U.S. Particle Accelerator School
Education in Beam Physics and Accelerator Technology

Self-Consistent Simulations of Beam and Plasma Systems
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A2. Mesh Refinement in Field Solvers

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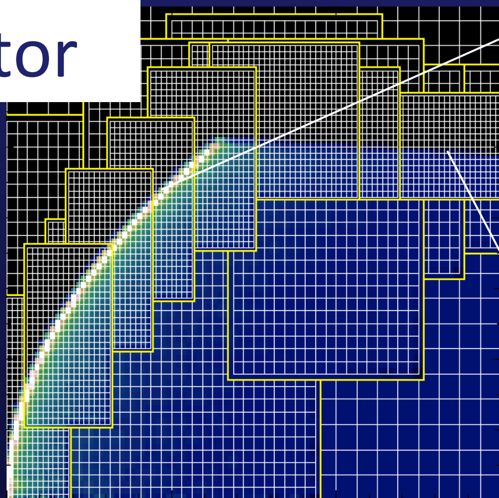
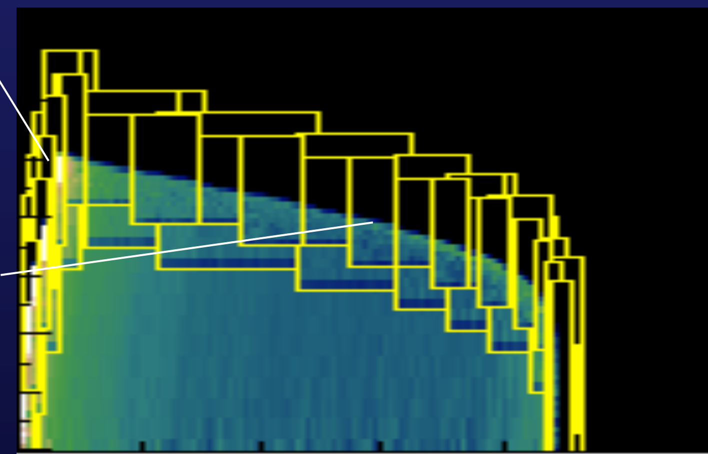
Outline

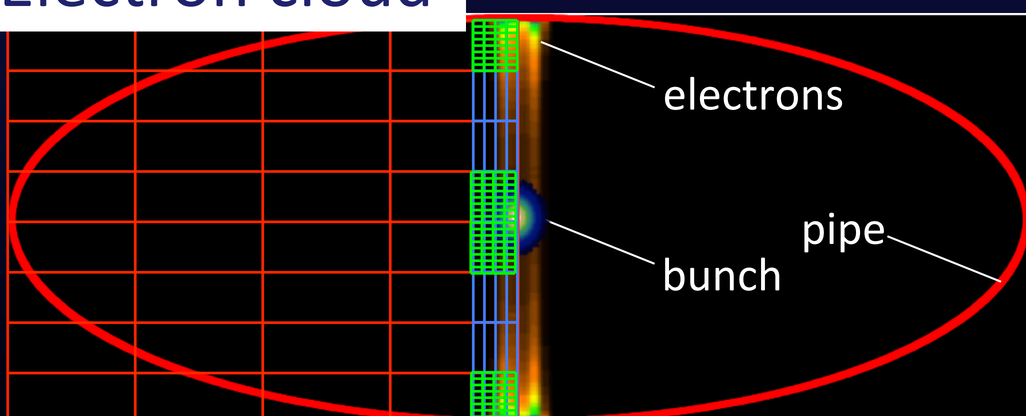
- Why mesh refinement?
- Potential issues
- Electrostatic mesh refinement
  - spurious self-force example
  - spurious self-force mitigation
  - application to the modeling of HCX injector
- Electromagnetic mesh refinement
  - spurious reflection of waves
  - spurious reflection of waves mitigation
  - Application to the modeling beam-induced plasma wake
- Special mesh refinement for particle emission
- Summary

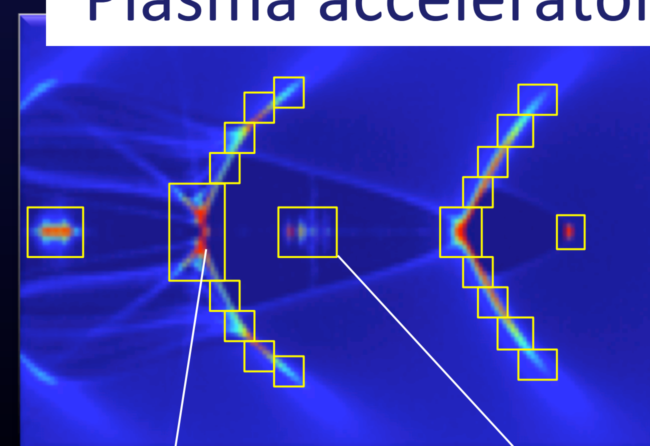
Why mesh refinement?

To resolve density spikes & gradients.







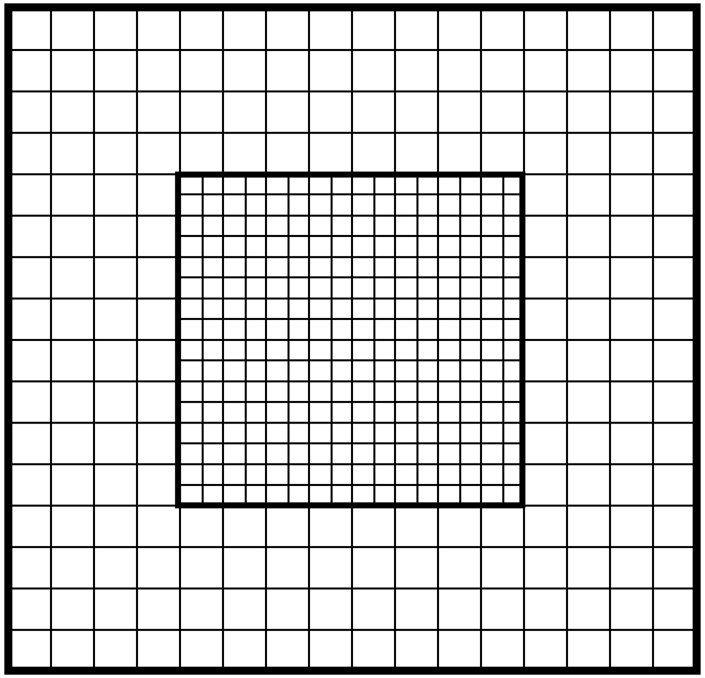


Coupling of AMR to PIC: issues

Mesh refinement implies:

➔ jump of resolution at coarse-fine interface,

➔ some procedure for coupling the solutions at the interface.



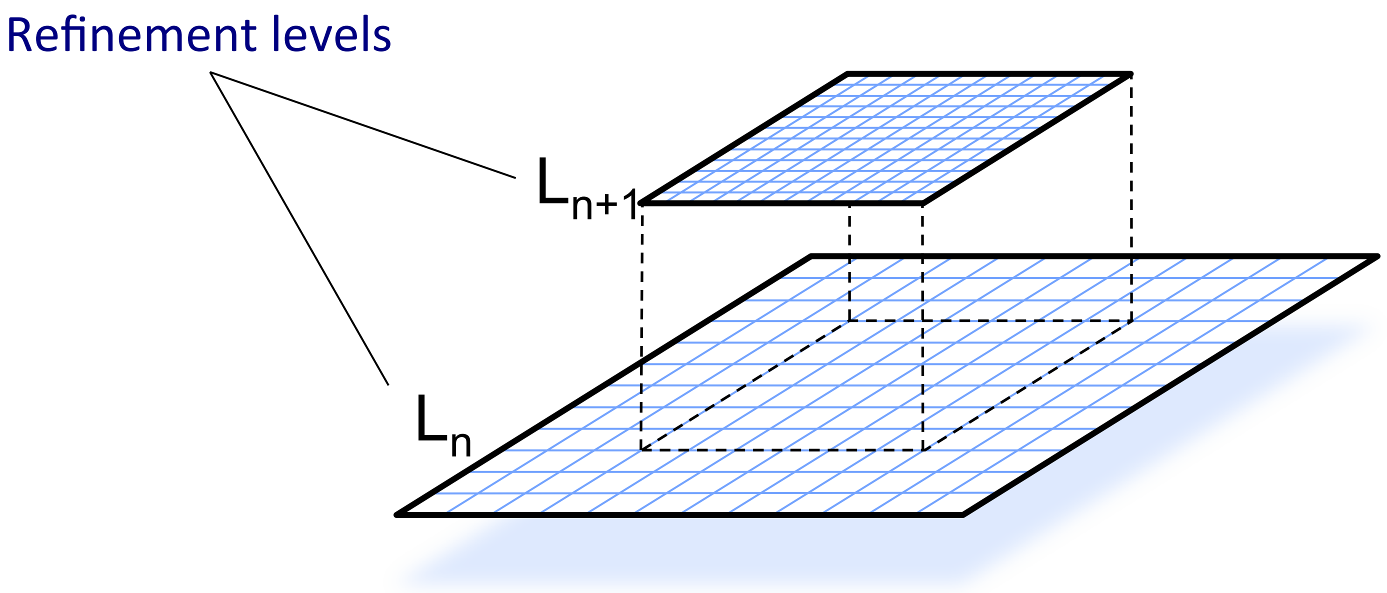
Consequences:

- loss of symmetry: self-force,

- loss of conservation laws,

- EM: waves reflection.

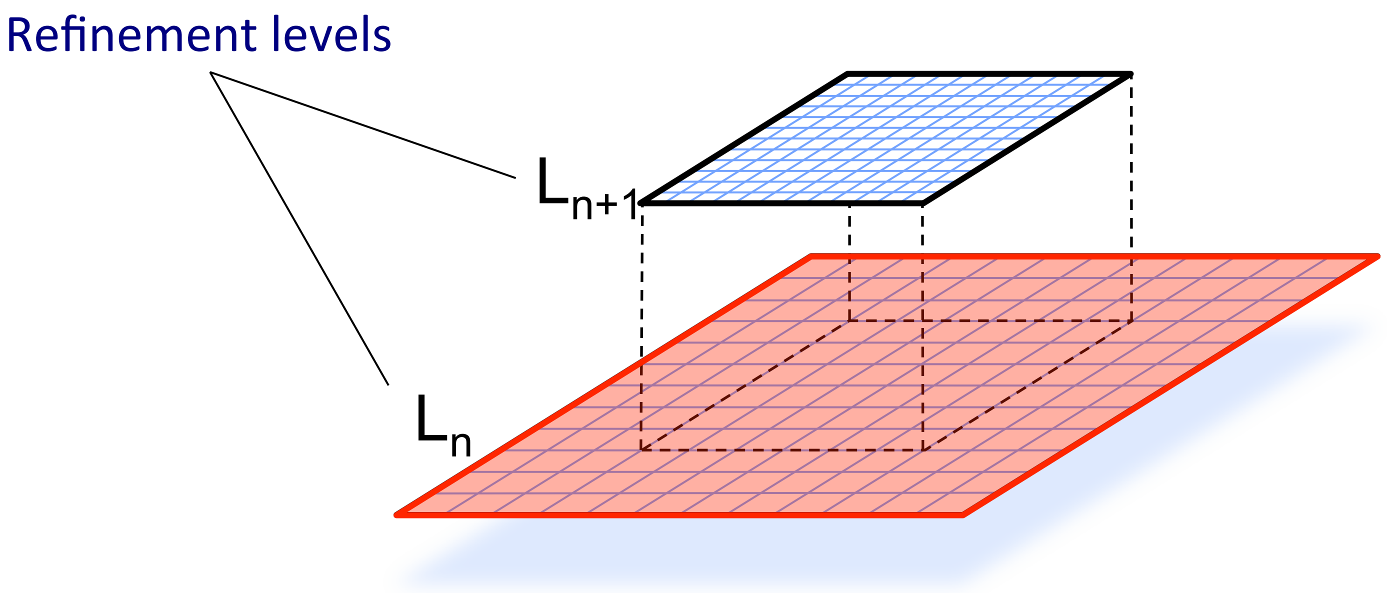
Electrostatic mesh refinement



Solution to Poisson is a boundary value problem.
We can define the following simple procedure:

1. solve on coarse grid,
2. interpolate on fine grid boundaries,
3. solve on fine grid.

Electrostatic mesh refinement



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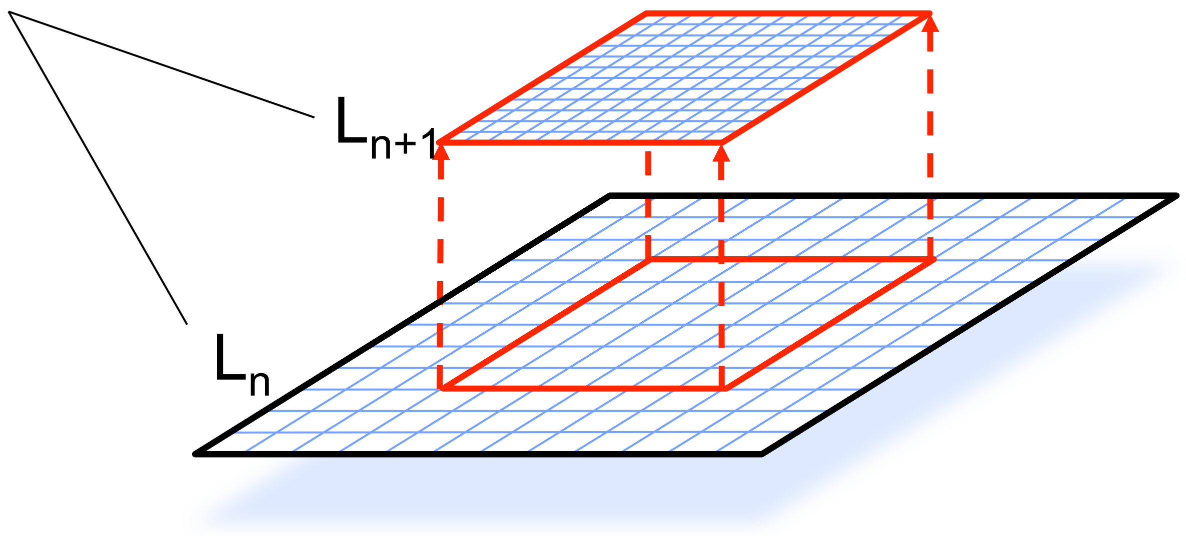
Electrostatic mesh refinement

Refinement levels



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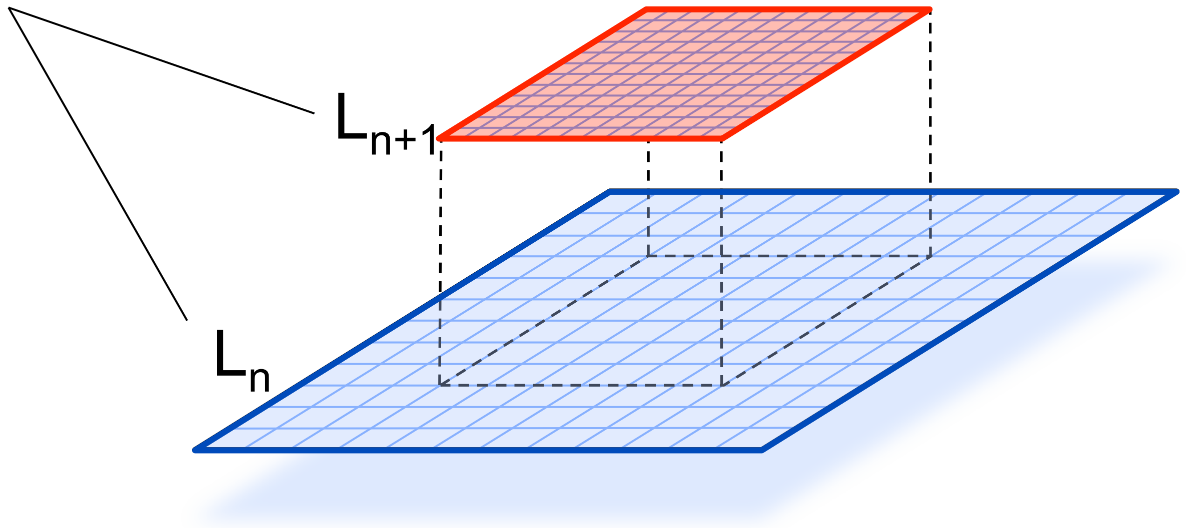
Electrostatic mesh refinement

Refinement levels



Solution to Poisson is a boundary value problem.
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1. solve on coarse grid,
2. interpolate on fine grid boundaries,
3. **solve on fine grid.**

Illustration potential problem: spurious self-force

Assume one charged macroparticle in a box with metallic BC

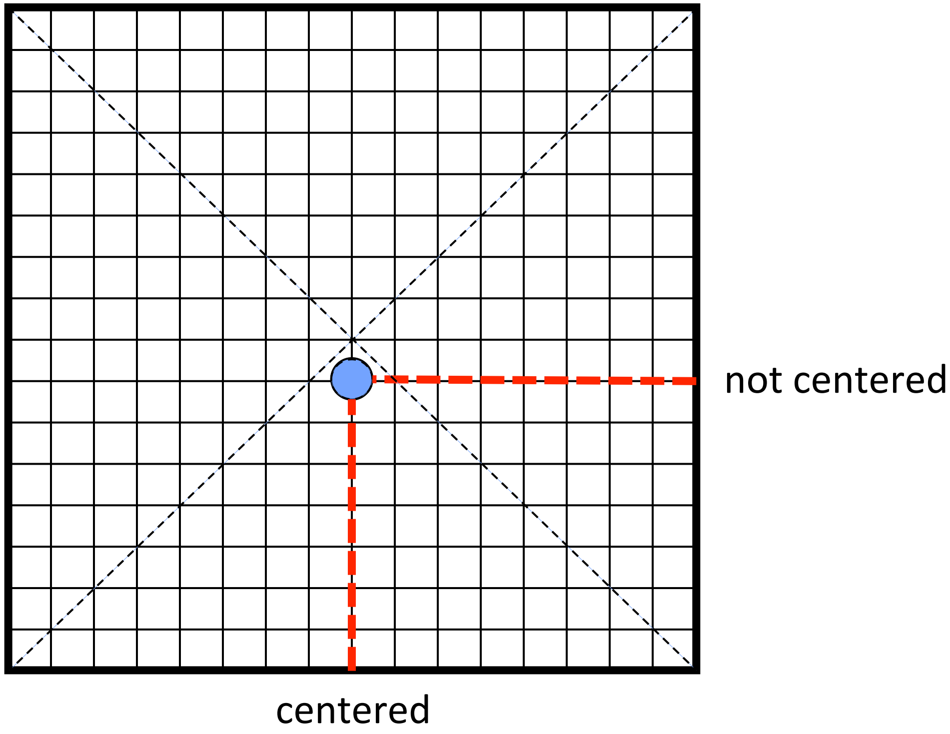


Illustration potential problem: spurious self-force

The macroparticle is attracted by its image from the closest metallic wall.

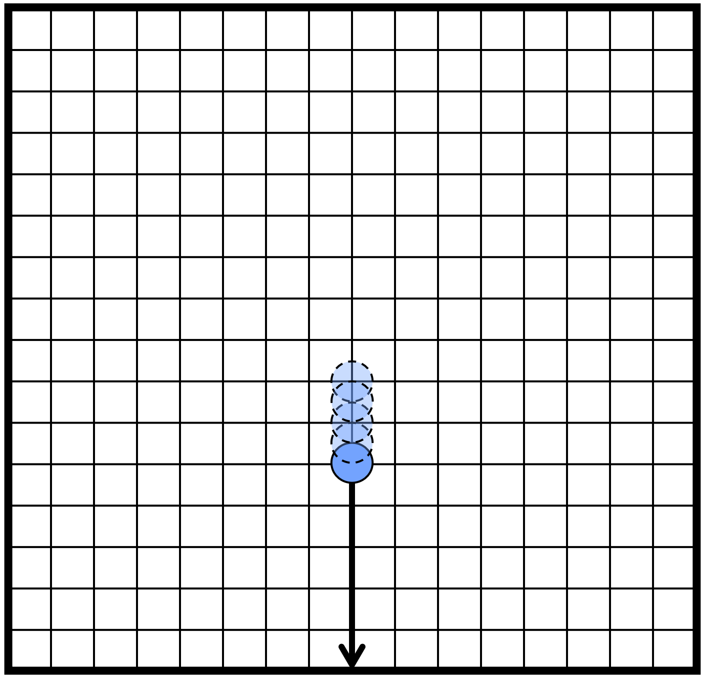


Illustration potential problem: spurious self-force

We apply specular reflection
at the boundary.

