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Intrabeam collisions, gas and electron effects in intense beams



1. Beam/beam coulomb collisions
2. Beam/gas scattering
3. Charge changing processes
4. Gas pressure instability
5. Electron cloud processes
6. Electron-ion instability

Gas and electron effects

-Effects are quite different depending on q , m of species being accelerated

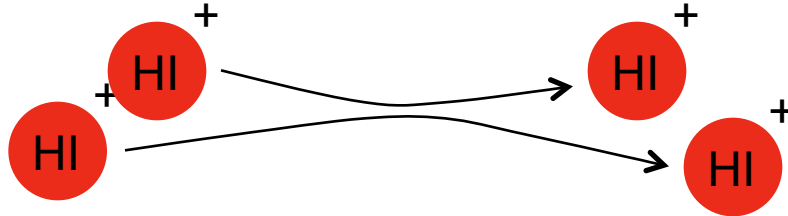
-Circular accelerators vs. Linacs
($t_{\text{residence}} \sim \text{ms to days vs. } 10\text{'s of } \mu\text{s}$)

-Long pulse vs. short pulse
($t_{\text{pulse}} \sim 10\text{'s of } \mu\text{s vs. } 10\text{'s of ns}$)

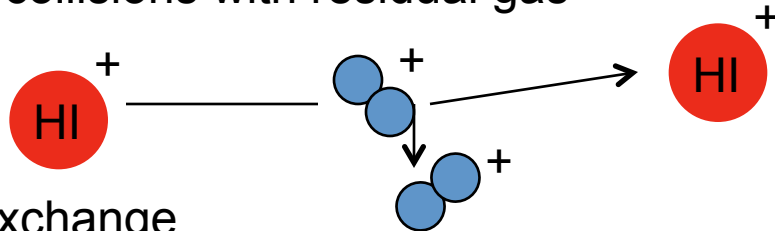
 HI^+ Heavy ion	 Residual gas molecule	e^- electron
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Processes:

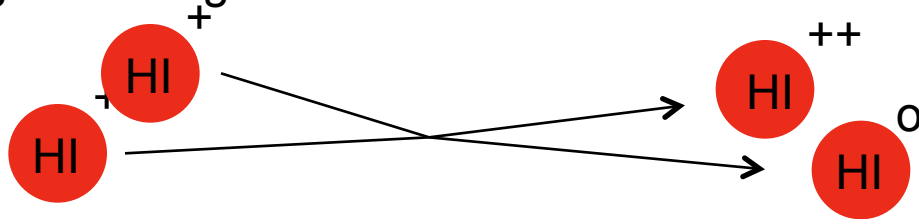
1. Coulomb collisions (intra-beam)



