superallowed weak nuclear decays are a powerful tool to search for oscillations of the Fermi matrix element to be

\[ T \equiv \text{CKM matrix, and vector current conservation requires} \]

TZ

\[ \text{processes and isospin-symmetry-breaking corrections. It is } \]

handed currents. However, before the measured mixing matrix as well as to place limits on scalar and right-handed unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) mass matrix, derived from muon decay,

Here

K

/\hbar c \delta C \equiv 2 \pi^2 \ln 2 / (m_e c^2)^5, G_F is the Fermi coupling constant derived from muon decay, \[ |V_{ud}| \] is an element of the CKM matrix, and vector current conservation requires the Fermi matrix element to be \[ |M_{fi}|^2 \] is an element of the CKM matrix, and vector current conservation requires the Fermi matrix element to be \[ |M_{fi}|^2 \equiv T(T + 1) - T_Z(T_Z \neq 1), \] where \( T \) is the isospin of the multiplet and \( T_Z \equiv \frac{1}{2}(Z - N) \). The remaining parameters are theoretical corrections that account for departures from strict isospin symmetry. \[ \Delta_Y \] is the nucleus-independent radiative correction; \( \delta_R \) is the nucleus-dependent radiative correction, which is the sum of a structure-independent \( \delta_R \) and structure-dependent \( \delta_{NS} \) components; and \( \delta_C \) is the nucleus-dependent isospin-symmetry-breaking correction. These corrections are all of the order of a few percentages.

The standard model predicts identical \( fT \) values for all \( J^\pi = 0^+, T = 1 \) decays. It is gratifying that the \( fT \) values of the nine most precisely known \( T = 1 \) superallowed decays are constant to better than one part in \( 3 \times 10^4 \) [1]; the mean \( fT \) value for these \( T = 1 \) superallowed transitions is [2]:

\[ fT(T = 1) = 3072.3(1.2)s. \]

In this article we investigate the isospin-symmetry-breaking correction \( \delta_C \) by studying the \( T = 2 \) superallowed \( \beta^+ \) decay of \( ^{32}\text{Ar} \), which is expected [3] to have an unusually large correction as shown in Fig. 1. The isospin symmetry breaking corrections are customarily separated into a “configuration mixing” part, \( \delta_{CM} \), that accounts for the charge-dependent mixing with other \( 0^+ \) states, and a “radial overlap” part, \( \delta_{RO} \), that accounts for the fact that the mean-field for the “parent proton” and “daughter neutron” is modified by the isospin-non-conserving Coulomb interaction (see Refs. [6–9]). For the nine most precisely known \( T = 1 \) cases \( \delta_{CM} \) is the

\[ \delta_{CM} \]

\[ \delta_{RO} \]

\[ \delta_{RO} \]
dominant part of the isospin-symmetry-breaking correction. It is therefore valuable to check the calculations in systems where these "radial overlap" corrections are inherently large. We studied $^{32}$Ar because the radial overlap effect is enhanced near the proton drip line where the rms radius of the valence orbits changes most rapidly with $Z$. Similar effects are also present in the $T = 1 \rightarrow T = 1$ multiplets but are less significant because these states are more tightly bound. The predicted $^{32}$Ar correction, $\delta C = (2.0 \pm 0.4)\%$, can be decomposed into $\delta_{C,\text{nom}} = 0.6\%$ and $\delta_{C,\text{eff}} = 1.4\%$.

B. Experimental technique

We measured the absolute branching ratio for the $^{32}$Ar superallowed $\beta^+$ decay by implanting $^{32}$Ar ions in a silicon detector and counting their subsequent decays. Absolute $\beta$ branches can be determined in implantation experiments using either of the following two techniques: (1) one can continuously count implanted parent ions along with the decay products and at the end of the experiment compute the ratio of the total number of decay products to the total number of implanted parent ions or (2) one can correlate each implantation with its corresponding decay, ensuring a high signal-to-background ratio because decay products are counted only after an ion of interest (which can be defined using stringent constraints) has arrived. In the case of $^{32}$Ar with $t_{1/2} = 100.5(3)$ ms [10], the second technique would have limited the incoming rate to $\approx 2$ ions/s because in this case one must wait a substantial period (>5 half-lives) before implanting another ion. We therefore used the first method and implanted ions at a rate of $\approx 20$ s$^{-1}$ to obtain the statistics necessary for a determination of the branch with better than 1% uncertainty. The problem of sacrificing statistics for a good signal-to-background ratio can in principle be avoided by using a segmented counter, but concerns [11] about the efficiencies of the intersegment sites led us to use unsegmented detectors.
event in detectors D₂, D₃, or D₄ resulting in a β-detection efficiency of (95 ± 1)% (see Sec. VA). Because the Si wafers came from the factory attached toward one face of the ≈8.3-mm-deep Al mounting rings, D₄ could be placed close to D₃ (≈2.0 mm away), whereas D₂ was necessarily further away (≈8.3 mm). The array of HPGe detectors consisted of three 4-fold segmented “clover” detectors [12] (G₁, G₂, and G₃, each with efficiencies of ≈120%) and two monolithic crystals (G₄ and G₅, with efficiencies of 80 and 120%, respectively).

III. INCOMING ION EVENTS

A. Strategy

We separated events into incoming ions and decays. The former consisted of events in which D₁, D₂, and D₃ registered energies larger than ≈0.1 GeV, and no energy was deposited in D₄. Decays left between 114 keV and 11.5 MeV in D₃ and/or between 55 keV and 5.5 MeV in either D₂ or D₄; in all cases, decay events were required to have not deposited any energy in D₁. Incoming ions were further separated into three categories. Good ions stopped in D₃ and were clearly identified as ³²Ar ions in both the E₁ vs. TOF₁ (energy and time-of-flight measured with D₁) and the E₃ vs. E₂ spectra. We created buffer regions in the particle identification spectra that included ions we could not guarantee were ³²Ar ions; we made sure that these regions were large enough so that no ³²Ar ion whose proton emissions could have been detected by D₃ could lie outside the union of the good and buffer regions. The ions in the buffer regions were labeled as ambiguous. Contaminant ions were all those not contained in either of the above two groups. To avoid contaminating our delayed proton and γ spectra by decay products of ambiguous ions, we rejected all ambiguous ions and imposed a 500-ms software dead time (about five ³²Ar half-lives) on counting either incoming ions or proton decays following the implantation of an ambiguous ion. If another ambiguous ion was detected within 500 ms of the previous ambiguous ion, the software dead time was reset to count for another 500 ms.

B. Ion identification

Incoming ions were identified with the help of the code LISE [13]. Figure 3 shows the E₁ versus TOF₁ spectrum; the area labeled “Region 3.1” contains mainly ³²Ar ions but is not clearly separated from contaminants. Region 3.2, which surrounds the main ³²Ar group in Region 3.1, mostly contains ambiguous ions. Figure 4 shows the E₃ versus E₂ spectrum of all events in Fig. 3. Figure 5 is similar to Fig. 4 but contains only events in Region 3.1 of Fig. 3. Region 5.1 of Fig. 5 contains the good ³²Ar ions; the remaining events are ³²Ar ions that either reacted before reaching D₃ or whose full energy was not detected in D₃.

Before we describe our criteria for separating good ions from ambiguous ions, we explain some peculiar features in our particle identification spectra.

(i) Saturation effects. In Fig. 4 the horizontal line in the high E₁ region and vertical line in the high E₂ region are due to saturation of the preamplifier signals. To obtain the best possible energy resolution we used a single preamplifier on each detector; therefore the preamplifiers had to process an unusually wide range of energies.

