High-precision $B(E2)$ measurements of semi-magic $^{58,60,62,64}$Ni by Coulomb excitation


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High-precision reduced electric-quadrupole transition probabilities $B(E2; 0^+ \rightarrow 2^+)$ have been measured from single-step Coulomb excitation of semi-magic $^{58,60,62,64}$Ni ($Z = 28$) beams at 1.8 MeV per nucleon on a natural carbon target. The energy loss of the nickel beams through the carbon target were directly measured with a zero-degree Bragg detector and the absolute $B(E2)$ values were normalized by Rutherford scattering. The $B(E2)$ values disagree with recent lifetime studies that employed the Doppler-shift attenuation method. The present high-precision $B(E2)$ values reveal an asymmetry about $^{62}$Ni, midshell between $N = 28$ and 40, with larger values towards $^{56}$Ni ($Z = N = 28$). The experimental $B(E2)$ values are compared with shell-model calculations in the full pf model space and the results indicate a soft $^{56}$Ni core.

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I. INTRODUCTION

The standard shell closures of $Z$ or $N$ equal to 2, 8, 20, 28, 50, 82, or 126 [1,2] and subshell closures such as 40 and 64 are subject to change as one moves to exotic nuclei and extreme $N/Z$ ratios. For example, a breakdown of the $N = 20$ shell closure in $^{31}$Na [3] and $^{32}$Mg [4], and $N = 28$ shell closure in $^{44}$S [5–8] have been observed. On the other hand, the $Z = 50$, $N = 82$ double-shell closure of radioactive $^{132}$Sn is robust [9–15]. However, reduced quadrupole transition probabilities $B(E2; 0^+ \rightarrow 2^+)$ for the Sn isotopes have shown enhanced $2^+$ collectivity when moving from midshell, which neighbors a potential $N = 64$ subshell, to radioactive $Z = N = 50$ (cf. Bader et al. [16] and references within). The evolution of nuclear shell structure has been of immense interest since the advent of fast and reaccelerated radioactive ion beams. The Ni and Sn isotopes, which both span across two radioactive double-shell closures with potential subshell closures in-between, have been of particular interest in the past decade. The Ni isotopes span the radioactive double-shell closures of $^{56}$Ni ($Z = N = 28$) and $^{78}$Ni ($Z = 28$, $N = 50$) with a potential subshell closure at $N = 40$, $^{68}$Ni (cf. shell-model diagram in Fig. 1).

Radioactive $^{56}$Ni and $^{68}$Ni have comparatively large $2^+$ energies and small $B(E2)$ values [17,18], which alone suggest good $N = 28$ and $N = 40$ shell and subshell closures, respectively. However, discontinuities in neutron separation energies show weak to no evidence for a $N = 40$ subshell closure for $^{68}$Ni [19,20]. Furthermore, a $B(E2)$ maximum would be expected at midshell, $^{62}$Ni. The situation near midshell is complicated by $B(E2)$ discrepancies between recent Doppler-shift attenuation method (DSAM) experiments on the stable Ni isotopes [21–23] and the 2001 comprehensive data evaluation of Raman et al. [24]. In particular, $B(E2; 0^+ \rightarrow 2^+)$ values of $^{58,60,62,64}$Ni were reported by Kenn et al. [21] with a precision that ranges from 1.4 to 3.5% and values that are several standard deviations from the 2001 evaluation [24]. Other recent DSAM studies [22,23], which are not high precision, appear more consistent with the study by Kenn et al. [21].

Discrepancies in the stable Ni $B(E2)$ values are not only a problem for understanding the structure of the Ni isotopes near midshell, but also in having a dependable reference point in which to discuss trends of the radioactive Ni isotopes. Furthermore, $B(E2)$ values of radioactive $^{106,108,110}$Sn were measured relative to $^{56}$Ni [25,26], which would show much less enhancement if normalized to the value by Kenn et al. [21]. In this paper, high-precision absolute $B(E2; 0^+ \rightarrow 2^+)$ values are reported from single-step Coulomb excitation of semi-magic $^{58,60,62,64}$Ni in inverse kinematics, which disagree with the recent DSAM studies [21–23]. Similar discrepancies exist for the stable Sn isotopes [24,27], which will be discussed in a future publication [28].

