In-beam $\gamma$-ray spectroscopy of $^{35}$Mg and $^{33}$Na

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Excited states in the very neutron-rich nuclei $^{35}$Mg and $^{33}$Na were populated in the fragmentation of a $^{38}$Si projectile beam on a Be target at 83 MeV/u beam energy. We report on the first observation of $\gamma$-ray transitions in $^{35}$Mg, the odd-$N$ neighbor of $^{36}$Mg and $^{34}$Mg, which are known to be part of the ‘island of inversion’ around $N = 20$. The results are discussed in the framework of large-scale shell-model calculations. For the $A = 32$ nucleus $^{33}$Na, a new $\gamma$-ray transition was observed that is suggested to complete the $\gamma$-ray cascade $7/2^+ \to 5/2^+ \to 3/2^+$, connecting three neutron two-particle–two-hole intruder states that are predicted to form a close-to-ideal $K = 3/2$ rotational band in the strong-coupling limit.

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I. INTRODUCTION

The structure of nuclei in the so-called “island of inversion” [1,2], a region of the nuclear chart composed of neutron-rich Ne, Na, and Mg isotopes around neutron number $N = 20$, has provided much insight into the driving forces of shell evolution. In this region, driven largely by the spin-isospin parts of the nuclear force [3], neutron $n$-particle–$n$-hole ($np$-$nh$) “intruder” configurations of $v(sd)^{n}(fp)^{+n}$ character (labeled $nh\omega$) gain “correlation energy” [4] with respect to the normal-order configurations and dominate the wave functions of low-lying states, including the ground states, signaling the breakdown of $N = 20$ as a magic number in these nuclei. The resulting onset of collectivity was quantified with inelastic scattering experiments [5–10] and inferred from measurements of the $2^+_1$ excitation energies [11–14] on even-even nuclei out to $^{32}$Ne and $^{38}$Mg at $Z = 10$ and $Z = 12$, respectively. These collective properties of even-even nuclei are rather well described by state-of-the-art Monte Carlo shell model (MCSM) calculations (SDPF-M effective interaction) that allow for unrestricted mixing of neutron particle-hole configurations across the $N = 20$ shell gap. However, experimental information on their odd-$N$ neighbors, $^{31,33}$Mg [15–22] and $^{30}$Na [23], for example, has posed significant challenges for these calculations and led to controversial interpretations, indicating that odd-$A$ and odd-odd nuclei pose very stringent benchmarks for a shell-model description of this region.

In this paper we present results from the first $\gamma$-ray spectroscopy of $^{35}$Mg and report a new $\gamma$-ray transition observed in the $A/Z = 3$ nucleus $^{33}$Na ($N = 22$). Excited states in these very neutron-rich nuclei were populated in the fragmentation of an intermediate-energy $^{38}$Si rare-isotope beam on a $^9$Be target. The results are confronted with large-scale shell-model calculations.

II. EXPERIMENT

The $^{38}$Si secondary beam was produced by fragmentation of 140 MeV/nucleon $^{48}$Ca primary beam delivered by the Coupled Cyclotron Facility of the National Superconducting Cyclotron Laboratory onto a $^{75}$Zg mg/cm$^2$ primary $^9$Be fragmentation target. The isotope of interest was selected in the A1900 fragment separator [25], using a 750 mg/cm$^2$ Al wedge degrader for purification. The $^{38}$Si secondary beam interacted with a 376(4)-mg cm$^{-2}$-thick $^9$Be reaction target positioned at the pivot point of the large-acceptance S800 spectograph [26]. The reaction residues were identified on an event-by-event basis from the time of flight taken between two scintillators and the energy loss measured in the S800 ionization chamber. The flight times were corrected for the reaction residue’s trajectories as reconstructed from the position and angle information provided by the cathode-readout drift chambers of the S800 focal-plane detector system. The particle identification spectrum for the reaction residues produced in $^9$Be$(^{38}$Si,$A$Z)X is shown in Fig. 1.

