Spectroscopy of $^{36}$Mg: Interplay of Normal and Intruder Configurations at the Neutron-Rich Boundary of the “Island of Inversion”

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We report on the first spectroscopy study of the very neutron-rich nucleus $^{36}$Mg$_{24}$ using the direct two-proton knockout reaction $^8$Be($^{30}$Si, $^{36}$Mg + $\gamma$)X at 83 MeV/nucleon. The energy of the first excited $2^+$ state of $^{36}$Mg, $E(2^+_1)$ = 660(6) keV, was measured. The magnitude of the partial cross sections to the ground state and the $2^+_1$ state is indicative of strong intruder admixtures in the lowest-lying states as suggested by Monte Carlo shell-model calculations.

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The atomic nucleus is a strongly interacting fermionic quantum many-body system composed of protons and neutrons. The discrete single-particle energies are clustered, giving rise to the shell structure of the nucleus [1]. Large energy gaps between groups of single-particle orbits occur at certain fillings of these orbits with a “magic number” of protons or neutrons. The nuclear potential and resulting shell structure have been established for stable nuclei. Modifications to this shell structure in exotic nuclei with a large imbalance of protons and neutrons have already been observed: New magic numbers develop and conventional shell gaps break down [2]. Present experimental and theoretical efforts in nuclear physics are aimed at probing the driving forces behind those structural changes, which are most prominent in neutron-rich nuclei.

Historically, measurements of the mass of the neutron-rich $^{31,32}$Na isotopes revealed that they are more tightly bound than expected if one assumes a spherical shape and the validity of the conventional shell closure at neutron number $N = 20$ [3]. Further evidence for an onset of deformation in neutron-rich nuclei around $^{31}$Na emerged from the low excitation energy of the first $2^+$ state of $^{32}$Mg [4], and from the large $B(E2; 0^+ \rightarrow 2^+_1)$ excitation strength [5]. Shell-model calculations that allow for neutron particle-hole excitations across the $N = 20$ shell gap reproduced the experimentally observed onset of deformation in this region. The $\nu(sd)^{-2}(fp)^{1+2}$ particle-hole “intruder” configurations ($2\hbar\omega$) were proposed to dominate the ground states of neutron-rich nuclei with $10 \leq Z \leq 12$ and $20 \leq N \leq 22$ [6]. This region, where the deformation-driving $2\hbar\omega$ states are energetically favored over the normal—$0\hbar\omega$—shell-model configurations, is called the “island of inversion” [6]. The breakdown of the $N = 20$ shell closure in this region was later put into the broader scheme of shell evolution driven by the spin-isospin properties of the effective nucleon-nucleon interaction [2].

In the chain of Mg isotopes, experimental evidence [4,5,7–12] confirmed the onset of collectivity and the presence of intruder configurations and placed $^{31–34}$Mg inside the island of inversion. $^{30}$Mg was found to be less collective and well described by calculations involving only $0\hbar\omega$ configurations [8,13]. On the neutron-rich side, however, so far hardly any data exist on the structure of the most exotic nuclei with $N > 22$. Warburton et al. predict the island to extend to neutron number $N = 22$ with magnesium isotopes heavier than $^{34}$Mg outside [6], while Utsumo et al. predict $^{36}$Mg to have intruder configurations in the ground state wave function [14].

The recently developed Monte Carlo shell model (MCSM) with the SDPF-M effective interaction [14] allows unrestricted mixing of neutron particle-hole configurations across the $N = 20$ shell gap and thus provides for the first time a consistent picture of the island of inversion and its neighborhood. In particular, the probabilities of $0\hbar\omega$ configurations and $2\hbar\omega$ cross-shell excitations can be quantified [15]. Experimental information on the interplay of normal and intruder configurations thus poses a benchmark test for the sd-$pf$ cross-shell interaction and the validity of the theoretical model space.

In this Letter we report on the first nuclear spectroscopy study of $^{36}$Mg. At all present rare-isotope facilities, the production rates for this exotic nucleus are insufficient to perform inelastic scattering experiments or direct reactions employing a beam of $^{36}$Mg. Furthermore, $^{36}$Mg cannot be populated in $\beta$ decay since the parent $^{36}$Na is neutron...
unbound. Another approach has to be chosen to reveal the nuclear structure of $^{36}$Mg. In the present study, we used the $^{9}$Be($^{38}$Si,$^{36}$Mg + γ)X two-proton knockout reaction at 83 MeV/nucleon to populate $^{36}$Mg. This will be the most neutron-rich nucleus with $Z = 12$ accessible to γ-ray spectroscopy until next-generation rare-isotope facilities are operational.