II. EXPERIMENT

A method for measuring the Coulomb excitation of stable or radioactive ion beams using inverse kinematics...
(A_{\text{projectile}} > A_{\text{target}}) has been developed at the Holifield Radioactive Ion Beam Facility (HRIBF) [11,29]. With this method, scattered target nuclei are measured at forward laboratory angles relative to the beam direction (corresponding to backward angles in the center-of-mass frame) to provide a clean trigger for selecting the $\gamma$-ray transitions from the Coulomb-excited beam and to normalize the integrated beam current through Rutherford scattering. While this technique was primarily developed for radioactive ion beams, there are distinct advantages in employing this technique with stable beams including: (1) it can deliver isotopically pure beams and use relatively pure targets (e.g., $^{nat}C_{i}$ $^{80}C_{i}$), and (2) the recoiling target nuclei are measured at backward center of mass angles where the Rutherford cross section is less sensitive to angle. Back angles minimize uncertainties related to geometry, and also maximize the ratio of Coulomb excitation to Rutherford scattering, which minimizes the nonprompt (or random) particle-$\gamma$ component.

Semi-magic $^{58,60,62,64}Ni$ beams at an energy of 1.8 MeV per nucleon were Coulomb excited on a $\sim 1 \text{ mg/cm}^2$ natural carbon target over a period of 4 d. The beams were provided by the 25-MV tandem accelerator at the HRIBF. The energy loss of the $^{58}Ni$ and $^{60}Ni$ beams through the carbon target at 1.8 MeV per nucleon were measured by a Bragg detector at zero degrees and resulted in 42.7(8) MeV and 42.1(8) MeV energy loss, respectively. The Bragg detector was calibrated by measuring direct beam from the tandem at multiple energies, which was achieved quickly by dropping charge states while keeping the magnetic rigidity fixed.

Recoiling target nuclei were detected in the BareBall array [30] using two rings of CsI crystals with minimal absorbers. Ring 2 has 10 CsI crystals at angles of $14^\circ$–$28^\circ$ relative to the beam direction and ring 3 has 12 CsI crystals at angles of $28^\circ$–$44^\circ$. Coincident $\gamma$ rays were detected by the CLARION array [31] using nine segmented HPGe clover detectors at angles of $90^\circ$ (five clovers), $132^\circ$ (three clovers), and $154^\circ$ (one clover). The clover detectors were at a distance of 21.75 cm from the target with a total efficiency of 2.44(6)% at 1 MeV, 2.24(5)% at 1.173 MeV, and 2.08(5)% at 1.333 MeV. The experimental trigger required either a scaled-down particle event or a particle-$\gamma$ coincidence event. The trigger type was recorded for each event in a bit register to cleanly distinguish particles from the scaled-down trigger and particle-$\gamma$ trigger. A relatively low beam intensity of $\sim 5 \text{ pA}$ was used to prevent target damage and to maintain a data acquisition live time of $\gtrsim 99\%$. The particle-gated $2_1^+ \rightarrow 0_1^+$ $\gamma$-ray transitions of $^{58,60,62,64}Ni$ from Coulomb excitation are shown in Fig. 2. The relatively high efficiency of the particle-$\gamma$ coincidence trigger and high resolution of CLARION provide an excellent tag of the $2_1^+$ states.