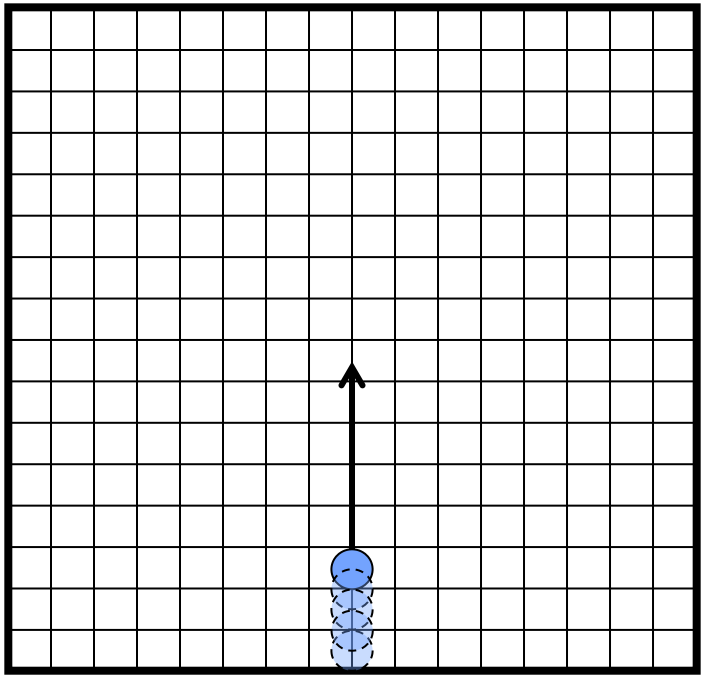
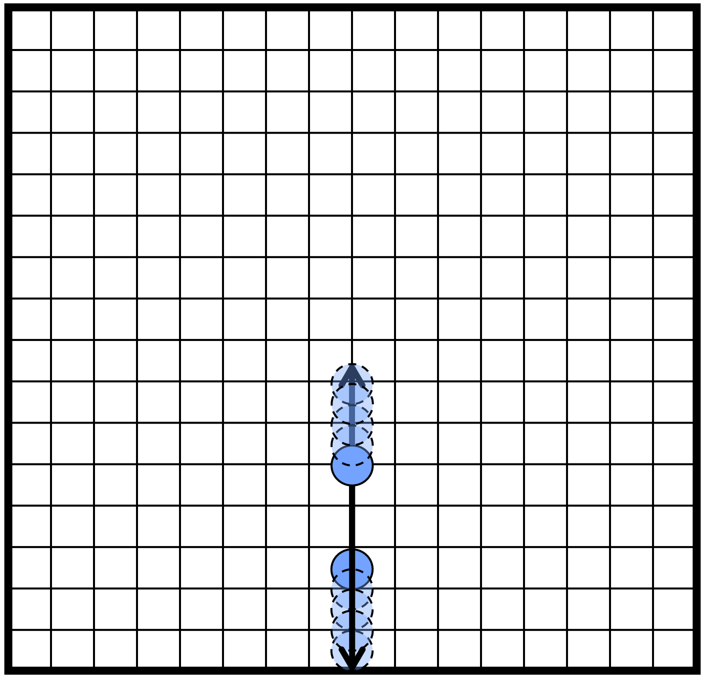


Illustration potential problem: spurious self-force

The particle moves up and down.



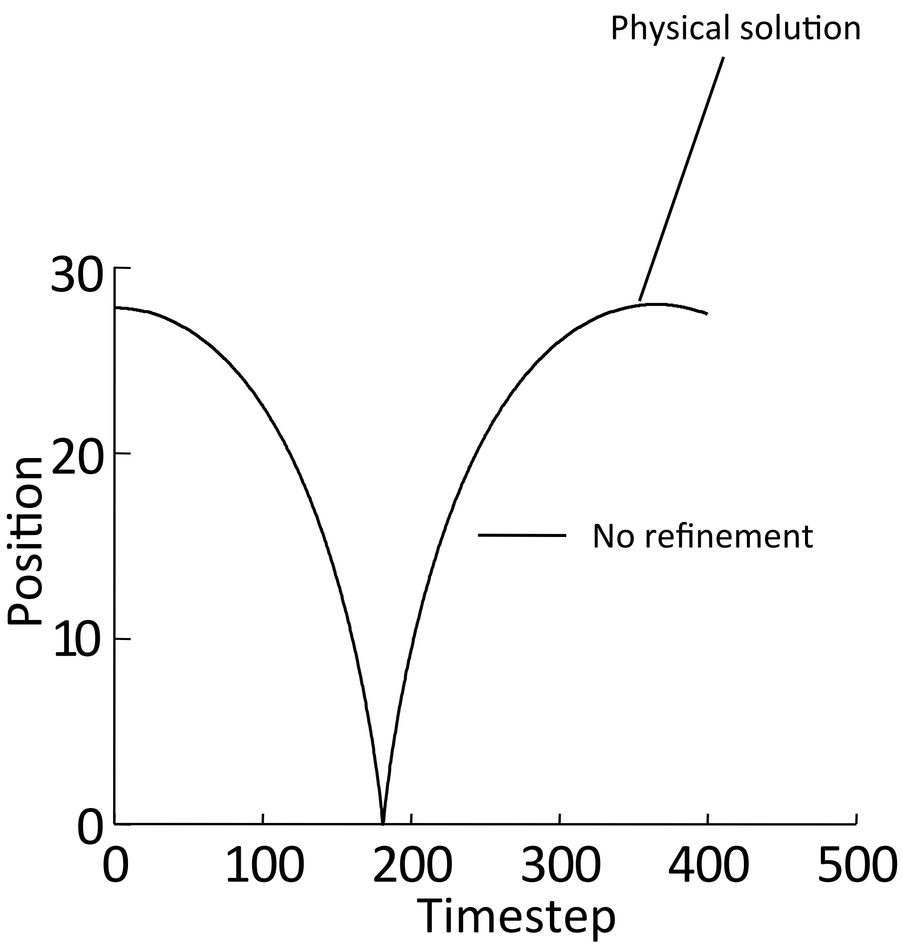
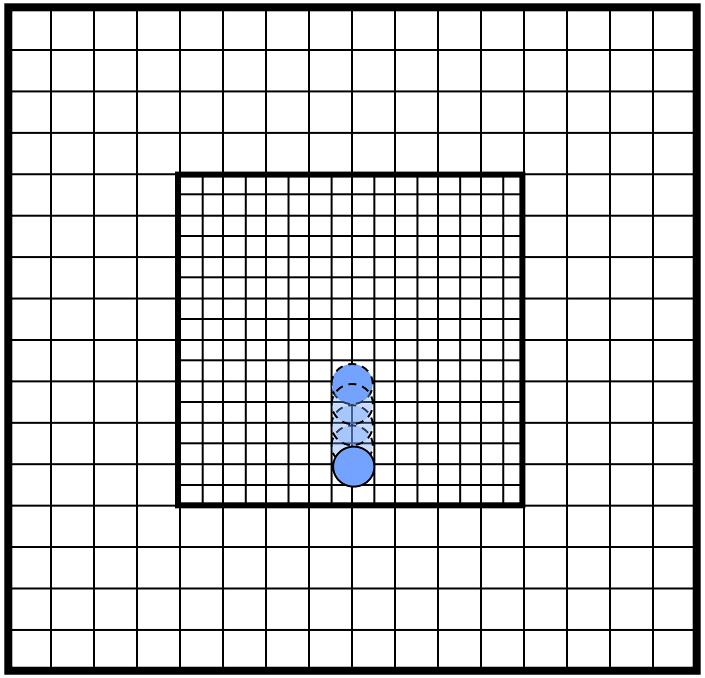
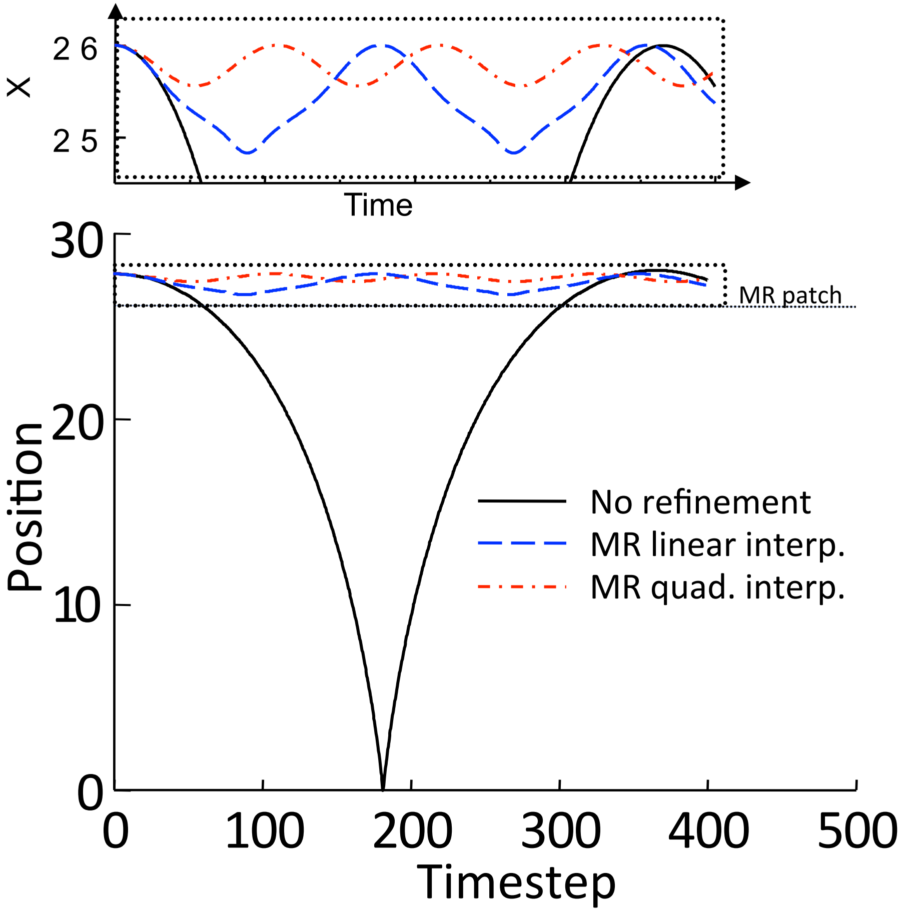


Illustration potential problem: spurious self-force

Now add a refinement patch.

➔ Particle is trapped in patch by "spurious self-force"

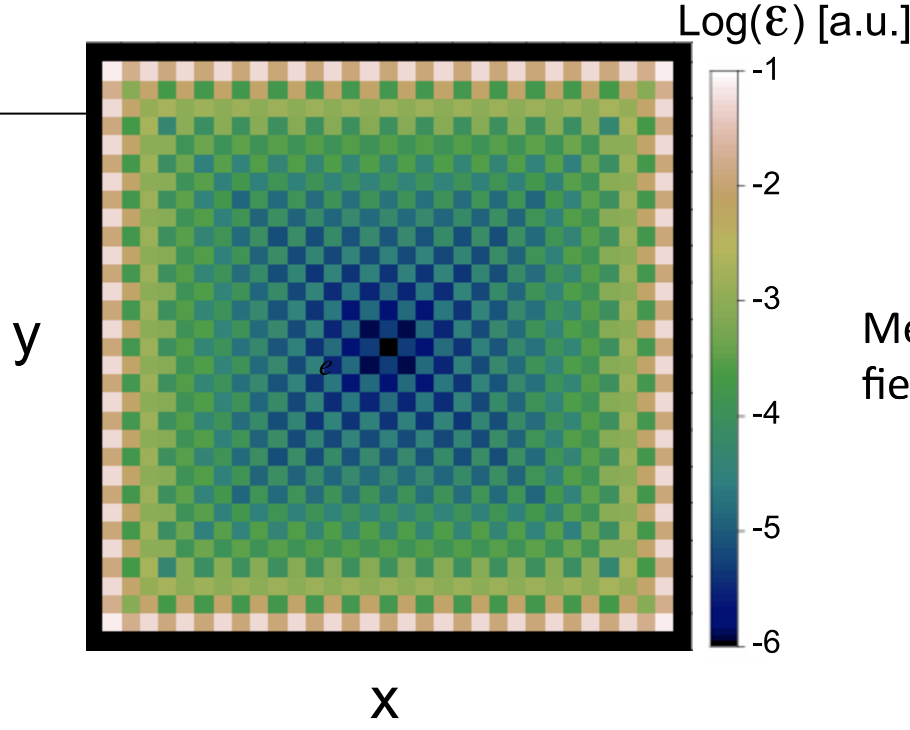




Spurious self-force: magnitude map

Map of spurious self-force as a function of particle position in refinement patch

32x32 refinement patch



$$\varepsilon = \sqrt{(E_x - E_x^{\text{ref}})^2 + (E_y - E_y^{\text{ref}})^2}$$

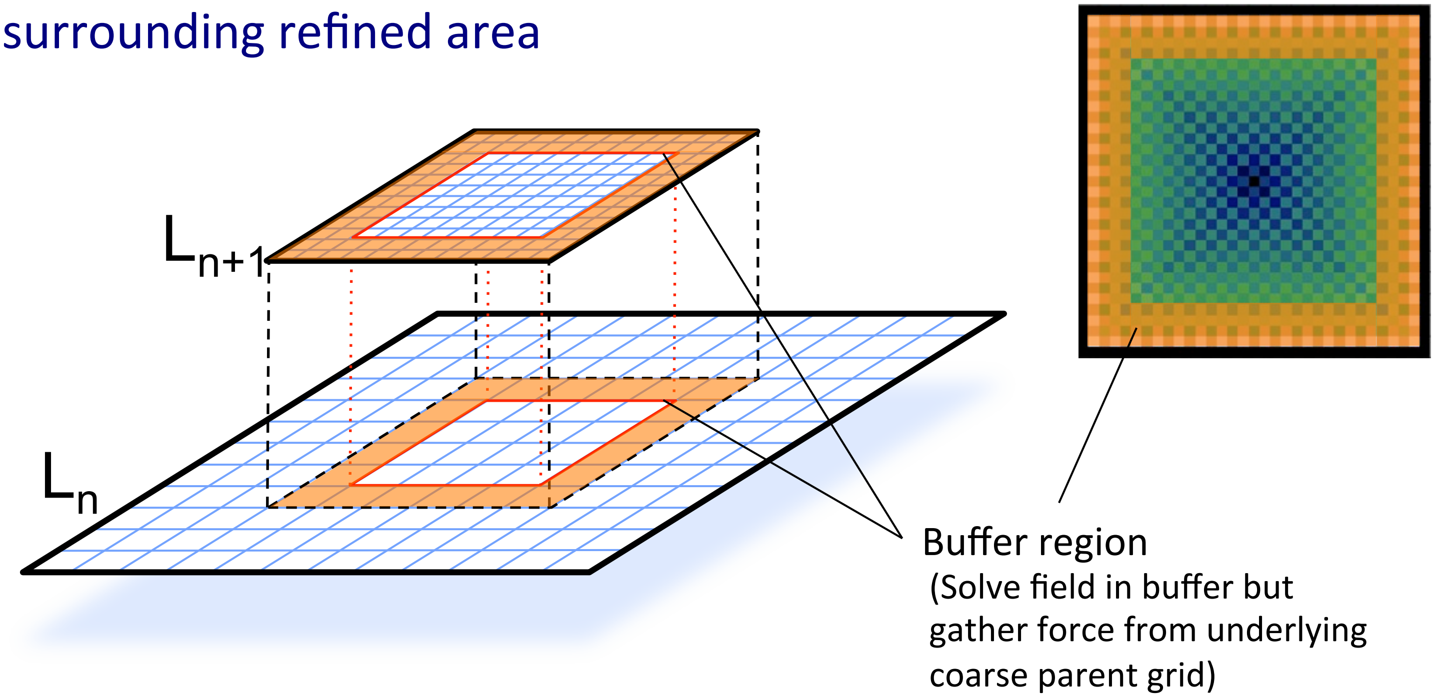
Mesh refinement field

Physical field

Spurious self-force decreases rapidly in patch

Spurious self-force: mitigation

Add buffer region surrounding refined area



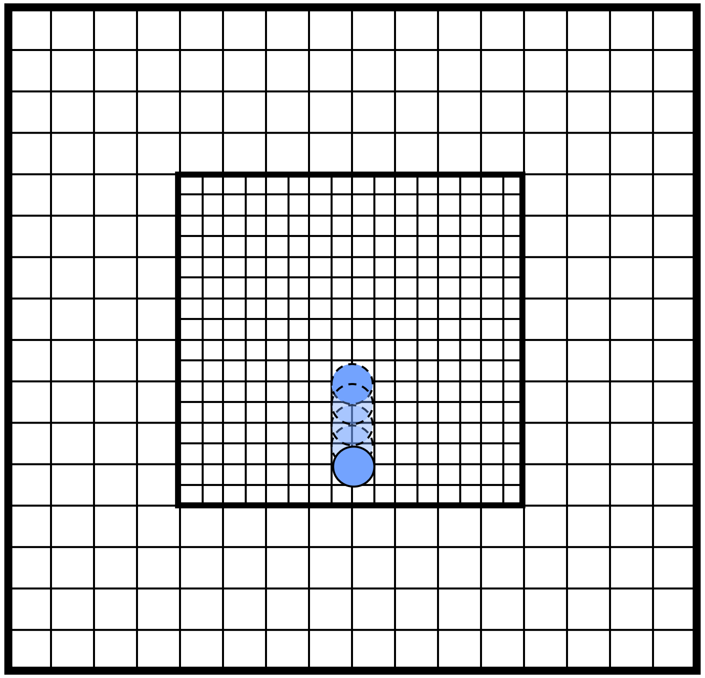
1 – solve on coarse grid,
2 – interpolate on fine grid boundaries,
3 – solve on fine grid,
4 – disregard fine grid solution close to edge when gathering force onto particles.

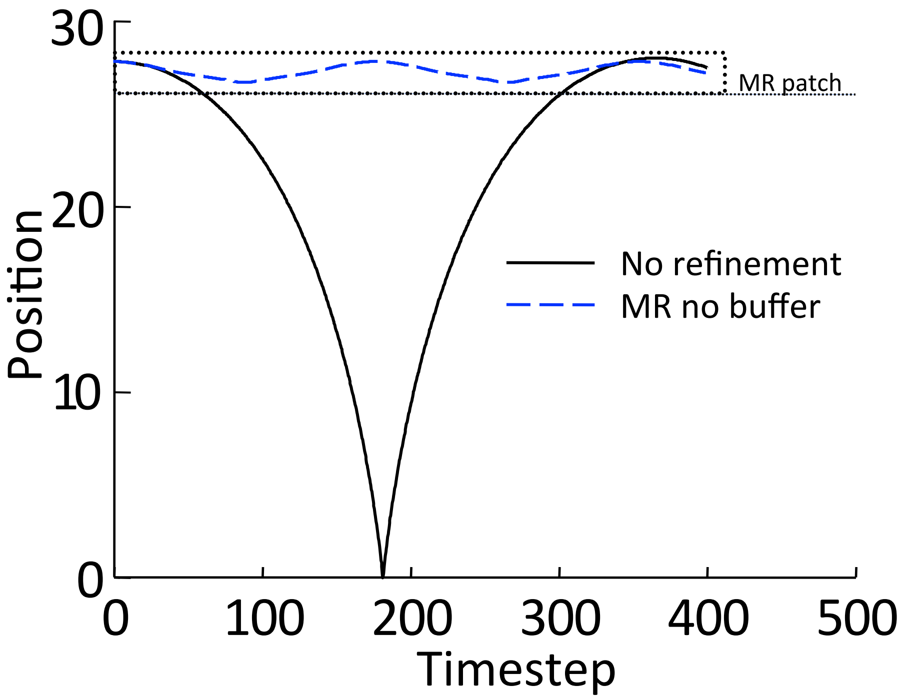
Thickness of buffer region provides user control of relative magnitude of spurious force.

