empirical tensor correction of Ref. [8] to the magnetic dipole operator leads to a reasonably consistent description of the data. Corrections from a microscopic calculation [7] overpredict the $B(M1)$ strength and cannot account for the asymmetry of the mirror GT decays. While the differences are quantitatively less pronounced and the analysis is complicated by the delicate interplay of the individual matrix elements contributing to the transition strength, the present results reinforce the findings for the $A = 39$ case. The failure of the otherwise successful microscopic approach to account for the tensor corrections of the $M1$ operator still asks for an explanation.

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