Since the measurement’s main focus was the study of $^{36}$Mg in the two-proton knockout from $^{38}$Si [13,27,28], the magnetic rigidity of the S800 spectograph was set to center the longitudinal momentum distribution of $^{36}$Mg in the S800 focal plane. Therefore, $^{35}$Mg and $^{33}$Na entered the focal plane off center and were impacted by acceptance cuts (estimated to be of the order of 20%–30%). The cross sections for the production of these nuclei from $^{38}$Si were measured to be $\sigma(^{36}$Mg) = 0.10(1) mb [13], $\sigma(^{35}$Mg) $\gtrsim$ 0.05 mb, and $\sigma(^{33}$Na) $\gtrsim$ 0.03 mb.
were arranged in two rings (at 90° emission angle relative to the target position. Sixteen detectors registered the largest energy deposition determines the residues in flight. For this, the location of the segment that event Doppler reconstruct the residues produced in 9Be(38Si, \[29\])X. The high degree of segmentation was used to event-by-event array of 32-fold segmented high-purity germanium detectors measured in coincidence with 35Mg and 33Na, respectively. In fully stripped 38Si passing through the reaction target, is the most intense constituent of the reacted beam.

The secondary 9Be target was surrounded by SeGA, an array of 32-fold segmented high-purity germanium detectors [29]. The high degree of segmentation was used to event-by-event Doppler reconstruct the \(\gamma\) rays emitted by the reaction residues in flight. For this, the location of the segment that registered the largest energy deposition determines the \(\gamma\)-ray emission angle relative to the target position. Sixteen detectors were arranged in two rings (at 90° and 37° central angles with respect to the beam axis). The 37° ring was equipped with seven detectors, while nine detectors occupied positions at 90°. The array was calibrated for energy and efficiency with \(^{152}\)Eu and \(^{226}\)Ra calibration standards.

### III. RESULTS

Figure 2 shows the Doppler-reconstructed \(\gamma\)-ray spectra measured in coincidence with 35Mg and 33Na, respectively. In the following sections we discuss the experimental results in comparison to Monte Carlo shell-model calculations using the SDPF-M effective interaction [24] that allows for unrestricted mixing of neutron particle-hole configurations across the \(N = 20\) gap and to large-scale shell-model calculations with the SDPF-U effective interaction [30] that does not include neutron intruder configurations in the model space.

#### A. 35Mg

For 35Mg, a \(\gamma\)-ray transition at 446 keV is clearly visible, and there are possibly indications of two other transitions at 621 and 670 keV. There is no evidence for \(\gamma\)-ray peaks at higher energies; in fact, the spectrum has remarkably few counts beyond 700 keV, likely indicative of a low neutron separation energy \([S_n(35\text{Mg}) = 1020(200)\text{ keV} \text{ reported in Ref. [31]}\] and beyond 700 keV, likely indicative of a low neutron separation energy \([S_n(35\text{Mg}) = 730(460)\text{ keV} \text{ estimated in the compilation by G. Audi et al. [32]}.\]

With its even-even neighbors 34Mg and 36Mg shown to be part of the island of inversion, one expects neutron particle-hole intruder excitations to dominate the structure of 35Mg as well. The Monte Carlo shell-model calculation with the SDPF-M effective interaction predicts eight excited states below 1.2 MeV, with a 5/2\(^-\) ground state almost degenerate with the first 3/2\(^-\) level at 30-keV excitation energy (see Figure 3 and Table I). The 446-keV transition observed in the experiment could correspond to the decay of any of the excited 7/2\(^+\), 3/2\(^+\), 1/2\(^-\), or 7/2\(^-\) states between 350 and 560 keV to either the ground state or the low-lying excited state predicted by theory. We note that it was impossible for the measurement presented here to detect a \(\gamma\)-ray transition below \(\sim 80\) keV due to the energy threshold settings of the SeGA electronics for this run. Assuming that the neutron separation energy is, indeed, low for 35Mg, the possible transitions at 621 and 670 keV likely proceed to one or both of the 5/2\(^-\) or 3/2\(^+\) near-degenerate states. It might be possible that the two transitions originate from the same level and that their energy difference of 49(11) keV corresponds to the energy spacing of the alleged 3/2\(^-\)–5/2\(^-\) doublet near the ground state. The possible scenarios of a decay level scheme are summarized in Fig. 3, assuming that the tentatively proposed 621- and 670-keV transitions are not feeding each other based on the assumption of a low neutron separation energy.
FIG. 3. (Color online) (right) Excited states below 1.2-MeV excitation energy predicted by the two shell-model calculations. (left) Possible arrangement of the experimentally observed transitions in the decay scheme of $^{35}$Mg. A low-lying excited state below 80 keV could not have been detected with the threshold setting of the SeGA array.

