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Injectors and longitudinal physics -- I

1. Fluid equations
2. Child-Langmuir Law
(Reiser 2.5.2, Appendix 1)
3. Pierce electrodes
4. Transients in injectors
5. Injector choices

I) FLUID EQUATIONS

START WITH VLASOV EQUATION FOR $f(x, \underline{p}, t)$

$$\frac{\partial f(x, \underline{p}, t)}{\partial t} + \underline{\dot{x}} \cdot \frac{\partial f(x, \underline{p}, t)}{\partial \underline{x}} + \underline{\dot{p}} \cdot \frac{\partial f(x, \underline{p}, t)}{\partial \underline{p}} = 0$$

$$\text{HERE } \underline{\dot{x}} = \frac{d\underline{x}}{dt} = \frac{\underline{p}}{\gamma m}$$

$$\underline{\dot{p}} = \frac{d\underline{p}}{dt} = q(\underline{E}(x, t) + \frac{\underline{p}}{\gamma m} \times \underline{B}(x, t))$$

$$\gamma^2 = (p/mc)^2 + 1$$

INTEGRATE OVER MOMENTUM AND MULTIPLY BY POWERS OF \underline{p}

a) CONTINUITY EQUATION

$$\int d^3 p \left\{ \frac{\partial f}{\partial t} + \underline{\dot{x}} \cdot \frac{\partial f}{\partial \underline{x}} + \left(q \underline{E}(x, t) + \frac{q \underline{p}}{\gamma m} \times \underline{B}(x, t) \right) \cdot \frac{\partial f}{\partial \underline{p}} \right\} = 0 \quad \text{①}$$

$$\text{DEFINE } n(x, t) = \int f(x, \underline{p}, t) d^3 p$$

$$n(x, t) \underline{v}(x, t) = \int \frac{\underline{p}}{\gamma m} f(x, \underline{p}, t) d^3 p$$

① FIRST INTEGRAL

$$\int d^3 p \frac{\partial f}{\partial t} = \frac{\partial}{\partial t} \int f d^3 p = \frac{\partial n(x, t)}{\partial t}$$

② SECOND INTEGRAL

$$\begin{aligned} \int d^3 p \underline{\dot{x}} \cdot \frac{\partial f}{\partial \underline{x}} &= \int d^3 p \frac{\underline{p}}{\gamma m} \cdot \frac{\partial f}{\partial \underline{x}} = \frac{\partial}{\partial \underline{x}} \cdot \int d^3 p \frac{\underline{p}}{\gamma m} f \\ &= \frac{\partial}{\partial \underline{x}} \cdot n \underline{v} \end{aligned}$$

(3) THIRD INTEGRAL

$$\int d^3p \left(q \underline{E} + \frac{q}{\gamma m} \underline{p} \times \underline{B} \right) \cdot \frac{\partial f}{\partial \underline{p}}$$

\downarrow
 $q \underline{E} f \Big|_{p=\infty}^{p=-\infty} = 0$

$$\int \frac{q}{\gamma m} (p_y B_z - p_z B_y) \frac{\partial f}{\partial p_x} \downarrow p_x \downarrow \frac{1}{\gamma} \downarrow \dots$$

$$\int \frac{q}{\gamma^3 m^3 c^2} (p_y B_z - p_z B_y) p_x$$

$$+ \frac{q}{\gamma^3 m^3 c^2} (p_z B_x - p_x B_z) p_y$$

$$+ \frac{q}{\gamma^3 m^3 c^2} (p_x B_y - p_y B_x) p_z \cdot f d^3p$$

= 0 !

$$\int_a^b uv' dx = uv \Big|_a^b - \int_a^b u'v dx$$

$$u = \frac{q}{\gamma m} (p_y B_z - p_z B_y)$$

$$u' = \frac{q}{\gamma^3 m} (v_y B_z - p_z B_y) \frac{\partial \gamma}{\partial p_x}$$

$$v = p_x$$

$$v' = \frac{\partial p}{\partial p_x}$$

$$\gamma^2 = \frac{p_x^2 + p_y^2 + p_z^2}{m^2 c^2} + 1$$

$$\Rightarrow 2\gamma \frac{\partial \gamma}{\partial p_x} = \frac{2 p_x}{m^2 c^2}$$

So $\int d^3p \left\{ \frac{\partial f}{\partial t} + \underline{x} \cdot \frac{\partial f}{\partial \underline{x}} + \underline{p} \cdot \frac{\partial f}{\partial \underline{p}} \right\} = 0$

$$\Rightarrow \left[\frac{\partial n(\underline{x}, t)}{\partial t} + \frac{\partial}{\partial \underline{x}} \cdot n(\underline{x}, t) \underline{v}(\underline{x}, t) = 0 \right]$$

CONTINUITY EQUATION \uparrow $q n(\underline{x}, t) \underline{v}(\underline{x}, t) = \underline{J}(\underline{x}, t)$
 CURRENT DENSITY \uparrow

ALTERNATIVELY $\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{J} = 0$

BALANCED
LUND

b) MOMENTUM EQUATION

(FOR SIMPLICITY: ASSUME NON-RELATIVISTIC)

$$\dot{\underline{x}} = \frac{\underline{p}}{m} \quad \dot{\underline{p}} = q(\underline{E}(x,t) + \frac{\underline{p}}{m} \times \underline{B}(x,t))$$

MULTIPLY BY $\dot{\underline{x}}$ & INTEGRATE OVER MOMENTUM ($\int d^3p$)

$$\int d^3p \left\{ \dot{\underline{x}} \frac{\partial f}{\partial t} + \dot{\underline{x}} \dot{\underline{x}} \cdot \frac{\partial f}{\partial \underline{x}} + \dot{\underline{x}} \left(q\underline{E} + \frac{\underline{p}}{m} \times \underline{B} \right) \frac{\partial f}{\partial \underline{p}} \right\} = 0$$

DEFINE $\underline{P} \equiv m \int d^3p (\underline{x} - \underline{v})(\underline{x} - \underline{v}) f(x, p, t)$

(\underline{P} = pressure tensor)

$$\begin{aligned} &= m \int d^3p \dot{\underline{x}} \dot{\underline{x}} f - 2m \underline{v} \int \dot{\underline{x}} f d^3p + m \underline{v} \underline{v} \int f d^3p \\ &= m \int d^3p \dot{\underline{x}} \dot{\underline{x}} f - m n \underline{v} \underline{v} \end{aligned}$$

① FIRST INTEGRAL:

$$\int d^3p \dot{\underline{x}} \frac{\partial f}{\partial t} = \frac{\partial}{\partial t} \int d^3p \dot{\underline{x}} f = \frac{\partial}{\partial t} n \underline{v}$$

