Two-proton decay of $^{45}$Fe has been observed with a peak energy of $(1.14 \pm 0.05)$ MeV and a half-life of $(4.7^{+3.4}_{-1.4})$ ms [1]. An independent experiment with less statistics and a lower energy resolution gave an energy of $(1.1 \pm 0.1)$ MeV and a half-life of $(3.2^{+2.6}_{-0.8})$ ms [2]. The average half-life from the two experiments is $(3.8^{+2.0}_{-0.8})$ ms [3]. The two-proton decay branch is experimentally estimated to be $70-80\%$ [3]. A 75% two-proton branch would give a two-proton decay half-life of 4–8 ms and a beta decay half-life of 12–23 ms. The lower beta decay half-life is not far from the prediction of 7 ms [4]. The energy is in good agreement with predictions of $(1.15 \pm 0.09)$ MeV [5], $(1.28 \pm 0.18)$ MeV [4], and $(1.22 \pm 0.05)$ MeV [6].

In this paper we give a first quantitative calculation for the di-proton decay of $^{45}$Fe in terms of a recent extension of the $R$ matrix [7], which includes the $s$-wave $p+p$ interaction as an intermediate state. This model allows us to make a connection to the interesting questions about pairing correlations. The results presented here supersede the previous $R$-matrix results of Ref. [1] in which an approximation was made for the reduced width, and those of Ref. [5] in which also the $s$-wave $p+p$ interaction was ignored. The half-life is $T_{1/2} = (4.56 \times 10^{-19}/\Gamma)$ MeV ms, where $\Gamma$ is the decay width, taken in this model to be the $R$-matrix observed width $\Gamma^0$. This is given by the formula [7]

$$\Gamma^0 = 2 \gamma^2 \tilde{P} \Gamma (1 + \gamma^2 \tilde{S} \tilde{S})$$

(1)

with

$$\tilde{P} = \int_0^Q P(Q-U)\rho(U)dU,$$

(2)

$$\tilde{S} = \int_{E=Q}^{\infty} \frac{dS(E-U)}{dE} \rho(U)dU,$$

(3)

where $Q$ is the available decay energy, $P$ and $S$ are the $R$-matrix penetration and shift factors [8], and $\rho$ is the density-of-states function, which is expressed in terms of the $p+p$ $s$-wave phase shift by Eq. (3) of Ref. [7], with $a_2 = 2.90$ fm, $\epsilon = 0.25$ fm, $A = -0.0045$ fm$^{-1}$, and $B = 1.073$ fm.

The reduced width $\gamma^2$ is related to the spectroscopic factor $S$ and the dimensionless reduced width $\theta^2_{sp}$ by [9]

$$\gamma^2 = S \theta^2_{sp} \frac{\hbar^2}{M a^2},$$

(4)

where $M$ is the reduced mass and $a$ is the channel radius. This is given as in Ref. [7] by the conventional formula

$$a = 1.45(A_1^{1/3} + A_2^{1/3}) \text{ fm},$$

(5)

where $A_1$ is the mass number of the daughter nucleus and $A_2 = 2$ for the di-proton, which is $a = 6.91$ fm for $A_1 = 43$ and $A_2 = 2$.

The single-particle dimensionless reduced width $\theta^2_{sp}$ is given by

$$\theta^2_{sp} = \frac{(a/2)u^2(a)}{\int_0^a u^2(r)dr},$$

(6)

where $u(r)/r$ is the single-particle radial wave function. $\theta^2_{sp}$ depends upon the potential parameters for the di-proton nucleus interaction. For the potential we take the Woods-Saxon form cutoff at radius $r = a$ plus a uniform-sphere Coulomb potential with radius $R_C = r_C A_1^{1/3}$. The Woods-Saxon parameters are $R = r_0 A_1^{1/3}$ for the radius, $a_0$ for the diffuseness, and a well depth adjusted to reproduce the resonance energy. The potential parameters are taken from an analysis of low-energy deuteron scattering [10]: $r_0 = 1.17$ fm, $a_0 = 0.72$ fm, and $r_C = 1.30$ fm.