Spurious self-force: mitigation

Example with 2 and 4 guard cells buffer region

No buffer: particle trapped in patch.

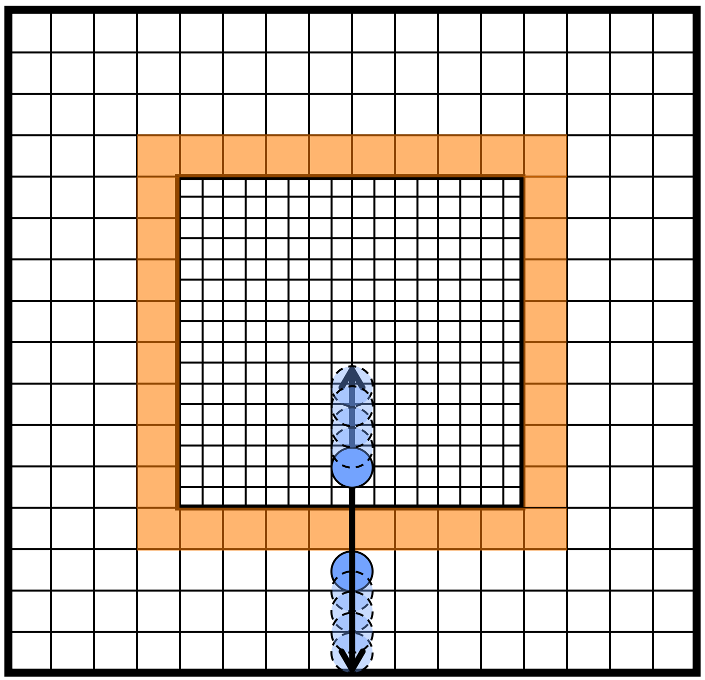


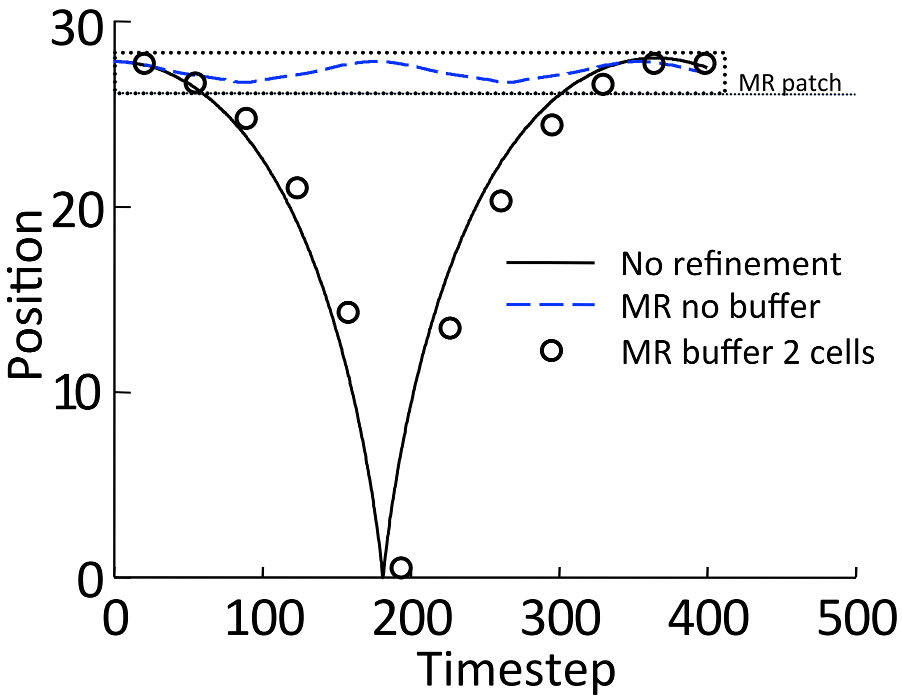


Spurious self-force: mitigation

Example with 2 and 4 guard cells buffer region

With buffer: no more trapping



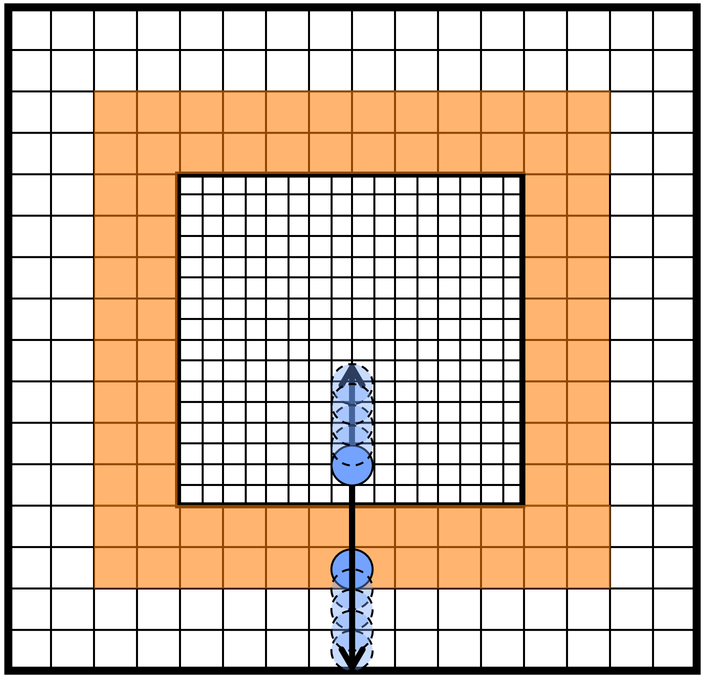


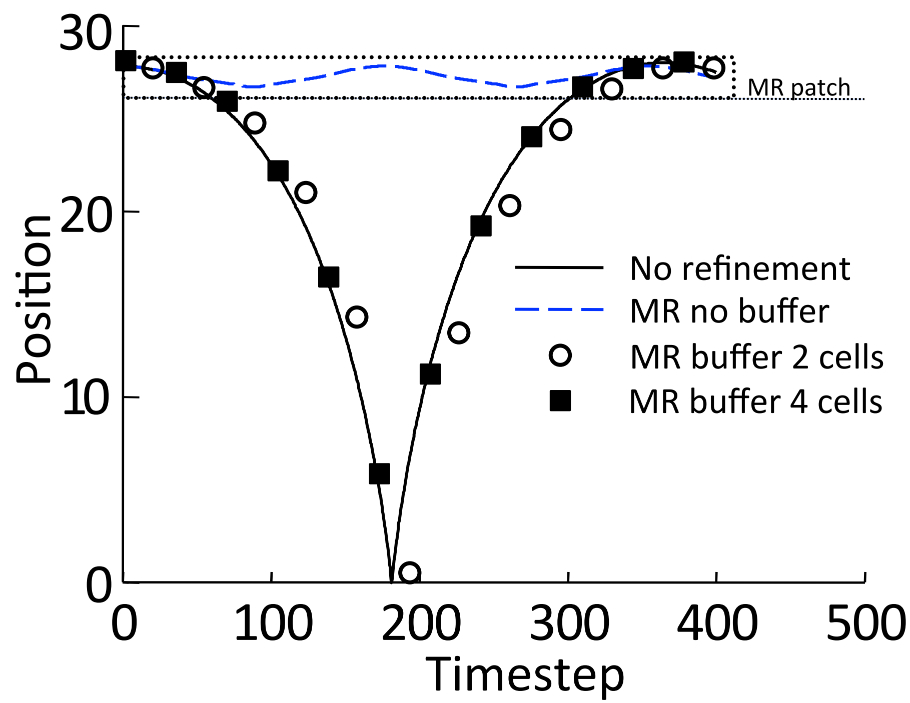
Spurious self-force: mitigation

Example with 2 and 4 guard cells buffer region

With buffer: no more trapping

4 guard cells better than 2

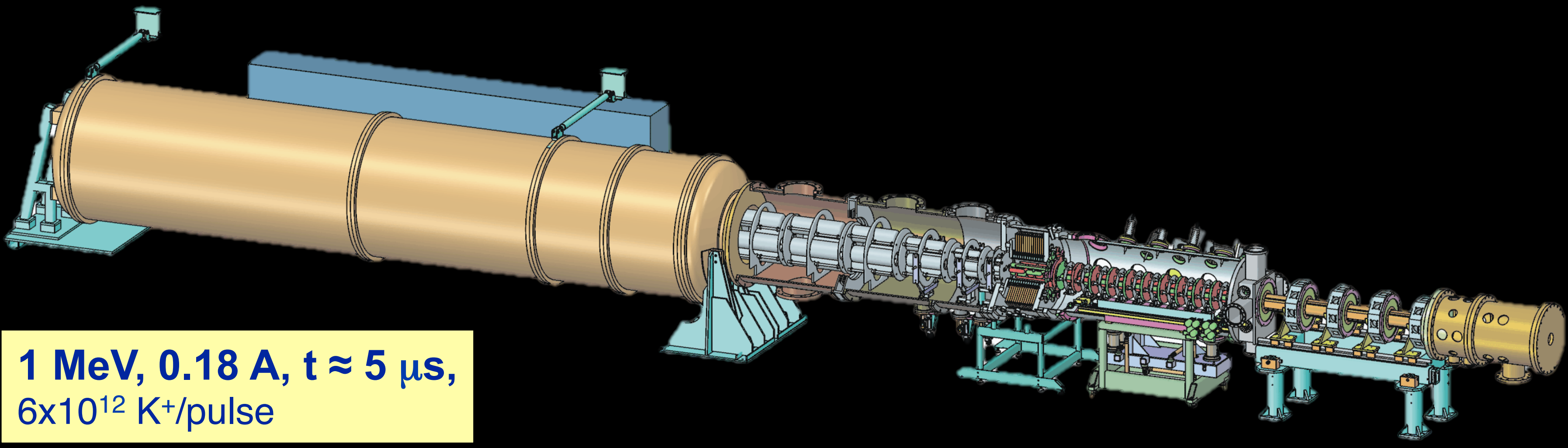




Buffer region is very effective.

Electrostatic AMR PIC example: HCX

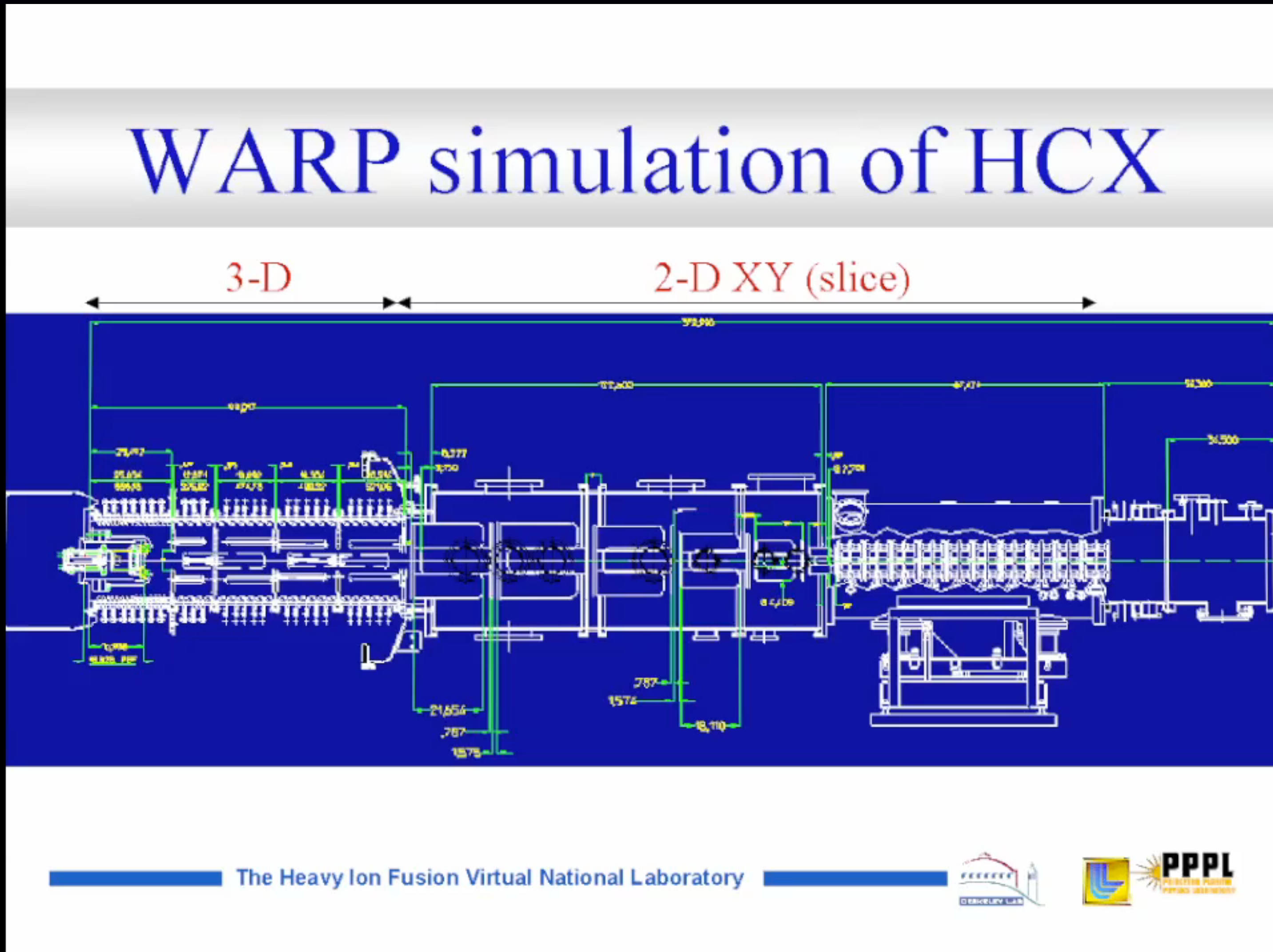
High Current Experiment
(High Brightness Beam Transport Campaign, 2005)



1 MeV, 0.18 A, t ≈ 5 μs,
$6x10^{12}$ K^{+}/pulse

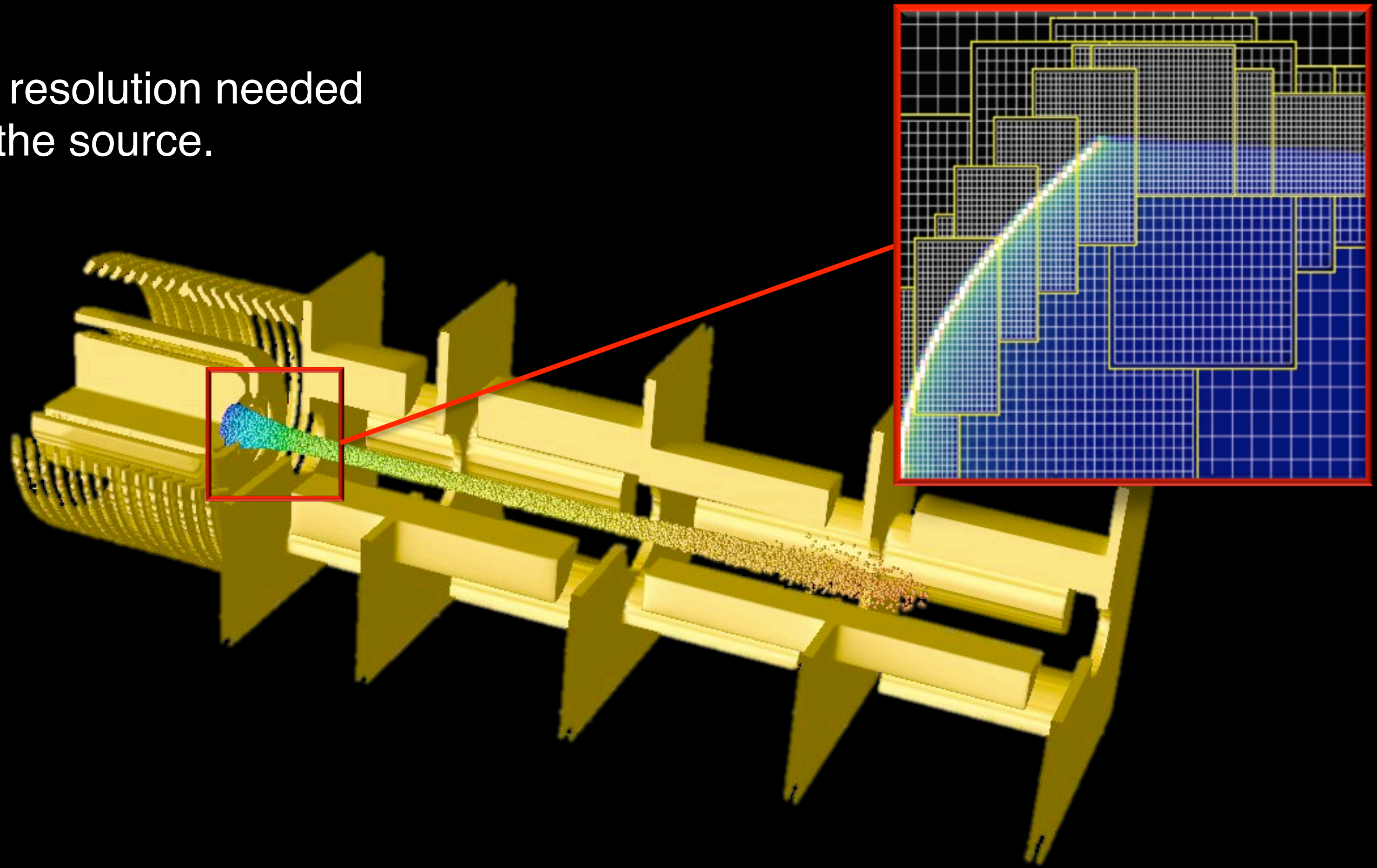
Heavy Ion Fusion program, LBNL

Electrostatic AMR PIC example: HCX



Electrostatic AMR PIC example: HCX

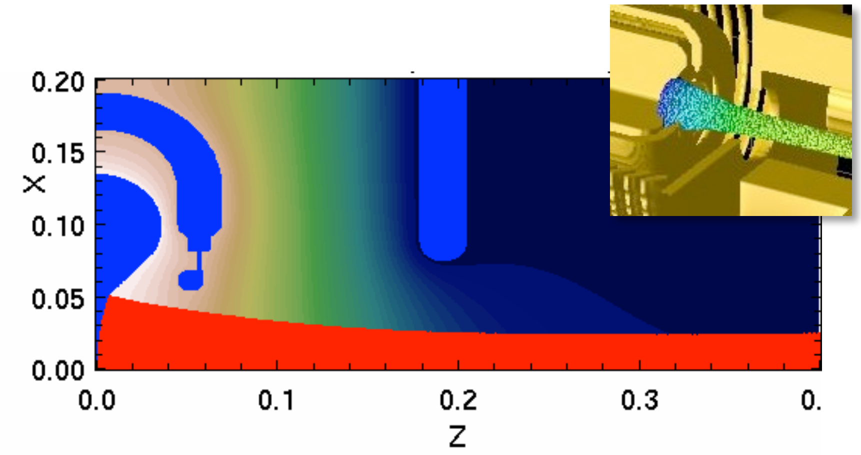
Very high resolution needed
to model the source.



Source region is axisymmetric and is well captured with RZ simulations.