(ii) Events in Region 4.1. Figure 6 shows the E₁ versus TOF₁ spectrum for events in Region 4.1, showing that these events originate in a high-energy tail in the beam profile. These ions left less energy in D₁ and D₂ but more in D₃ than the good ³²Ar ions, as one would expect from a high-energy tail. Other effects, such as channeling in D₁, could lead to the same features as seen in Fig. 4, but then E₁ would be independent of TOF₁ which is not what we observe (see Fig. 6).

(iii) Vertical line descending from the main ³²Ar group in Fig. 5. These events are the combined results of ³²Ar
ions that landed near the edge of D3 after scattering in D2 and those that reacted before coming to rest. This was confirmed in our Monte Carlo calculations (which are described below).

(iv) 45° line descending from main 32Ar group in Fig. 5. We are able to select this structure by requiring events to register as a heavy-ion event but to give signals below the threshold of the high-gain ADCs for D2 and D3. Figure 7 shows that this is an efficient vetoing strategy. These events apparently resulted from high-voltage breakdowns in the RPMS that occurred at about the same time as a heavy-ion was passing through. The breakdown produced intense x-ray bursts that loaded down our particle counters. Events in this 45° line are from ions that arrived before the detectors had fully recovered.

C. Number of implanted 32Ar ions

We used the following criteria to tag 32Ar ions as good or ambiguous. Good 32Ar ions had to appear in Regions 3.1 and 5.1, without depositing any energy in D4. Ambiguous ions, however, had to appear in either

(i) Region 3.2 and either Region 5.1 or 5.2,
(ii) Region 3.1 and Region 5.2 (which excludes Region 5.1),
(iii) Regions 3.1 and 5.1 (much like a good ion) but also depositing energy in D4,

or to fall under the categories of subsections 2 and 4 defined in the previous section (the high-momentum tail of the incoming beam and the “45° line”). Thus, all of the features described

FIG. 4. (Color online) $E_3$ versus $E_2$ spectrum of events shown in Fig. 3. Region 4.1 was defined to show that the events in this region arise from a high-energy tail of the beam and to define Region 6.1 (see text and Fig. 6).

FIG. 5. (Color online) $E_3$ versus $E_2$ distribution of events in Region 3.1 in Fig. 3. Region 5.1 contains good 32Ar ions and has 61 times more events than the surrounding Region 5.2 which contains ambiguous ions.

FIG. 6. (Color online) $E_1$ versus TOF1 distribution of events in Region 4.1. The correlation between TOF1 and $E_1$ shows that these events arise from a high-momentum tail of the implanted ion distribution.

FIG. 7. (Color online) Same as Fig. 5, except that events corresponding to sparks in the RPMS (see text) have been rejected.

065503-4
in Sec. III B are encompassed by our definition of ambiguous ions.

To check if the non-Gaussian features in the momentum distribution of the beam profile could have caused $^{32}$Ar ions to appear in the contaminant regions of the $E_1$ vs. TOF$_1$ spectrum, we generated an $E_3$ vs. $E_2$ spectrum gated by contaminant ions. The resulting spectrum contained no events that corresponded to mass 32.

A total of $N_{\text{Ar}} = 2.241 \times 10^5$ good $^{32}$Ar ions (not preceded by an ambiguous ion in 500 ms) were implanted, along with $N_c = 158 \times 10^4$ ambiguous ions and $N_c \approx 1.000 \times 10^6$ contaminant ions.

If any of the ambiguous events were $^{32}$Ar ions, then at most $2^{-500/\tau_{1/2}} = (3.18 \pm 0.03)%$ of their decays would occur after the 500-ms veto period had ended and would be indistinguishable from the decay products of good $^{32}$Ar ions. We corrected for this effect by averaging the two extreme cases: that all ambiguous ions were $^{32}$Ar ions and that no ambiguous ions were $^{32}$Ar ions. We added this average to $N_{\text{Ar}}$ with a 100% uncertainty to obtain:

$$N_{\text{Ar}}^{\text{incorr}} = 2.244(3) \times 10^6,$$

for the number of incoming $^{32}$Ar ions without corrections for fragmentation. We discuss the effect of fragmentation in subsection A1 and show it requires a (0.2 ± 0.2)% correction. We consequently adopt

$$N_{\text{Ar}} = 2.239(5) \times 10^6.$$

Because the gates defined by Regions 3.1 and 5.1 are so narrow (note the purity of $^{32}$Ar in Fig. 5 compared to Fig. 4), the probability that a contaminant ion satisfied both gate conditions was negligible.

### IV. $\beta$-DELAYED PROTON BRANCHES

Figure 8 shows the energy spectrum of decay events in D$_1$ for one of our 21$^{32}$Ar runs (approximately 3% of the total data set). The prominent peak at $E_p \approx 3500$ keV was produced by delayed protons following the superallowed decay of $^{32}$Ar (see Fig. 9 for a simplified decay scheme). The proton lines have pronounced high-energy tails (cf. Fig. 10) from the summing with the energy deposited by the escaping positrons (i.e., $E_3 = E_p + E_\gamma$). The structure below $E_3 \approx 1.2$ MeV is dominated by $\beta$ decays that did not produce protons (such as $^{32}$Ar decays to particle-bound states of $^{32}$Cl, or implanted $^{31}$Cl ions that decay mainly to the $^{31}$S ground state). The spectrum is dominated by $\beta$-delayed proton decays to the ground and first excited states of $^{31}$S (the $p_0$ and $p_1$ groups, respectively). We did not find any evidence for $\beta$-delayed proton decay to the third excited state of $^{31}$S ($p_2$ branch), and only a weak branch for the Fermi transition followed by protons leaving $^{31}$S in its second excited state ($p_2$ branch). The other peaks, most prominently at $E_3 = 2.3$ and 2.6 MeV in Fig. 8 and $E_p = 2.1$ and 2.4 MeV in Fig. 10, originate from Gamow-Teller transitions. These transitions are not relevant for isospin mixing of the isobaric analog state (IAS), but will be used to calculate the total feeding of the $^{31}$S first excited state and are

![Figure 8](image8.png)

**FIG. 8.** (Color online) (Top panel) Singles delayed proton spectrum in D$_1$ for a typical run (histogram) along with the fit to the Monte Carlo simulation (solid line). The filled curves correspond to proton emission following the superallowed transition and the dashed lines are the backgrounds. (Bottom panel) Same as above but all runs added together and gated by $E_p = 1249$ keV which selects the $p_1$ group at $E_p \approx 2.3$ MeV. In both cases, the ratio of the residuals to the standard deviation for each point is shown below the corresponding spectrum.

![Figure 9](image9.png)

**FIG. 9.** Simplified $^{32}$Ar decay scheme showing observed transitions that produce $\gamma$ rays.
The ISOLDE spectrum was fitted using an R-matrix formalism for overlapping, interfering daughter states [15] that parameterized the intrinsic delayed proton spectrum in terms of the transition matrix elements, energies and proton widths of 19 daughter states. The R-matrix spectra were separated into four noninterfering groups corresponding to proton emission leaving the $^{31}\text{S}$ in its ground and first three excited states for Fermi transitions and another four corresponding to Gamow-Teller transitions. These R-matrix intrinsic shapes were folded with two exponentials as described in Ref. [14] of fitting the MSU data in Fig. 8 by feeding our R-matrix fit. (Lower panel) Ratio of residuals to standard deviation for each point. This spectrum was taken by implanting 60-keV $^{32}\text{Ar}$ ions into a 20 $\mu$g/cm$^2$ carbon foil and observing the $\beta$-delayed proton groups with cooled PIN diodes. The detection setup was immersed in a 3.5-T magnetic field that prevented the $\beta^+$'s from reaching the detectors and summing with protons.