The reaction process leading to $^{35}$Mg from $^{38}$Si will not predominantly proceed as the direct removal of two protons and a neutron from $^{38}$Si but is likely dominated by the two-proton knockout into the continuum of $^{36}$Mg and subsequent neutron emission. Therefore, one cannot expect the final-state and nuclear structure selectivity of a strictly direct reaction and would rather expect all bound low-lying states to be populated without preference for certain configurations. This consideration together with the predicted high-level density in the MCSM and the exceptionally clean $\gamma$-ray spectrum, showing evidence for at most three $\gamma$-ray transitions, suggests that the neutron separation energy of $^{35}$Mg is, indeed, low, likely at the lower end of $S_n = 1020(200)$ keV, and that the $\gamma$-ray transitions we observe are not in coincidence and depopulate excited states at $\approx 500$ and $\approx 700$ keV to the ground state or the alleged near-degenerate excited state below 100-keV excitation energy (this corresponds to the scenario in Fig. 3 with $x \lesssim 80$ keV and $y = 0$ keV or $z \lesssim 80$ keV).

The neutron-separation energy predicted in the MCSM calculation is too low with $S_n \approx 300$ keV. This is likely related to the pairing matrix elements in the interaction as evidenced by an overestimation of the odd-even staggering in the calculation of $S_n$ for the odd-$A$ isotopes by several hundred keV.

Table I gives the $\hbar \omega$ decomposition of the wave functions of all negative and positive-parity states at energies $E_x \leq 1.29$ MeV calculated with the MCSM. We note that the first state, with $J^z = 3/2^-$, that is dominated (56%) by $0\hbar \omega$ (nonintruder) configurations occurs at 790-keV excitation energy. All other states calculated in the MCSM in this energy window are almost pure intruder states with $1\hbar \omega$ (positive-parity states) or $2\hbar \omega$ (negative-parity states) neutron intruder configurations.

For comparison, the conventional shell-model calculations with the SDPF-U effective interaction predict only four excited states with $J^z = 5/2^-, 7/2^-, 3/2^-$, and $1/2^-$ on top of a $3/2^-$ ground state.

B. $^{33}$Na

The $\gamma$-ray spectrum taken in coincidence with $^{33}$Na shows two clear full-energy peaks that correspond to $\gamma$-ray transitions at 429(5) and 688(6) keV in the most neutron-rich Na isotope ($A = 3Z$) studied with $\gamma$-ray spectroscopy to date. A pioneering experiment at the Radioactive Ion Beam Factory (RIBF) at RIKEN populated one excited state at 467(13) keV.

TABLE I. Composition of the $^{35}$Mg wave functions with respect to $\hbar \omega$ probabilities calculated within the MCSM. Only excited states with $E_x \leq 1.29$ MeV are listed.

<table>
<thead>
<tr>
<th>$J^+$</th>
<th>$E$ (MeV)</th>
<th>$1\hbar \omega$ (%)</th>
<th>$3\hbar \omega$ (%)</th>
<th>$5\hbar \omega$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3/2^+$</td>
<td>0.36</td>
<td>96.5</td>
<td>3.5</td>
<td>0.0</td>
</tr>
<tr>
<td>$5/2^+$</td>
<td>0.81</td>
<td>97.1</td>
<td>2.9</td>
<td>0.0</td>
</tr>
<tr>
<td>$7/2^+$</td>
<td>1.29</td>
<td>97.4</td>
<td>2.5</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$J^-$</th>
<th>$E$ (MeV)</th>
<th>$0\hbar \omega$ (%)</th>
<th>$2\hbar \omega$ (%)</th>
<th>$4\hbar \omega$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2^-$</td>
<td>0.40</td>
<td>1.7</td>
<td>97.8</td>
<td>0.4</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>0.03</td>
<td>6.9</td>
<td>92.5</td>
<td>0.6</td>
</tr>
<tr>
<td>$5/2^-$</td>
<td>0.79</td>
<td>56.1</td>
<td>43.5</td>
<td>0.4</td>
</tr>
<tr>
<td>$7/2^-$</td>
<td>0.00</td>
<td>4.6</td>
<td>94.9</td>
<td>0.5</td>
</tr>
<tr>
<td>$9/2^-$</td>
<td>1.13</td>
<td>10.1</td>
<td>89.6</td>
<td>0.4</td>
</tr>
<tr>
<td>$5/2^-$</td>
<td>0.35</td>
<td>12.4</td>
<td>87.0</td>
<td>0.6</td>
</tr>
<tr>
<td>$7/2^-$</td>
<td>0.56</td>
<td>4.7</td>
<td>94.6</td>
<td>0.7</td>
</tr>
<tr>
<td>$9/2^-$</td>
<td>1.22</td>
<td>7.8</td>
<td>91.8</td>
<td>0.4</td>
</tr>
</tbody>
</table>
We propose that the 429(6)-keV $\gamma$-ray originating from their two different states and feeds the first excited $5/2^+$ state. The MCSM calculations with the SDPF-M effective interaction predict the first excited state in $^{33}$Na, with good agreement with the data as well when assuming that the $\gamma$-ray branch for the decay to the ground state is estimated to be only 4.2% of the $\gamma$-decay branch to the $5/2^+$ state, which is below the sensitivity limit of our measurement due to low statistics. Assuming that these states form a rotational band with $K = 3/2$ in the strong-coupling limit, the intrinsic quadrupole moments $Q_0$ were deduced. As shown in Table II, all quadrupole moments and $B(E2)$ values are well described with a common $Q_0 \sim 70$ fm$^2$. This indicates that in the MCSM calculation this structure is close to the ideal case of a well-deformed $K = 3/2$ rotational band than at $K = 2$ rotational band within the MCSM calculation.