### III. ANALYSIS AND RESULTS

The reduced $E2$ matrix elements for $^{58,60,62,64}Ni$ can be obtained approximately from the data using the following relation in second-order perturbation theory [32]:

$$
\frac{\sigma_{\text{Coulex}}(2_1^+)}{\sigma_{\text{Ruth}}} \approx N(0_1||M(E2)||2_1)^2 (1 + K(2_1||M(E2)||2_1))
$$

where $\sigma_{\text{Coulex}}(2_1^+)$ is the $2_1^+$ Coulomb-excitation cross section, $\sigma_{\text{Ruth}}$ is the Rutherford cross section, and $N$ and $K$ are scale factors dependent on the kinematics of the projectile/target combination. The reduced transition probability and static quadrupole moment are related to the reduced $E2$ matrix elements by

$$
B(E2; 0_1^+ \rightarrow 2_1^+) = \langle 0_1 || M(E2) || 2_1 \rangle^2
$$

and

$$
Q(2_1^+) = 0.7579 \langle 2_1 || M(E2) || 2_1 \rangle.
$$

At least two different targets, beam energies, or center-of-mass angles are generally required to solve for both $\langle 0_1 || M(E2) || 2_1 \rangle$ and $\langle 2_1 || M(E2) || 2_1 \rangle$ in Eq. (1). However, the $2_1^+$ quadrupole moments are expected to be zero [33,34]. Furthermore, the scale factor $K$ in Eq. (1) is relatively small for the low-$Z$ carbon target data, $K \sim 0.17$. For instance, a quadrupole moment of 0.1 $eb$, a reasonable upper limit on the magnitude [33–35], would only have a $\sim 2\%$ effect on the extracted $B(E2)$. In the present study, the cross sections are calculated with the Coulomb excitation (Coulex) code GOSIA [36]. The GOSIA calculations are not limited to second-order perturbation theory, cf. Eq. (1). Furthermore, the GOSIA calculations include the following corrections to the Coulex cross sections and $\gamma$-ray angular distributions (cf. Fig. 3) [36]: dipole polarization correction, kinematic correction to the solid angle, nuclear
FIG. 3. The $2^+_1 \rightarrow 0^+_1$ $\gamma$-ray angular distributions for $^{64}$Ni, gated on the carbon target recoils in (a) BareBall ring 2 and (b) BareBall ring 3.

deorientation correction, and finite-size $\gamma$ detector correction. The deorientation correction was essentially negligible. The $\Delta \phi$ particle-$\gamma$ angular correlations were analyzed using the recoil-in-vacuum (RIV) technique (see Refs. [12,14]) but the low recoil velocity, low average charge state, and small $2^+_1$ lifetime and $g$ factor [34] resulted in no observed attenuation.

The extracted $\langle 0_1^+ | M(E2)| 2_1^+ \rangle$ matrix elements for $^{58,60,62,64}$Ni are given per BareBall ring in Table I with only the statistical uncertainties. The two rings show excellent consistency in the extracted $E2$ matrix elements with no systematic difference. A nonzero quadrupole moment or nuclear interference would have resulted in a systematic difference between the extracted matrix elements from each ring, which is not observed. A summary of the final $E2$ matrix elements and $B(E2)$ values for $^{58,60,62,64}$Ni is also given in Table I. The total error includes systematic uncertainties from the efficiency, energy loss of the beam, detector geometry, and static quadrupole moment. Each systematic error was roughly 1% for the $E2$ matrix elements and 2% for $B(E2)$ values. The Bragg detector measurements of the beam energy-loss through the target were particularly critical in achieving high precision and controlling systematic errors from target thicknesses and stopping powers.