The final result depends upon the channel radius $a$ through the penetration factor $\tilde{P}$, the energy derivative of the shift factor $\tilde{S}^2$, and the reduced width $\gamma^2$. We find that when the channel radius $a$ is chosen to be large enough (beyond the range of the strong interaction) the final result is insensitive to the choice of $a$.

To calculate the spectroscopic factor we project the shell-model wave function onto the $0s$ internal (relative) wave...
function for a di-proton in the $pf$ shell by using harmonic oscillator wave functions with the general formalism of Ref. [11] to obtain

\[ S = \left( \frac{A}{A-2} \right)^{\lambda} G^2(pf)C(A,Z), \]

(7)

where $G^2 = 5/16$, $A$ is the mass of parent nucleus ($A = 45$ in our case), and

\[ C(A,Z) = \langle \langle \Psi(A-2,Z-2) | \Psi_\text{c} | \Psi(A,Z) \rangle \rangle^2 \]

is the cluster overlap for the di-proton cluster wave function $\Psi_\text{c}$ in the $pf$ shell with $L=0, S=0$, and $T=1$ in the SU3 basis.

The spectroscopic factor depends on the model space. In Ref. [5] it was assumed that the $^{45}$Fe to $^{43}$Cr matrix element could be approximated by that for the $^{46}$Fe to $^{44}$Cr matrix element. That is, the $sd$-shell neutron hole is an inactive spectator in the transition. We have checked this assumption by calculating the cluster overlaps for $^{45}$Fe and $^{46}$Fe in the $sd$-$pf$ model space with the full $pf$ shell for the protons and one-neutron hole for $^{45}$Fe. With the $sd$-$pf$ Hamiltonian from Ref. [12] the ratio $C(^{45}$Fe)/$C(^{46}$Fe) is 0.96. Thus we confirm that the $^{45}$Fe di-proton overlap is essentially the same as that for $^{46}$Fe. We use the FPD6 interaction from Ref. [13] to obtain $C(^{46}$Fe) = 0.480, which gives $S = 0.197$. Thus we use $S = 0.20$ for the present calculation, which is essentially the same as the original value of 0.195 obtained in Ref. [5].

It is interesting to note that the two-proton overlaps that enter into the $^{46}$Fe decay are the same as those that enter into the interpretation of the $^{46}$Ca($p,t$)$^{44}$Ca reaction on the mirror nuclei. The two-proton transition density matrix elements are $-1.19, -0.28, -0.23$, and $-0.12$ for $(0f_{7/2})^2$, $(0f_{5/2})^2$, $(1p_{3/2})^2$, and $(1p_{1/2})^2$, respectively. The amplitudes are dominated by the $0f_{7/2}$ orbit, but the coherent mixture is important. If only the $0f_{7/2}$ orbit is kept, the cluster overlap is reduced by a factor of 0.28.

With $Q = 1.14$ MeV [1], we find $\theta_{sp}^2 = 0.097$ for the di-proton in a 3s state (three zeros of the wave function between 0 and $\infty$), leading to $\gamma^2 = 8.6 \times 10^{-3}$ MeV. Also $P = 0.638 \times 10^{-18}$ and $S'^2 = 0.286$ MeV$^{-1}$, so that the $S'$ term in the denominator of Eq. (1) is negligible. This value of $P$ is $3 \times 10^{-4}$ times the value of $P(Q)$, the penetration factor used in the simple model [1]. The factor is small due to the integrand in Eq. (2) peaking at $U \approx 0.06$ MeV, while $P$ is equal to $P(Q-U)$ for $U \approx 0.25$ MeV. From Eq. (1), $10^0 = 1.10 \times 10^{-26}$ MeV, and $T_{1/2} = 41$ ms. The result is sensitive to the $Q$ value. If we take the experimental upper range of 1.19 MeV we would obtain $T_{1/2} = 10$ ms, which is in reasonable agreement with experiment. Thus there is a large uncertainty in the calculated half-life due to the error in $Q$ value. An improved experimental $Q$ value that will be obtained in future experiments will reduce this source of error.