② SECOND INTEGRAL:

$$\begin{aligned} \int d^3p \dot{\underline{x}} \dot{\underline{x}} \cdot \frac{\partial f}{\partial \underline{x}} &= \frac{\partial}{\partial \underline{x}} \cdot \int d^3p \dot{\underline{x}} \dot{\underline{x}} f \\ &= \frac{1}{m} \frac{\partial}{\partial \underline{x}} \cdot \underline{P} + \frac{\partial}{\partial \underline{x}} \cdot n \underline{v} \underline{v} \\ &= \frac{1}{m} \frac{\partial}{\partial \underline{x}} \cdot \underline{P} + \left(\frac{\partial}{\partial \underline{x}} \cdot n \underline{v} \right) \underline{v} + n \underline{v} \cdot \frac{\partial \underline{v}}{\partial \underline{x}} \end{aligned}$$

③ THIRD INTEGRAL:

$$\begin{aligned} &\int d^3p \frac{\underline{p}}{m} \left(q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B} \right) \cdot \frac{\partial f}{\partial \underline{p}} \\ &= \underbrace{f \frac{\underline{p}}{m} \left(q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B} \right)}_{=0} \Big|_{-\infty}^{\infty} - \int d^3p \left(q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B} \right) f \\ &= -\frac{nc(x,t)}{m} (q\underline{E} + q\underline{v} \times \underline{B}) \end{aligned}$$

$$\begin{aligned} u &= \frac{\underline{p}}{m} \left(q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B} \right) \\ u' &= \frac{1}{m} \left(q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B} \right) \\ v &= \frac{\partial f}{\partial \underline{p}} \\ v' &= \frac{\partial f}{\partial \underline{x}} \end{aligned}$$

BALANCED
LUND

b) MOMENTUM EQUATION

(FOR SIMPLICITY: ASSUME NON-RELATIVISTIC)

$$\dot{\underline{x}} = \frac{\underline{p}}{m} \quad \dot{\underline{p}} = q(\underline{E}(x,t) + \frac{\underline{p}}{m} \times \underline{B}(x,t))$$

MULTIPLY BY $\dot{\underline{x}}$ & INTEGRATE OVER MOMENTUM ($\int d^3p$)

$$\int d^3p \left\{ \dot{\underline{x}} \frac{\partial f}{\partial t} + \dot{\underline{x}} \dot{\underline{x}} \cdot \frac{\partial f}{\partial \underline{x}} + \dot{\underline{x}} \left(q\underline{E} + \frac{\underline{p}}{m} \times \underline{B} \right) \frac{\partial f}{\partial \underline{p}} \right\} = 0$$

DEFINE $\underline{P} \equiv m \int d^3p (\underline{x} - \underline{v})(\underline{x} - \underline{v}) f(x, p, t)$

(\underline{P} = pressure tensor)

$$\begin{aligned} &= m \int d^3p \dot{\underline{x}} \dot{\underline{x}} f - 2m \underline{v} \int \dot{\underline{x}} f d^3p + m \underline{v} \underline{v} \int f d^3p \\ &= m \int d^3p \dot{\underline{x}} \dot{\underline{x}} f - m n \underline{v} \underline{v} \end{aligned}$$

① FIRST INTEGRAL:

$$\int d^3p \dot{\underline{x}} \frac{\partial f}{\partial t} = \frac{\partial}{\partial t} \int d^3p \dot{\underline{x}} f = \frac{\partial}{\partial t} n \underline{v}$$

② SECOND INTEGRAL:

$$\begin{aligned} \int d^3p \dot{\underline{x}} \dot{\underline{x}} \cdot \frac{\partial f}{\partial \underline{x}} &= \frac{\partial}{\partial \underline{x}} \cdot \int d^3p \dot{\underline{x}} \dot{\underline{x}} f \\ &= \frac{1}{m} \frac{\partial}{\partial \underline{x}} \cdot \underline{P} + \frac{\partial}{\partial \underline{x}} \cdot n \underline{v} \underline{v} \\ &= \frac{1}{m} \frac{\partial}{\partial \underline{x}} \cdot \underline{P} + \left(\frac{\partial}{\partial \underline{x}} \cdot n \underline{v} \right) \underline{v} + n \underline{v} \cdot \frac{\partial \underline{v}}{\partial \underline{x}} \end{aligned}$$

③ THIRD INTEGRAL:

$$\begin{aligned} &\int d^3p \frac{\underline{p}}{m} \left(q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B} \right) \cdot \frac{\partial f}{\partial \underline{p}} \\ &= \underbrace{f \frac{\underline{p}}{m} \left(q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B} \right)}_{=0} \Big|_{-\infty}^{\infty} - \int d^3p \left(q\underline{E} + q \frac{\underline{p}}{m} \times \underline{B} \right) f \\ &= -\frac{nc(x,t)}{m} (q\underline{E} + q\underline{v} \times \underline{B}) \end{aligned}$$

$$\begin{aligned} u &= \frac{\underline{p}}{m} \left(q\underline{E} + \frac{\underline{p}}{m} \times \underline{B} \right) \\ u' &= \frac{1}{m} \left(q\underline{E} + \frac{\underline{p}}{m} \times \underline{B} \right) \\ v' &= \frac{\partial f}{\partial \underline{p}} \cdot \frac{\partial \underline{p}}{\partial \underline{x}} \end{aligned}$$

Summary of fluid equations

$$\text{Let } n(\underline{x}, t) = \int d^3p f(\underline{x}, \underline{p}, t)$$

PARTICLE DENSITY

$$\underline{v}(\underline{x}, t) \equiv \frac{1}{n(\underline{x}, t)} \int d^3p \frac{\underline{p}}{\gamma m} f(\underline{x}, \underline{p}, t)$$

FLUID VELOCITY

$$\underline{P}(\underline{x}, t) \equiv \frac{1}{n(\underline{x}, t)} \int d^3p \underline{p} f(\underline{x}, \underline{p}, t)$$

FLUID MOMENTUM

$$\underline{P}(\underline{x}, t) \equiv \int d^3p (\underline{p} - \underline{P}) \left(\frac{\underline{p}}{\gamma m} - \underline{v} \right) f(\underline{x}, \underline{p}, t)$$

PRESSURE TENSOR

$$\frac{d\underline{x}}{dt} = \frac{\underline{p}}{\gamma m} \quad \frac{d\underline{p}}{dt} = q(\underline{E}(\underline{x}, t) + \frac{\underline{p}}{\gamma m} \times \underline{B}(\underline{x}, t)) \quad \gamma^2 = \frac{\underline{p} \cdot \underline{p}}{(mc)^2} + 1$$

CONTINUITY EQUATION:

$$\frac{\partial n(\underline{x}, t)}{\partial t} + \frac{\partial}{\partial \underline{x}} \cdot n(\underline{x}, t) \underline{v}(\underline{x}, t) = 0$$