We determined the areas of the individual delayed proton peaks with the aid of previous data—a high-resolution ($\approx$5-keV FWHM) proton spectrum obtained at ISOLDE [10,14] shown in Fig. 10, and branching ratios (relative to the superallowed group) of nine weakly populated states with $E_p > 4$ MeV as measured in Ref. [16].

The ISOLDE spectrum was fitted using an R-matrix formalism for overlapping, interfering daughter states [15] that parameterized the intrinsic delayed proton spectrum in terms of the transition matrix elements, energies and proton widths of 19 daughter states. The R-matrix spectra were separated into four noninterfering groups corresponding to proton emission leaving the $^{31}\text{S}$ in its ground and first three excited states for Fermi transitions and another four corresponding to Gamow-Teller transitions. These R-matrix intrinsic shapes were folded with a detector response function consisting of a Gaussian folded with two exponentials as described in Ref. [14] to fit the data in Fig. 10, yielding the relative intensities, energies, and intrinsic widths of the proton groups with energies up to 4 MeV.

We fitted the MSU data in Fig. 8 by feeding our R-matrix intrinsic shapes, extracted from the ISOLDE data, into a Monte Carlo simulation of the MSU experiment. The level structure above $E_p = 4$ MeV, which was not determined by the ISOLDE data, was varied to fit Fig. 8 giving the results in Table I. Including a broad ($\approx$360-keV FWHM) Gaussian peak at 6.05 MeV significantly improved the local $\chi^2$. This peak was not reported in the tables of Ref. [16] but could arguably be present in their spectrum. However, we see no evidence of this broad peak when gating the $D_3$ spectrum on a $\beta$ event in either $D_2$ or $D_4$. Therefore, the peak is not produced in an $^{32}\text{Ar} \beta^+$ decay, but we cannot exclude the possibility of EC decay or a background. The uncertainties we quote below will include whether we assume this peak are $^{32}\text{Ar}$ or not. The weak extra peaks in the lower panel of Fig. 8 at 2.6 and 3.6 MeV originate from random coincidences with the Ge detectors.

We accounted for detector responses and for scattering by using GEANT [17] to track the decay products (protons, $\beta$s, and their associated annihilation radiation). These simulated spectra were then fitted with the following free parameters—the gain of the energy calibration, the Gaussian noise of the detectors, an overall normalization, and two parameters describing the $\beta$ background (discussed below). The data were divided into 49 blocks of approximately equal numbers of implanted ions and then fitted separately. This separation was made because the extremely sharp rise of the $p_0$ peak made the results very sensitive to small changes in the gain of $D_3$ and, as shown in the top plot of Fig. 11, the level of background/contamination changed as the run progressed.

The largest background in the proton singles spectrum of Fig. 8 came from $\beta$s that were not followed by delayed protons. We simulated the shape and magnitude of this background using the decay scheme outlined in Fig. 9; we expect two of such $\beta$ events per incoming $^{32}\text{Ar}$, with end-point energies ranging from 1.4 to 12.2 MeV. The dashed curve in Fig. 8 shows the combined simulated $\beta$ energy losses from $^{32}\text{Ar}$, $^{31}\text{Cl}$, and $^{31}\text{S}$ decays. Figure 11 shows that this background level changed somewhat as the run progressed, indicating earlier runs had a significantly worse $^{31}\text{Cl}$ contamination level than later ones.

### Table I. $p_0$ groups with $E_p > 4$ MeV

<table>
<thead>
<tr>
<th>$E_p$ (keV)</th>
<th>$E_\gamma$ (keV) (in $^{32}\text{Cl}$)</th>
<th>$\Gamma_p$ (%)</th>
<th>$E_p$ (keV)</th>
<th>$\Gamma_p$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3984(5)</td>
<td>5659(5)</td>
<td>1.1(1)</td>
<td>3994</td>
<td>1.2(9)</td>
</tr>
<tr>
<td>4340(5)</td>
<td>6063(5)</td>
<td>0.71(7)</td>
<td>4341</td>
<td>0.70(9)</td>
</tr>
<tr>
<td>4529(8)</td>
<td>6259(8)</td>
<td>0.54(5)</td>
<td>4521</td>
<td>0.52(4)</td>
</tr>
<tr>
<td>4997(10)</td>
<td>6742(10)</td>
<td>0.10(2)</td>
<td>4975</td>
<td>0.06(2)</td>
</tr>
<tr>
<td>5567(5)</td>
<td>7330(5)</td>
<td>0.76(8)</td>
<td>5552</td>
<td>0.57(4)</td>
</tr>
<tr>
<td>5699(10)</td>
<td>7467(10)</td>
<td>0.18(8)</td>
<td>5675</td>
<td>0.05(3)</td>
</tr>
<tr>
<td>5833(4)</td>
<td>7605(5)</td>
<td>0.54(5)</td>
<td>5817</td>
<td>0.44(4)</td>
</tr>
<tr>
<td>6097(10)</td>
<td>7878(10)</td>
<td>0.11(2)</td>
<td>6060</td>
<td>0.09(2)</td>
</tr>
<tr>
<td>6396(10)</td>
<td>8186(10)</td>
<td>0.06(2)</td>
<td>6347</td>
<td>0.06(2)</td>
</tr>
</tbody>
</table>

*From the spectrum in the top panel of Fig. 8; $p_1$ groups are shown in Table II.

*Relative to the superallowed $p_0$ group.

### A. Total proton branch in $^{32}\text{Ar} \beta$ decay

The proton branching ratios were found by fitting each of the 49 blocks of $\beta$-delayed proton spectra individually and then averaging the results. The number of ions was simultaneously broken up, and the corrections described in Sec. III C were applied in each instance. Summing up the contributions from all R-matrix levels associated with $^{32}\text{Ar}$ decay and dividing by $N_A$ gave the results shown in the bottom plot of Fig. 11. The fit to the average of all 49 blocks had a $\chi^2$ per 48 degrees of freedom.
strength, \( B(F) = 4 \), resides in the superallowed transition. Nevertheless, we will show in Sec. V that the \( T = 2 \) state has an \( \approx 10\% \gamma \) branch. In addition the end-point energy is now known to be lower than the value used in Ref. [16] by about 21 keV; using the correct end-point energy would decrease their deduced branch by \( \approx 2\% \). Finally, isospin symmetry breaking effects are expected to reduce \( B(F) \) by \( \approx 2\% \). When these corrections are included the number quoted above translates to \( b^\beta_{\gamma\gamma} = (37 \pm 3\%) \), in agreement with our value.

### B. Delayed proton branches following the superallowed decay

The proton branching ratio of the \( ^{32}\text{Cl} \) isobaric analog state (IAS) to the \( ^{31}\text{S} \) ground state (\( p_1 \) group), deduced from \( b^\beta_{\gamma\gamma} \) using the relative intensities from the ISOLDE data, is

\[
b^\beta_{p_1} = (20.50 \pm 0.03 \pm 0.12\%) .
\]

The peak at 2.3 MeV, clearly visible in the bottom of Fig. 8, corresponds to proton emission from the IAS to the first excited level in \( ^{31}\text{S} \) (the \( p_1 \) group). This peak appears in the ISOLDE spectrum (Fig. 10) as a partially resolved shoulder on the right of the structure at 2.1 MeV. We obtained \( N_{p_1}/N_{p_0} = (1.25 \pm 0.10\%) \) from the NSCL data and \( N_{p_1}/N_{p_0} = (1.29 \pm 0.04\%) \) from the ISOLDE data. We adopted the weighted average\(^1 \)

\[
N_{p_1}/N_{p_0} = (1.28 \pm 0.04\%).
\]

We see no evidence for \( p_2 \) and \( p_3 \) decays to \( ^{31}\text{S} \) states at \( E_\gamma = 2235.6 \) keV and \( E_\gamma = 3079 \) keV, respectively. We used the ISOLDE spectrum to obtain: \( N_{p_1}/N_{p_0} = (0.12 \pm 0.04\%) \) and \( N_{p_3}/N_{p_0} = (0.07 \pm 0.07\%) \). These numbers are consistent with the upper limits obtained from this work.

