Proton Reaction Cross Sections as Measures of the Spatial Distributions of Neutrons in Exotic Nuclei

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Proton and neutron densities from Skyrme-Hartree-Fock calculations of a number of nuclei with masses ranging from 28 to 58 have been used to generate optical potentials for proton elastic scattering. Those potentials, generated by folding the structure functions with effective in-medium nucleon-nucleon (NN) interactions, have been used to evaluate proton total reaction cross sections; cross sections that reveal signatures of the structures.


There is much current interest in the properties of, and reactions with, exotic (radioactive) nuclei. Many can be formed as radioactive ion beams (RIBs) at modern experimental facilities throughout the world, which has spurred interest from the viewpoint of new studies in nuclear physics. However, interest in those systems is more widespread, with the role they play in the quiescent as well as explosive burning processes in stellar systems being one example. All data from reactions must be analyzed with a scattering theory. The most useful of those theories require that all input, other than that of the structure of the nuclei involved in the reaction to be assessed, be preset. Then, with an assumed model structure, as there are no adjustable parameters in such a theory, the results are predictions. Only with such analyses can physical properties of exotic nuclei that may be involved be determined with confidence.

One of the best experiments to consider for the purpose of probing matter densities of exotic nuclei is RIB scattering from hydrogen targets. By inverse kinematics this equates to proton scattering from RIB nuclei as targets. As the external proton interacts more strongly with bound neutrons than it does with bound protons, the scattering is somewhat more, but not exclusively, sensitive to neutron distributions in the nucleus. However, most current research projects with RIBs have been of reactions with complex nuclei as targets, experiments which only probe the long range radial properties of the incident projectile [1,2]. Indeed, breakup occurs readily and that correlates with a strongly absorptive optical potential through most of the nuclear volume. As there are no electron scattering data from (most) radioactive nuclei, RIB scattering from hydrogen targets essentially is the only current sensitive means to probe the character of the ions within the nuclear surface. That proton-nucleus scattering does so probe the nucleus is evident from the results reported in Ref. [3] for the stable nucleus $^{208}$Pb (for which there is a corroborating electron scattering form factor), and in Ref. [4] for the radioactive nucleus $^6$He. That sensitivity can be used to discriminate between competing models of the structure of the nucleus involved, but only when a credible scattering theory is used in the analysis of the scattering data. Defining a credible scattering theory to use, however, is dependent upon the incident energy. For low energies, coupled channel effects involving low-lying discrete states of the nucleus need be considered. How much, and with what additional requirements, has been indicated in recent studies made using a multichannel algebraic scattering theory [5,6]. For energies typically 25 to 250 MeV, a $g$-folding method [7] of data analysis has proved appropriate [4], as explicit channel coupling is not of prime importance.

As the $g$-folding method analyses of RIB-hydrogen scattering are highly sensitive to the character of nuclei in and through the nuclear surface, and as the associated relative motion wave functions are required in any distorted wave analysis of other reaction data measured, it is to be hoped that program advisory committees at RIB producing facilities will see much merit to proposals involving elastic scattering from hydrogen, and ascribe greater importance to them than has been the case to date. Such should also be considered very carefully with transfer reaction proposals, especially those involving the transfer of a single nucleon. In any event, a more favorable view of making elastic scattering measurements with RIB’s would be most welcome. To extract physics information from any reaction studied invariably requires specification of the relative motion wave functions, so far only defined by an analysis of elastic scattering. Relying upon global forms of optical potentials, especially since they are...
Clearly the isotope sets vary from that optimal curve with, as expected, the proton rms values slowly increasing as neutrons are added to form neighboring isotopes. Those increases are not uniform or even parallel. For the nickel isotopes the increase is quite well matched by the \( A \) dependence of the minimal-\( T \) curve.

The SkX model calculations gave rms values for matter radii that are shown in Fig. 2. Those results are compared with the \( A^{1/3} \) values (dashed curve) and with the minimal-\( T \) curve (solid) derived from

\[
\sqrt{\langle r_{\rm mass}^2 \rangle} = [1.054 - 0.0017A]A^{1/3}.
\]

For this quantity, the increase in neutron rms radii complementing the much slower increase in the proton rms radius makes for a net mass rms radius increase more like the ubiquitous \( A^{1/3} \) value. The minimal-\( T \) curve indicates that the isotope shifts steadily increase from that curve. In this case it is most evident with the nickel isotope values. Thus the more or less steady values of proton rms radii are compensated by a marked increase in the neutron values. That is clear from the skin thickness, which we define as the difference between the neutron and proton radii.

FIG. 2 (color online). The matter rms radii from the SHF models versus mass number. The notation is as in Fig. 1.
The skin thicknesses of the nuclei considered and as given by the SkX model of their structure, are plotted in Fig. 3. From this model, the minimal-\(T\) set of six nuclei all have proton radii larger than the neutron values, while each isotope set has skin thickness values that form almost parallel lines with nuclear mass but with decreasing increments as the charge of the isotope increases.

Good agreement with experiment for electron scattering [8] justifies using the proton and neutron radial wave functions defined by the SHF studies in forming optical model potentials to analyze proton elastic scattering. Specifically, we have formed optical potentials for the model potentials to analyze proton elastic scattering.