The $^{38}$Si secondary beam was produced by fragmentation of a 140 MeV/nucleon $^{40}$Ca primary beam delivered by the Coupled Cyclotron Facility of the National Superconducting Cyclotron Laboratory. The primary beam was directed onto a 752 mg/cm² $^{9}$Be production target. $^{38}$Si of 95% purity was selected in the A1900 fragment separator [16] at 1.66% momentum acceptance. The $^{38}$Si secondary beam interacted with a 376(4) mg/cm² thick $^{9}$Be reaction target placed at the reaction target position of the large-acceptance S800 spectrograph [17]. At $2.5 \times 10^{11}$/s $^{40}$Ca primary beam rate, the typical rate of $^{38}$Si on target was 1500 per second. The projectiles were identified using the time-of-flight difference measured between two scintillators before the reaction target. The reaction residues were identified on an event-by-event basis in the focal plane of the S800 spectrograph from the energy loss measured in the ionization chamber and the time of flight measured between plastic scintillators (see Fig. 1).

The secondary $^{9}$Be target was surrounded by the high-resolution γ-ray detection system SeGA, an array of 32-fold segmented HPGe detectors [18]. The segmentation of the detectors allows an event-by-event Doppler reconstruction that deduces the angle of the γ-ray emission from the position of the segment that registered the highest energy deposition. Sixteen detectors were arranged in two rings (90° and 37° central angles with respect to the beam axis). The 37° ring was equipped with seven detectors while nine detectors were located at 90°. The energy-dependent photopeak efficiency of the array was determined with a $^{152}$Eu source (2.6% at 1 MeV).

An inclusive cross section of 0.10(1) mb for the $^{9}$Be($^{38}$Si,$^{36}$Mg + γ)X reaction was derived from the yield of $^{36}$Mg reaction residues divided by the number of incoming $^{38}$Si projectiles relative to the number density of the reaction target. This corresponds to one $^{36}$Mg ion for each $4 \times 10^5$ incoming $^{38}$Si projectiles. Systematic uncertainties of 4% and 2% attributed to the stability of the incoming beam and the choice of the software gates, respectively, are assumed to be independent and have been added in quadrature to the statistical uncertainty.

In coincidence with the $^{36}$Mg residues, a γ-ray transition at 660(6) keV was detected in SeGA (see Fig. 2). This γ-ray transition is attributed to the decay of the $2^+_1$ state of $^{36}$Mg to the $0^+$ ground state. From the efficiency-corrected γ-ray peak area, the number of reaction residues produced in this excited state was determined. The $2^+_1 \rightarrow 0^+$ γ-ray angular distribution was calculated from the $m$-substate distribution predicted by the eikonal reaction theory of the two-proton knockout [19] and found to have a negligible effect for the angular range covered by SeGA (see Fig. 2). A 5% uncertainty was assumed for the in-beam

![FIG. 1 (color online). Particle identification spectrum (energy loss versus time of flight) of reaction residues detected in the S800 spectrograph. $^{36}$Mg is well separated. The most intense constituent of the reacted beam is the $^{38}$Si$^{13+}$ charge state produced by electron pickup in the reaction target.](image1.png)

![FIG. 2 (color online). Upper panel: Doppler-reconstructed γ-ray spectrum in coincidence with $^{36}$Mg. The photopeak at 660 keV corresponds to the $2^+_1 \rightarrow 0^+$ transition of $^{36}$Mg. The γ-ray angular distribution for this transition (solid red curve) is calculated from the $m$-state distribution provided by the reaction theory and compared to an isotropic, Lorentz-boosted distribution. The effect is very small at 37° and 90°. Lower panel: Experimental $2^+_1$ energies in the chain of Mg isotopes compared to theory [20,22,23,37].](image2.png)
γ-ray efficiency, which was corrected for the Lorentz boost of the γ-ray distribution emitted by nuclei moving at 40% of the speed of light. 42(6)% of the measured inclusive two-proton knockout cross section leads to the first excited 2+ state of 36Mg; 58(6)% of the two-proton removal reactions populate the ground state. This corresponds to partial cross sections of $\sigma(0^+) = 0.058(9)$ mb and $\sigma(2^+) = 0.042(8)$ mb for the population of final states in 36Mg. There is no experimental evidence for the population of other excited states with appreciable cross section.