We find a total proton branch for the superallowed transition of

\[
b^\beta_{SA} = b^\beta_{p_0} \left( 1 + \sum_{i=1,3} \frac{N_{p_i}}{N_{p_0}} \right) = (20.79 \pm 0.07 \pm 0.12\%)
\]

Additional potential systematic effects are discussed in Sec. A2 and are shown to be negligible compared to the total uncertainty. This the first time a delayed proton branch has been measured with a precision better than 1%.

### C. Delayed proton transitions feeding the \( ^{31}\text{S} \) first excited state

In addition to the \( p_1 \) superallowed group described above, we found several proton groups corresponding to \( p_1 \) decays following Gamow-Teller transitions. The lower panel of Fig. 8 shows the \( E_\gamma \) spectrum of events in coincidence with the 1249-keV \( \gamma \) ray in any of the five Ge detectors. Once we

\(^1\)Although this Fermi peak does not interfere with its neighboring Gamow-Teller peaks, its intensity extracted from the ISOLDE spectrum varied by \( \approx 20\% \) depending on the assumed sign of the GT-GT interference between the state at \( E_\gamma = 2.1 \) MeV and tails from other resonances, mainly from the wide resonance at \( E_\gamma = 2.4 \) MeV. We removed this ambiguity by requiring the R-matrix parameters to fit simultaneously the spectra in Fig. 10 and both panels of Fig. 8.
identified the groups, we used the ISOLDE spectrum to obtain their relative intensities. This allowed us to infer the relative intensities of these groups without depending on the ambiguous ions. Table II lists the intensities of these proton groups relative to the intensity of the superallowed proton group populating the $^{31}\text{S}$ ground state. The intensities are only approximate and were obtained as areas under the R-matrix fits within a region corresponding to four times the width of the state.

### Table II. Proton groups in coincidence with a 1249-keV $\gamma$ ray from the coincidence spectrum in Fig. 8.

<table>
<thead>
<tr>
<th>$E_p$ (keV)</th>
<th>$E_i$ (keV) (in $^{32}\text{Cl}$)</th>
<th>$I_p$ (%)</th>
<th>Ref. [16]</th>
</tr>
</thead>
<tbody>
<tr>
<td>912(5)</td>
<td>3772(5)</td>
<td>0.074(4)</td>
<td>–</td>
</tr>
<tr>
<td>1218(5)</td>
<td>4087(5)</td>
<td>1.90(22)</td>
<td>1214(10)</td>
</tr>
<tr>
<td>2145(5)</td>
<td>5046(5)</td>
<td>1.28(4)</td>
<td>–</td>
</tr>
<tr>
<td>2394(5)</td>
<td>5302(5)</td>
<td>0.56(11)</td>
<td>–</td>
</tr>
<tr>
<td>2515(5)</td>
<td>5427(5)</td>
<td>2.93(11)</td>
<td>–</td>
</tr>
<tr>
<td>2870(5)</td>
<td>5794(5)</td>
<td>3(1)</td>
<td>–</td>
</tr>
<tr>
<td>3581(5)</td>
<td>6528(5)</td>
<td>0.24(4)</td>
<td>3592(10)</td>
</tr>
<tr>
<td>3649(5)</td>
<td>6599(5)</td>
<td>0.32(3)</td>
<td>3643(10)</td>
</tr>
<tr>
<td>3785(5)</td>
<td>6738(5)</td>
<td>0.52(5)</td>
<td>–</td>
</tr>
<tr>
<td>4529(5)</td>
<td>7507(5)</td>
<td>&lt;0.03%</td>
<td>4521(10)</td>
</tr>
<tr>
<td>4630(5)</td>
<td>7611(5)</td>
<td>0.16(5)</td>
<td>4621(10)</td>
</tr>
<tr>
<td>4869(5)</td>
<td>7857(5)</td>
<td>0.26(3)</td>
<td>4858(10)</td>
</tr>
</tbody>
</table>

$^a$From the spectrum in the bottom panel of Fig. 8, unless otherwise stated.

$^b$Relative to superallowed proton group leaving $^{31}\text{S}$ in its ground state.

$^c$This value comes mainly from the ISOLDE data, as discussed in Sec. IV B.

$^d$This is a very broad peak, and therefore it is difficult to accurately quote its intensity.

$^e$We observe much less strength at this proton energy in coincidence with a 1249-keV $\gamma$ ray compared to Ref. [16], but our intensities agree in the singles spectrum.

#### V. $\beta$-DELAYED $\gamma$ BRANCHES

Figure 12 shows the summed $\gamma$-ray spectrum from the five Ge detectors in coincidence with a decay event in $D_2$, $D_3$, or $D_4$. To optimize statistics we did not impose the ambiguous condition. Table II lists the corresponding sum peaks (with energy of 511 keV plus the photon energy) were clearly visible.

![Figure 12](image-url)  
**FIG. 12.** (Color online) Spectrum of $\gamma$ rays in coincidence with a $\beta$ signal in $D_2$, $D_3$, or $D_4$. Lines from $^{32}\text{Ar}$ decay are marked with a “*”. The remaining labeled lines are from $^{32}\text{Cl}$ decay. For all strong lines the corresponding sum peaks (with energy of 511 keV plus the photon energy) were clearly visible.

with a PENELOPE [18] simulation. We use PENELOPE in this case rather than GEANT because the former provides a better description of low-energy $\beta$ s [19]; GEANT was used elsewhere when the analysis involved protons and heavy ions, which are not available in PENELOPE. Using the measured energy thresholds and assuming a ±10 keV uncertainty in their values, the PENELOPE simulation indicates $\epsilon_{\beta} = 0.95(1)$. As we show below, our method for determining the $\gamma$-ray efficiencies does not depend significantly on $\epsilon_{\beta}$ which we present only for completeness.

#### B. HPGe detector $\gamma$-ray efficiencies

As Fig. 9 shows, $1 - b_{\beta\gamma}^{32}\text{Ar} = 64.5\%$ of the $^{32}\text{Ar}$ $\beta$ decays are not followed by proton emission but rather feed the ground state of $^{33}\text{Cl}$ which itself is unstable $\beta^+$ decaying with a half-life of 0.3 s. The $^{32}\text{Cl}$ and $^{33}\text{Ar}$ decays have $\gamma$ lines in the same energy range, so the known intensities of the $^{32}\text{Cl}$ lines [20] provide an in situ calibration of the HPGe detection efficiencies. The detection efficiency, $\epsilon_{\gamma}$, of $^{32}\text{Cl}$ $\gamma$ rays with energy $E_{\gamma}$ registered in the $i$th Ge detector is given by

$$N_{\text{Ar}} \epsilon_{\gamma}^{(i)}(E_{\gamma}) \epsilon_{\beta} = \frac{\tilde{N}_{\gamma}^{(i)}}{(1 - b_{\beta\gamma}^{32}\text{Cl})},$$

where $\tilde{N}_{\gamma}^{(i)}$ is the photopeak area corrected for summing effects and $b_{\beta\gamma}^{32}\text{Cl}$ is the known $^{32}\text{Cl}$ $\gamma$ branch. The factor $\epsilon_{\beta}$ is the $\beta$ detection efficiency described in the previous section. We show in Eq. (9) below that the factor on the left side of Eq. (8), rather than $\epsilon_{\gamma}^{(i)}$ alone, is needed to compute the $\gamma$ branches following $^{32}\text{Ar}$ superallowed decay. This minimizes systematic uncertainties from the geometrical size and distribution of the source, as well as uncertainties from $^{32}\text{Ar}$ ions that could have escaped detection (i.e., ions that land outside the active area in $D_3$ but whose $\gamma$s and $\beta$s could have been detected). The $\gamma$-ray efficiencies depend on the $\beta$ thresholds
due to the different end points. However, our simulations indicate that this contribution to the total uncertainty of the $\gamma$ branch is negligible.