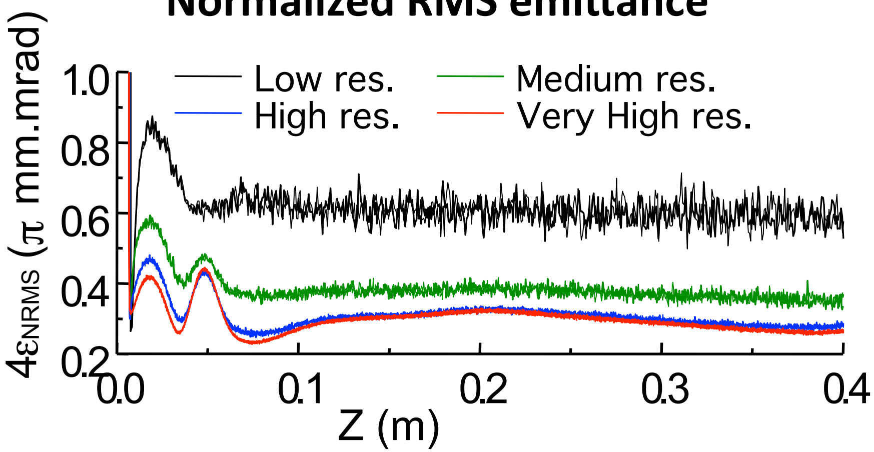
Modeling of source critical - determines initial shape of beam.

Axisymmetric (RZ) time-dependent simulations.



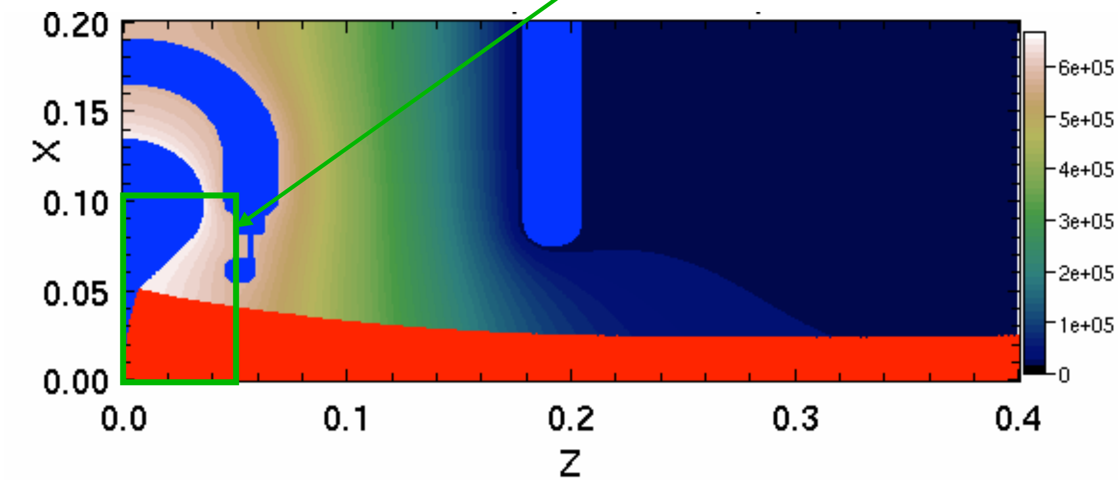
Run	Grid size	Nb particles
Low res.	56x640	~1M
Medium res.	112x1280	~4M
High res.	224x2560	~16M
Very High res.	448x5120	~64M

Normalized RMS emittance

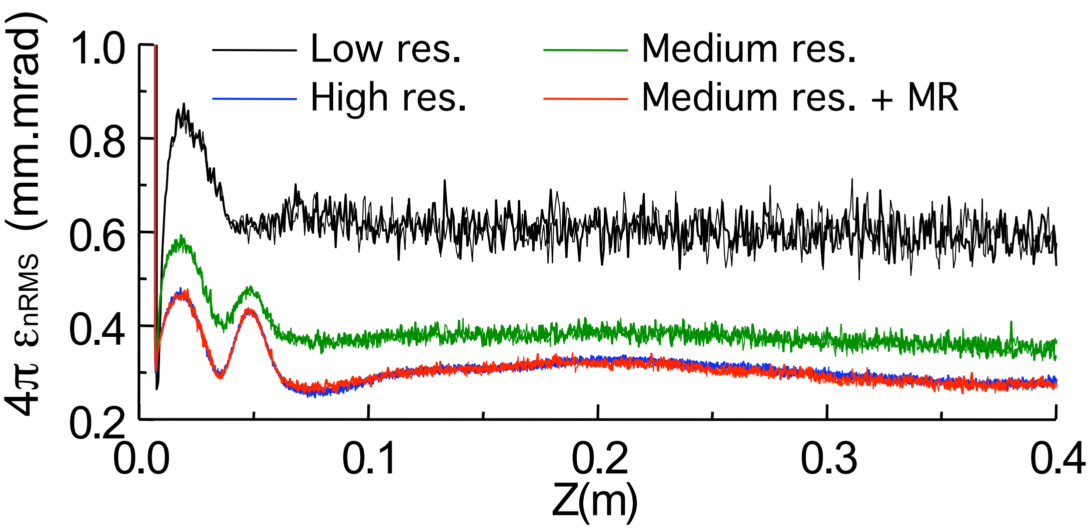


A fairly high resolution is needed to reach convergence

First MR attempt - 1 MR block surrounding emitter.



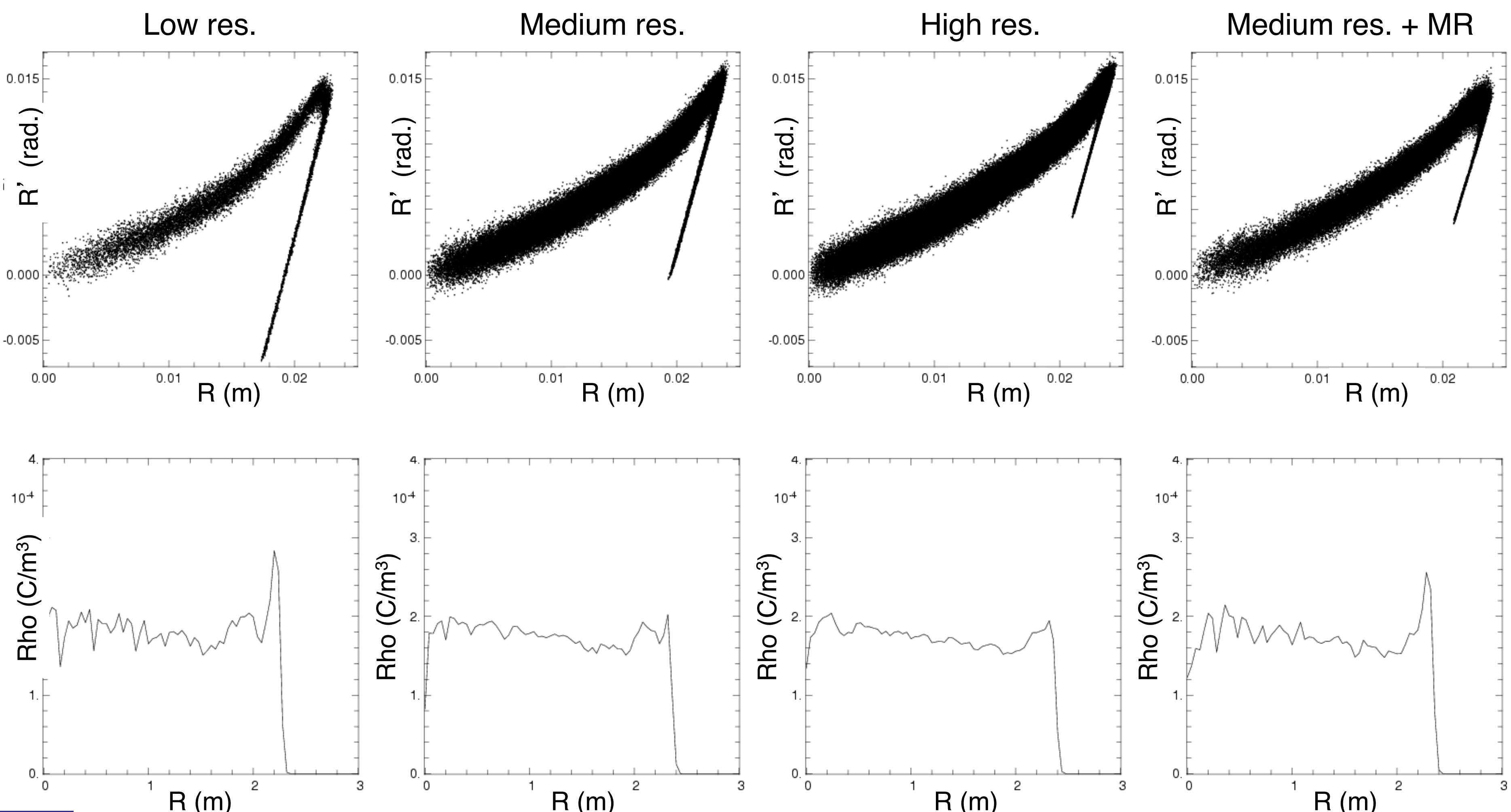
Refining around the emitter area is enough to recover emittance from converged high-resolution case.



Run	Grid size	Nb particles
Low res.	**56x640**	**~1M**
Medium res.	**112x1280**	**~4M**
High res.	**224x2560**	**~16M**
Medium res. + MR	**112x1280**	**~4M**

First MR attempt - 1 MR block surrounding emitter (2).

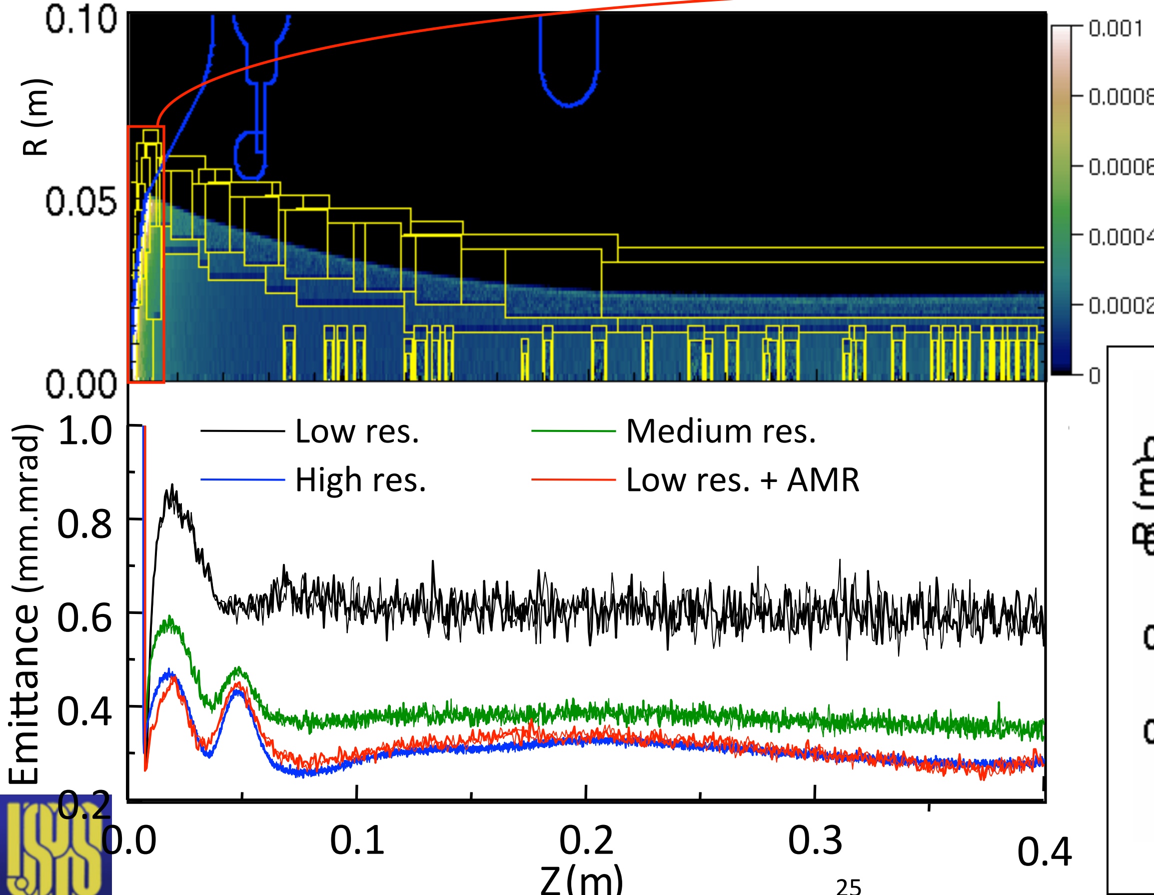
However, it is not enough for recovering details of distribution.

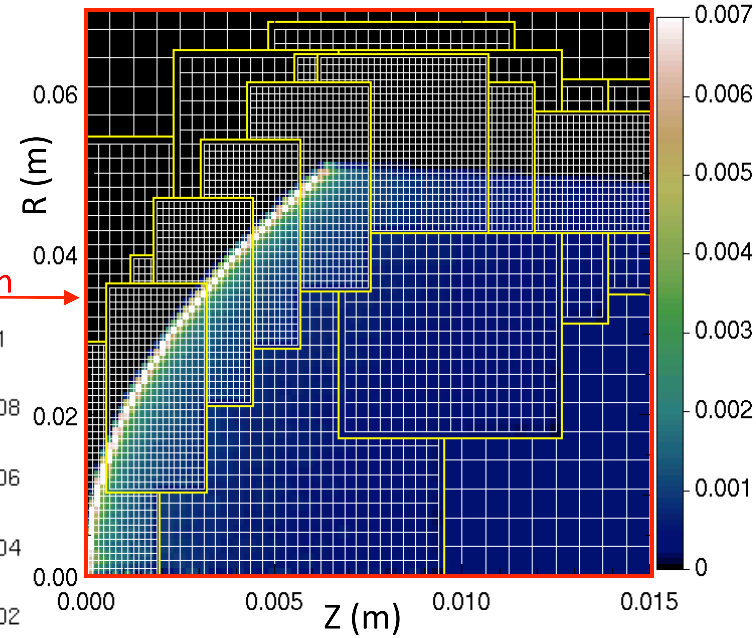


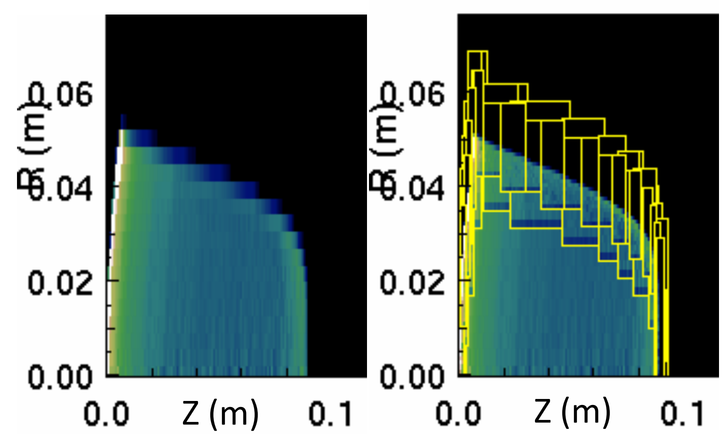
(plots from data at z=0.4m)

Full adaptive mesh refinement implementation
--speedup from AMR: x10

Run	Grid size	Nb particles
Low res.	56 x 640	~1M
Medium res.	112 x 1280	~4M
High res.	224 x 2560	~16M
Low res. + AMR	56 x 640	~1M





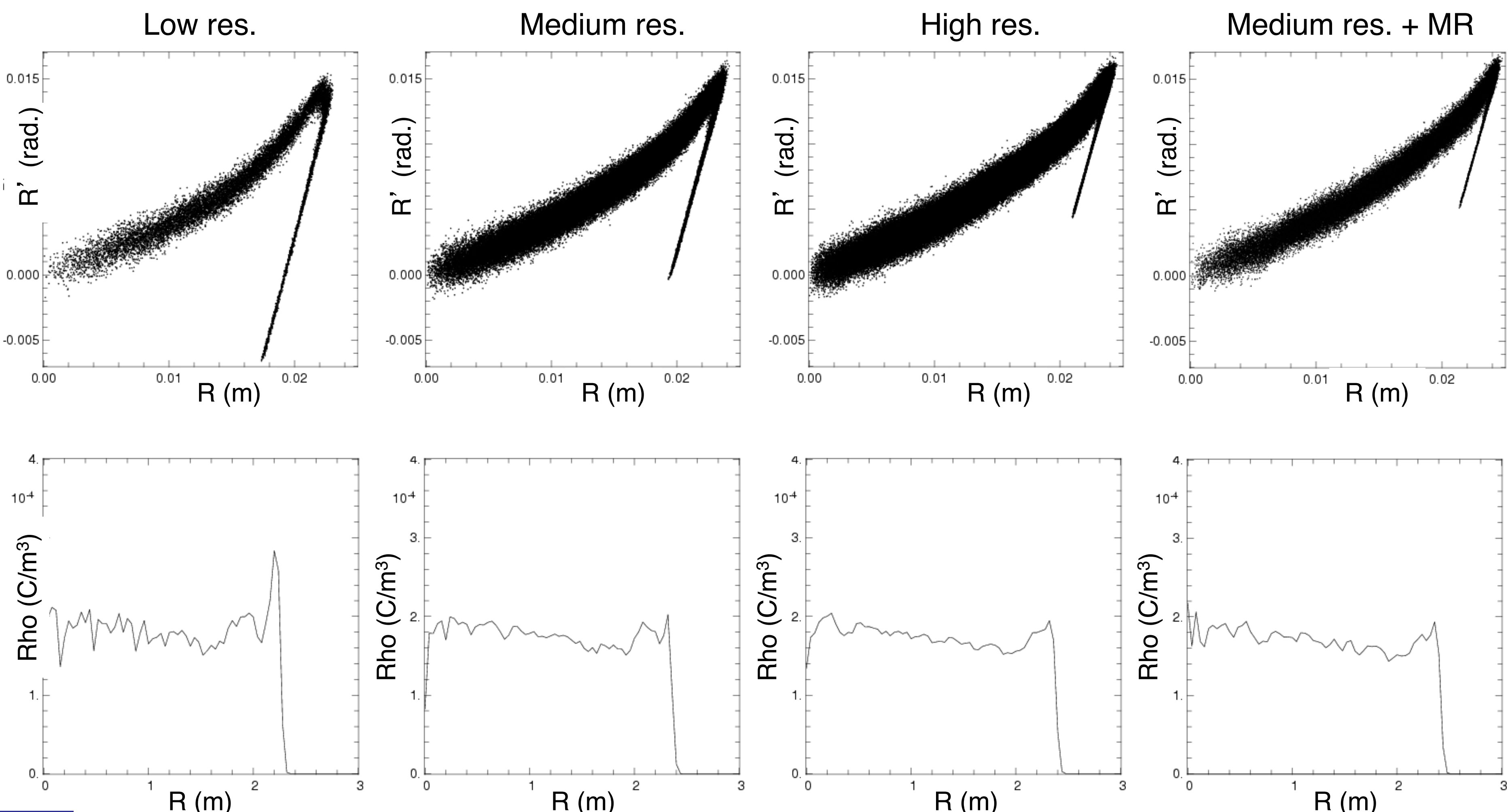


Automatic mesh refinement follows gradients: emitting area, beam edge and front.