Figure 4 shows the present $B(E2)$ values compared with the 2001 evaluation of Raman et al. [24] and the DSAM studies by Kenn et al. [21], Orce et al. [22], and Chakraborty et al. [23]. The recent DSAM studies indicate a $B(E2)$ maximum at $^{62}$Ni, midshell between $N = 28$ and 40, which disagree with the 2001 evaluation of Raman et al. [24] and the present results. These DSAM studies were not included in the 2001 evaluation of Raman et al. [24] but have been included in a 2012 evaluation of the $N \sim Z \sim 28$ region [37]. The $B(E2)$ values of the present study, which are consistent with the Raman evaluation [24] but provide a much more precise $B(E2)$ for $^{64}$Ni, indicate a smoother profile with an asymmetry about $^{62}$Ni, where $^{60}$Ni and $^{62}$Ni have essentially the same electric quadrupole transition strength; $^{64}$Ni has a significantly lower $B(E2)$ than $^{60}$Ni. The simple expectation is that if $N = 28$ and $N = 40$ are robust shell and subshell closures, then the maximum $B(E2)$ should be at midshell where the valence space is ideally maximized. The $B(E2)$ values of $^{58,60}$Ni, approaching $Z = N = 28$, are enhanced with respect to $^{62}$Ni (midshell) and $^{64}$Ni, similar to the data on the Sn $B(E2)$ values approaching $Z = N = 50$ (cf. Bader et al. [16] and references within).

IV. DISCUSSION

Experimental and theoretical $B(E2)$ values for the $N = 28$ isotones and Ni ($Z = 28$) isotopes are compared in Fig. 5. The experimental data are from the present study and the recent $N \sim Z \sim 28$ evaluation [37], which includes the recent Coulomb excitation studies of radioactive $^{66,68}$Ni by Sorlin et al. [18], $^{54,56}$Ni by Yurkevitz et al. [17], $^{70}$Ni by Perru et al. [38], and $^{66}$Ni by Bree et al. [39]; only model-independent

TABLE I. Summary of $E2$ matrix elements and $B(E2)$s.

| Z = 28 | N   | $<0_1^+ | M(E2)| 2_1^+ >$ | $<0_1^+ | M(E2)| 2_1^+ >$ | $B(E2; 0^+_1 \rightarrow 2^+_1)c^2b^2$ |
|--------|-----|-----------------|-----------------|------------------|
|        |     | Ring 2          | Ring 3          |                  |
| $^{58}$Ni | 30  | (+) 0.253(279)  | (+) 0.2506(49)  | (+) 0.251(8)     | 0.0630(40) |
| $^{60}$Ni | 32  | (+) 0.3008(47)  | (+) 0.3018(30)  | (+) 0.301(7)     | 0.0906(41) |
| $^{62}$Ni | 34  | (+) 0.3038(37)  | (+) 0.302(24)   | (+) 0.301(6)     | 0.0906(37) |
| $^{64}$Ni | 36  | (+) 0.2626(47)  | (+) 0.2708(30)  | (+) 0.268(5)     | 0.0718(29) |

$a$Statistical uncertainties.

$B(E2)$ values in Table I are given in units of $a.u.$

$\sigma_{Rhe} = \sigma_{Ruth} / \sigma_{Ruth}$

$\sigma_{Rhe} = \sigma_{Ruth} / \sigma_{Ruth}$

Neutron Number (N)

Mass Number (A)

$B(E2; 0^+_1 \rightarrow 2^+_1)c^2b^2$
The experimental results are given here. The theoretical results were obtained with the GXPF1A Hamiltonian [40,41] for protons and neutrons in the full $pf$ model space ($0f_{7/2}$, $0f_{5/2}$, $1p_{3/2}$, $1p_{1/2}$) using two different sets of effective charges. Wave functions with $J$-scheme dimensions up to $10^8$ were obtained with the code NUSHELLX [42]. The GXPF1 Hamiltonian [40] was obtained starting with a set of two-body matrix elements derived from the Bonn-C potential [43]. Seventy linear combinations of the four single-particle energies and 195 two-body matrix elements were then fitted to 699 binding energies and excitation energies for nuclei in the $pf$ shell. For the GXPF1A Hamiltonian [41], five of the $T = 1$ GXPF1 two-body matrix elements were modified to improve the energies for the neutron-rich isotopes of Ca, Ti, and Fe. For comparison, the $pf$ shell results obtained when no excitations are allowed across the $N = 28$ gap are also shown, i.e., the “$t = 0$” truncation. For $A \geq 66$ the $t = 0$ results were also obtained in the $0f_{7/2}$, $1p_{3/2}$, $1p_{1/2}$, $0g_{9/2}$ model space for neutrons with the Hamiltonian obtained in Ref. [44]. This Hamiltonian was obtained starting with a set of two-body matrix elements derived from the Bonn-C potential [43]. Twenty linear combinations of the four single-particle energies and the 65 two-body matrix elements were then fitted to 104 binding energies and excitation energies for $^{57-78}$Ni [44]. In the NUSHELLX interaction library [42], this model space is called jj44pn and the Hamiltonian is called jj44pna.