Figure 13 shows the calculated efficiencies from the known $^{32}$Cl lines along with PENELOPE calculations that describe the detectors' responses. These simulations accounted for matter outside the detectors that could attenuate or scatter $\gamma$-rays and included the radial and depth distribution of the $^{32}$Ar ions described in subsection A2. The 2230-keV peak from $^{32}$Cl appears near a line at 2236 keV that is fed by decays of both $^{31}$Cl and $^{32}$Ar. Figure 13 shows the efficiency calculated at this energy even though the 2230-keV line was not used in fitting the efficiency curve because it was not clearly resolved in all of the detectors.

The measured $^{32}$Cl points were fitted to the PENELOPE calculations with only the normalization free to vary. Once determined, the same normalization was applied to simulations at energies corresponding to the $^{32}$Ar lines. Corrections for $\gamma$ summing are dominated by summing with 511-keV annihilation radiation; however, both $^{32}$Ar and $^{32}$Cl undergo $\beta^+$ decay so this summing does not affect the ratios of peak areas. The summing with cascade $\gamma$ rays, however, depends on the multiplicity and correlations of $\gamma$ rays. We used the PENELOPE calculation to estimate these effects and calculate $\tilde{N}_Y^{(i)}$ from the fitted photopeak areas. As an example of the magnitude of the summing corrections, $\tilde{N}_Y^{(2)}$ for the 2230-keV $\gamma$ ray from $^{32}$Cl (which is always part of a cascade) is $\approx 5\%$ larger than the measured number of counts. In the less efficient 80% detector, the summing correction is only $\approx 2\%$.

C. $\gamma$ decays of the $^{32}$Cl IAS

The lowest $T = 2$ state in $^{32}$Cl decays predominantly by emitting protons. However, the proton decay of this state violates isospin symmetry, and the total width of this state is only $\approx 20$ eV [10,14]. As a result, $\gamma$ decays of this state cannot be neglected. Based on the decays of the isobaric analog states in $^{32}$P and $^{32}$S we expect $\gamma$ decays to three $1^+$ levels (see Fig. 9).

The absolute $\gamma$-ray branches were computed as:

$$b^{\beta \gamma} = \frac{\sum_{i=1}^{5} \tilde{N}_Y^{(i)}}{N_{Ar} \sum_{i=1}^{5} \epsilon_\gamma^{(i)} \epsilon_\beta},$$

where the sum runs over all five Ge detectors.

The denominator on the right side of Eq. (9) comes from calibrations using lines from $^{32}$Cl decay; this procedure makes the calculation of the branches rather independent of the distribution of parent ions and summing with 511-keV $\gamma$-rays. The $\beta-\gamma$ coincidence spectrum shown in Fig. 12 exhibits a peak at $E_\gamma = 3877.5(3)$ keV that is a candidate for the analog of the $^{32}$P $5072$ keV $\rightarrow 1149$ keV transition. Figure 14 shows the spectrum of $\gamma$ rays in coincidence with a 3878-keV $\gamma$ ray in the $\beta-\gamma$ coincidence spectrum (Fig. 12). The spectrum clearly shows the $\gamma$ rays expected from de-excitation of the 1168.5(2) keV state. This leads us to conclude that the decays originate in a state at $E_\gamma = 5046.3(4)$ keV, after correcting for the nuclear recoil (see Sec. VI C4 for implications on the mass of $^{32}$Cl). We have thus identified the isobaric analog of the $^{32}$P $5072$ keV $\rightarrow 1149$ keV transition and we find its branch to be $b^{\beta \gamma}(E_\gamma = 3878$ keV) = $(1.58 \pm 0.08)$%.

The $^{32}$P $T = 2$ state also decays directly to the ground state, implying there should be a 5046-keV $\gamma$ transition in $^{32}$Cl. Unfortunately, the first escape peak of the 5550-keV $\gamma$ ray from $^{32}$Cl appears as a strong peak in the region of interest in the spectrum. Figure 15 shows data from $G_5$ (the 120% HPGe detector) and the corresponding fit. The 5046-keV peak was fitted with a fixed centroid, width and line shape.

FIG. 13. (Color online) The absolute $\epsilon_\gamma, \epsilon_\beta$ from $^{32}$Cl decays (points) and the normalized PENELOPE curves for each detector (line). The point at 2.2 MeV was not used in normalizing the curves as explained in the text.
FIG. 15. (Color online) The region of interest where the \( \gamma \) ray corresponding to the decay from the \( T = 2 \) state to the ground state should appear at \( E_{\gamma} = 5046 \) keV. The main 5038-keV peak is the first escape from the 5550-keV \( \gamma \) ray from \( ^{32}\text{Cl} \).

allowing only the background and area parameters to vary to obtain the photopeak yields from each detector. We obtain \( b_{\gamma}^{\text{SA}}(E_{\gamma} = 5046 \text{ keV}) = (0.098 \pm 0.021)\% \).

The analog of the 5072 keV \( \rightarrow 2230 \) keV \( \gamma \) transition in \( ^{32}\text{P} \) is observed at \( E_{\gamma} = 2836(1) \) keV, close to another peak at 2839 keV. As shown in Fig. 16, the identification was done by observing the corresponding coincident proton spectra. The energy of the proton group implies that it is emitted from a state at 2212(5) keV which, when added to 2836(1) keV, yields \( E_x = 5048(5) \) keV, consistent with the energy of the \( T = 2 \) state in \( ^{32}\text{Cl} \). We obtain

\[ b_{\gamma}^{\text{SA}}(E_{\gamma} = 2838 \text{ keV}) = (0.24 \pm 0.03)\% \]. This is significantly smaller than the \( (0.39 \pm 0.07)\% \) intensity of 607-keV protons observed in singles by Bjornstad et al. [16], which correspond roughly to the same excitation energy. This can be explained by direct \( \beta \) feeding of the 2212-keV level in addition to \( \gamma \) feeding from the \( T = 2 \) state. The singles data from the NSCL experiment shows a small peak at \( \approx 610 \) keV, but it is difficult to extract its area accurately because it sits on top of the intense \( \beta \) tails from decays that are not followed by particle emission. Our IsOLDE data yields \( (0.385 \pm 0.008)\% \) for the intensity of this proton group, in agreement with Ref. [16].

The absolute intensities of the three \( \gamma \) decays of the \( T = 2 \) state are listed in Table III. We find the total \( \beta \)-delayed \( \gamma \) branch for the superallowed transition to be

\[ b_{\gamma}^{\text{SA}} = (1.92 \pm 0.08 \pm 0.04)\% \]. (10)

The sources contributing to the uncertainty are summarized in Table IV and discussed in subsection A3.