Full adaptive mesh refinement implementation

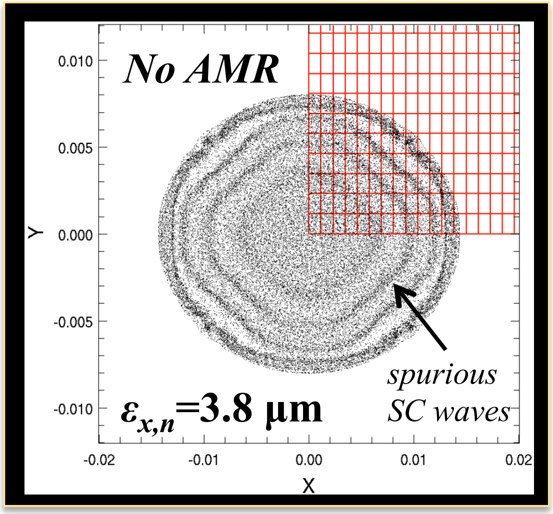
--speedup from AMR: x10

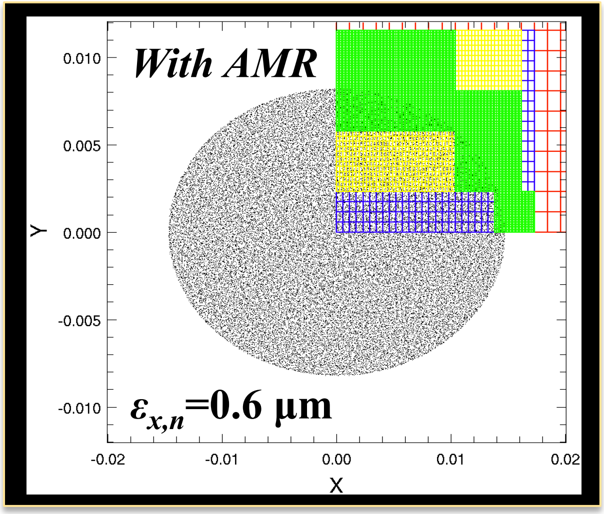
Full AMR enable recovery of details of distribution.



(plots from data at z=0.4m)

Example of AMR at edge of beam





Test using script testxy_amr.py:

- **Run with case='lowres', then 'highres' and 'AMR'.**
- **Observe how using AMR enables accurate simulation at reduced CPU cost.**

Summary of electrostatic AMR-PIC

- Simple method for electrostatic AMR-PIC was presented.
- Buffer region mitigates spurious self-force effect very effectively.
- Speedups of x10 demonstrated on simulation of injector.
- Alternate methods such as multipole expansions have other advantages/drawbacks.

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1-D FDTD EM wave equation

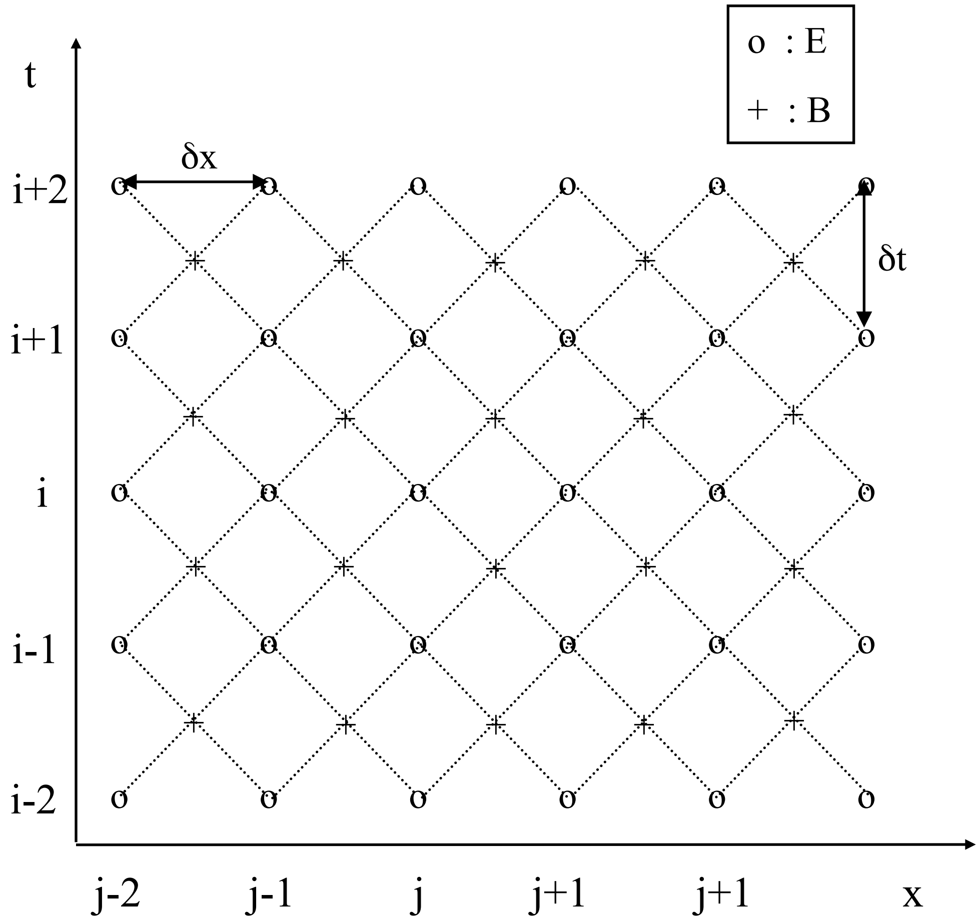
- We consider 1d wave equation (natural units)

$$\frac{\partial E}{\partial t} = \frac{\partial B}{\partial x}; \quad \frac{\partial B}{\partial t} = \frac{\partial E}{\partial x}$$

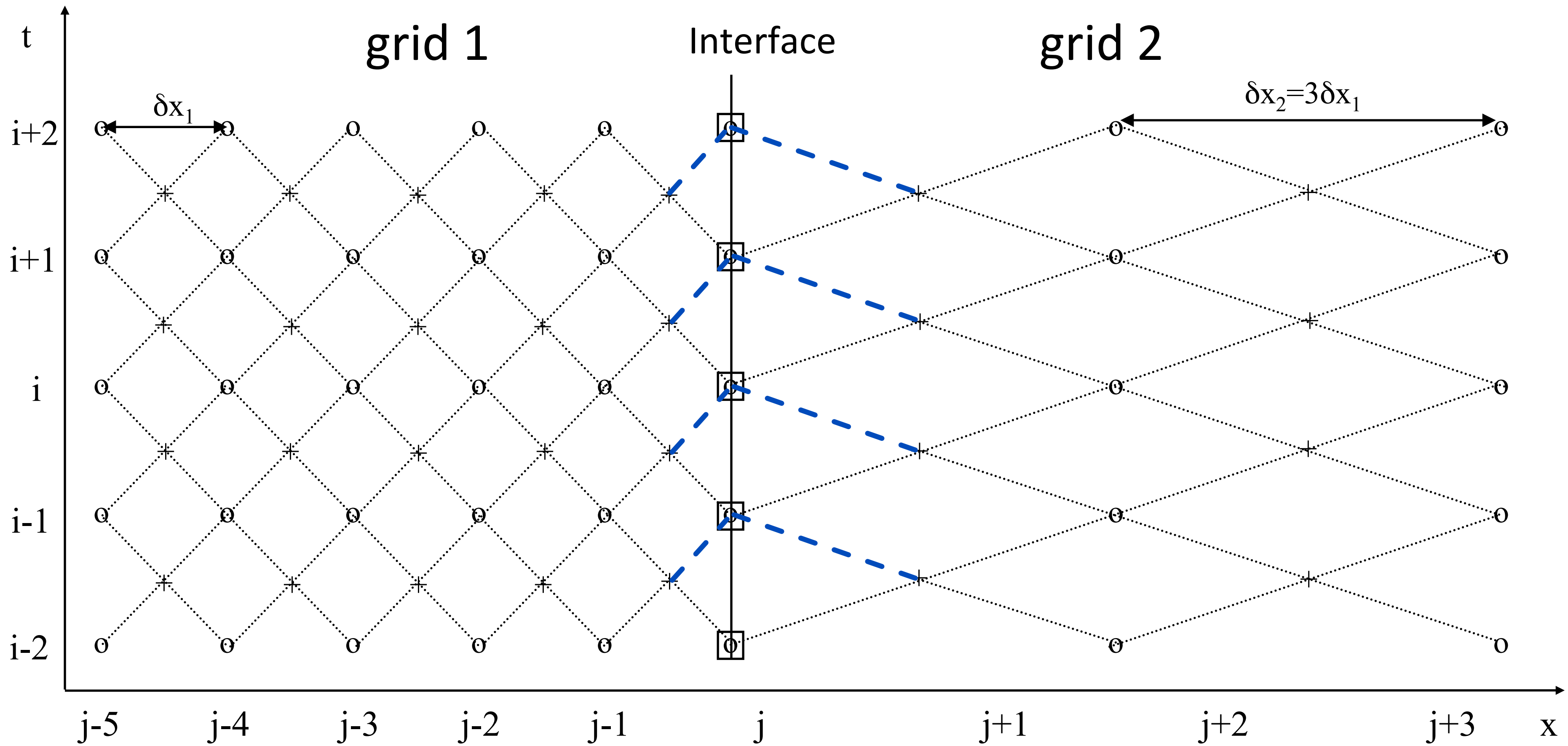
- staggered on a regular space time grid using finite-difference time-domain (FDTD) centered scheme

$$\frac{E_j^{i+1} - E_j^i}{\delta t} = \frac{B_{j+1/2}^{i+1/2} - B_{j-1/2}^{i+1/2}}{\delta x}$$

$$\frac{B_{j+1/2}^{i+1/2} - B_{j+1/2}^{i-1/2}}{\delta t} = \frac{E_{j+1}^i - E_j^i}{\delta x}$$



1-D MR-EM: space refinement
uncentered finite-difference



o, + : finite-difference at positions ≠j

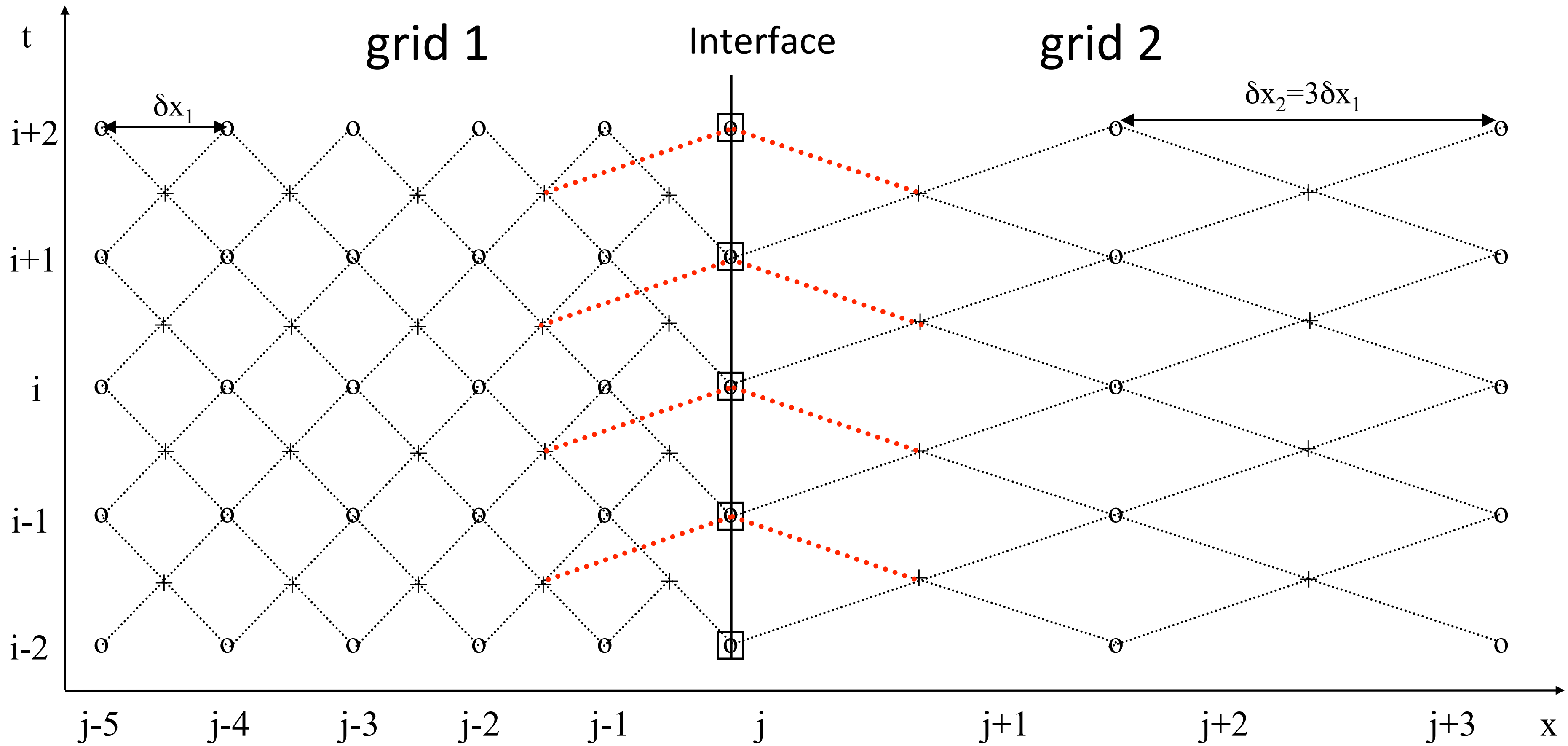
▣ : finite-volume (=uncentered FD) at j

$$\frac{E_j^{i+1} - E_j^i}{\delta t} = 2\frac{B_{j+1/2}^{i+1/2} - B_{j-1/2}^{i+1/2}}{\delta x_1 + \delta x_2} \quad \text{(method 1)}$$

or

$$\frac{E_j^{i+1} - E_j^i}{\delta t} = \frac{B_{j+1/2}^{i+1/2}}{\delta x_2} - \frac{B_{j-1/2}^{i+1/2}}{\delta x_1} \quad \text{(method 2)}$$

1-D MR-EM: space refinement
centered finite-difference



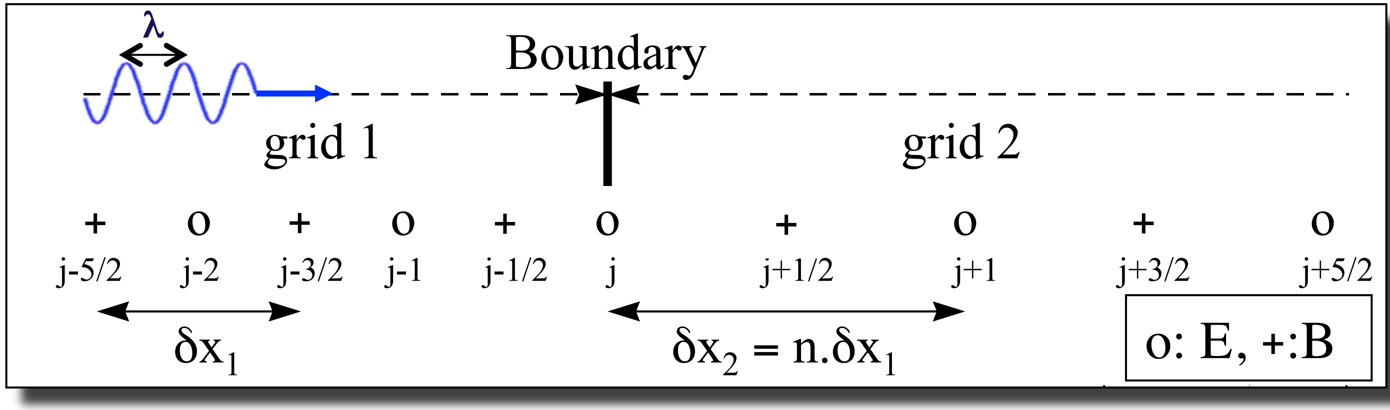
o, + : finite-difference at positions ≠j

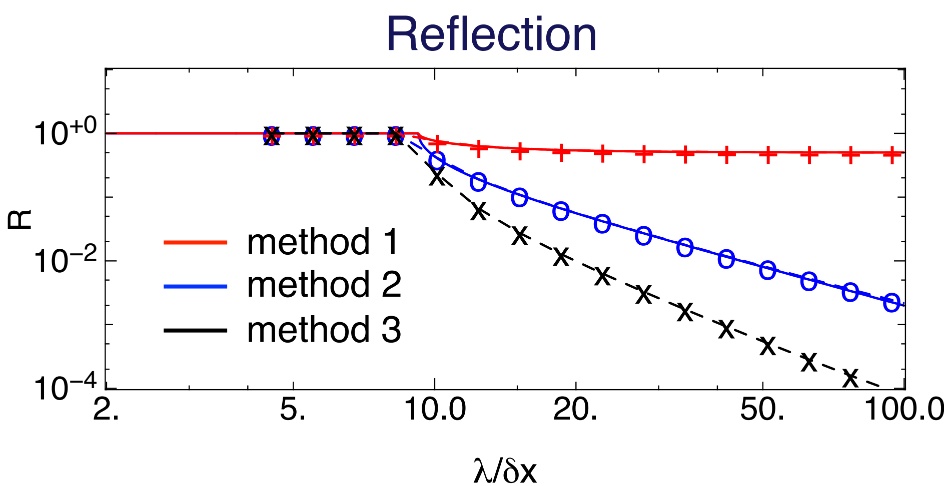
▣ : 'jump' inside fine grid at j

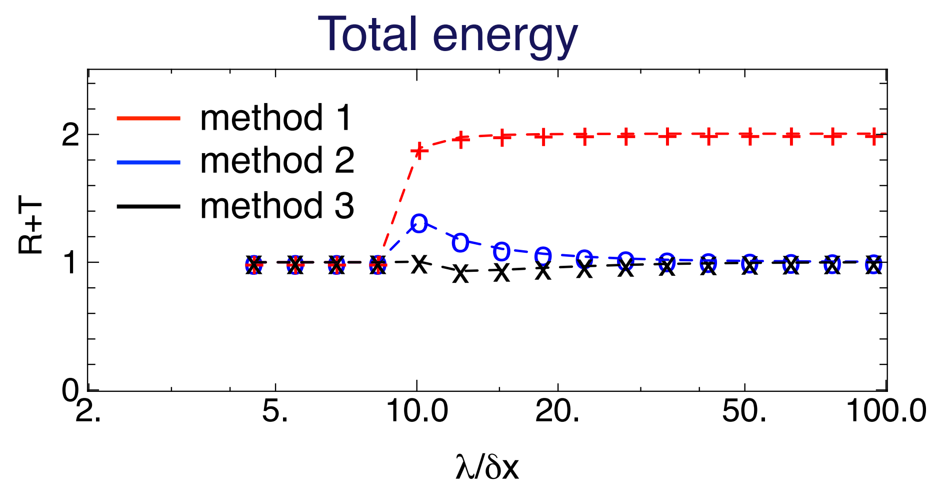
$$\frac{E_j^{i+1} - E_j^{i}}{\delta t} = \frac{B_{j+1/2}^{i+1/2} - B_{j-n/2}^{i+1/2}}{\delta x_2}$$ (method 3)

1-D MR-EM: coefficients of spurious reflection

Test to measure spurious reflection R at interface at j of signal injected on fine grid.