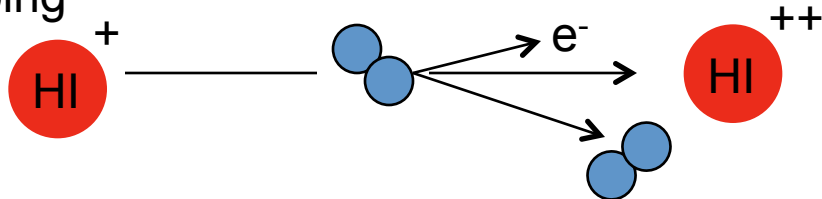
2. Coulomb collisions with residual gas



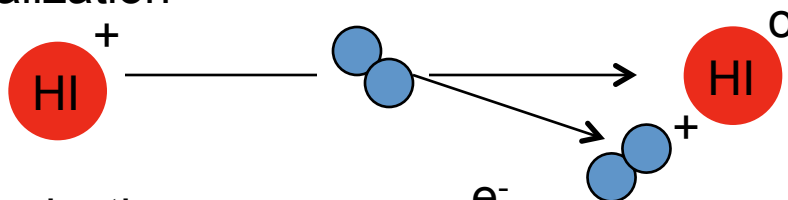
3. Charge exchange



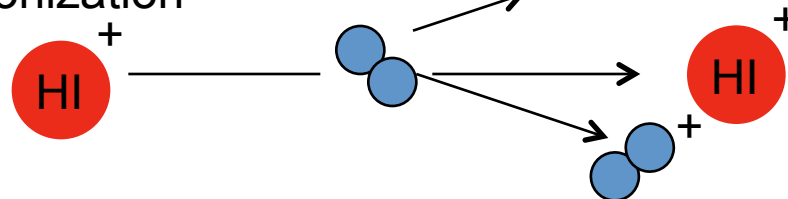
4. Stripping



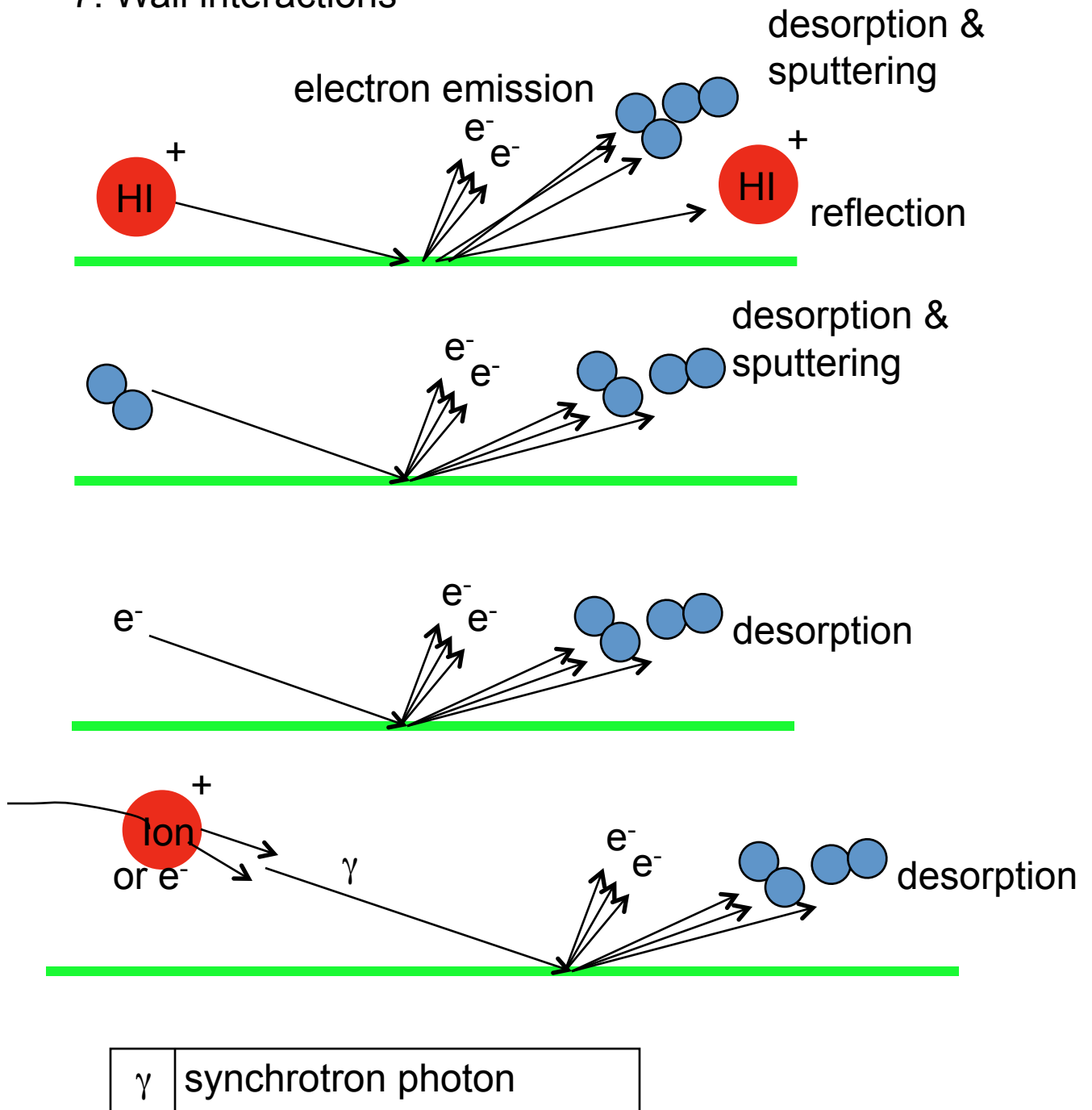
5. Neutralization



6. Gas Ionization



7. Wall interactions



1. COLLISIONS WITHIN BEAM REISER 6.4

CONSIDER EFFECTS OF COULOMB COLLISIONS
IN A CONTINUOUS BEAM PROPAGATING THROUGH
A CONTINUOUS FOCUSING CHANNEL WITH $T_{\perp 0} \neq T_{\parallel 0}$

(IF $T_{\perp 0} = T_{\parallel 0} \Rightarrow$ BEAM ALREADY KEULED)

FROM ICHIMARU & ROSENBLUTH, Phys Fluids 13, 2718, (1970):

$$\frac{dT_{\perp}}{dt} = -\frac{1}{2} \frac{dT_{\parallel}}{dt} = \frac{-(T_{\perp} - T_{\parallel})}{\uparrow}$$

(since $T_x = T_y = T_{\perp}$, T_{\parallel} CHANGES AT TWICE THE RATE OF T_{\perp})
(since $2k_B T_{\perp} + k_B T_{\parallel} = \text{const}$)

\uparrow = RELAXATION TIME

$$= \frac{15 (k_B T_{\text{eff}} / m c^2)^{3/2} (4\pi \epsilon_0)^2 m^2 c^3}{8\pi^{1/2} q^4 \ln \Lambda n} = \left(\frac{15 \pi^{1/2}}{8 \ln \Lambda} \right) v_c^{-1}$$

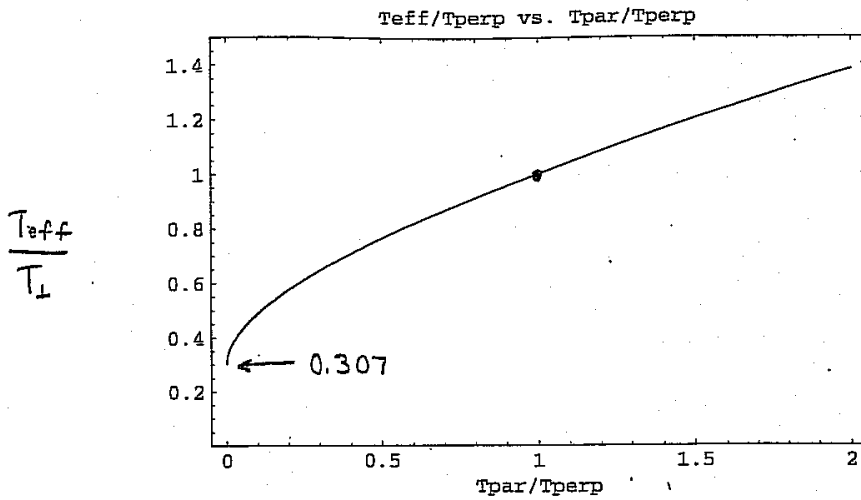
$$\ln \Lambda = \begin{cases} \ln \frac{(4\pi \epsilon_0 k_B T_{\parallel})^{3/2} 12\pi}{q^3 n^{1/2}} & \text{for } \lambda_D < r_b \\ \ln \frac{12\pi \epsilon_0 k_B T_{\parallel} r_b}{q^2} & \text{for } \lambda_D > r_b \end{cases}$$

COULOMB COLLISION RATE FOR LARGE ANGLES
(PAGE 9 OF INTRODUCTION NOTES)

$$v_c \sim \pi \left(\frac{q^2}{4\pi \epsilon_0 k_B T} \right)^2 n_0 \left(\frac{k_B T}{m} \right)^{1/2}$$

$$T_{\text{eff}} = T_{\perp} \left[\frac{15}{4} \int_{-1}^1 \frac{\mu^2 (1 - \mu^2) d\mu}{[(1 - \mu^2) + \mu^2 (T_{\parallel} / T_{\perp})]^{3/2}} \right]^{-2/3}$$

T_{eff} is an appropriate average of T_{\perp} & T_{\parallel}
and depends only on $T_{\perp} / T_{\parallel}$



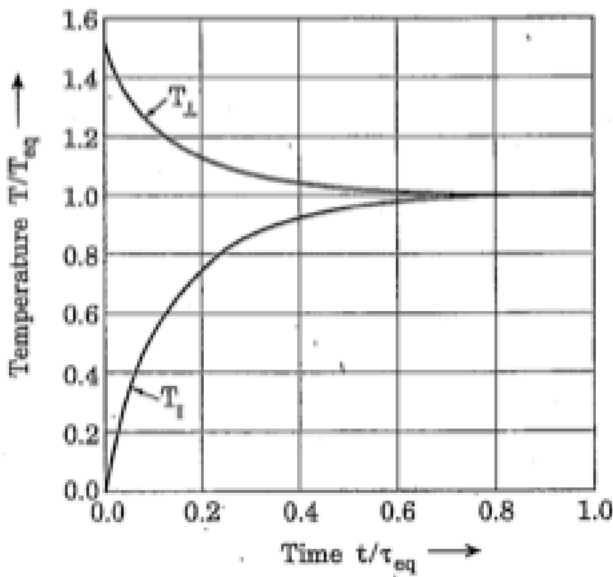
For $T_{\parallel 0} = 0$

$$T_{\perp} = \frac{2}{3} T_{\perp 0} \left(1 + \frac{1}{2} e^{-3t/\tau_{eff}} \right), \quad (6.156a)$$

$$T_{\parallel} = \frac{2}{3} T_{\perp 0} (1 - e^{-3t/\tau_{eff}}), \quad (6.156b)$$