**TABLE III. Absolute \( \beta \gamma \) branches for \( ^{32}\text{Ar} \) \( \beta \) decays to the lowest \( T = 2 \) level of \( ^{32}\text{Cl} \) followed by \( \gamma \) emission.**

<table>
<thead>
<tr>
<th>( E_{\gamma} ) (keV)</th>
<th>Absolute ( \beta \gamma ) branch ( b_{\gamma}^{\text{SA}}(%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3878</td>
<td>1.58(8)</td>
</tr>
<tr>
<td>2838</td>
<td>0.24(3)</td>
</tr>
<tr>
<td>5046</td>
<td>0.10(2)</td>
</tr>
</tbody>
</table>

**TABLE IV. Uncertainties contributing to the absolute superallowed branch in \( ^{32}\text{Ar} \) decay.**

<table>
<thead>
<tr>
<th>Component</th>
<th>( \Delta b_{\gamma}^{\text{SA}}/b_{\gamma}^{\text{SA}}(%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton branch</td>
<td></td>
</tr>
<tr>
<td>Implanted ( ^{32}\text{Ar} ) ions</td>
<td>( \pm 0.23 )</td>
</tr>
<tr>
<td>Number of ( p_0 ) protons</td>
<td>( \pm 0.53 )</td>
</tr>
<tr>
<td>Ratio of ( p_1/p_0 ) protons</td>
<td>( \pm 0.04 )</td>
</tr>
<tr>
<td>Ratio of ( p_2/p_0 ) protons</td>
<td>( \pm 0.04 )</td>
</tr>
<tr>
<td>Ratio of ( p_3/p_0 ) protons</td>
<td>( \pm 0.07 )</td>
</tr>
<tr>
<td>Proton decays near detector surface</td>
<td>(&lt; 0.01 )</td>
</tr>
<tr>
<td>Sub-total</td>
<td>( \pm 0.58 )</td>
</tr>
<tr>
<td>( \gamma ) branch</td>
<td></td>
</tr>
<tr>
<td>Statistics in IAS decay peaks</td>
<td>( \pm 0.34 )</td>
</tr>
<tr>
<td>Statistics in ( ^{32}\text{Cl} ) decay peaks</td>
<td>( \pm 0.12 )</td>
</tr>
<tr>
<td>( ^{32}\text{Cl} ) branching ratios (from Ref. [20])</td>
<td>( \pm 0.11 )</td>
</tr>
<tr>
<td>Ge detector efficiency</td>
<td>( \pm 0.09 )</td>
</tr>
<tr>
<td>Sub-total</td>
<td>( \pm 0.39 )</td>
</tr>
<tr>
<td>Total</td>
<td>( \pm 0.70 )</td>
</tr>
</tbody>
</table>
VI. CONCLUSIONS

A. \( ft \) value of the superallowed decay

Adding the proton and \( \gamma \) branches, we obtain

\[
b_{SA}^{\beta \gamma} = (b_{SA}^{\beta p} + b_{SA}^{\beta \gamma}) = (22.71 \pm 0.101 \pm 11)\%,
\]

where \( b_{SA}^{\beta \gamma} \) and \( b_{SA}^{\beta p} \) are given by Eq. (10) and Eq. (7), respectively. Table IV summarizes the error budget for this superallowed decay branch.

A recent determination of the \( ^{32}\text{Ar} \) mass [21], along with our determination of the \( \beta \)-delayed proton energy [10] and the known masses of \( ^{31}\text{S} \) and the proton, implies

\[
Q_{\text{EC}} = 6.0913(25) \text{ MeV},
\]

which in turn yields a statistical phase-space factor [22]

\[
f = 3506(8).
\]

Using the \( ^{32}\text{Ar} \) half-life from an ISOLDE measurement [10],

\[
t_{1/2} = 100.5(3) \text{ ms},
\]

we find that the superallowed decay has

\[
ft(^{32}\text{Ar}) = 1552(12) \text{ s}.
\]

B. Experimental value for \( \delta_c \)

We can now use our \( ft \) value from Eq. (15) in combination with Eq. (1) to obtain:

\[
\delta_c^{\text{exp}} = 1 - \frac{\mathcal{F}t(T = 1)}{2(1 + \delta_R) ft(^{32}\text{Ar})} = (2.1 \pm 0.8)\%,
\]

where the factor of 2 corresponds to the ratio of squared matrix elements for \( T = 2 \) and \( T = 1 \) decays and we used \( \delta_R = (1.145 \pm 0.041)\% \) [22]. This is in agreement with the prediction \( \delta_c = (2.0 \pm 0.4)\% \).

A more stringent test of the prediction requires an experiment with higher accuracy as well as a more careful assessment of the theoretical uncertainty in \( \delta_c \). The predicted configuration mixing correction \( \delta_c^{\text{exp}} \) arises mainly from mixing of the \( ^{32}\text{Cl} \) \( T = 2 \) state with a \( T = 1 \) state 0.26 MeV lower in energy. A more meaningful comparison to the theory can be made if the radial-overlap and configuration-mixing contributions to \( \delta_c \) could be separated experimentally. This can be done using the \( \beta - \nu \) correlation to determine where the rest of the Fermi strength lies. This information should come from a full analysis of the ISOLDE data [10].

Using the calculated value for \( \delta_c \), we obtain:

\[
\mathcal{F}t = 1538(14) \text{ s}.
\]

C. Spectroscopic information

1. Total proton branch

We obtained \( b_{SA}^{\beta p} = (35.58 \pm 0.22)\% \) for the total of all proton branches following \( ^{32}\text{Ar} \) decay.

TABLE V. \( \gamma \) branches of the lowest \( T = 2 \) states in the \( A = 32 \) multiplet. The \( ^{32}\text{Cl} \) results were extracted from this work; \( ^{32}\text{S} \) and \( ^{32}\text{P} \) branches from other work.

<table>
<thead>
<tr>
<th>( E_{\gamma} ) (MeV)</th>
<th>Relative ( \gamma ) branch (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{32}\text{Cl} )</td>
<td>( ^{32}\text{S} ) [23]</td>
</tr>
<tr>
<td>3.9</td>
<td>83(4)</td>
</tr>
<tr>
<td>2.8</td>
<td>12(2)</td>
</tr>
<tr>
<td>5.1</td>
<td>5(1)</td>
</tr>
</tbody>
</table>

2. Width and branches for \( \gamma \) decays of \( T = 2 \) state

Table V compares the \( \gamma \)-ray branches of analogous decays of the lowest \( T = 2 \) states in the \( A = 32 \) multiplet. There is rough agreement with isospin conservation, which predicts that the branches should be independent of \( T_Z \).

We combine our value for the \( \beta \gamma \) branch of the \( T = 2 \) state (which we obtained as the ratio of \( b_{SA}^{\beta \gamma}/b_{SA}^{\beta p} \)) and the total width \( \Gamma = 20(5) \text{ eV} \) from the ISOLDE data [10] to obtain \( \Gamma_{\gamma} = 1.7(4) \text{ eV} \). The shell-model prediction using the USD interaction [25] yields \( \Gamma_{\gamma} \approx 1.1(1) \text{ eV} \).

3. \( \beta \)-delayed proton emission to first excited state of \( ^{31}\text{S} \)

Table II presents the intensities of the proton groups corresponding to decays leaving \( ^{31}\text{S} \) in its first excited state \( (p_1 \) groups) relative to the intensity of the superallowed \( p_0 \) group. We found several new groups corresponding to \( p_1 \) decays.

4. Mass of the \( ^{32}\text{Cl} \) ground state

The superallowed proton energy and the \( \gamma \) energy following de-excitation from the IAS yield a more precise determination of \( Q_p \), the proton separation energy of the \( ^{32}\text{Cl} \) ground state. The ISOLDE data [10] show that the total kinetic energy of the daughter \( p \) and \( ^{31}\text{S} \) in the superallowed decay is \( E_{\text{cm}} = 3465.0(4) \text{ keV} \). Combining this with \( E_{s}(T = 2) = 5046.3(4) \text{ keV} \) (from Sec. VC) gives

\[
Q_p = E_s(T = 2) - E_{\text{cm}} = 1581.3(6) \text{ keV}.
\]

This result, along with the mass excesses of hydrogen and \( ^{31}\text{S} \) [26], yields a \( ^{32}\text{Cl} \) mass excess of

\[
\Delta M(^{32}\text{Cl}) = \Delta M(^{1}\text{H}) + \Delta M(^{31}\text{S}) - Q_p = -13337.0 \pm 1.6 \text{ keV}
\]

which is more precise than the previously accepted value of \(-13329.8 \pm 6.6 \text{ keV} \) [26].