MOMENTUM EQUATION:

$$\frac{\partial \underline{P}}{\partial t} + \underline{v} \cdot \frac{\partial \underline{P}}{\partial \underline{x}} = q(\underline{E} + \underline{v} \times \underline{B}) - \frac{1}{n(\underline{x}, t)} \frac{\partial}{\partial \underline{x}} \cdot \underline{P} = 0$$

THE ABOVE EQUATIONS ARE RELATIVISTICALLY CORRECT.
IN THE NON-RELATIVISTIC LIMIT THE CONTINUITY EQUATION REMAINS UNCHANGED & THE MOMENTUM EQUATION MAY BE WRITTEN:

$$\text{NON RELATIVISTIC} \rightarrow \frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \frac{\partial \underline{v}}{\partial \underline{x}} = \frac{q}{m} (\underline{E} + \underline{v} \times \underline{B}) - \frac{1}{m n} \frac{\partial}{\partial \underline{x}} \cdot \underline{P}$$

THESE EQUATIONS ARE SUPPLEMENTED WITH MAXWELL'S EQUATIONS:
for $\underline{E}(\underline{x}, t)$ & $\underline{B}(\underline{x}, t)$

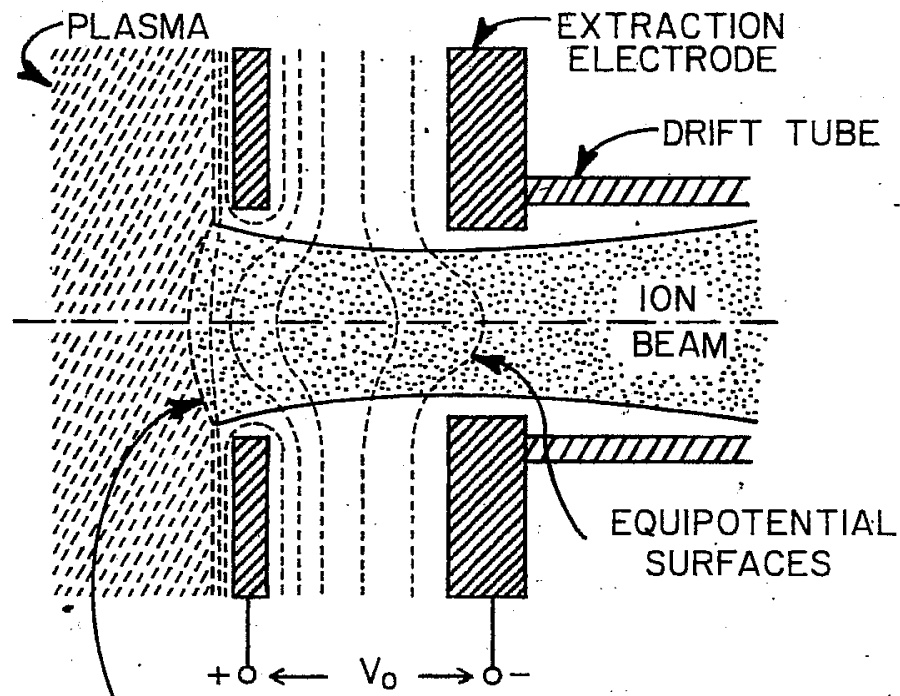
$$\frac{\partial}{\partial \underline{x}} \cdot \underline{E} = \frac{q n(\underline{x}, t)}{\epsilon_0} \quad \frac{\partial}{\partial \underline{x}} \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\frac{\partial}{\partial \underline{x}} \cdot \underline{B} = 0 \quad \frac{\partial}{\partial \underline{x}} \times \underline{B} = \mu_0 \underline{J} + \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} \quad \underline{J}(\underline{x}, t) = q n(\underline{x}, t) \underline{v}(\underline{x}, t)$$

NEED ADDITIONAL EQUATIONS SUCH AS $\underline{P} = 0$ OR ENERGY EQUATION TO TERMINATE SET OF EQUATIONS.

INJECTORS

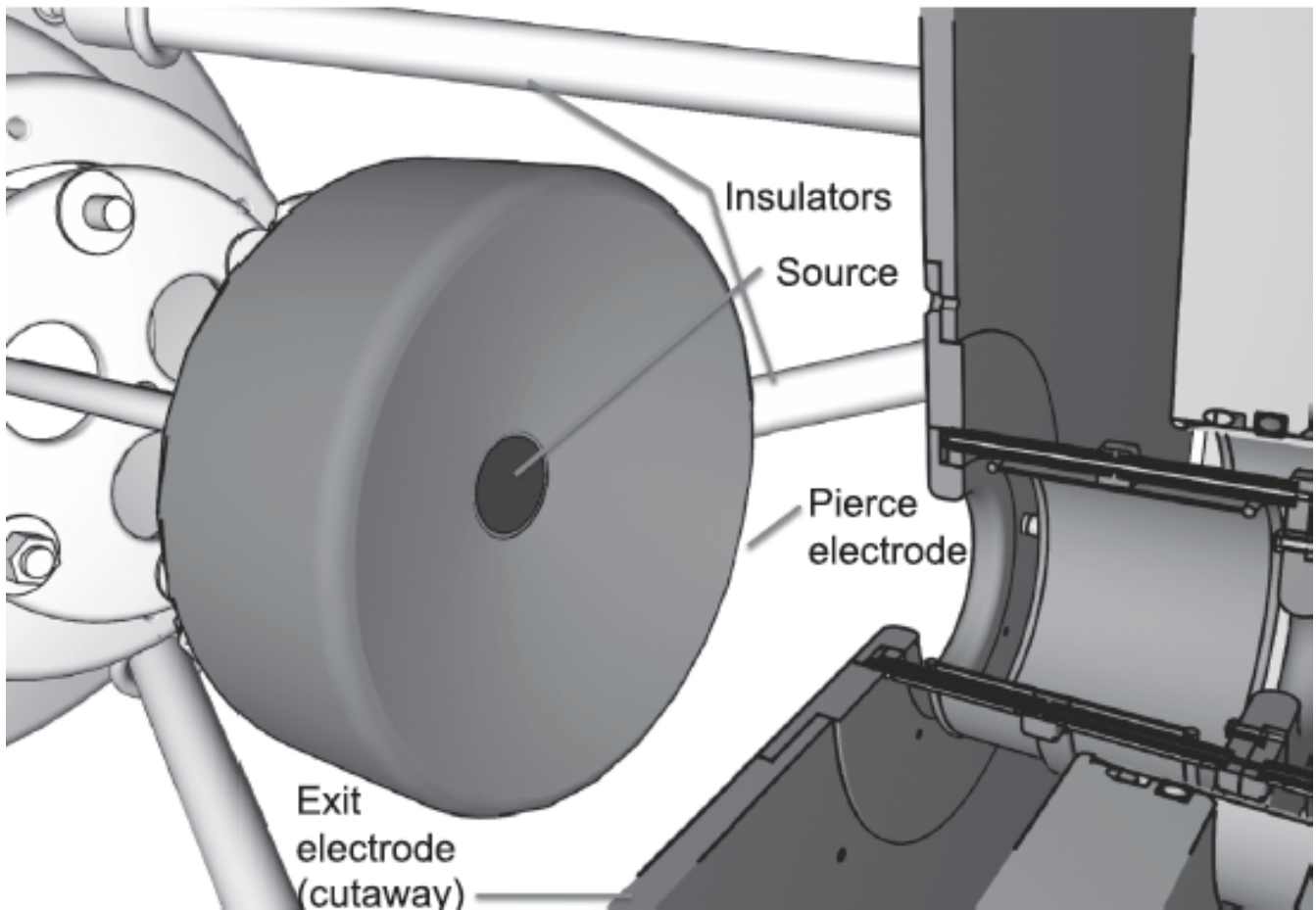
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EMITTING SURFACE
(PLASMA "SHEATH" OR "MENISCUS")
OR "HOT PLATE"