APPROXIMATE SOLUTIONS

$$\tau_{eff} = 0.42 \tau_{eq}$$



FROM REISER p. 527

$$\tau = \left(\frac{15\pi^{1/2}}{8 \ln \lambda} \right) \frac{m^{1/2}}{\pi n_0} \left(\frac{4\pi\epsilon_0}{q^2} \right)^2 (k_B T_{eff})^{3/2}$$

$$\tau_{eq} = \tau(T_{eff} = T_{eq})$$

$$T_{eff} = T_{\perp} \left[\frac{15}{4} \int_{-1}^1 \frac{\mu^2 (1 - \mu^2) d\mu}{[(1 - \mu^2) + \mu^2 (T_{\parallel} / T_{\perp})]^{3/2}} \right]^{-2/3}$$

BOERSCH EFFECT

ARENT COLLISIONS NEGLIGIBLE? (NOT ALWAYS)

POTTING IN NUMBERS:

FOR IONS:

$$\tau_{\text{eff}} = 4.3 \cdot 10^{-4} \text{ s} \left(\frac{A^{1/2}}{Z^2} \right) \left(\frac{kT_{\text{eff}}}{1 \text{ eV}} \right)^{3/2} \left(\frac{15}{\ln \lambda} \right) \left(\frac{10^{10} \text{ cm}^{-3}}{n} \right)$$

$$\ln \lambda = \ln \left[\frac{1.5 \cdot 10^5 (kT/1 \text{ eV})^{3/2}}{Z^2 (n/10^{10} \text{ cm}^{-3})} \right]$$

EXAMPLE: 2 MeV INJECTOR

$$\begin{aligned} \tau_{\text{eff}} &\approx 8.8 \cdot 10^{-4} \text{ s} & \text{for } A &= 39 & kT_{\text{eff}} &= 0.3 \text{ eV} \\ & & Z &= 1 & \ln \lambda &= 8.5 \\ & & n &= 10^{10} \text{ cm}^{-3} & & \end{aligned}$$

$$t_{\text{transit}} \approx \frac{zd}{v} \approx \frac{z(zm)}{(0.1) 3 \cdot 10^8} = 1.3 \mu\text{s}$$

So $\tau_{\text{eff}} \gg t_{\text{transit}} \Rightarrow$ collisions are rare BUT

$$T_{\text{accel}}^{\text{coll}} = \frac{1}{2} \left(\frac{kT_0}{qV} \right) kT_0 = 2.5 \cdot 10^{-9} \text{ eV} \quad \text{for } \begin{matrix} kT_0 = 0.1 \text{ eV} \\ qV = 2 \text{ MeV} \end{matrix}$$

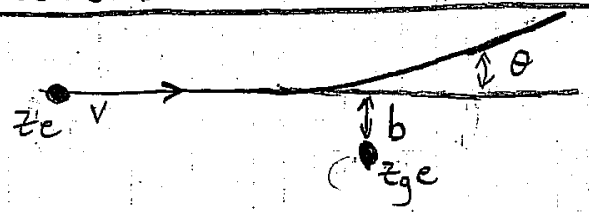
$$T_{\text{collisions}} \approx \frac{2}{3} T_{10} (1 - \exp(-3t/\tau_{\text{eff}})) \approx 2T_{10} \left(\frac{t_{\text{transit}}}{\tau_{\text{eff}}} \right) = .006 \text{ eV} \quad \text{for } T_{10} = 1 \text{ eV}$$

So T_H FROM "BOERSCH EFFECT"

>> T_H FROM LONGITUDINAL COOLING



COULOMB COLLISIONS IN RESIDUAL GAS (REISER 6.4.3) JACKSON CHAPTER 13



(RUTHERFORD SCATTERING)

$$\frac{dp_x}{dt} = \frac{z z_g e^2}{4\pi\epsilon_0 r^2} \frac{b}{r} \Rightarrow \Delta p = \int_{-\infty}^{\infty} \frac{dp_x}{dt} dt dz$$

$$= \frac{z z_g e^2 b}{4\pi\epsilon_0 v} \int_{-\infty}^{\infty} \frac{dz}{(z^2 + b^2)^{3/2}}$$

$$= \frac{2 z z_g e^2}{4\pi\epsilon_0 v b}$$

$$\theta \cong \frac{\Delta p}{p} = \frac{2 z z_g e^2}{4\pi\epsilon_0 p v b} \Rightarrow \frac{db}{d\theta} \sim \frac{1}{\theta^2}$$

DIFFERENTIAL CROSS SECTION FOR SCATTERING WITH IMPACT PARAMETER b INTO SOLID ANGLE $d\Omega$ AT ANGLE θ SATISFIES

$$\underbrace{2\pi b db}_{\text{AREA}} = \underbrace{\frac{d\Omega}{4\pi} 2\pi \sin\theta d\theta}_{\text{SOLID ANGLE}}$$

$$\Rightarrow \frac{d\Omega}{4\pi} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right| = \left(\frac{z z_g e^2}{4\pi\epsilon_0 p v} \right)^2 \frac{1}{\theta^4}$$

ELECTRON SCREENING PUTS CUTOFF AT SMALL θ (LARGE b) SO BETTER TO USE

$$\frac{d\Omega}{dR} = \left(\frac{z z_g e^2}{4\pi\epsilon_0 p v} \right)^2 \frac{1}{(\theta^2 + \theta_{\min}^2)^2}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 p v} \right)^2 \frac{1}{(\theta^2 + \theta_{\min}^2)^2}$$

(10)

AVERAGE ANGLE SQUARED FOR A SINGLE SCATTERING IS:

$$\begin{aligned} \bar{\theta}^2 &= \frac{\int \theta^2 \frac{d\sigma}{d\Omega} 2\pi \sin\theta d\theta}{\int \frac{d\sigma}{d\Omega} 2\pi \sin\theta d\theta} \approx \frac{\int_0^{\theta_{\max}} \frac{\theta^3}{(\theta^2 + \theta_{\min}^2)^2} d\theta}{\int_0^{\theta_{\max}} \frac{\theta}{(\theta^2 + \theta_{\min}^2)^2} d\theta} \\ &\approx 2 \theta_{\min}^2 \ln\left(\frac{\theta_{\max}}{\theta_{\min}}\right) \end{aligned}$$

ASSUMES $\theta_{\max}^2 \gg \theta_{\min}^2$
 $\ln\left(\frac{\theta_{\max}}{\theta_{\min}}\right) \gg 1$

MULTIPLE COLLISIONS

AFTER TRAVERSING DISTANCE s ,
 AND UNDERGOING N_s COLLISIONS, THE
 MEAN SQUARE ANGLE $\bar{\theta}^2$

$$\begin{aligned} \bar{\theta}^2 &= N_s \bar{\theta}^2 = n_0 \sigma_s s \bar{\theta}^2 \\ &= 8 \pi n_0 \left(\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 m c^2 \gamma \beta^2} \right)^2 \ln\left(\frac{\theta_{\max}}{\theta_{\min}}\right) s \end{aligned} \quad \left[\sigma_s = \pi \left(\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 p v} \right)^2 \frac{1}{\theta_{\min}^2} \right]$$