ACKNOWLEDGMENTS

We thank Ian Towner for calculating the nuclear-dependent radiative correction and the phase-space factor, Dick Seymour for computing support and Maria J. G. Borge for comments that...
improved the clarity of the text. The University of Washington researchers were supported by the DOE under grant DE-FG02-97ER41020. The Notre Dame, MSU, and FSU researchers were supported by the NSF under grants PHY-9901133, PHY-9528844, PHY-0244453, and PHY-9970991.

APPENDIX: SYSTEMATIC UNCERTAINTIES IN THE SUPERRADLOVED BRANCHING RATIO

D. Number of implanted 32Ar ions

1. Nuclear fragmentation

If a 32Ar ion fragmented before coming to rest in D3, it could be misidentified as a good 32Ar ion. Two independent parametrizations of the total reaction cross sections by Shen et al. \[27\] and Tarasov et al. \[28\] agree to within 10%. Both agree with data in a wide variety of cases. In particular, Shen et al.’s parametrization reproduces P and 16O reactions in Si, and Tarasov et al.’s reproduces 32,34,36S ions in C and Au. Shen’s parametrization predicts that 0.3% of the incoming ions react in D1, 0.4% in the Al foil, 0.6% in D2, and 0.2% in D3. Altogether, we therefore expect that a total of \( \approx 1.5\% \) of incoming 32Ar ions should react.

However, the good ion cuts veto a significant fraction of those ions that would otherwise have been misidentified. For example, some of the fragmentation products would deposit energy in D4. Predicting the exact fraction that is vetoed, however, requires a reliable model of the partial cross sections. Models that do predict the partial cross sections include EPAX Version 2 \[29\], Silberberg and Tsao \[30\], and HZEFRG1 \[31\]. We found very poor agreement between these models, even under conditions where all three are expected to be valid.

The most difficult fragmentation channel to reject is 32Ar + x n. Fortunately, all these models predict that a negligible fraction goes into this channel, as expected for fragmenting a proton-rich nucleus. Supporting evidence comes from a calculation \[32\] of the single neutron knock-out reaction cross section of \( \approx 12 \) mb, which is \( \approx 1\% \) of the total calculated reaction cross section of \( \approx 1500 \) mb. This negligible contribution is below systematic uncertainties, so we neglect neutron channels in the following analysis.

We have included reactions in our simulation using Shen et al.’s parametrization of the total reaction cross section. For the partial cross sections, one common aspect of all of the models considered is that the strongest channels are the ones with larger mass asymmetries, i.e., a heavy fragment close to the original 32Ar mass and a light partner. We have considered a number of different reaction channels:

\[
32\text{Ar} + A_i Z_i \rightarrow A_j Z_j + \begin{cases} \text{31Cl + p} \\ \text{28S + \alpha} \\ \text{29Si + 3Be} \end{cases} \quad \text{(A1)}
\]

Here \( A_i Z_i = (28Si, 27Al) \) is the target material (in the detectors and foil, respectively) where the reaction occurs. Our simple model assumes the target nuclei are not modified in the reaction and so are treated as spectators to the reaction. However, these nuclei do recoil, taking energy from the daughter fragments.

This simple model predicts that two of our particle identification conditions can be used to veto events that correspond to reacted ions. According to LISE calculations and the results of subsection A2, we should not see any 32Ar ions implanted in D4. However, the \( E_3 \) vs. \( E_4 \) scatter plot gated by events in Region 3.1, Fig. 17, shows that some of these events result in a significant amount of energy deposited in D4. Our simulations show that these events correspond to ions that reacted before coming to rest and that the resulting light fragment—with up to tens of MeV per nucleon—has enough energy to penetrate D4 and deposit >800 keV in D4. Our first condition, therefore, is to veto events for which \( E_4 > 800 \) keV. To efficiently tag reacted events in this way, we would need \( 4\pi \) coverage around the detectors. We estimate that our apparatus gave a vetoing efficiency of (50–75)\%, depending on which reaction channel is considered (the least efficient being the \( p \) and \( \alpha \) channels). Of the 1.5\% incoming ions that react, simulations predict that an \( E_4 < 800 \) keV cut should reduce the contamination to \((0.4–0.8)\%\): a reduction of \((1.1–0.7)\%\). In fact, we observed that this cut eliminated 0.8\% of the good ion candidates.

Region 5.1 of the \( E_2 \) vs. \( E_3 \) spectrum is the other particle identification condition we use to preferentially veto reacted ion events. When applying this extra condition to the data, we observe that the change in the branch with and without the \( E_4 < 800 \) keV condition goes to 0.15\%. This is also roughly consistent with the Monte Carlo prediction of \((0.2–0.3)\%\). Figure 18 shows a comparison similar to the one shown in Fig. 17, but with the additional condition that ions were in Region 5.1.
We estimated the fragmentation correction and its uncertainty by taking two extreme cases. As one extreme, we assume no reactions survive our cuts. As the other extreme, we assume all of the reactions proceed through the $^{31}\text{Cl} + p$ channel because this is the one with the poorest vetoing efficiency. The contamination level that remains after all our cuts are made is the maximum value from scheme 4 of Table VI, i.e., 0.2%. We assign a 100% uncertainty that yields

$$N_{\text{Ar}} = N_{\text{Ar}}^{\text{uncorr}} \times 0.998(2).$$  \hspace{1cm} (A2)

**E. Delayed proton detection efficiency**

Ions that are implanted too close to the surface of the detector (the range of a 3.35-MeV proton in Si is $\approx 0.1$ mm) may emit protons outside the active area of the detector, leaving too little energy to be clearly identified, thereby making it appear as though the proton branch is smaller than it is. We will first consider implantation close to the rim and then near the upstream and downstream surfaces.

**TABLE VI.** Effect of particle identification conditions on the superallowed proton branch [see Eq. (7)]. The results are normalized to $b_{\Delta A}^{\beta p} = 20.83\%$, the result using our nominal analysis scheme without any corrections for contamination from reacted ions. For comparison, we also list the Monte Carlo calculated prediction of the fraction of events misidentified as $^{32}\text{Ar}$ ions.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Conditions</th>
<th>$\Delta b_{\Delta A}^{\beta p}$</th>
<th>Monte Carlo contamination level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reg.</td>
<td>Reg. $E_4 &lt;$ 0.8 MeV</td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>x</td>
<td>−1.00%</td>
<td>1.5%</td>
</tr>
<tr>
<td>2.</td>
<td>x</td>
<td>x</td>
<td>−0.19% (0.4–0.8)%</td>
</tr>
<tr>
<td>3.</td>
<td>x</td>
<td>x</td>
<td>−0.05% (0.3–0.5)%</td>
</tr>
<tr>
<td>4.</td>
<td>x</td>
<td>x</td>
<td>+0.10% (0.1–0.2)%</td>
</tr>
</tbody>
</table>

**FIG. 18.** (Color online) $E_4$ spectrum (roughly calibrated) of events in Region 3.1. This is similar to Fig. 17 but with the extra condition on Region 5.1.

We estimated the uncertainty from possible implantations of $^{32}\text{Ar}$ ions landed just outside the active region of the detector. These ions would not be labeled as good ions, yet they could have emitted protons into the active area that would be counted as a valid decay event. Our simulations predicted that a negligible fraction of such events leave enough energy to be counted as decay events.