With the extrapolated one-proton $Q$ value of Ref. [5], it was estimated in Ref. [1] that the sequential decay should be negligible. Although the effective penetration factor is very small, there could be a contribution to the denominator of Eq. (1) coming from the sequential decay channel, but this is estimated to be less than 0.1, and so to have at most a 10% effect on the calculated $T_{1/2}$.

In Ref. [1] the half-life obtained with the $R$-matrix model (with the $p+p$ resonance) was about ten times longer than the present result. The reason is that the channel radius of $a = 4.2$ fm used in that calculation is too small (not sufficiently outside the strong interaction potential), and it was assumed that $\theta_{sp}^2 = 1$. The half-life estimates given in Ref. [5] ignored the $p+p$ resonance (which increases the half-life by a factor of about 3000), used a small value of $a = 4.0$ fm, and also assumed $\theta_{sp}^2 = 1$. Thus the present theoretical result replaces those of Refs. [1,5].

$^{48}$Ni is also a good candidate for the observation of two-proton decay [5]. The $pf$-shell spectroscopic factor is 0.14. Using the extrapolated two-proton decay $Q$ value of 1.36(13) MeV from Ref. [5] together with the same set of values for the other parameters discussed above for $^{45}$Fe, we obtain $T_{1/2} = 0.4, 8$, and 260 ms for $Q = 1.49, 1.36$, and 1.23 MeV, respectively.

In the present model there is some sensitivity to the potential parameters. For $r_0$ or $a_0$ increased by 0.05 fm, $T_{1/2}$ is further decreased by 20% and 18%, respectively. Thus, the assumption of our use of the deuteron-scattering potentials must be checked. There is of course a sensitivity to the spectroscopic factor. We have used one of the best configuration mixing models available, but the interaction and model-space sensitivity should be examined and also compared with the information inferred from the $(p,t)$ reactions on mirror nuclei. The $(p,t)$ reactions are interpreted in terms of enhancement factors $\epsilon$ relative to a given model space. For wave functions that are dominated by the $0f_{7/2}$ orbit, the experimental enhancement factor relative to the full $pf$ shell for $L=0$ transfer is about 2.4 [14]. [When the $^{46}$Ca($p,t$) data of Ref. [15] are analyzed with the FPD6 wave functions one obtains an enhancement factor of 2.2 for the transition to the $^{46}$Ca ground state. This transition is the mirror of the $^{48}$Ni di-proton decay transition.] This enhancement can be qualitatively understood in perturbation theory from the admixtures of correlated $J=0, T=1$ components from the major shell below and above the $pf$ shell [14]. Qualitatively, one might apply the same enhancement factor to increase the di-proton decay spectroscopic factors by 2.4, giving $T_{1/2} = 17$ ms with $Q = 1.14$ MeV and $T_{1/2} = 4$ ms with $Q = 1.19$ MeV, which brings the result into better agreement with experiment (the new result for $^{48}$Ni would be 3.3 ms with $Q = 1.36$ MeV). However, the relationship between the $(p,t)$ cross-section enhancements and those for di-proton decay needs to be quantified in terms of overlap functions for the two-proton removal process.

The work of Grigorenko et al. [16,17] provides a more general formulation for the three-body asymptotics for two-proton decay in terms of a solution of a three-body Hamiltonian. However, in this formulation the connection to the many-body nuclear structure remains at the level of the single-particle wave functions, and does not include pairing correlations. It would be useful to find a way to compare our
$R$-matrix results (a correlated di-proton decay through an intermediate state resonance of the two protons) with those of Grigorenko.

In summary, an $R$-matrix model, which includes the $s$-wave resonance of the two protons, provides a basis for using di-proton decay as a quantitative spectroscopic tool.

When new experimental results are available for $^{45}$Fe and $^{48}$Ni, our results will provide a means of extracting unique information on the pairing correlations of protons in the nucleus.

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