REISER, FIGURE 1.2

- DOPED TUNGSTEN
- ALUMINO-SILICATE

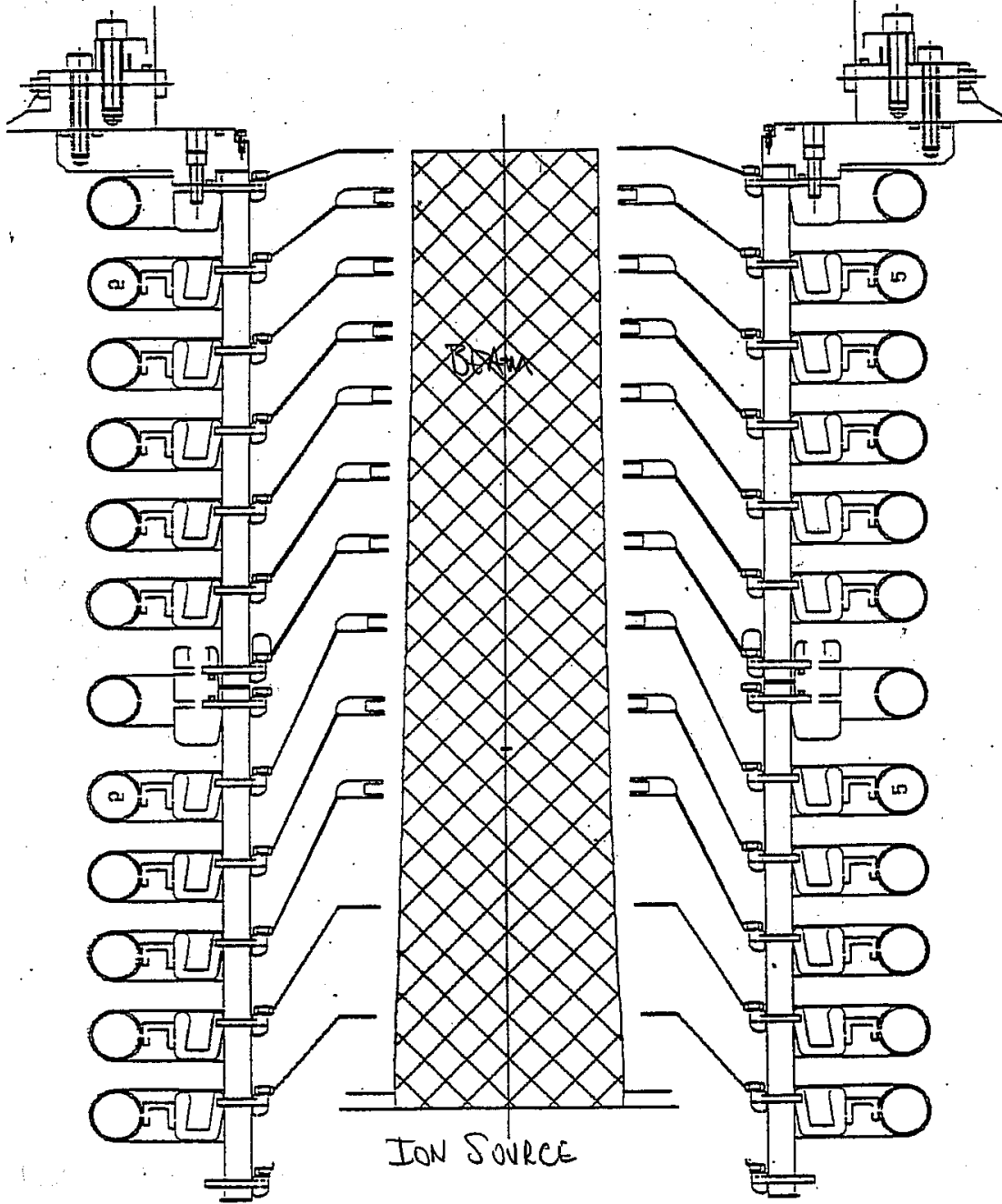


A mechanical drawing of a hot-plate diode used on the NDCX-1 experiment at LBNL. One quarter of the exit electrode is cut away for viewing the source geometry

PIERCE COLUMN

$V \sim z^{4/5}$

$E \sim z^{1/3}$



~~Derive~~ ^{Derive} WE A THE PARAXIAL RAY EQUATION FOR PARTICLES IN AXISYMMETRIC SYSTEMS:

$$r'' + \underbrace{\frac{\gamma'}{\beta^2 \gamma}}_{\text{INERTIAL}} r' + \underbrace{\frac{\gamma''}{2\beta^2 \gamma}}_{E_r} r + \underbrace{\left(\frac{\omega_c}{2\gamma\beta c}\right)^2}_{V_0 B_z - \text{CENTRIFUGAL}} r - \underbrace{\left(\frac{p_0}{\gamma\beta m c}\right)^2 \frac{1}{r^3}}_{\text{CENTRIFUGAL}} - \underbrace{\frac{q}{\gamma^3 m v_z^2} \frac{\lambda(r)}{2\beta^2 \gamma}}_{\text{SELF-FIELD}} = 0$$

$$\theta' = \frac{p_0}{\gamma m r^2 \beta c} - \frac{\omega_c}{2\gamma\beta c} \quad \leftarrow \text{CONSTANCY + DEFINITION OF CANONICAL MOMENTUM}$$