**TABLE VII.** Ratios of heavy-ion count rates in detectors D$_1$, D$_2$, and D$_3$. We used these measurements to put an upper limit on the fraction of $^{32}\text{Ar}$ ions that could have been implanted in a region where protons would not be detected with high efficiency.

<table>
<thead>
<tr>
<th></th>
<th>$R_2/R_1$</th>
<th>$R_3/R_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{32}\text{Ar}$ without collimator</td>
<td>0.9370</td>
<td>0.9065</td>
</tr>
<tr>
<td>$^{28}\text{Al}$ without collimator</td>
<td>0.9504</td>
<td>0.9107</td>
</tr>
<tr>
<td>$^{28}\text{Al}$ with collimator</td>
<td>0.9194</td>
<td>0.9170</td>
</tr>
</tbody>
</table>

**1. Implantation near the edges of D$_3$**

In the extreme case where the distribution of $^{32}\text{Ar}$ beam over the surface of D$_3$ was uniform, the fraction of ions implanted in the region where the proton detection efficiency is less than unity would be $dr/r \approx 8 \times 10^{-3}$. However, the distribution of the $^{32}\text{Ar}$ beam was far from uniform. This was verified during the latter part of the experiment when we produced a $^{28}\text{Al}$ beam for calibration purposes and introduced a collimator upstream of D$_2$ that blocked 40% of its active area. The fractional changes in the counting rates of incoming ions in this setup (listed in Table VII) showed that the implantation profile was concentrated near the center of the detector. Simulations based on a combination of LISE and GEANT [17] were used to estimate the characteristics of the incoming ion beam. The energy of the $^{28}\text{Al}$ and the $^{32}\text{Ar}$ ion beams were taken from LISE, and GEANT was used to track the ions through the system. We varied the beam width and divergence until the results listed in Table VII were reproduced. The simulations imply that $7 \times 10^{-4}$ of the implanted ions landed near the rim. Further simulations that also tracked the emitted protons from these events suggest that only $\approx 1/7$ of them would have had their signals degraded enough to fall outside the peak window. We conclude that the effect is negligible.

We also studied the effect of $^{32}\text{Ar}$ ions that landed just outside the active region of the detector. These ions would not be labeled as good ions, yet they could have emitted protons into the active area that would be counted as a valid decay event. Our simulations predicted that a negligible fraction of such events leave enough energy to be counted as decay events.

**2. Implantation near the up- and downstream surfaces of D$_3$**

We estimated the uncertainty from possible implantations near the downstream and upstream surfaces of the detectors using the D$_1$ decay energy spectra in coincidence with D$_2$ or D$_3$ shown in Fig. 19. These spectra provide information both about the mean of the implantation profile and its width because the peak energies are the sums of the energy of the $\beta$-delayed protons and the energy left in D$_3$ by the preceding $\beta$s. Because D$_2$ and D$_3$ subtended different solid angles at D$_3$ the spectrum in coincidence with D$_2$ has only $\approx 65\%$ of the events as compared to that in coincidence with D$_3$ and the widths of the proton groups are different for the two spectra. However, our simulations show that the energies corresponding to half of the peak intensity on the lower energy side should coincide if the $^{32}\text{Ar}$ ions had been implanted exactly at the center of D$_3$. This is expected because this point would correspond to the proton energy plus the energy left by $\beta$s traveling through...
the minimum amount of detector, i.e., along the beam axis, and so is independent of the solid angle. The observed energy difference of $\Delta E = 4.5(3)$ keV between the two spectra at the half peak-intensity points (indicated by the arrows) implies that the mean implantation depth is 9(1) $\mu$m from the center of $D_3$ in the direction of $D_4$. The widths of the peaks in Fig. 19 are sensitive to the width of the implantation distribution. The distribution of ions within $D_3$ can be estimated from a LiSE calculation using the fragment separator settings from Sec. II; Monte Carlo simulations using this nominal distribution (shown in Fig. 19) agree well with the data, whereas profiles with double that width—which would be wide enough that some protons could escape before stopping—are strongly excluded. We conclude that the probability for a 3.5-MeV proton escaping $D_3$ without depositing all of its energy is negligible ($\approx 8 \times 10^{-5}$).

**F. $\gamma$ branching ratio**

Gamma feeding of the $^{32}$Cl $T = 2$ state is highly unlikely. Given that the $T = 2$ state lies at $E_\gamma \approx 5$ MeV, and the $\beta$-endpoint energy is at $E_\beta \approx 10$ MeV, the phase space for $\beta$ decays to levels above the $T = 2$ state decreases very rapidly. Also, the first $T = 2$ state will have a vastly larger $\beta$-decay matrix element than any other daughter state. Finally, the only states above the IAS that can be fed by allowed $\beta$ decay are $T = 1$ levels, whose proton decays, unlike those of the IAS, are not inhibited by isospin selection rules; as a result their $\gamma$ decays are unlikely to compete with proton decay.

We estimated the uncertainty in the shape of the predicted efficiency curve by performing simulations with significantly different geometries for the Ge crystals. In particular, we increased dead layer thicknesses by 20 $\mu$m and 2 mm for the outer and inner surfaces, respectively, and increased the lengths of the crystals by 2 mm. The change in the shape of the efficiency curve between the highest statistics peak from $^{32}$Ar (3.9 MeV) and the nearest high-statistics peak from $^{32}$Cl (4.8 MeV) was negligible.

The absolute $\gamma$-ray efficiencies of the Ge detectors depend on the solid angle subtended by the detectors. Our method of efficiency calibration relative to the $^{32}$Cl decay lines is self-correcting, so the effects of variations in solid angle end up being negligible. This is not true for summing corrections because $^{32}$Cl and $^{32}$Ar have different multiplicities; we investigated the sensitivity of the summing correction with respect to the solid angle by comparing the calculations to one with an additional 5 mm offset in the position of all the Ge detectors. This uncertainty in the position of the detectors led to a summing correction uncertainty of $\pm 0.02\%$ in $b^{\beta\gamma}_{\text{SA}}$.

The uncertainty in the efficiency normalizations that result from the fits shown in Fig. 13 arises from three sources:

1. the statistical uncertainty due to the number of counts in the peaks corresponding to $\gamma$-decays from the isobaric analog state in $^{32}$Cl following the superallowed $\beta$ decay of $^{32}$Ar. Only 1.92% of $^{32}$Ar $\beta$ decays result in one of these $\gamma$s, so the statistics is relatively small and contributes $\Delta b^{\beta\gamma}_{\text{SA}} = 0.08\%$.

2. the statistical uncertainty from the number of counts in the $\gamma$-calibration peaks. As the branching ratio is much higher in this case (only $\sim 1\%$ of $^{32}$Cl $\beta$ decays are directly to the ground state), this uncertainty ends up being only $\pm 0.03\%$ in $b^{\beta\gamma}_{\text{SA}}$.

3. the precision of the measured branching ratios of the calibration peaks [20] contributes another $\pm 0.03\%$ to the uncertainty in $b^{\beta\gamma}_{\text{SA}}$.

The total uncertainty in $b^{\beta\gamma}_{\text{SA}}$ from the $\gamma$ branching ratio, dominated by the small statistics in the $^{32}$Ar $\gamma$ peaks, is $\Delta b^{\beta\gamma}_{\text{SA}} = \pm 0.09\%$.

---

[17] CERN GEANT Detector Description and Simulation Tool, Oct. 1994 version, 1993. We use GEANT instead of PENELOPE only when we are required to track protons or heavy ions that are not available in PENELOPE.
[22] I. S. Towner (private communication).
[32] We thank Jeff Tostevin for contributing to this calculation (private communication).