The competing two-step process is due to two-nucleon correlation effects in the nuclear wave function. The composition of the ground state and the nuclear structure details on the parentage and phase of the participating two-nucleon configurations in the ground state of the projectile with respect to the final states of the projectilelike reaction residue. The reaction $^9\text{Be}(^{36}\text{Si}, ^{36}\text{Mg})X$ is of particular interest since it proceeds from $^{38}\text{Si}$, with a nonintruder ground state that is well described by 0hω configurations [30], to $^{36}\text{Mg}$ which is expected to exhibit very significant 2hω components in the wave functions of the lowest-lying states. Thus, in the direct removal of two protons from the ground state of $^{38}\text{Si}$, only 0hω configurations in $^{36}\text{Mg}$ can be reached. The cross sections measured in this reaction thus allow one to quantify these nonintruder components in the wave functions of the final states of $^{36}\text{Mg}$.

The question arises whether $^{36}\text{Mg}$ can be considered to lie inside or outside of the island of inversion. The MCSM calculation with the SDPF-M effective interaction provides an excellent description of the $2^+$ excitation energies in the chain of Mg isotopes. The large-scale conventional shell-model calculations by Caurier et al. [21] overpredict the $2^+$ energy of $^{36}\text{Mg}$ only by 200 keV although it uses the $0h\omega$ model space only. For comparison, another shell-model calculation that does not allow for intruder configurations (using the interaction of [24]) places the $2^+$ of $^{36}\text{Mg}$ at 970 keV and thus underestimates the collectivity more. It becomes clear that the excitation energy alone may not be indicative of intruder contributions in $^{36}\text{Mg}$.

The question arises whether $^{36}\text{Mg}$ can be considered to lie inside or outside of the island of inversion. The MCSM calculation using the SDPF-M effective interaction predicts strong mixing of $2h\omega$ intruder and $0h\omega$ normal configurations in the wave functions of the lowest-lying states of $^{36}\text{Mg}$ and thus makes this nucleus a testing ground. The composition of the ground state and the $2^+$ excited state are predicted to be (40.8%, 58.6%) and (39.9%, 59.5%) of ($0h\omega, 2h\omega$), respectively. $4h\omega$ components are negligible at 0.7% or less. In the following we outline how the two-proton removal from $^{38}\text{Si}$ to $^{36}\text{Mg}$ can be used to assess this composition experimentally.

In the collision of a neutron-rich projectile beam with a light target, for example, $^9\text{Be}$, the sudden removal of two protons has been shown to proceed as a direct reaction not only providing information on nuclear structure but also on two-nucleon correlation effects in the nuclear wave functions [19,25–29]. The competing two-step process is energetically strongly hindered for the present case. A nondirect population of the bound states of $^{36}\text{Mg}$ by one-proton removal and subsequent proton evaporation would involve states more than 14 MeV above the low neutron separation energy of $^{37}\text{Al}$. A reaction theory formalism, using eikonal dynamics and microscopic, correlated two-nucleon transition densities from shell-model calculations, has been developed to include two-nucleon stripping (inelastic breakup) and diffraction dissociation [19,29]. The two-nucleon amplitudes carry the nuclear structure details on the parentage and phase of the participating two-nucleon configurations in the ground state of the projectile with respect to the final states of the projectilelike reaction residue. The reaction $^9\text{Be}(^{36}\text{Si}, ^{36}\text{Mg})X$ is of particular interest since it proceeds from $^{38}\text{Si}$, with a nonintruder ground state that is well described by 0hω configurations [30], to $^{36}\text{Mg}$ which is expected to exhibit very significant 2hω components in the wave functions of the lowest-lying states. Thus, in the direct removal of two protons from the ground state of $^{38}\text{Si}$, only 0hω configurations in $^{36}\text{Mg}$ can be reached. The cross sections measured in this reaction thus allow one to quantify these nonintruder components in the wave functions of the final states of $^{36}\text{Mg}$.

The eikonal S matrices for elastic scattering of protons and $^{36}\text{Mg}$ on $^9\text{Be}$ were calculated as described in [19]. The protons in the ground state of $^{38}\text{Si}$ were assumed to occupy $sd$-shell orbitals with two-nucleon amplitudes in the $0h\omega$ space obtained [31] using the SDPF-M effective interaction. The proton single-particle wave functions were calculated from Woods-Saxon potentials. Based on $[1d_{5/2}]^2$ calculations using a range of binding potentials, with $1.20 < r_0 < 1.35$ fm, $0.60 < a < 0.75$ fm, and $0 < V_{so} < 6$ MeV, that generate single-particle states with rms radii in the range 3.25–3.57 fm, the computed cross sections scale linearly (each to better than 3%) with the mean squared radius of the orbital. There is negligible dependence on potential parameters beyond their effect on this radial extent. We thus constrain the orbital radii by choosing $r_0$ to reproduce the rms radii of the $d_{5/2}, d_{3/2}$, and $s_{1/2}$ orbits from the HF calculation and use a standard potential set with $a = 0.7$ fm and $V_{so} = 6$ MeV. The depths of the potentials were chosen to match the ground and excited-state two-proton separation energies, respectively. With this prescription, we calculate cross sections of 0.36 mb at the 0+ ground state and of 0.28 mb to the first 2+ state in $^{36}\text{Mg}$. The calculated cross sections are most sensitive to the choice of the rms radius of the core. A change of 0.05 fm results in a change of 7.5% in the cross section. To constrain the geometry best, the density of $^{36}\text{Mg}$ was taken from a spherical Skyrme Hartree-Fock calculation [32], having a rms matter radius of 3.34 fm.