Harmonic oscillator radial wave functions with $\hbar \omega = 45A^{-1/3} - 25A^{-2/3}$ are adequate to use at the level of about 10% accuracy. For the effective charges $e_p = 1 + \delta e_p$ and $e_n = \delta e_n$, the “standard” isoscalar core-polarization effective charges of $\delta e_p = \delta e_n = 0.5$ were used first (cf. original calculations with GXPF1A [40,41]). These were combined with the model space $E2$ amplitudes $A_p$ and $A_n$ to give the total $B(E2; I_i \to I_f) = [A_p e_p + A_n e_n]^2/(2I_i + 1)$ [45]. The effective charges arise from the perturbative coupling of the nucleons in the model space with $\Delta N = 2$ particle-hole excitations, where $N = 2n + t$ in the harmonic oscillator.

The $t = 0$ calculations are far below the data (cf. dashed lines in Fig. 5). With the $t = 0$ truncation, many of the $\Delta N = 0$ configurations that are important for the $B(E2)$ are left out. In the early analysis of $B(E2)$ data below $A = 56$ in the $f_{7/2}$ model space [46], a large proton effective charge of about $e_p \approx 2.2$ for $N = 28$ was needed to compensate for the $t = 0$ truncation. A neutron effective charge of $e_n = 1.0$ was used for the heavy nickel [44] and tin isotopes [47]. However, near $^{56}$Ni (and $^{100}$Sn [16]), the experimental $B(E2)$ values are even larger than those obtained with $t = 0$ and $e_n = 1.0$. For the nickel isotopes this enhancement is explained by the expansion to the full $pf$ model space. Within the full $pf$ model space, the enhancement can be traced to two mechanisms: (A) mixing with one-particle one-hole excitations across the $N = 28$ gap that can approximately be treated as a core-polarization contribution to the effective charges for the entire chain of nickel isotopes, and (B) mixing with low-lying deformed bands around $^{56}$Ni that arise from many-particle many-hole excitations across the $N = 28$ gap. For example, there is a “$4p$-$4h$” band in $^{56}$Ni starting around 4.5 MeV [48]. It is low in energy due to the two-proton two-neutron $\alpha$-type correlation in this configuration. For this deformed band in $^{56}$Ni, $B(E2; 0^+_1 \to 2^+_1) = 0.21 \text{ e}^2\text{b}^2$ (off the scale of Fig. 5). The calculated $B(E2)$ for low-lying states near $^{56}$Ni are enhanced due to the partial mixing of spherical and deformed configurations.

The calculated results depend on the effective charges. The $B(E2)$ data for the mirror $27/2^-$ to $23/2^-$ transitions in $^{51}$Fe and $^{51}$Mn can be used to determine the $t$, dependence of the effective charges [49]. In order to reproduce the experimental $B(E2)$ values of 0.00413(24) $\text{e}^2\text{b}$ and 0.00467(14) $\text{e}^2\text{b}$ [49], respectively, in the full $pf$ model space, $\delta e_p = 0.12$ and $\delta e_n = 0.67$ are required. The results obtained with these effective charges are given by the solid black lines in Fig. 5. The agreement with experiment improves for most cases.