JACKSON ARGUES θ_{\max} ARISES FROM DISTRIBUTED
 NATURE OF NUCLEUS (NOT POINT CHARGE)
 AND θ_{\min} ARISES FROM SCREENING OF ELECTRONS
 OR UNCERTAINTY PRINCIPLE

$$\ln \frac{\theta_{\max}}{\theta_{\min}} \approx \ln[(204 Z_2^{-1/3})^2] = 2 \ln[204 Z_2^{-1/3}]$$

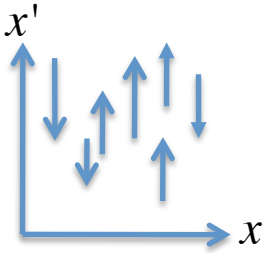
$$\begin{aligned} \text{So } \bar{\Theta}^2 &= N_s \bar{\theta}^2 = n_g \sigma_s s \bar{\theta}^2 \\ &= 8\pi n_g \left(\frac{ZZ_g e^2}{4\pi\epsilon_0 mc^2 \gamma\beta^2} \right)^2 \ln\left(\frac{\theta_{\max}}{\theta_{\min}}\right) s = 16\pi n_g \left(\frac{ZZ_g e^2}{4\pi\epsilon_0 mc^2 \gamma\beta^2} \right)^2 \ln(204 Z_g^{-1/3}) s \end{aligned}$$

$$\text{Now } \Theta^2 = \langle x'^2 \rangle + \langle y'^2 \rangle = 2\langle x'^2 \rangle$$

$$\begin{aligned} \Rightarrow \frac{d}{ds} \langle x'^2 \rangle &= 4\pi n_g \left(\frac{ZZ_g e^2}{4\pi\epsilon_0 mc^2 \gamma\beta^2} \right)^2 \ln\left(\frac{\theta_{\max}}{\theta_{\min}}\right) \\ &\equiv C_{sc} \end{aligned}$$

How does scattering change the envelope equations?

We assume the scattering locally changes the transverse momentum, without directly changing the position (thin lens).



So after an incremental distance δs

$$x \rightarrow x_0 \quad x' \rightarrow x_0' + \delta x'$$

$$\delta \langle x'^2 \rangle = \langle (x_0' + \delta x')^2 - x_0'^2 \rangle = 2\langle x_0' \delta x' \rangle + \langle \delta x'^2 \rangle = \langle \delta x'^2 \rangle = C_{sc} \delta s$$

$$\delta \langle x x' \rangle = \langle (x_0' + \delta x') x_0 - x_0' x_0' \rangle = \langle x_0' \delta x' \rangle = 0$$

$$\delta \langle x^2 \rangle = \langle x_0^2 - x_0^2 \rangle = 0$$

And the moment equations become:

$$\frac{d}{ds} \langle x^2 \rangle = 2\langle x x' \rangle$$

$$\frac{d}{ds} \langle x x' \rangle = \langle x x'' \rangle + \langle x'^2 \rangle \quad \text{For } x'' = -K(s)x + \frac{2Q}{r_x + r_y} \frac{x}{r_x} \text{ and if energy loss}$$

$$\frac{d}{ds} \langle x'^2 \rangle = 2\langle x' x'' \rangle + C_{sc} \quad \text{is negligible the envelope equation becomes:}$$

$$r_x'' + \frac{2Q}{r_x + r_y} + K(s)r_x + \frac{\epsilon_x^2}{r_x^3} = 0$$

$$\frac{d\epsilon_x^2}{ds} = 4r_x^2 C_{sc}$$

(Envelope equation

unchanged, but $\frac{d\epsilon_x^2}{ds} \neq 0$)

For a beam undergoing acceleration or deceleration or if both stopping and scattering are not negligible:

$$r_x'' + \frac{(\gamma\beta)'}{\gamma\beta} r_x' + \frac{2Q}{r_x + r_y} + K(s)r_x + \frac{\varepsilon_{nx}^2}{\gamma^2 \beta^2 r_x^3} = 0$$

$$\frac{d\varepsilon_{nx}^2}{ds} = 4\gamma^2 \beta^2 r_x^2 C_{sx}$$

$$mc^2 \frac{d\gamma}{ds} = qE_z(s) - \frac{dE_{stopping}}{ds}$$

Example:

$$\frac{d\varepsilon_{nx}^2}{ds} = 4\gamma^2 \beta^2 r_x^2 C_{sx} = 32\pi n_g r_x^2 \left(\frac{ZZ_g e^2}{4\pi\varepsilon_0 mc^2 \beta} \right)^2 \ln(204 Z_g^{-1/3})$$

$$n_g = 10^{-7} \text{ torr} = 3.5 \times 10^9 \text{ cm}^{-3} = 3.5 \times 10^{15} \text{ m}^{-3}$$

$$r_x = 0.01 \text{ m}; Z_g = 7; Z = 19; A = 39; \beta = 0.01; \varepsilon_N = 1 \times 10^{-6} \text{ m-rad}$$

$$\frac{d\varepsilon_{nx}^2}{ds} = 4.6 \times 10^{-17} \text{ m}^2 \text{-rad}^2 / \text{m}$$

$$\Rightarrow \varepsilon_{nx}^2 / \frac{d\varepsilon_{nx}^2}{ds} = 22,000 \text{ m}$$

So 22 km needed to equal original emittance! (So more important for rings and/or low mass particles).

BEAM LOSS FROM CHARGE CHANGING COLLISIONS

REFERENCE: WORKSHOP ON BEAM INDUCED PRESSURE RISE IN LINGS, BNL, Dec. 2003.

σ_s = STRIPPING CROSS SECTION

σ_{ce} = CHARGE EXCHANGE CROSS SECTION

σ_i = IONIZATION CROSS SECTION

v_{cm} = mean ion velocity in ion beam frame

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① BEAM LOSS

$$\frac{dN_b}{dt} = -\sigma_s v_i N_b \bar{n} - \sigma_{ce} v_{cm} N_b^2 - \left. \frac{dN_b}{dt} \right|_{HALO}$$

② GAS EVOLUTION

\bar{n} = average gas density

$$\begin{aligned} \frac{d\bar{n}}{dt} = & \eta_s \sigma_i v_i N_b \bar{n} \left(\frac{V_{beam}}{V_{PIPE}} \right) + \eta_{HT} \sigma_s v_i N_b \bar{n} \left(\frac{V_{beam}}{V_{PIPE}} \right) \\ & + \eta_{CE} \sigma_{ce} v_{cm} N_b^2 \left(\frac{V_{beam}}{V_{PIPE}} \right) + q - (S/A_p) \bar{n} \end{aligned}$$

IONIZATION (under σ_i) STRIPPING (under σ_s) VOLUME OF BEAM (under V_{beam})
 CHANGE EXCHANGES (under σ_{ce}) OUTGASSING (under q) PUMPING (under (S/A_p)) VOLUME OF PIPE (under V_{PIPE})