This theoretical approach, when applied to five test cases of two-nucleon knockout from $sd$-shell configurations, overestimates the measured cross sections [33]. A fit to those data requires a suppression of the shell-model spectroscopic strengths by a factor $R_s(2N) = 0.5$ [19]. This reduction, the two-nucleon analog of the suppressions ($R_s$) observed in electron- and nuclear-induced single-nucleon knockout reactions [34], has to be taken into account in any quantitative comparison. Here, we use the weighted mean of the five test cases with an added 15% systematic uncertainty; that is, $R_s(2N) = 0.50(8)$. In the single-nucleon
case, a dependence of the $R_s$ on the asymmetry at the neutron and proton Fermi surfaces has been reported [35,36]. Any such dependence of the $R_s(2N)$ has yet to be established. With $R_s(2N) = 0.50(8)$ we obtain partial cross sections of $\sigma^{0}\theta(0^+) = 0.15(2)\text{ mb}$ and $\sigma^{0}\theta(2^+_1) = 0.13(2)\text{ mb}$.

The ratio of the measured partial cross sections $b = \sigma(2^+_1)/\sigma(0^+) = 0.72(15)$ is reproduced by the calculation with $b_{th} = 0.87$. Given that the $2\hbar\omega$ components of the wave functions of $^{36}\text{Mg}$ have no overlap with the $0\hbar\omega$ configurations in the ground state of $^{38}\text{Si}$, these partial cross sections can now be compared with the measured values to quantify the $0\hbar\omega$ components. Thus, the ratios $\sigma(0^+)/\sigma^{0}\theta(0^+) = 0.38(8)$ and $\sigma(2^+_1)/\sigma^{0}\theta(2^+_1) = 0.32(8)$ imply $38(8)\%$ and $32(8)\%$ of $0\hbar\omega$ components in the ground and $2^+_1$ states of $^{36}\text{Mg}$, respectively. These are compared to MCSM calculations in the lower panel of Fig. 3, showing that the measured results are reproduced.

Our estimate, based on the mismatch between the experimental and theoretical cross sections, is consistent with the $0\hbar\omega$ fractions predicted by the MCSM, placing $^{36}\text{Mg}$ inside the island of inversion. Figure 3 also shows the predicted composition of the wave function of the second excited $2^+_1$ state of $^{36}\text{Mg}$. Both the very small $0\hbar\omega$ component in this $2^+_1$ state and the small calculated cross section to the first $4^+_1$ state, an order of magnitude smaller than those for the $0^+$ and $2^+_1$ states, are consistent with the nonsurvival of these states in the present experiment. The second $0^+$ state is predicted to lie above the neutron separation energy and thus is not accessible by $\gamma$-ray spectroscopy.

In summary, we have used the direct two-proton knockout reaction $^9\text{Be}(^{38}\text{Si}, ^{36}\text{Mg} + \gamma)X$ at 83 MeV/nucleon to perform spectroscopy of the very neutron-rich nucleus $^{38}\text{Mg}$ for the first time. The $2^+_1$ state was observed at 660(6) keV excitation energy. Partial cross sections to the ground and first $2^+_1$ state were measured and the wave functions of these lowest-lying states were found to be dominated by intruder configurations, as suggested by state-of-the-art large-scale MCSM calculations. This places $^{36}\text{Mg}$ within the island of inversion.

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[21] E. Caurier et al. performed the $0\hbar\omega$ calculation for $^{30,36}\text{Mg}$ [22,23] and the $2\hbar\omega$ calculations for $^{32,34}\text{Mg}$ [23].
[33] $^9\text{Be}(^{44}\text{Si}, ^{42}\text{Si}X$ [25] cannot be considered a test case since the shell structure of $^{44}\text{Si}$ and $^{42}\text{Si}$ is not yet understood.