To use the \(g\)-folding method in data analyses requires specification of an effective (medium modified) \(NN\) interaction, single-nucleon bound-state wave functions, and ground-state one-body density matrix elements (OBDME). The latter, mainly, are the shell occupancies of nucleons in the target ground state. The effective \(NN\) interactions have been developed from \(NN\) \(g\) matrices, which are solutions of the Brueckner-Bethe-Goldstone (BBG) equations for infinite nuclear matter [7]. Those we use have been built upon the Bonn-B free \(NN\) interaction. The other items required in the method must be defined using specific nucleon-based models of structure for the target nucleus. When all such details are predetermined, and other information has shown them to be credible, the \(g\)-folding method gives predictions of observable quantities in very good agreement with measured values. With stable nuclei targets, consistent checks are found when results are interpreted and compared against electron scattering form factors.

The \(g\)-folding method is implemented by using the code DWBA98 [13]. A crucial element in that code is explicit evaluation of exchange (knock out) amplitudes in determining total and differential cross sections as well as spin observables.

Herein we illustrate, with two cases, that the \(g\)-folding method does successfully predict cross-section and analyzing power data for cases in the mass range of our SHF structures as well as at the two energies (65 and 200 MeV). Much more data at many more energies are to be the subjects of a longer subsequent presentation. In particular, in Fig. 4, the differential cross-section and analyzing power data taken at 65 MeV of proton elastic scattering from \(^{40}\text{Ar}\) [14] and at 192 MeV from \(^{58}\text{Ni}\) [15] (solid squares), supplemented with data from Ref. [16] (solid circles), are compared with the results found using the \(g\)-folding model. These two sets of results are in excellent agreement with the data notwithstanding that predicted analyzing powers drift slightly from measured results as the scattering angle increases.

It is important to note that the results given here are predictions in that the effective \(NN\) interactions were predetermined from solutions of BBG equations with the chosen nuclear density of each target defining the effective Fermi momenta of the \(NN\) interaction to be used at each radius. Thus there is no parameter to be adjusted and a single calculation of scattering cross sections, etc., was made. The excellent agreement between our predictions and measured data is direct justification that the chosen prescription of the target matter distributions is a credible one.

![FIG. 3 (color online). The skin thickness \(S_n\) from the SHF model structures used. The notation is as defined previously.](image)

![FIG. 4 (color online). The differential cross section (top) and analyzing power (bottom) for the elastic scattering of 65 MeV protons from \(^{40}\text{Ar}\) (left) and of 200 MeV protons from \(^{58}\text{Ni}\) (right).](image)
tion cross sections. We included isotopes that are radioactive and, specifically, we considered the reaction cross sections for 65 and 200 MeV protons scattering from isotopes of S, Ar, and Ca, and for scattering from \(^{28}\)Si, \(^{54}\)Fe, and \(^{58}\)Ni. Results are plotted in Fig. 5. Therein some experimental values from Ingemarrson et al. [17] are shown by the crosses with error bars. The lines for each isotope set, and for the minimal-\(T\) set of nuclei, are values predicted by the Carlson model [18],

\[
\sigma_R = \pi (R_p + r_0 A^{1/3})^2. \tag{4}
\]

The values of \(R_p\) and \(r_0\) required are listed in Table I. The total reaction cross sections of the sets of isotopes all lie on smooth curves defined by the Carlson model as do the minimal-\(T\) set of \(N = Z\) nuclei, \(^{54}\)Fe, and \(^{58}\)Ni. It is clear that as neutrons are added there is a steady progression in the total reaction cross sections. There is also a Coulomb shift effect that is more pronounced at 65 MeV than at 200 MeV.

For both energies, the total reaction cross sections evaluated for the minimal-\(T\) nuclei as well as for each isotope set satisfy the Carlson model very well. The parameter values to fit the minimal-\(T\) nuclei properties also agree quite well with the universal set found by Carlson [18]. But the parameter values required to fit the isotope sets are quite different. It is noteworthy that our calculated values tend to be parallel (for each energy), with the 200 MeV results closer to those of the minimal-\(T\) set. Also the curves defined from the reaction cross sections for those nuclei are characterized by values of parameters very similar to those assessed by Carlson using a much larger data set. We then anticipate that measurement of total reaction cross sections from the scattering of radioactive beams from hydrogen is a means to assess the reliability of the structure assumed.

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\begin{table}
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\caption{Carlson model parameter values that give the curves plotted in Fig. 5.}
\begin{tabular}{|c|c|c|c|}
\hline
Set & Energy (MeV) & \(r_0\) (fm) & \(R_p\) (fm) \\
\hline
S & 65 & 2.133 & -2.255 \\
Ar & 65 & 2.144 & -2.416 \\
Ca & 65 & 2.094 & -2.322 \\
minimal-\(T\) & 65 & 1.301 & +0.347 \\
S & 200 & 1.608 & -1.346 \\
Ar & 200 & 1.763 & -1.913 \\
Ca & 200 & 1.718 & -1.806 \\
minimal-\(T\) & 200 & 1.281 & -0.332 \\
\hline
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