S = Effective linear pump rate $[m^3 s^{-1} / m]$

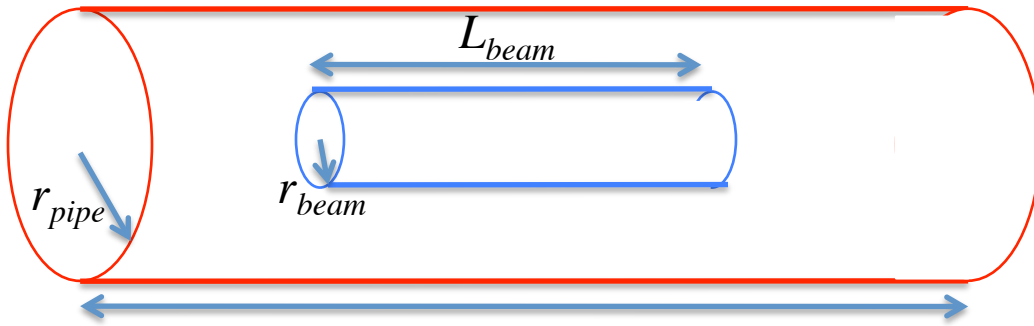
$A_p = \pi r_p^2$ = AREA OF PIPE

q = OUTGASSING rate = $\frac{2\pi r_p Q}{\pi r_p^2} = \frac{2Q}{r_p}$; $Q = \frac{\#}{m^2 s}$

η_s = GAS MOLECULES DESORBED FOR INCIDENT RESIDUAL GAS ION

η_{HT} = GAS MOLECULES DESORBED FOR INCIDENT IONIZATION STRIKING WALL

$(V_{beam} / V_{PIPE}) \rightarrow \left(\frac{V_{beam}}{V_{PIPE}} \right) v_{rel} \Delta t_{beam}$ for a rep rated linac



$$V_{beam} = \pi r_{beam}^2 L_{beam} \quad V_{pipe} = \pi r_{pipe}^2 L_{pipe} \quad A_p = \pi r_{pipe}^2$$

Gas evolution equation:

$$\frac{d\bar{n}}{dt} = \eta_G \sigma_i v_i n_b \bar{n} \left(\frac{V_{beam}}{V_{pipe}} \right) + \eta_{HI} \sigma_s v_i n_b \bar{n} \left(\frac{V_{beam}}{V_{pipe}} \right) + \eta_G \sigma_{ce} v_{cm} n_b^2 \left(\frac{V_{beam}}{V_{pipe}} \right) +$$

$$+ q - \left(\frac{S}{A_p} \right) \bar{n}$$

outgassing
pumping

Here S = effective linear pumping rate $m^3/s/m = m^2/s$

q = effective linear outgassing rate

$$= 2\pi r_{pipe} Q_{outgassing} / (\pi r_{pipe}^2) = 2Q_{outgassing} / r_{pipe}$$

where $Q_{outgassing} = \#/cm^2/s$

η_G = gas molecules desorbed per incident ionized gas molecule

η_{HI} = gas molecules desorbed per incident ionized heavy ion

$V_{beam}/V_{pipe} \rightarrow (r_{beam}^2/r_{pipe}^2)(v_{rep}\Delta t)$ for a rep rated linac

Δt = pulse duration; v_{rep} = repetition rate

If we take $n_b \approx \text{constant}$

then we may express gas evolution equation as:

$$\frac{d\bar{n}}{dt} = \frac{\bar{n}}{\tau} + q_{\text{eff}}$$

with solution:

$$\bar{n} = (\bar{n}_0 + \tau q_{\text{eff}}) \exp[t/\tau] - \tau q_{\text{eff}}$$

$$\text{HERE } \tau = \frac{1}{(\eta_g \sigma_i + \eta_{HI} \sigma_s) \left(\frac{V_{\text{beam}}}{V_{\text{plc}}} \right) n_b V_i - S/A_p}$$

$$q_{\text{eff}} = q + \eta_{HI} \sigma_{CE} V_{\text{cm}} n_b^2 \left(\frac{V_{\text{beam}}}{V_{\text{plc}}} \right)$$

EQUILIBRIUM REACHED IF $\tau < 0$ (i.e. pumping exceeds desorption).

$$\Rightarrow \bar{n} = -\tau q_{\text{eff}} = \frac{q + \eta_{HI} \sigma_{CE} V_{\text{cm}} n_b^2 \left(\frac{V_{\text{beam}}}{V_{\text{plc}}} \right)}{S/A_p - (\eta_g \sigma_i + \eta_{HI} \sigma_s) \left(\frac{V_{\text{beam}}}{V_{\text{plc}}} \right) n_b V_i}$$

INSTABILITY IF

$$n_b V_i \gg \frac{S/A_p \left(\frac{V_{\text{plc}}}{V_{\text{beam}}} \right)}{\eta_g \sigma_i + \eta_{HI} \sigma_s}$$

Instability first observed on the ISR proton storage ring, limiting current in μg , in 1970's.

$$\text{If } l_{\text{beam}} = l_{\text{pipe}}$$

INSTABILITY CRITERION MAY BE WRITTEN

$$I > \frac{zeS}{\eta_g \mathcal{O}_1 + \eta_{HI} \mathcal{O}_2}$$

EXAMPLE: If $S = 100 \text{ l s}^{-1} \text{ m}^{-1} = 0.1 \text{ m}^3 \text{ s}^{-1} \text{ m}^{-1}$

ISR

$$\eta_g = 4$$

$$\mathcal{O}_1 = 10^{-22} \text{ m}^2 = 10^{-16} \text{ cm}^2; \quad \mathcal{O}_2 = 0$$

$$z = 1 \quad (\text{protons})$$

$$\Rightarrow I \leq 40 \text{ Amperes}$$

(PRESSURE RUNAWAYS WERE OBSERVED ON THE ISR AT 14-18A,
(BENVENUTI et al, IEEE Trans. on Nuc. Sci. NS-24, 1773, 1977)

SEE "BEAM INDUCED PRESSURE RISE IN RINGS"

13th ICFA BEAM DYNAMICS MINI WORKSHOP, BNL, Dec. 9-12, 2003.

WEBSITE: <http://www.c-ad.bnl.gov/icfa>

"ELECTRON CLOUD EFFECTS"

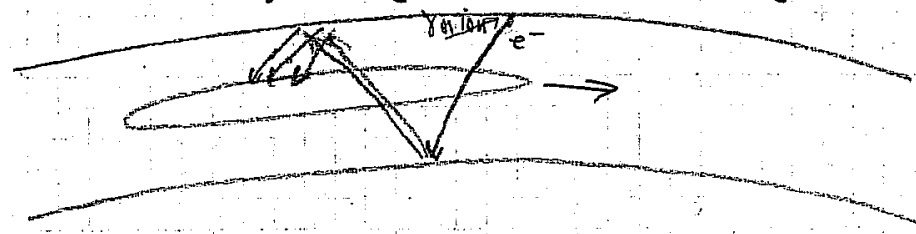
REFERENCE: CERN e-CLOUD WORKSHOP

<http://wwwslap.cern.ch/collective/ecloudφz/>
→ proceedings.html

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BASIC IDEA

IN ION STORAGE RINGS OR COLLIDER RINGS:



ELECTRONS ARE ATTRACTED TO POSITIVE POTENTIAL OF BEAM & ACCUMULATE

SOME SYMPTOMS:

1. BEAM LOSS & pressure rise
2. HIGH FREQUENCY centroid oscillations

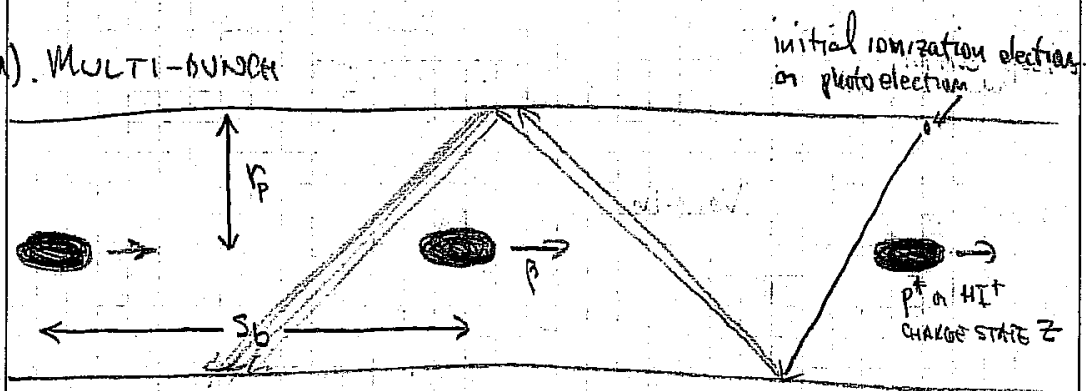
SOME ACCELERATORS WHICH SHOW EVIDENCE OF e- EFFECTS

1. LANL PSR
2. CERN PS & SPS
3. BNL RHIC

cf. "Electron-cloud effects in HIGH INTENSITY PROTON ACCELERATORS" J. Wei & R. Maccek, CERN

BEAM INDUCED MULTIACTING

a) MULTI-BUNCH



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Using COULOMB COLLISION FORMULA FROM PAGE 9:

$$\Delta p_x \approx \frac{2 Z N_b e^2}{4 \pi \epsilon_0 v \beta r_p}$$

N_b = Number of ions of charge Z in bunch

$$\Delta E_e = m_e c^2 \left[\sqrt{\frac{\Delta p_x^2}{m_e^2 c^2} + 1} - 1 \right] = m_e c^2 \left[\sqrt{\frac{(2 r_e Z N_b)^2}{\beta^2 r_p^2} + 1} - 1 \right]$$

(where $r_e = \frac{e^2}{4 \pi \epsilon_0 m_e c^2} \approx 2.8 \times 10^{-15} \text{ m}$)

$$\approx 2 r_e^2 m_e c^2 \frac{Z^2 N_b^2}{\beta^2 r_p^2} \quad \text{for } \Delta E_e \ll m_e c^2 \quad \left(\text{or } \frac{2 r_e Z N_b}{\beta r_p} \ll 1 \right)$$

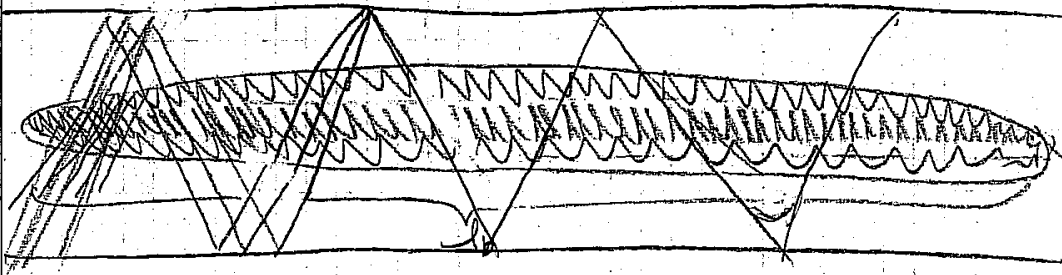
DEFINE A MULTIACTING PARAMETER J_m

$$J_m = \frac{\text{TIME FOR ELECTRON TO CROSS LIFE}}{\text{TIME BETWEEN BUNCHES}} = \frac{2 r_p \beta}{S_b \beta_e}$$

$$\approx \frac{\beta^2 r_p^2}{Z N_b r_e S_b} \quad \text{RESONANCE CONDITION: } J_m = 1$$

S_b = distance between bunches

b). SINGLE-BUNCH BEAM-INDUCED MULTIPLYING



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$$\int_s = \frac{r_p \beta}{l_b \rho_e} = \frac{\text{time for electrons to cross pipe}}{\text{passage time for half of the bunch}}$$

Recall:

$$\phi = \begin{cases} \frac{\lambda}{2\pi\epsilon_0} \left[\frac{1}{2} \left(1 - \frac{v^2}{v_b^2} \right) + \ln \frac{r}{r_b} \right] & 0 < r < r_b \\ \frac{\lambda}{2\pi\epsilon_0} \left[\ln \frac{r}{r} \right] & r_b \leq r < r_p \end{cases}$$

$$\frac{1}{2} m_e v_e^2 + q\phi \approx \text{const} \approx 0$$

(AVERAGE e^- VELOCITY)

$$\beta_0 \sim \frac{1}{2} \sqrt{\frac{2q\phi}{m_e c^2}} \sim \sqrt{\frac{N_0 z e^2}{l_b 4\pi\epsilon_0 m_e c^2}} \sim \sqrt{\frac{z v_e N_0}{l_b}}$$

$$\Rightarrow \int_s = \frac{\beta v_p}{\sqrt{v_e l_b N_0 z}} = \frac{\text{pipe crossing time}}{.5 * \text{pulse duration}} \quad \left(r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2} \right)$$

THE ENERGY GAIN OF THE ELECTRON, RELIES ON THE DENSITY CHANGING OVER THE COURSE OF THE BUNCH.

$$\Delta E_e \sim \frac{m_e c^2}{2} \left[\frac{z v_e N_0(z)}{l_b} - \frac{m_e c^2}{2} \left[\frac{z v_e N_0(z+\Delta z)}{l_b} \right] \right]$$

$$\sim \frac{m_e c^2}{2} \left(\frac{\partial N_0}{\partial z} \Delta z \right) \left(\frac{z v_e}{l_b} \right)$$

$$\Delta E_e \sim \frac{m_e c^2}{2} \left(\frac{\partial N_0}{\partial z} \Delta z \right) \left(\frac{z r_e}{l_b} \right)$$

$$\Delta z = \frac{v_p}{v_e} \beta_e = \beta r_p \sqrt{\frac{l_b}{z r_e N_0}}; \quad \frac{\partial N_0}{\partial z} \sim \frac{N_0}{l_b}$$

So
$$\Delta E_e \sim m_e c^2 \left(\frac{z N_0 r_e}{l_b^3} \right)^{1/2} \beta r_p$$

$$\left(r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2} \right)$$

$\beta \leq 1 \Rightarrow$ Electron build up possible within bunch

WHAT IS STEADY STATE ELECTRON DENSITY?