$B(E2) = 447 e^2$ fm$^4$ (in agreement with experiment [7,9]) and $Q(2^+)$ = −21.4 fm$^2$, yielding an intrinsic quadrupole moment of $Q_0 = 75$ fm$^2$, which is very similar to $^{33}$Na. At $N = 20$, $^{32}$Mg, the calculated $B(E2; 0^+ \rightarrow 2^+_1) = 447 e^2$ fm$^4$ (in agreement with experiment [5–7,9]) and $Q(2^+_1)$ = −17.1 fm$^2$ lead to $Q_0$ = 67 fm$^2$ and 60 fm$^2$, respectively, with the small discrepancy potentially indicative of triaxiality or $\gamma$ softness.

The level scheme predicted by the conventional shell-model calculations based on the SDPF-U effective interaction, which does not allow for neutron intruder configurations, has the levels in the same order but predicts the $5/2^+$ first excited state to be within 175 keV of the $3/2^+$ ground state (see Fig. 5).
A strong $\gamma$-ray transition above 90–100 keV would have been observed in the present measurement.

**IV. SUMMARY**

In summary, in-beam $\gamma$-ray spectroscopy of the very neutron-rich nuclei $^{35}\text{Mg}$ and $^{33}\text{Na}$ was performed following the fragmentation of a $^{38}\text{Si}$ projectile beam on a $^{9}\text{Be}$ target at intermediate beam energy. In $^{35}\text{Mg}$, the odd-$N$ neighbor of the island of inversion nuclei $^{34}\text{Mg}$ and $^{36}\text{Mg}$, $\gamma$-ray transitions were measured and provided first information on excited states in this nucleus. In comparison to Monte Carlo shell-model calculations with the SDPF-M effective interaction, the transitions are interpreted as connecting excited states around $\sim$450 and $\sim$700 keV to the 5/2$^-$ ground state and/or the first excited 3/2$^-$ state that is predicted to be within 30 keV of the ground state. For the $N=22$ nucleus $^{33}\text{Na}$, the most neutron-rich Na isotope studied to date with $\gamma$-ray spectroscopy, a new $\gamma$-ray transition at 688(6) keV was measured in addition to the known transition at 429(5) keV. The two transitions, if in coincidence, are in very good agreement with MCSM calculations and are proposed to establish the $7/2^+ \rightarrow 5/2^+ \rightarrow 3/2^+$ cascade of the ground-state band in $^{33}\text{Na}$, predicted in the MCSM to be an almost-ideal $K=3/2$ rotational band structure (of intruder states) in the strong-coupling limit. Coulomb excitation or excited-state lifetime measurements are needed to confirm the degree of deformation and rotational character and remain a challenge for future experiments. All low-lying states in these two nuclei are predicted to be intruder states, putting $^{35}\text{Mg}$ and $^{33}\text{Na}$ inside the island of inversion.

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