ENVELOPE EQUATION FOR AXISYMMETRIC BEAM

$$r_b'' + \frac{\gamma' r_b'}{\beta^2 \gamma} + \frac{\gamma''}{2\beta^2 \gamma} r_b + \left(\frac{\omega_c}{2\gamma\beta c}\right)^2 r_b - \frac{4\langle p_0 \rangle^2}{(\gamma m \beta c)^2 r_b^3} - \frac{E_r}{V_b^3} - \frac{Q}{r_b} = 0$$

$$E_r \equiv 4(\langle r^2 \rangle \langle v_{\perp}^2 \rangle - \langle r v_{\perp} \rangle^2) + \langle v_z^2 \rangle \langle r^2 \theta'^2 \rangle - \langle r^2 \theta' \rangle^2$$

RETURNING TO PARAXIAL ENVELOPE EQUATION:

$$(for \beta \ll 1) \quad v_b'' + \frac{\beta'}{\beta} v_b' + \left[\frac{1}{2} \frac{\beta'^2}{\beta^2} + \frac{1}{2} \frac{\beta''}{\beta} \right] v_b - \frac{Q}{v_b} = 0$$

$$for \quad v_b'' = \frac{\beta'}{\beta} v_b' = 0$$

$$if \quad \Phi = v_b \left(\frac{z}{d} \right)^{1/3}$$

$$v = C z^{1/3}$$

$$v' = \frac{1}{3} C z^{-2/3}$$

$$v'' = -\frac{2}{9} C z^{-5/3}$$

$$\Rightarrow \left[\frac{1}{2} \frac{\beta'^2}{\beta^2} + \frac{1}{2} \frac{\beta''}{\beta} \right] v_b^2 = Q$$

$$\left[\frac{2}{9} \frac{1}{z^2} \quad -\frac{1}{9} \frac{1}{z^2} \right]$$

$$\Rightarrow Q(z) = \frac{1}{9} \frac{v_b^2}{z^2}$$

So Child-Langmuir flow satisfies the

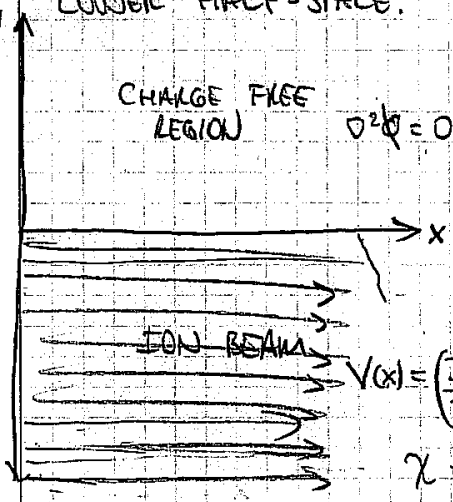
PARAXIAL ENVELOPE EQUATION FOR

A CONSTANT BEAM CURRENT (AS IT SHOULD!)

PIERCE'S ELECTRODES: GOING BEYOND 1D

CONSIDER THE CASE A BEAM WHICH FILLS THE LOWER HALF-SPACE.

42-182 100 SHEETS
National Brand
Made in U.S.A.



$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

FIND SOLUTION

SUCH THAT

$$\frac{\partial \phi}{\partial y}(x, y=0) = 0$$

$$\phi(x, y=0) = V(x)$$

PIERCE'S SOLUTION: LET THE POTENTIAL BE THE REAL PART

OF
$$\phi + iW = V(x+iy) \equiv V(z) \quad z = x+iy$$

NOTE THAT FOR ANY $V(z)$ WITH DERIVATIVES THAT EXIST INDEPENDENT OF DIRECTION (ANALYTIC) THE REAL PART OF $V(z)$ SATISFIES LAPLACE'S EQUATION:
$$\frac{\partial^2 \text{Re}[V]}{\partial x^2} + \frac{\partial^2 \text{Re}[V]}{\partial y^2} = 0$$

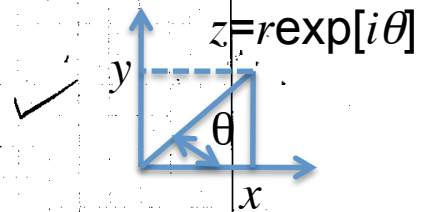
$$\frac{\partial \phi}{\partial x} = \text{Re} \left[\frac{dV}{dz} \right] \quad \frac{\partial \phi}{\partial y} = \text{Re} \left[i \frac{dV}{dz} \right]$$

$$\frac{\partial^2 \phi}{\partial x^2} = \text{Re} \left[\frac{d^2 V}{dz^2} \right] \quad \frac{\partial^2 \phi}{\partial y^2} = -\text{Re} \left[\frac{d^2 V}{dz^2} \right] \Rightarrow \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \text{Re} \left[\frac{d^2 V(z)}{dz^2} \right] - \text{Re} \left[\frac{d^2 V(z)}{dz^2} \right] = 0$$

(12)

CHOOSE $V(z) = \left(\frac{J}{\lambda}\right)^{2/3} (x+iy)^{4/3}$

By construction $\phi(x, y=0) = V(x)$



$$\begin{aligned}\phi &= \text{Re} \left[\left(\frac{J}{\lambda}\right)^{2/3} (x+iy)^{4/3} \right] \\ &= \left(\frac{J}{\lambda}\right)^{2/3} (x^2+y^2)^{2/3} \text{Re} \left[\exp \left[i \frac{4}{3} \tan^{-1} \left(\frac{y}{x} \right) \right] \right]\end{aligned}$$

Let $x+iy = r \exp[i\theta]$
 $(x+iy)^{4/3} = r^{4/3} \exp \left[i \frac{4\theta}{3} \right]$

$$\phi(x, y) = \left(\frac{J}{\lambda}\right)^{2/3} (x^2+y^2)^{2/3} \cos \left[\frac{4}{3} \tan^{-1} \left(\frac{y}{x} \right) \right]$$

Note that $\phi(x, y) = \phi(x, -y) \Rightarrow \frac{\partial \phi}{\partial y}(x, y=0) = 0$

$\phi = 0$ EQUIPOTENTIAL:

$$\Rightarrow 0 = \cos \left[\frac{4}{3} \tan^{-1} \left(\frac{y}{x} \right) \right]$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x} \right) = \frac{3}{4} \left(\frac{\pi}{2} \right) = 67.5^\circ$$

So line $\theta = 67.5^\circ$ is a $\phi=0$ equipotential.