Electrons can build up until E_r at pipe ~ 0 .

$$\Rightarrow \lambda_e = \lambda_J$$

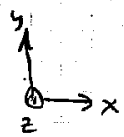
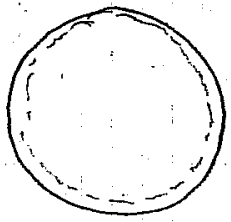
$$\pi r_p^2 n_e = \pi n_b^2 z n_i$$

$$n_e = \left(\frac{n_b}{r_p} \right)^2 z n_i$$

ELECTRON-ION INSTABILITY

(SEE ALSO R.C. DAVIDSON
& H. QIN, PHYSICS OF INTENSE
CHARGED PARTICLE BEAMS IN
HIGH ENERGY ACCELERATORS, P. 503
FOR KINETIC TREATMENT)

CONSIDER A UNIFORM DISTRIBUTION OF ELECTRONS (AT REST)
WHICH HAS THE SAME RADIUS (OR SLIGHTLY SMALLER RADIUS)
AS A UNIFORM DENSITY ION BEAM, THAT IS MOVING AT VELOCITY
 v_z (OUT OF THE PLANE OF THE PAPER)



Electric field from ions:

$$E_x = \frac{\lambda_i(r)(x-x_i)}{2\pi\epsilon_0 r} = \frac{\rho_i(x-x_i)}{2\epsilon_0}$$

THE EQUATION OF MOTION FOR THE CENTROID OF THE
ELECTRONS IS OBTAINED FROM THE EQUATION OF MOTION FOR SINGLE ELECTRON:

$$m_e \ddot{x} = -\frac{e\rho_i}{2\epsilon_0}(x-x_i) + \frac{e\rho_e}{2\epsilon_0}(x-x_e)$$

Taking statistical average:

$$\frac{d^2 x_e}{dt^2} = -\frac{\omega_{pi}^2}{2} \left(\frac{m_i}{q} \frac{e}{m_e} \right) (x_e - x_i)$$

$$\text{here } \omega_{pi}^2 = \frac{q^2 n_i}{\epsilon_0 m_i} = \frac{q \rho_i}{\epsilon_0 m_i}$$

(THE CENTER OF OSCILLATION FOR THE ELECTRONS IS THE
CENTER OF THE ION BEAM),

x_e = centroid of electron beam

x_i = centroid of ion beam

THE EQUATION OF MOTION FOR THE CENTROID OF THE IONS IS GIVEN BY

$$\frac{d^2 x_i}{dt^2} = -\omega_{po}^2 x_i - \left[\frac{m_e N_e}{m_i N_i} \right] \left(\frac{\omega_{pi}^2}{2} \frac{m_i}{q} \frac{e}{m_e} \right) (x_i - x_e)$$

↑
THE TOTAL MOMENTUM KICK TO EACH SPECIES MUST BE EQUAL & OPPOSITE

$$\omega_{\beta 0} \equiv v_z k_{\beta 0}$$

$$\Rightarrow \frac{d^2 x_i}{dt^2} = -\omega_{po}^2 x_i - f \frac{\omega_{pi}^2}{2} (x_i - x_e)$$

HERE $f \equiv \frac{e N_e}{q N_i}$ = fractional neutralization

Now $\frac{d}{dt}$ = total derivative = $\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z}$

⇒ THE ION & ELECTRON EQUATIONS MAY BE WRITTEN

$$\left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z} \right)^2 x_i = -\omega_{po}^2 x_i - f \frac{\omega_{pi}^2}{2} (x_i - x_e)$$

$$\frac{d^2}{dt^2} x_e = -\frac{\omega_{pe}^2}{2} \left(\frac{m_i}{q} \frac{e}{m_e} \right) (x_e - x_i)$$

Now let $X_e = X_e \exp[i(\omega t - kz)]$; $X_i = X_i \exp[i(\omega t - kz)]$

$$\Rightarrow (-\omega^2 + 2\omega kv_z - k^2 v_z^2) X_i = -\omega_{pi}^2 X_i - f \frac{\omega_{pi}^2}{z} (X_i - X_e)$$

$$-\omega^2 X_e = -\frac{\omega_{pi}^2}{z} \left(\frac{m_i e}{m_e q} \right) (X_e - X_i)$$

$$\Rightarrow \left[(\omega - kv_z)^2 - \omega_{po}^2 - f \frac{\omega_{pi}^2}{z} \right] X_i = -\frac{f \omega_{pi}^2}{z} X_e$$

$$\left[\omega^2 - \frac{\omega_{pi}^2}{z} \left(\frac{m_i e}{m_e q} \right) \right] X_e = -\frac{\omega_{pi}^2}{z} \left(\frac{m_i e}{m_e q} \right) X_i$$

Multiplying the above equations and dividing by $X_e X_i$, yields the dispersion relation:

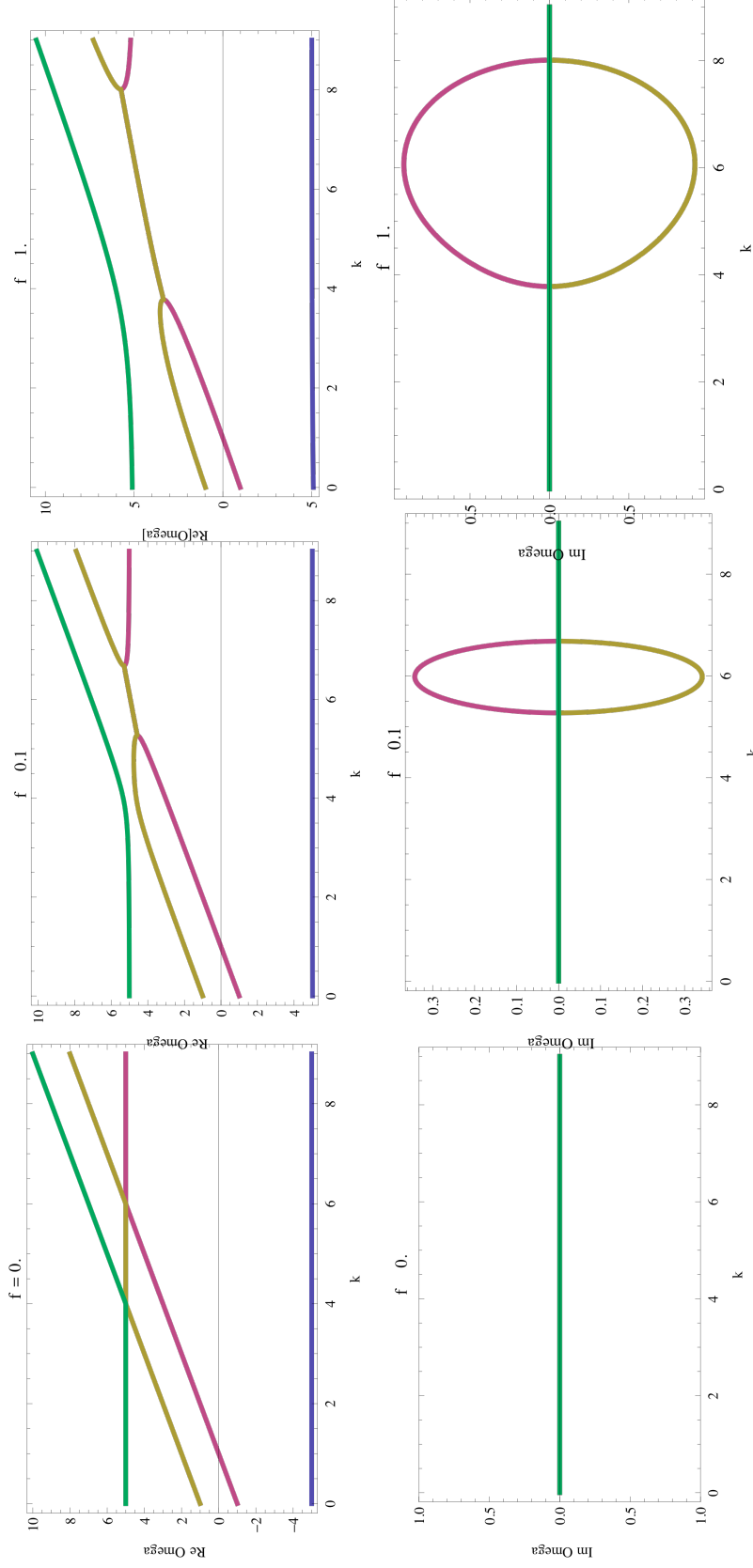
$$\underbrace{\left[(\omega - kv_z)^2 - \omega_{po}^2 - f \frac{\omega_{pi}^2}{z} \right]}_{\text{ION BETATRON FREQUENCY (INCREASED BY SIGN CHANGE OF ELECTRON)}} \underbrace{\left[\omega^2 - \frac{\omega_{pi}^2}{z} \left(\frac{m_i e}{m_e q} \right) \right]}_{\text{ELECTRON OSCILLATING IN POTENTIAL WELL OF ION}} = \underbrace{\frac{f \omega_{pi}^4}{4} \left(\frac{m_i e}{m_e q} \right)}_{\text{COUPLING}}$$