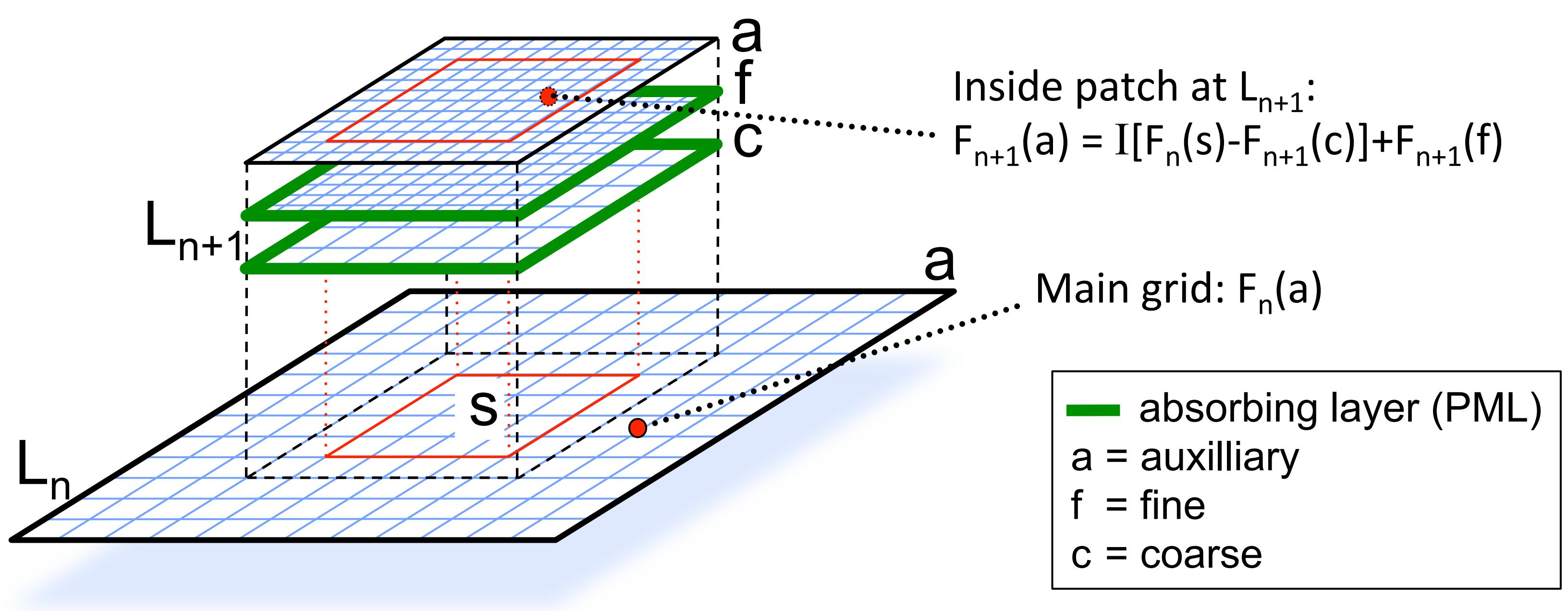


$\lambda \leq \lambda_{Nyquist}$ of coarse grid are reflected with amplification of total energy!

Warp's Electromagnetic MR uses PML and substitution to prevent reflections

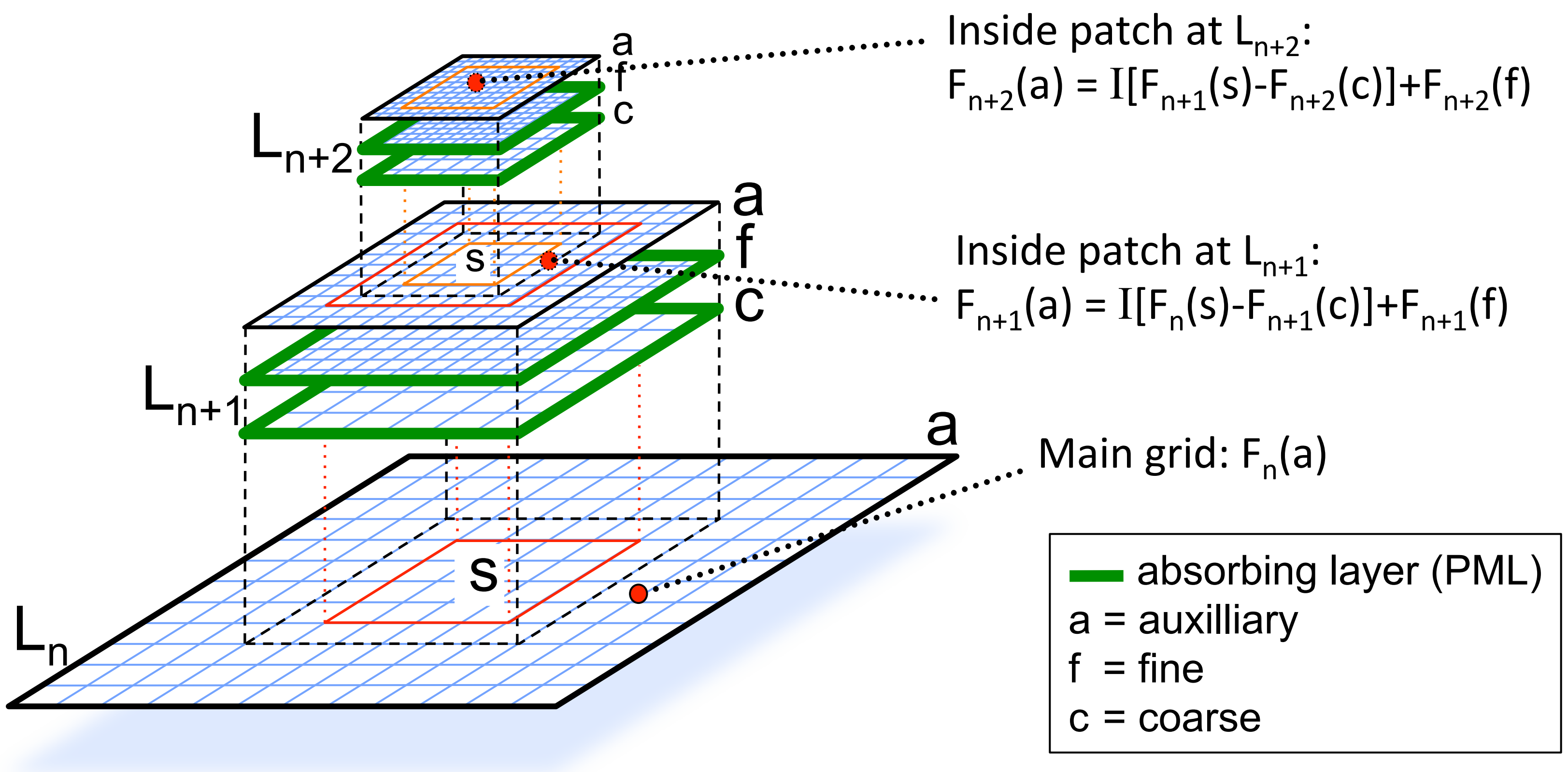
Warp's electromagnetic MR solver

- Termination of patches with Perfectly Matched Layers (PML) to avoid spurious reflections
- Buffer zone used for mitigating spurious self-force

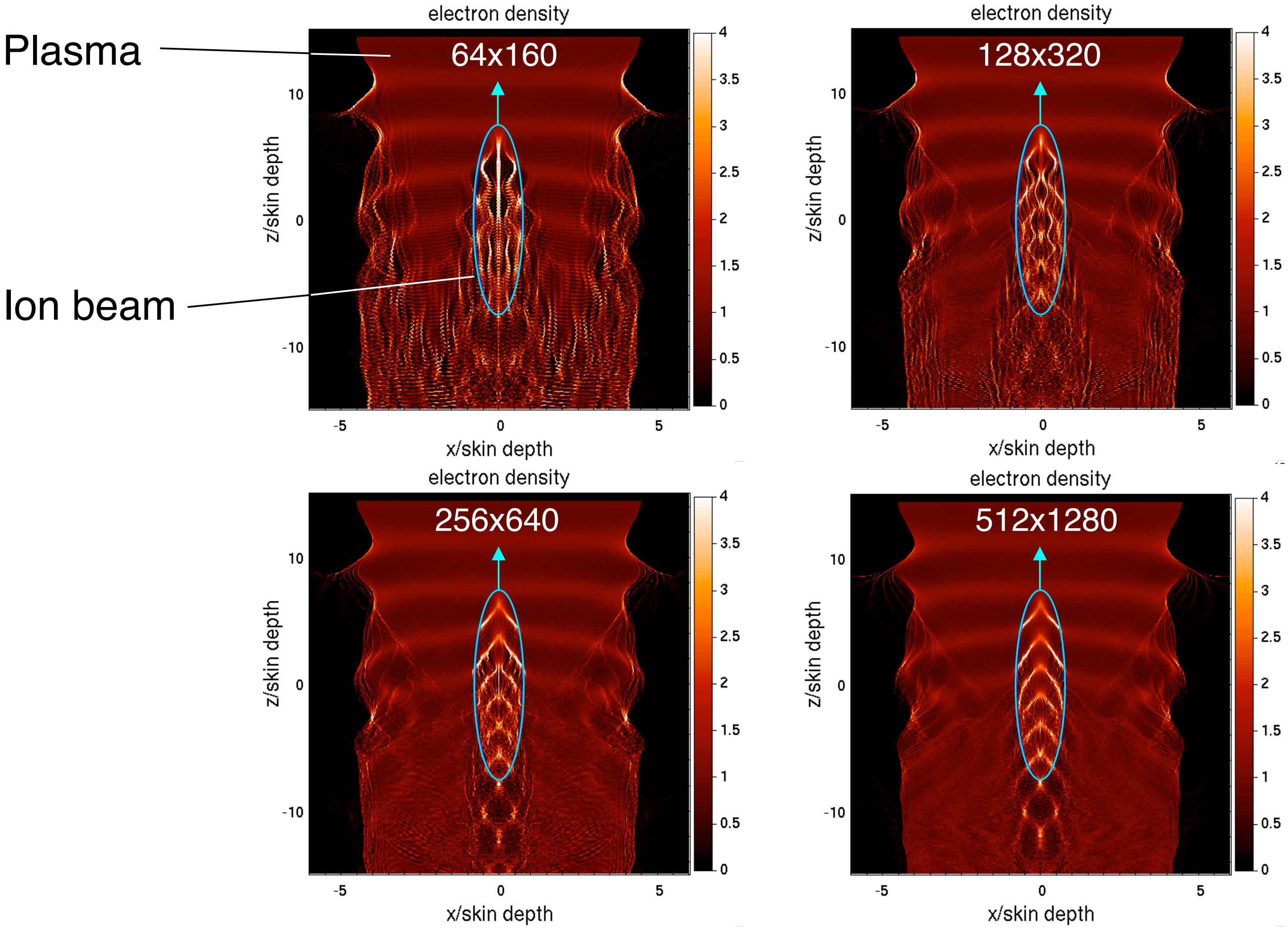


MR procedure is recursive, accommodating an arbitrary number of levels

Example with two levels of refinement



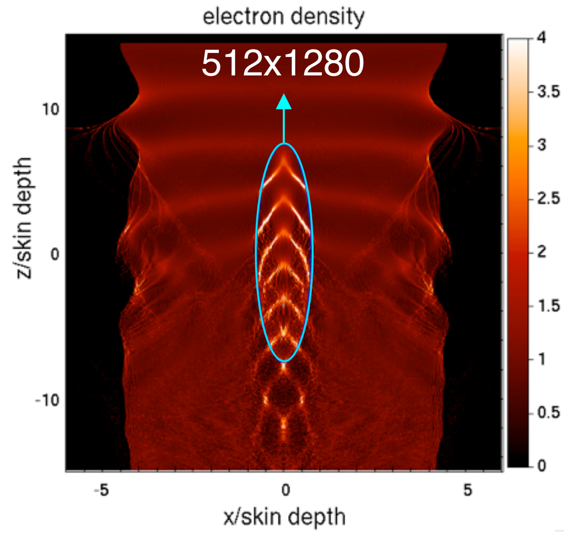
Example: simulation of beam-induced plasma wake

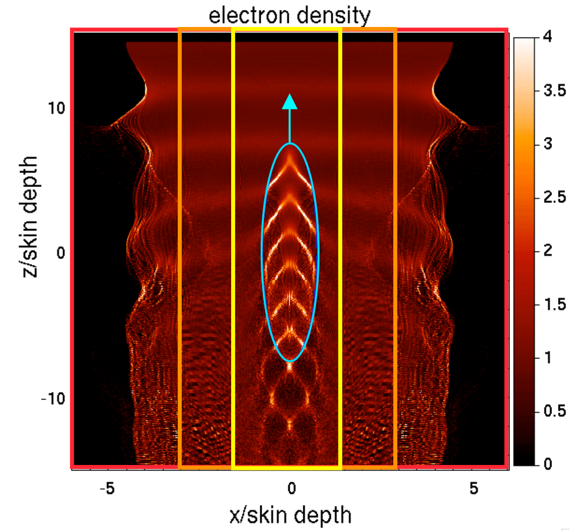


Slab XZ simulations

High resolution is needed to capture details.

Example: simulation of beam-induced plasma wake



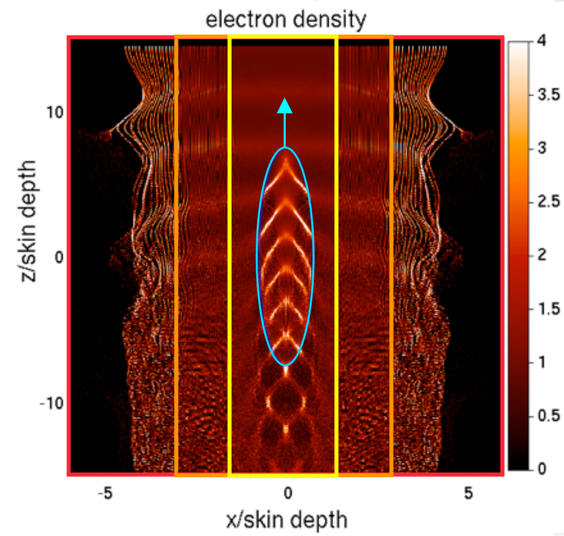


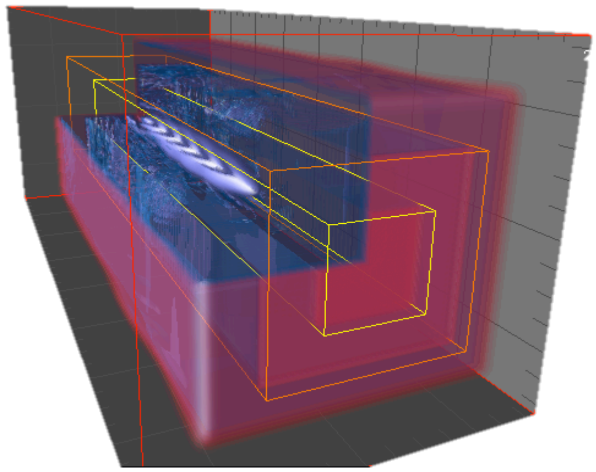
Mesh ref.

- 2 levels
- fields only

low resolution + MR

- 2 levels
- fields+ particles





3-D

Speedup x10 in 3D (using the same time steps for all refinement levels).

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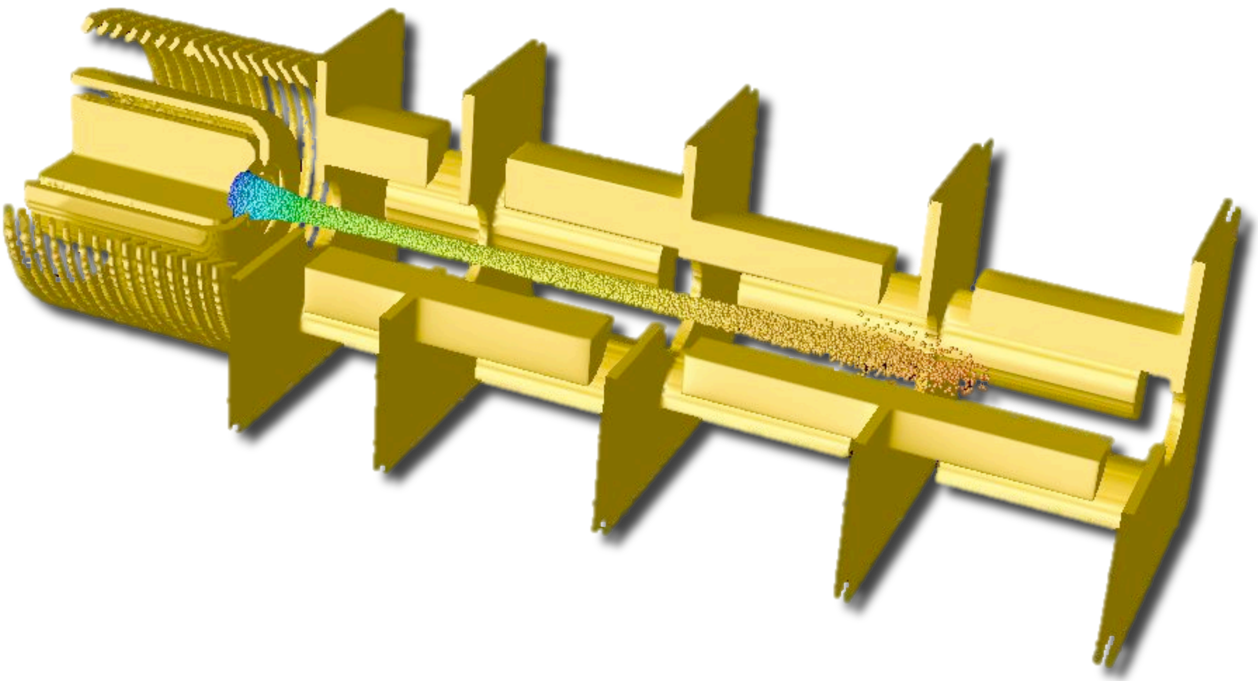
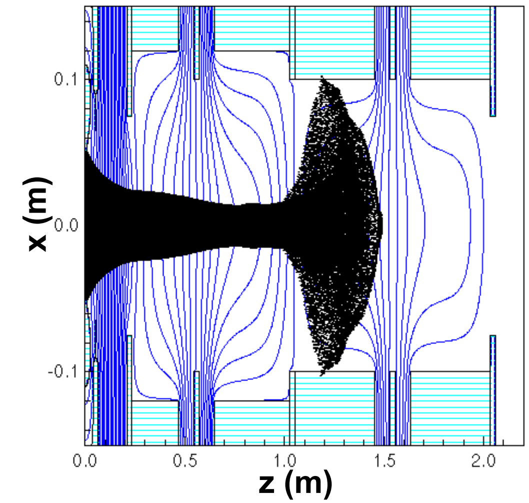
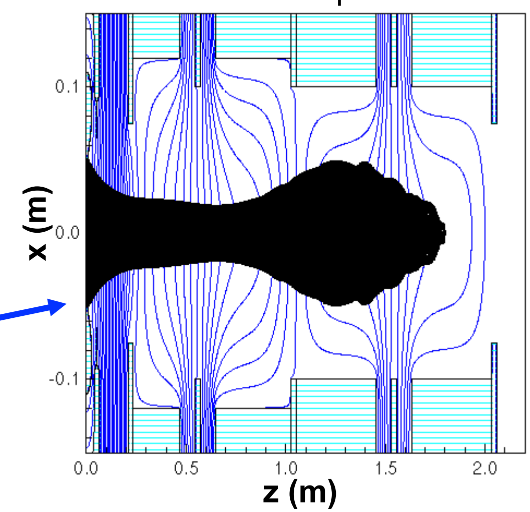
3-D WARP simulation of HCX showed beam head scrapping



beam head particle loss < 0.1%

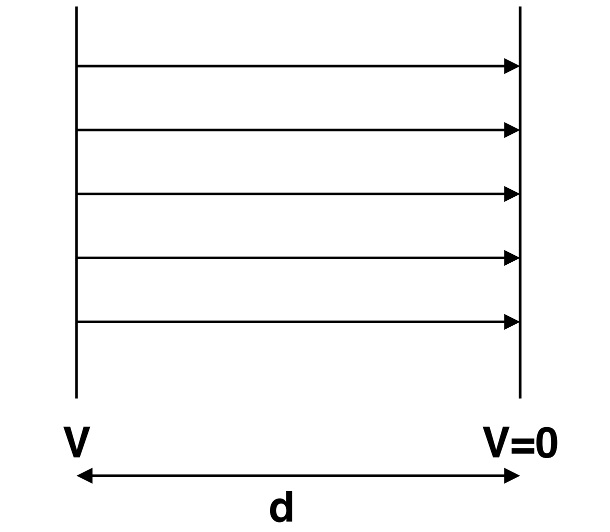


Rise-time τ = 400 ns
zero beam head particle loss

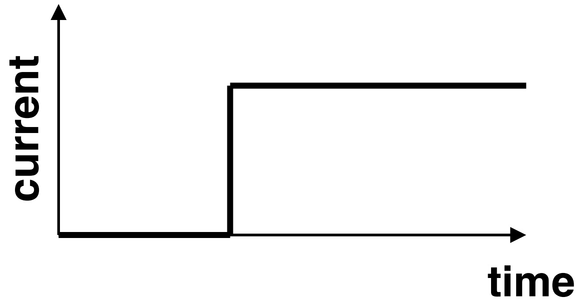


- Head cleaner with shorter voltage rise-time.
- Questions:
  - what is the optimal rise-time?
  - can we produce and model very-fast rise-time?