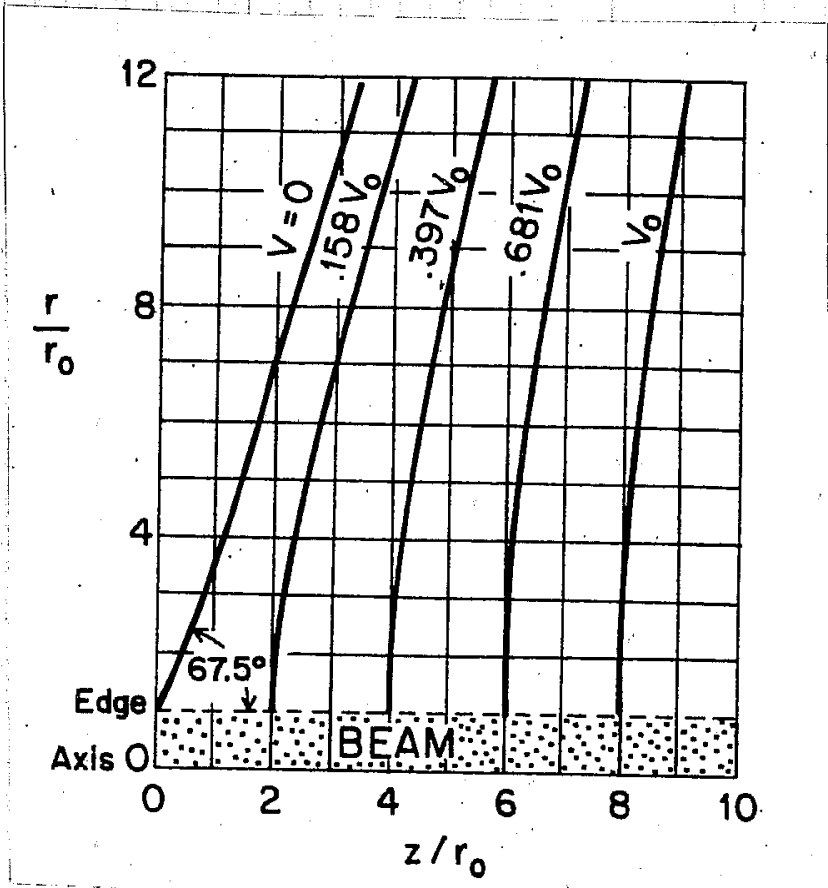
FOR A GENERAL EQUIPOTENTIAL PASSING THROUGH x_0 :

$$x_0^{4/3} = (x^2+y^2)^{2/3} \cos \left[\frac{4}{3} \tan^{-1} \left(\frac{y}{x} \right) \right]$$

$$\phi(x_0) = \left(\frac{J}{\lambda}\right)^{2/3} x_0^{4/3}$$

Equipotential that passes through point $(x_0, 0)$

PIERCE ELECTRODES FOR CIRCULAR BEAMS



- SOLUTION is similar, but must be done numerically
- $\phi = 0$ is same as planar case

FIGURES FROM A.T. FORRESTER,
 "LARGE ION BEAMS,"
 Wiley, 1988

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So if we know $z_0(t)$ we can determine $\Phi(t)$.

$$\frac{1}{2} m \dot{z}_0^2 = q V_0 \left(\frac{z_0}{d}\right)^{4/3}$$

(SINCE BY CONSTRUCTION, HEAD OF BEAM TRAVELS AT CHILD-LANGMUIR VELOCITY LIKE ALL PARTICLES),

$$\dot{z}_0 = \left(\frac{2qV_0}{m}\right)^{1/2} \left(\frac{z_0}{d}\right)^{2/3}$$

$$\frac{dz_0}{z_0^{2/3}} = \left(\frac{2qV_0}{m}\right)^{1/2} \frac{dt}{d^{2/3}}$$

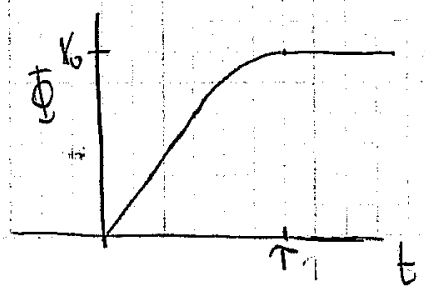
$$\Rightarrow 3 z_0^{1/3} = \left(\frac{2qV_0}{m}\right)^{1/2} \frac{t}{d^{2/3}} \Rightarrow t = \frac{3(z_0 d^2)^{1/3}}{\left(\frac{2qV_0}{m}\right)^{1/2}}$$

Let $\uparrow = \frac{3d}{\left(\frac{2qV_0}{m}\right)^{1/2}}$ = transit time across diode

$$\Rightarrow \frac{t}{\uparrow} = \left(\frac{z_0}{d}\right)^{1/3}$$

$$\Phi(d, z_0) = V_0 \left[\frac{4}{3} \left(\frac{z_0}{d}\right)^{1/3} - \frac{1}{3} \left(\frac{z_0}{d}\right)^{4/3} \right]$$

$$\Rightarrow \Phi(d, t) = \begin{cases} V_0 \left[\frac{4}{3} \left(\frac{t}{\uparrow}\right) - \frac{1}{3} \left(\frac{t}{\uparrow}\right)^4 \right] & \text{for } 0 < t < \uparrow \\ V_0 & \text{for } t > \uparrow \end{cases}$$



AV 24 54 75
V =

from A. Faltens:

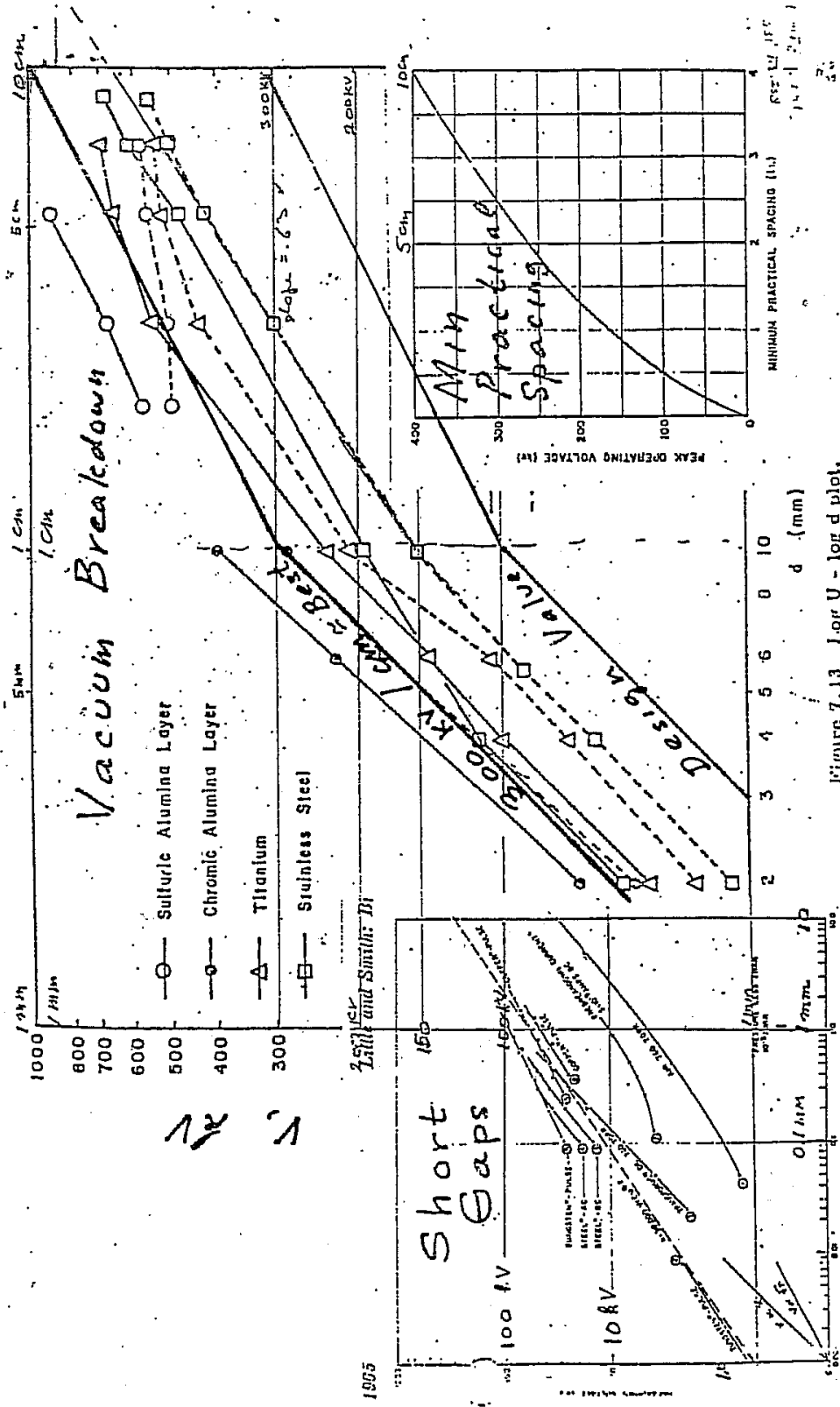
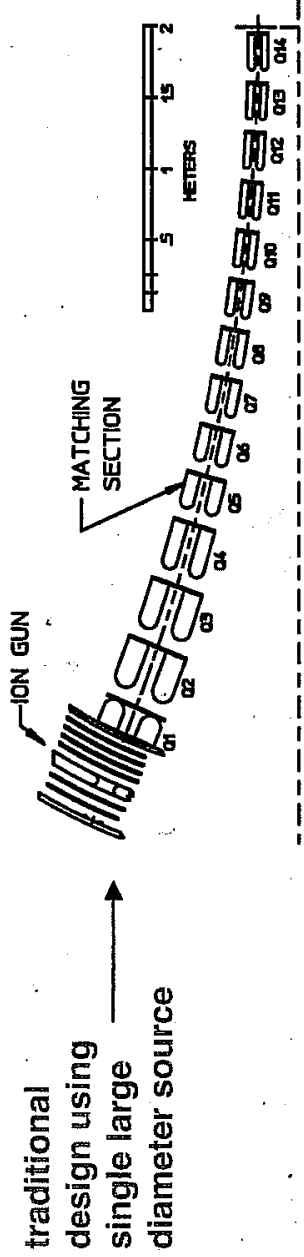


Figure 7.13 $\log U - \log d$ plot,

Fig. 1. Breakdown voltage-vs.-gap length for uniform-field and non-uniform-field geometry. Numbers on curves indicate the

MULTIPLE BEAMLET INJECTORS CAN HAVE HIGHER CURRENT DENSITY
 DECREASING SIZE OF INJECTOR



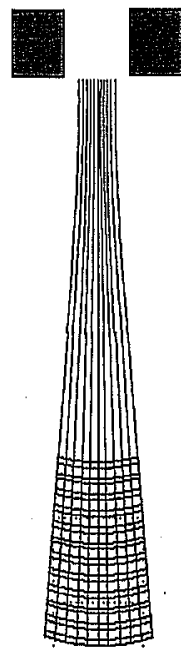
advanced design using multiple beamlets

Each beamlet carries higher current density; But merging beamlets increases thermal spread.

Child-Langmuir $J_{CL} \propto \frac{V^{3/2}}{d^2}$

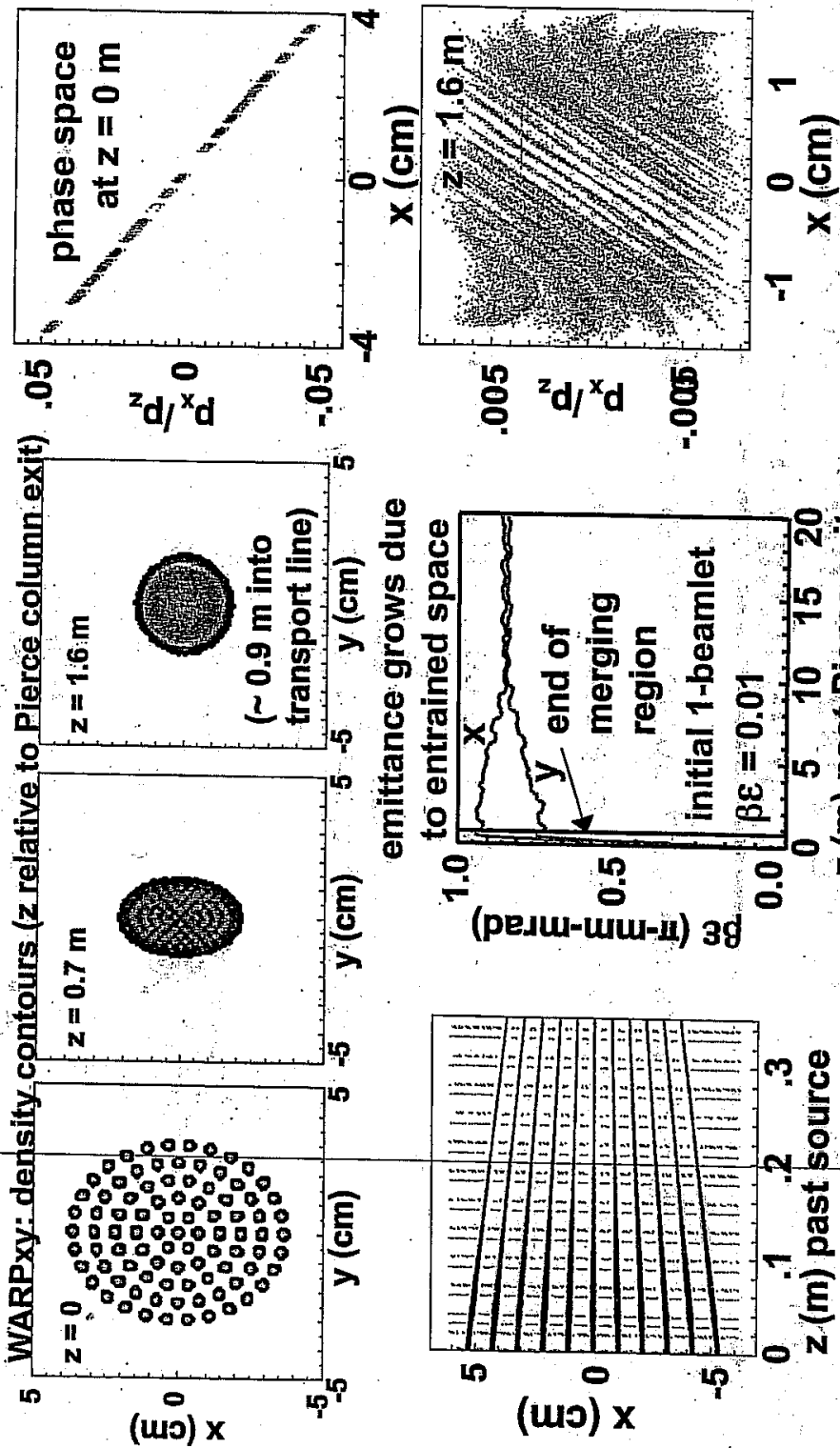
$J \propto V^{-1/2}$ to $-5/2 \propto d^{-1/2}$ to $-5/4$