The $B(E2)$ data for mirror transitions in $A = 43-45$ are in best agreement with effective charges of $e_p = 1.20$ and $e_n = 0.55$ [50]. One of the earliest analyses of $E2$ transitions in the lower part of the $pf$ shell gave $e_p = 1.16(16)$ and $e_n = 0.45(5)$ [46]. It is expected that $\delta e_n > \delta e_p$ due to the repulsive contribution of the giant isovector quadrupole resonance [45]. Microscopically derived $\Delta N = 2$ effective charges are nucleus and orbital dependent [51]. For the dominant contributions involving the $f_{7/2}$-$f_{5/2}$ and $f_{1/2}$-$p_{3/2}$ orbital combinations in the titanium isotopes, Ma et al. [51] obtained $e_p \approx 1.30$ and $e_n \approx 0.4-0.6$.

There is a relatively large disagreement between experiment and theory for $^{54}$Ni and $^{56}$Ni, but the experimental uncertainties are large and the data should be confirmed. For $A \geq 66$ the calculations in the jj44 model space require at least a neutron effective charge of $e_n = 1.0$ (as needed for the $8^+$ to $6^+$ transition in $^{70}$Ni [44]) whose increase over $e_n = 0.6$ could be interpreted in terms of a core-polarization contribution of one-particle one-hole protons across $N = 28$ (mechanism A above). The theoretical $B(E2; 0^+_1 \to 2^+_1)$ for $^{70}$Ni obtained with $e_n = 1.0$ is a factor of 2–3 smaller than experiment. Tsunoda et al. [52] have expanded the jj44 model space by adding $1d_{5/2}$ orbital for neutrons and allowing excitations from $0f_{7/2}$ for both protons and neutrons. Their $B(E2)$ results
shown in Fig. 1 of Ref. [52] are consistent with our results up to \( N = 38 \) (\(^{66}\text{Ni}\)). Their results for \(^{68,70}\text{Ni}\) are a factor of 2–3 smaller than experiment. The experimental \( B(E2) \) for \(^{70}\text{Ni}\) should be confirmed.

Coraggio et al. [53] have also added 1ds\(_{5/2}\) orbital for neutrons and allow excitations from 0f\(_{7/2}\) only for protons to the 1p\(_{3/2}\) orbital. Their results for \( N \geq 34 \) are similar to those of Tsunoda et al. [52]. Their calculation does not conserve isospin and this particularly affects \(^{56}\text{Ni}\) where both proton and neutron excitations from 0f\(_{7/2}\) should be included. For \( N = 30–32 \) their \( B(E2) \) values are smaller than experiment.

In conclusion, high-precision absolute \( B(E2; 0^+ \rightarrow 2^+ \) values were measured from single-step Coulomb excitation of semi-magic \(^{58,60,62,64}\text{Ni}\), which disagree with recent DSAM studies [21–23]. The present \( B(E2) \) results are consistent with the 2001 evaluation of Raman et al. [24], which preserves the \( B(E2) \) enhancement of radioactive \(^{106,108,110}\text{Sn} \) [25,26], measured relative to \(^{58}\text{Ni}\). However, the present results provide a much more precise \( B(E2) \) for \(^{64}\text{Ni}\). The high-precision Ni \( B(E2) \) values reveal an asymmetry about \(^{62}\text{Ni}\), midshell between \( N = 28 \) and 40, with larger values towards \(^{58}\text{Ni}\) (\( Z = N = 28 \)). Large-basis shell-model calculations indicate that the full \( pf \) shell is required to explain the overall magnitude of the Ni \( B(E2) \) values, with the excitation of several nucleons out of the \(^{56}\text{Ni}\) (\( Z = N = 28 \)) core. The calculations reproduce the observed \( B(E2) \) asymmetry about midshell but maintain a pronounced maximum at midshell, which disagrees with experiment.

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