Test: 1-D time-dependent modeling of ion diode



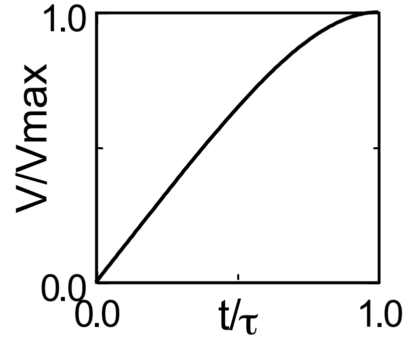
Applied voltage for Heavyside current history?

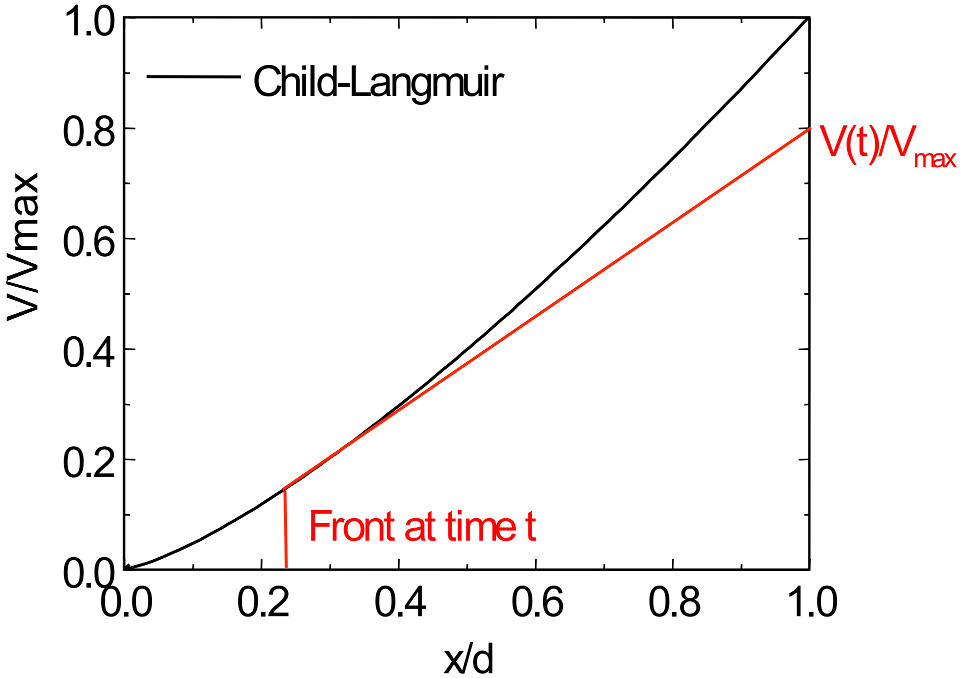


Analytic solution from Lampel-Tiefenback

$$V(t) = \frac{t}{3\tau}\left[4 - \left(\frac{t}{\tau}\right)^3\right]V_{\max}$$

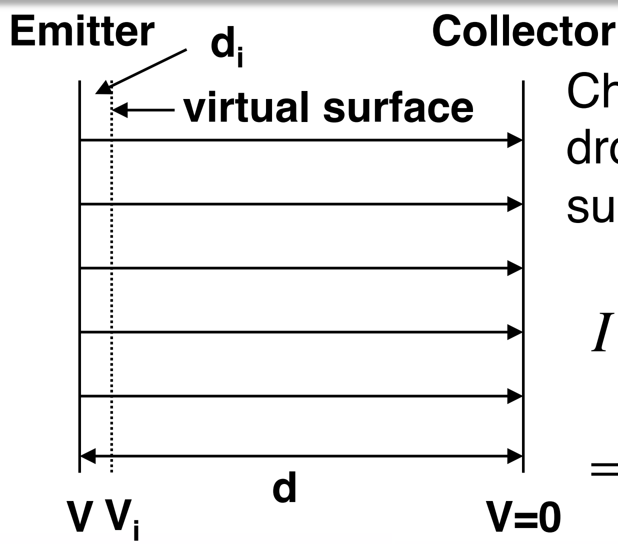
(τ: transit time)





Test: 1D time-dependent modeling of ion diode (algo 1)

Injection algorithm

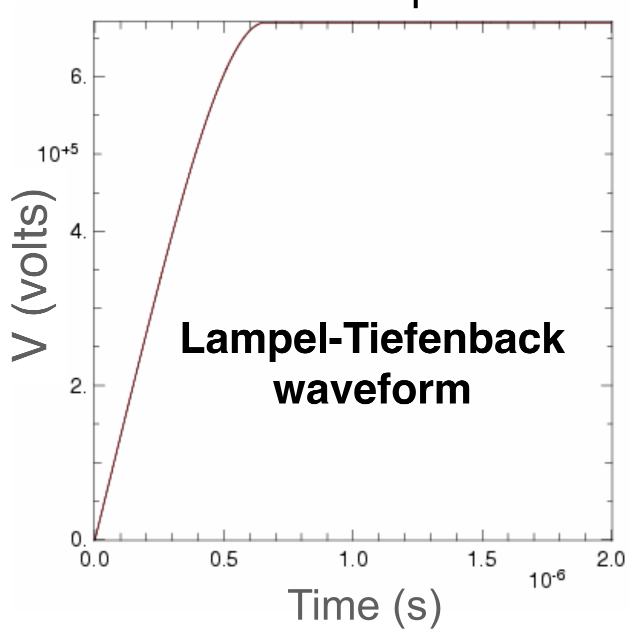


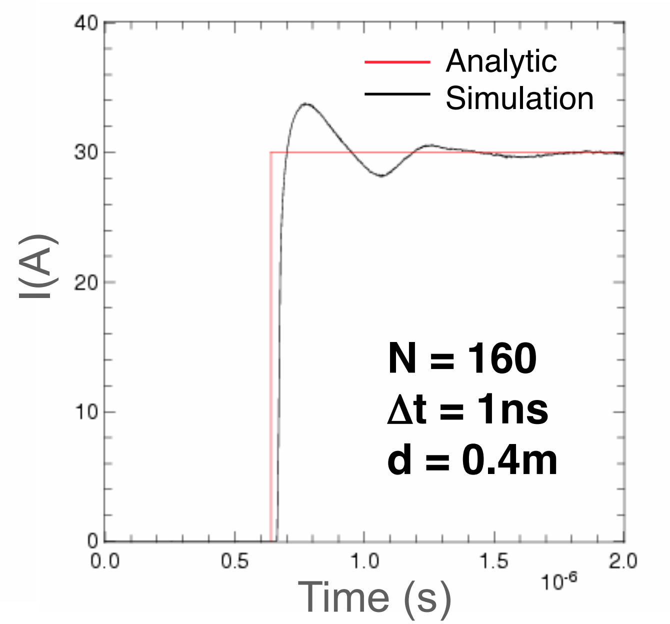
Child-Langmuir solution* + voltage drop between emitter and virtual surface determines current to inject.

$$I = \chi \frac{(V - V_i)^{3/2}}{d_i^2}; \ \chi = \frac{4}{9}\varepsilon_0 \sqrt{\frac{2q}{m}}$$

$$\Rightarrow \Delta Q = Nq = I\Delta t$$

Result

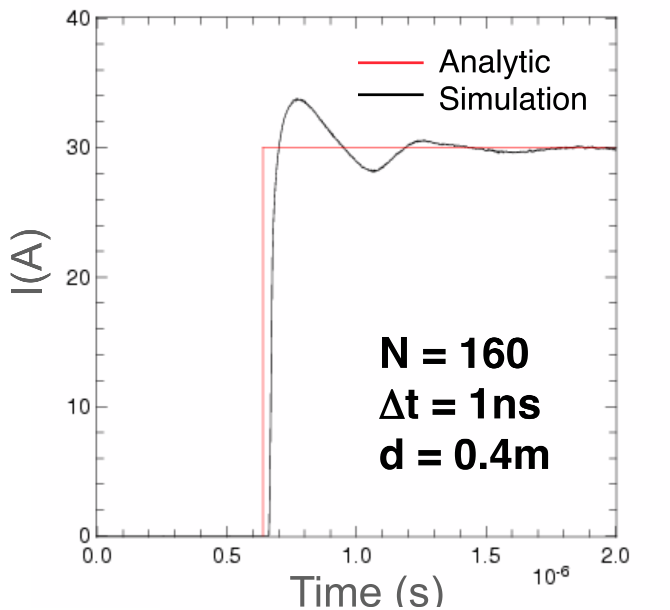


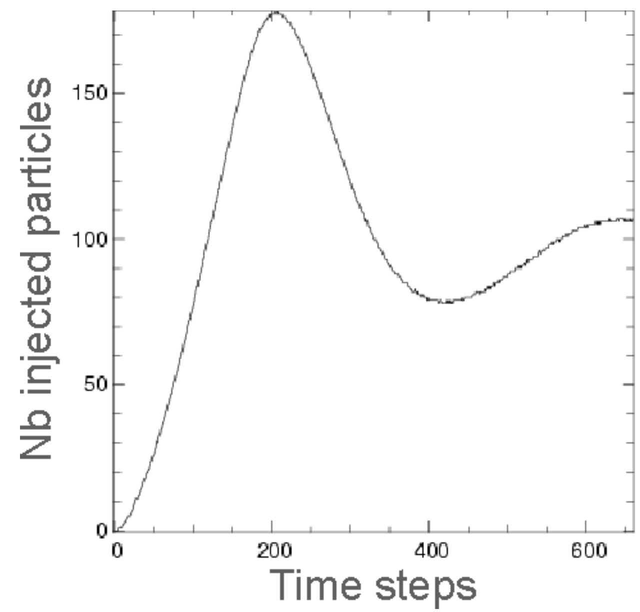


Simulation result exhibits large unphysical oscillation.

*1-D; => J≡I (J=I/S, S=1)

Unphysical oscillation related to Nb particles injected/time step (N_i)

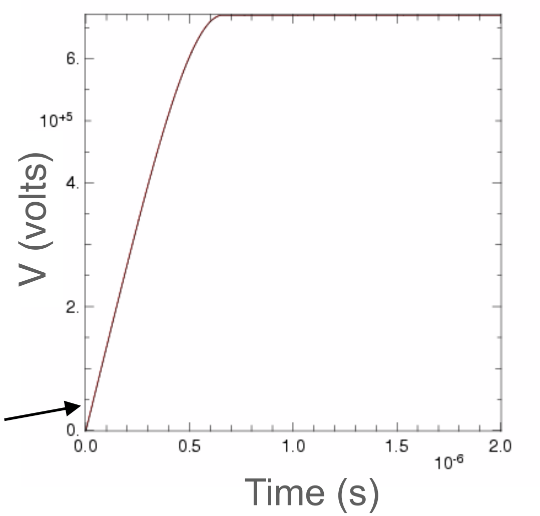




Ideally,

$$\frac{N_i}{\Delta t} = \chi \frac{(V - V_i)^{3/2}}{q d_i^2} = Cste$$

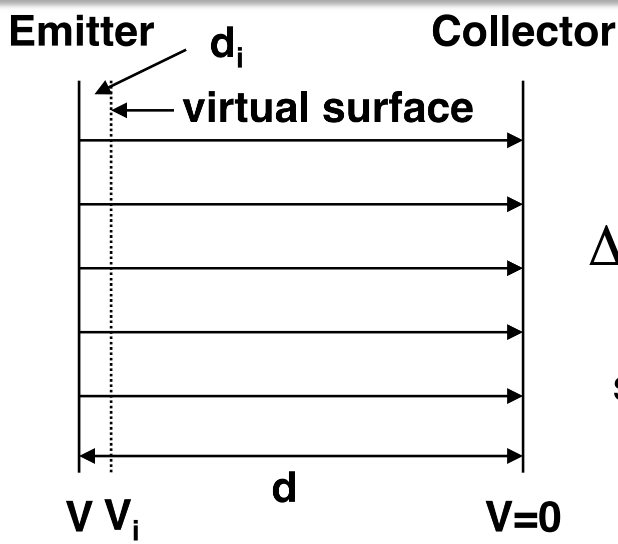
but the driving voltage is a continous function derived analytically.



⇒ Inconsistency due to infinitesimal solution applied in discrete world.

Cure: derive voltage history numerically

Injection algorithm



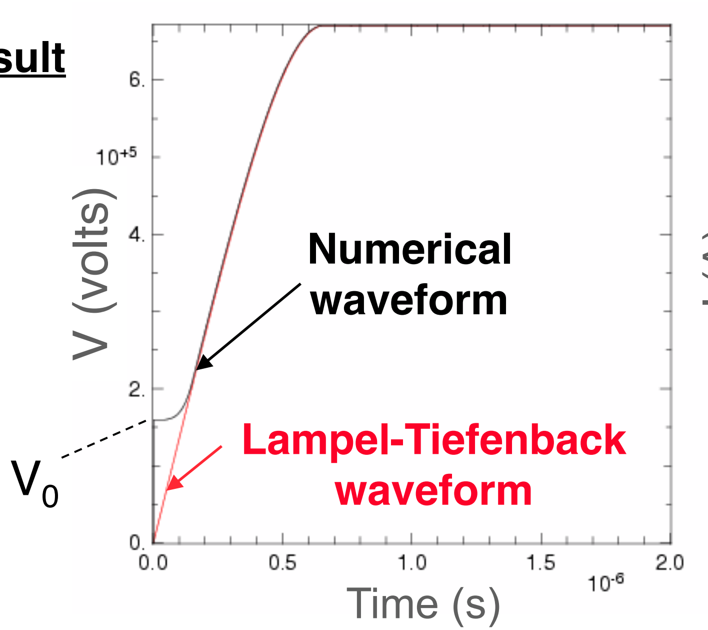
We apply Lampel-Tiefenback method at the discrete level

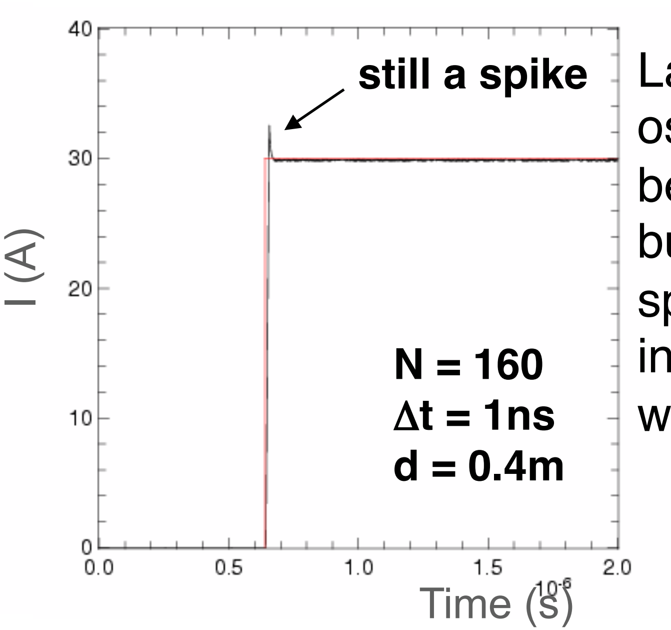
$$\Delta Q = Nq = I\Delta t \Rightarrow V - V_i = \left(\frac{I d_i^2}{\chi}\right)^{2/3}$$

solve for V using linearity of Poisson

$$(V\text{-}V_i) = (V\text{-}V_i)_{V=0} + (V\text{-}V_i)_{\rho=0}$$

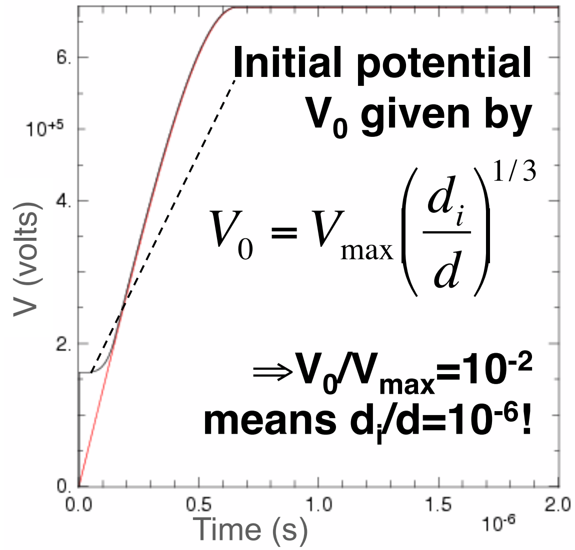
Result

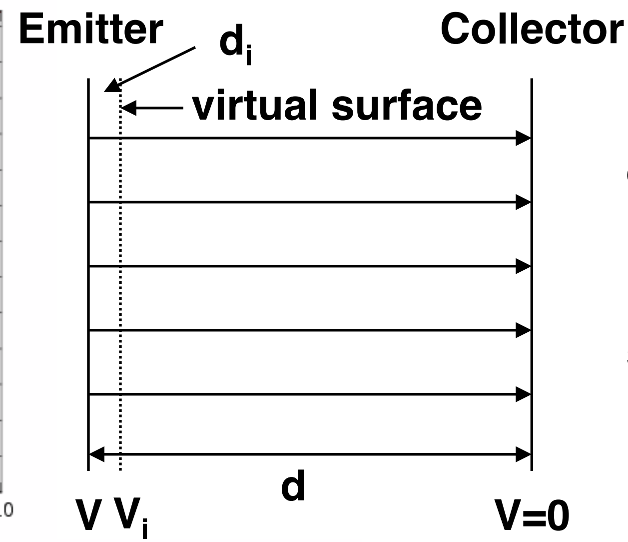




Large unphysical oscillation has been suppressed but there is still a spike. Is it due to initial step V_0 in waveform?

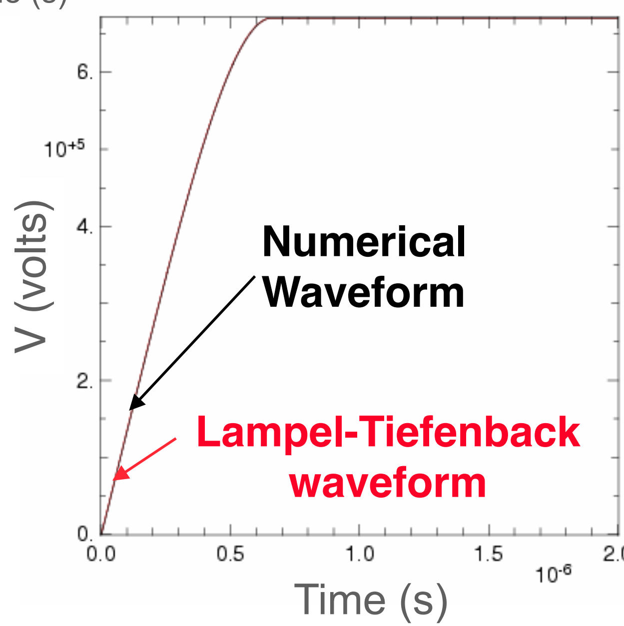
Cure #2: apply irregular gridded patch around emitter.