Breakdown limit $V \propto d^{1.0}$ to 0.5



Merge and match beamlets into an ESQ channel

Simulations of merging-beamlet injector



The Heavy Ion Fusion Virtual National Laboratory

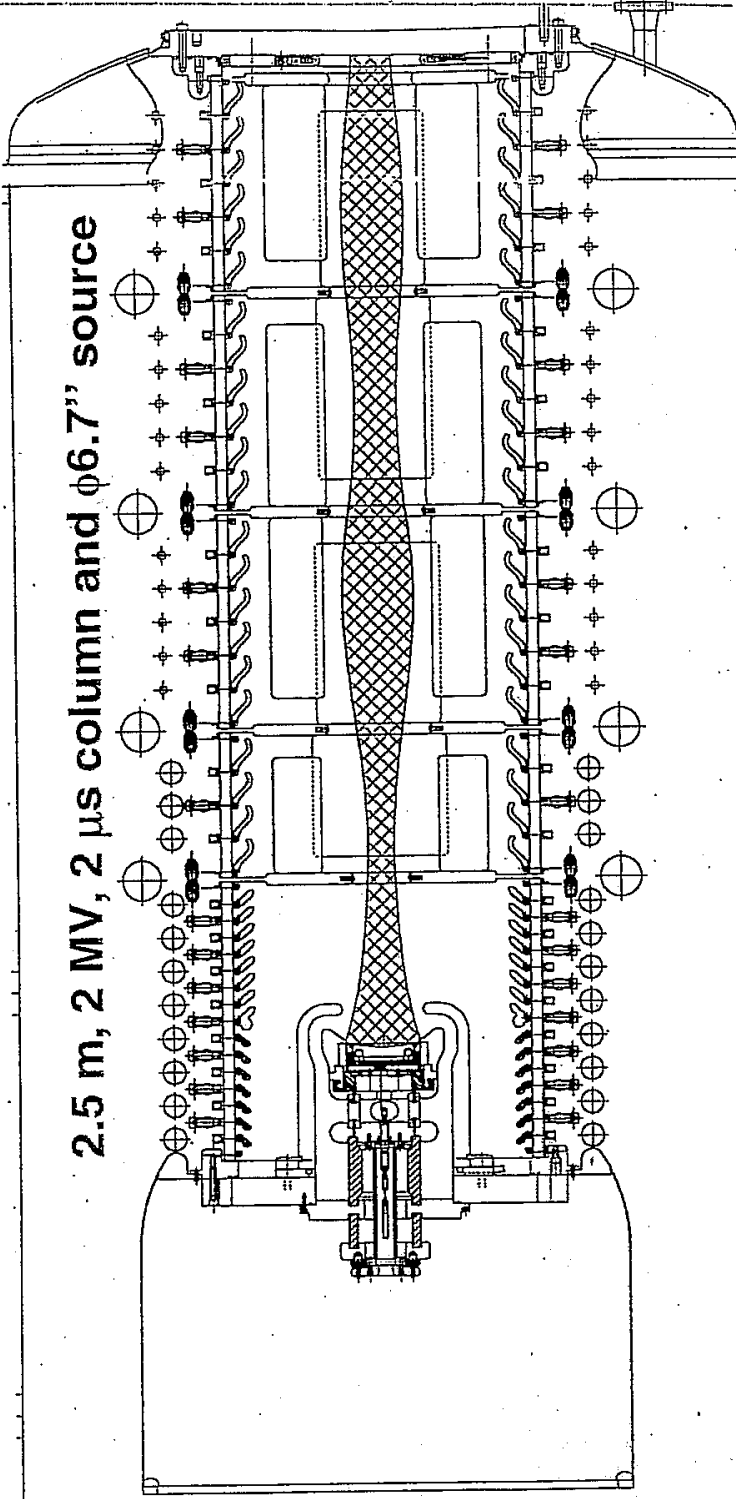
PPPL
PRINCETON PLASMA PHYSICS LABORATORY
PRINCETON UNIVERSITY

FROM D.L. GROTE, E. HENESTROZA, J.W. KOHN, "DESIGN & SIMULATION OF THE MULTIBEAMLET SIMULATOR IN FACTA (2007)"



0.8 Ampere, 2 MV K^+ Injector produced a $\lambda=0.25\mu C/m$ beam

Electrostatic Quadrupole Accelerator for simultaneous
focusing and acceleration of ion beams to 2 MV.



LAWRENCE BERKELEY NATIONAL LABORATORY

FIGURE FOR CASE 01 1989/90

SCALING OF BRIGHTNESS IN INJECTORS

$$G_N = 4 \beta \langle x^2 \rangle^{1/2} \langle x'^2 \rangle^{1/2} = \frac{4}{c} \left(\frac{v_b}{2} \right) \langle v_x^2 \rangle^{1/2}$$

$$C_1 = 2 \pi \beta \sqrt{\frac{kT}{mc^2}}$$

$$\frac{1}{2} m v_x^2 = \frac{1}{2} kT$$

$$\Rightarrow B = \frac{I}{\epsilon_N^2} = \frac{\pi J}{4(kT/mc^2)} \sim \frac{J}{T}$$

\Rightarrow FOR HIGH BRIGHTNESS & HIGH CURRENT
 MAY WISH TO ACCELERATE MANY BEAMLETS
 AND THEN MERGE TO FORM SINGLE BEAM.

MANY ISSUES NOT DISCUSSED HERE!

- SOURCES
- ELECTRON TRAPPING
- CONVERGING BEAMS
- MATCHING TO AN ESQ (e.g.)
- rf
- ...