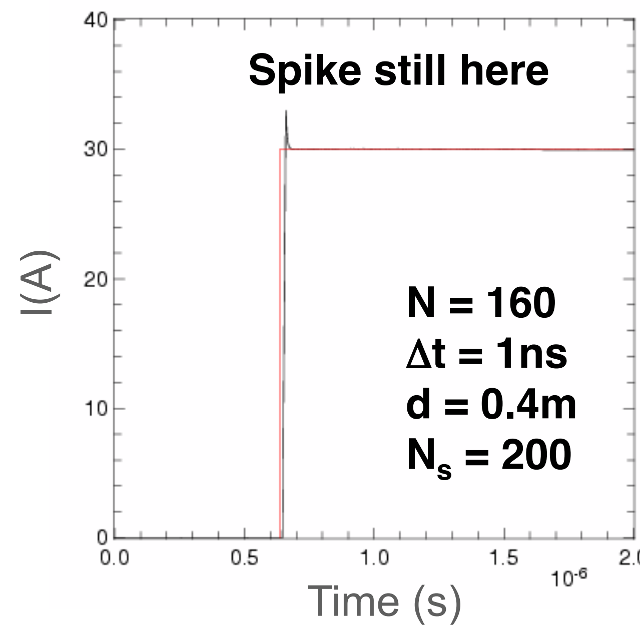




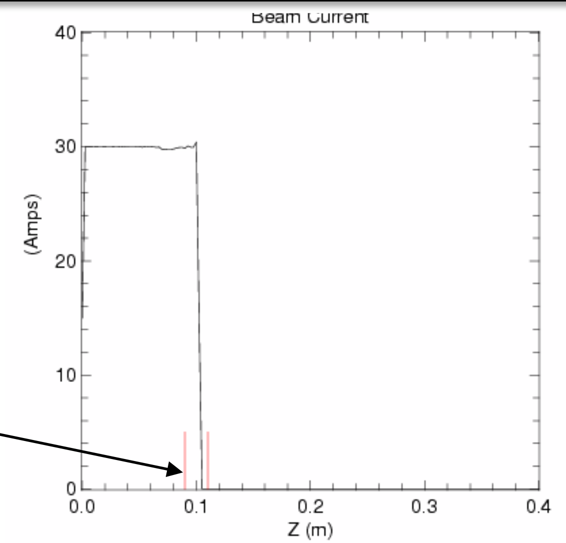
- Apply irregular gridded patch covering d_i
- Mesh sizes such that number of particles per cell is a constant in patch assuming Child-Langmuir solution for $\rho(z)$
- Apply same injection algorithm as before in patch

<u>Result</u>



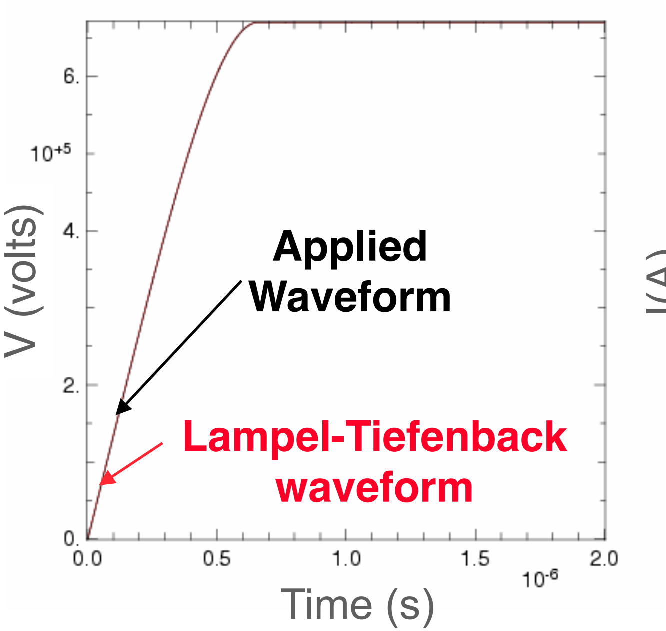


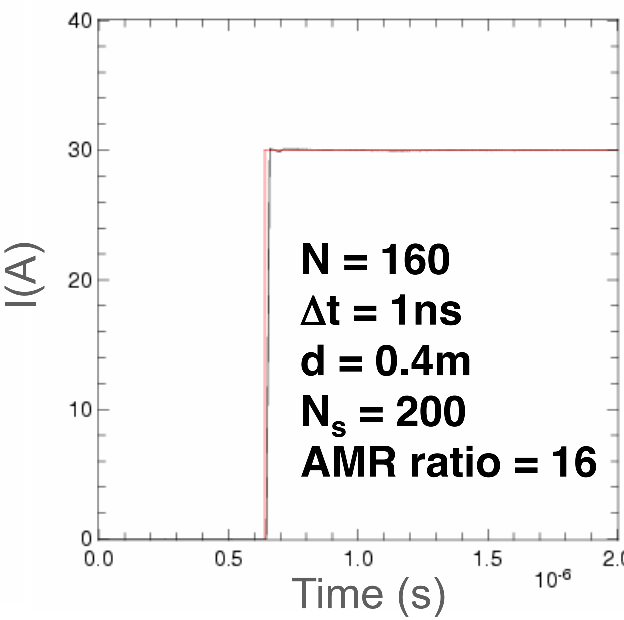
Cure #3: apply regularly gridded patch following front.



An Adaptive-Mesh-Refinement patch
Follows the front

Result

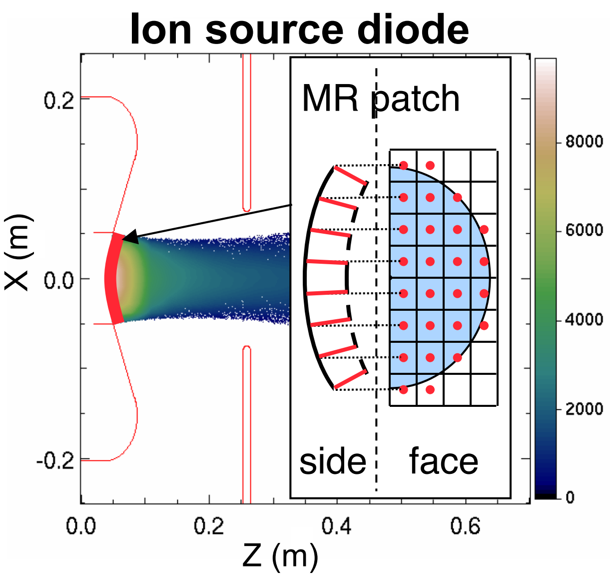




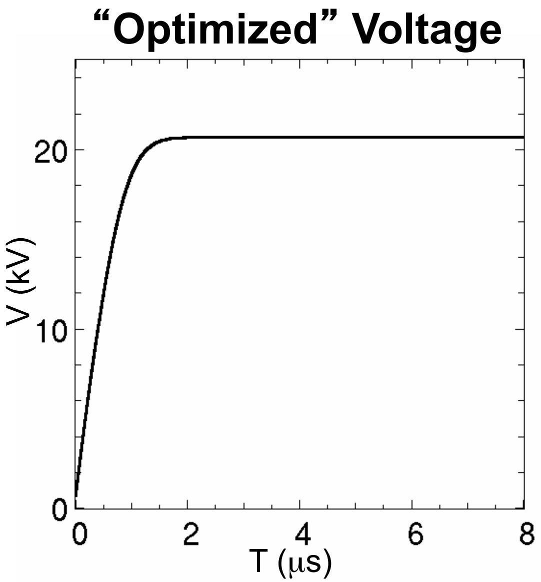
At this point, we declared victory!

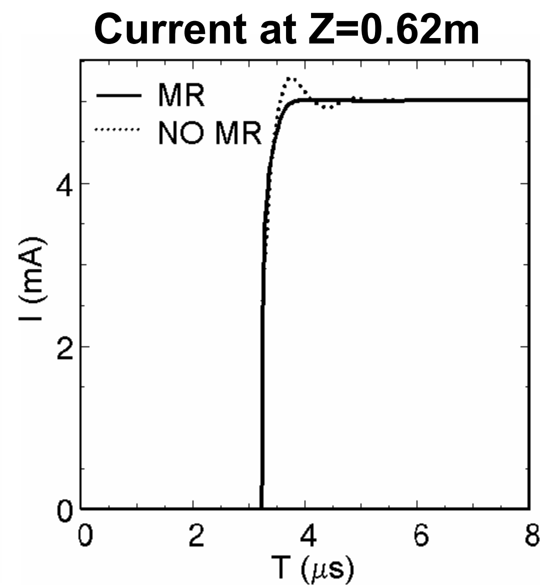
Extension to three dimensions

- Specialized 1-D patch implemented in 3-D injection routine, as a 2-D array of 1-D patches.



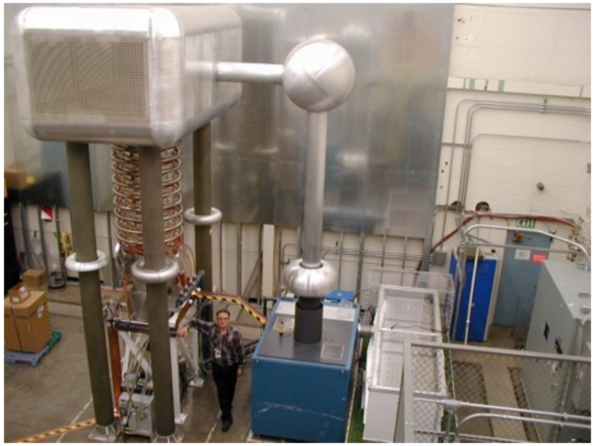
- Extended Lampel-Tiefenback technique to 3-D, and implemented in WARP
  - ➢ predicts a voltage waveform which extracts a nearly flat current at emitter

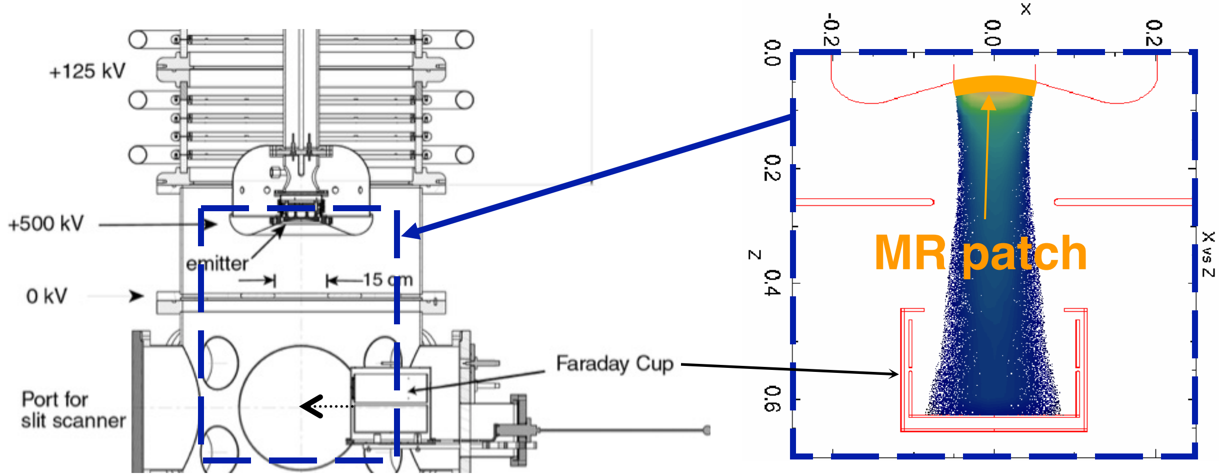


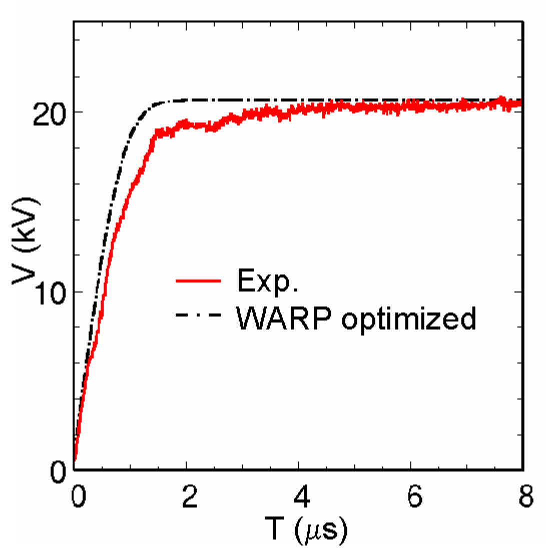


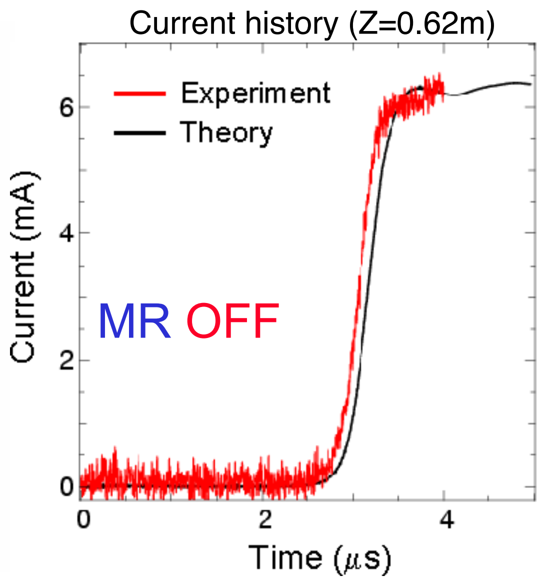
- Without MR, WARP predicts overshoot
- Run with MR predicts very sharp risetime (not square due to erosion)

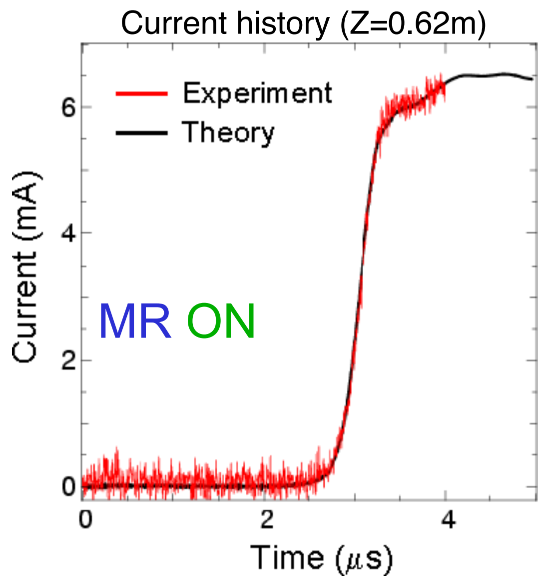
Test of MR patch on modeling of STS500 Experiment.







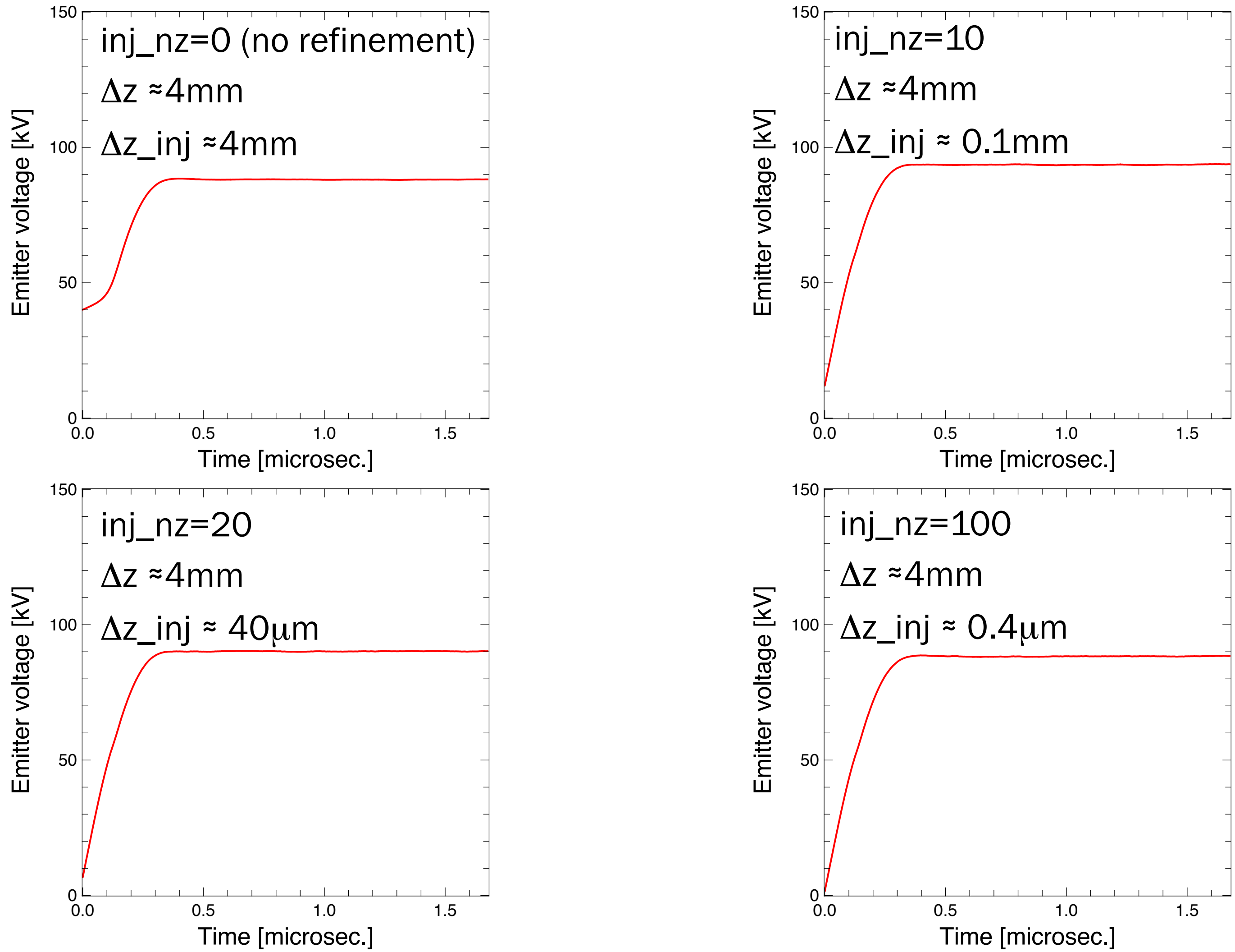




Pierce diode: exercise

① Open Pierce_diode.py. Run with w3d_injnz = 0, 10, 20 and 100.

② Observe convergence of voltage at t=0 toward 0. Notice very small dz required!



AMR-PIC summary

- Mesh refinement (static or adaptive) can reduce simulation time by several.
- Care is needed to avoid spurious effects (spurious charge & reflections).
- Warp implementation has validated methods, but maintenance is lacking sufficient manpower:
  - ➔ To be used with great care by experience users.
  - ➔ Novel implementation with external AMR package (BoxLib) is planned.

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2. Vay, J-L.; Friedman, A,; Grote, D.P; "Application of Adaptive Mesh Refinement to PIC Simulations in Inertial Fusion", Nuclear Inst. and Methods in Physics Research A, 544, pp. 347-352 (2005)
3. Vay J.-L., Colella P., Kwan JW., McCorquodale P., Serafini DB., Friedman A., Grote DP., Westenskow G., Adam JC., Heron A., Haber I., "Application of adaptive mesh refinement to particle-in-cell simulations of plasmas and beams" Physics of Plasmas., 11, pp. 2928-2934 (2004)
4. Vay J.-L., Colella P, Friedman A, Grote DP, McCorquodale P, Serafini DB, "Implementations of mesh refinement schemes for particle-in-cell plasma simulations.", Computer Physics Comm., 164, pp. 297-305 (2004)
5. Vay J.-L., Adam JC, Héron A, "Asymmetric PML for the absorption of waves. Application to mesh refinement in electromagnetic particle-in-cell plasma simulations.", Computer Physics Comm., 164, pp. 171-177 (2004)