So a beam perturbation with high spatial frequency undergoing betatron oscillations in the comoving frame, $kv_z - \omega \approx \sqrt{\omega_{po}^2 + f \frac{\omega_{pi}^2}{z}}$ will resonate with electrons oscillating in the ion well if

$$\omega \approx \frac{\omega_{pi}}{\sqrt{z}} \sqrt{\frac{m_i e}{m_e q}}$$

Giving rise to instability!

Dispersion relation for two stream instability ($m_e/m_i=0.04$; $\omega_{\beta 0} = \omega_{\beta i}/2^{1/2}=1$; $\nu=1$)



FROM DAVIDSON & QIN (20)
 PHYSICS OF INTENSE CHARGED
 MULTIPLE BEAMS, 2001.

10.4] Instability in Intense Particle Beams 513

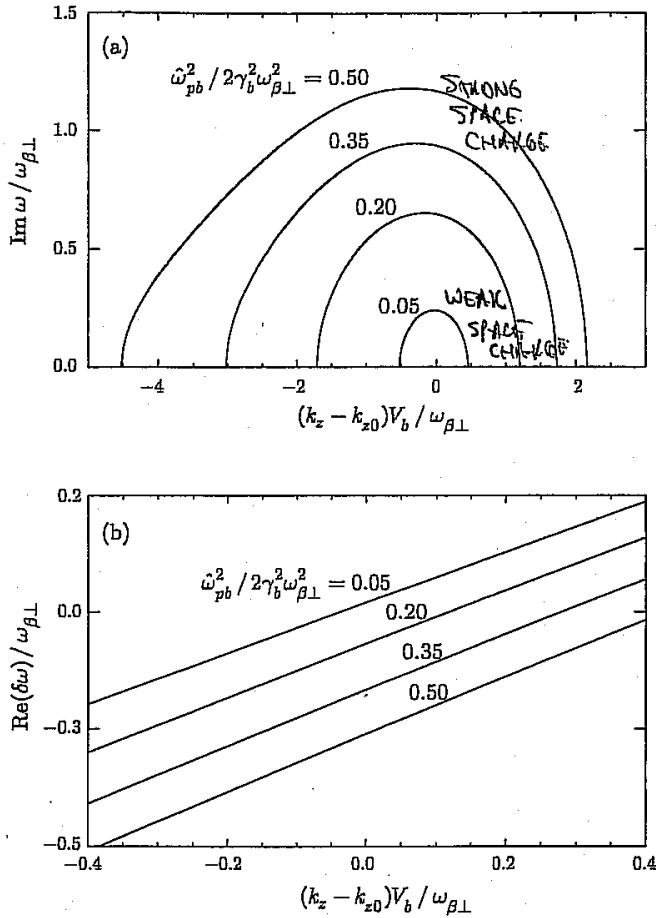


Figure 10.11. Plots of (a) normalized growth rate ($Im\omega/\omega_{\beta\perp}$), and (b) normalized real frequency ($Re\omega - \omega_e$)/ $\omega_{\beta\perp}$ versus shifted axial wavenumber $(k_z - k_{z0})V_b/\omega_{\beta\perp}$ obtained from the dispersion relation (10.103) for the unstable branch with positive real frequency. System parameters correspond to $v_{T\parallel b} = 0 = v_{T\parallel e}$, $m_b/m_e = 1836$ (protons), $(\gamma_b - 1)m_b c^2 = 800$ MeV, $r_b/r_w = 0.5$, and $f = 0.1$. Curves are shown for several values of normalized beam intensity $\omega_{pb}^2/2\gamma_b^2\omega_{\beta\perp}^2$ ranging from 0.05 to 0.5.

$$k_{z0} V_z = \omega \mp \sqrt{\omega_{p0}^2 + f\omega_i^2/2}; \quad \omega = \frac{\omega_{pe}}{2} \sqrt{\frac{m_i}{m_e}}$$

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From Dawson & Qin, 2001

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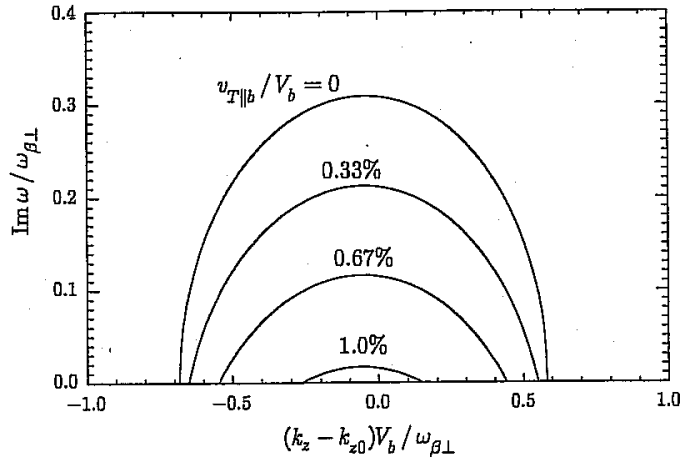


Figure 10.12. Plot of normalized growth rate ($Im\omega/\omega_{\beta\perp}$), and normalized real frequency ($Re\omega - \omega_e$)/ $\omega_{\beta\perp}$ versus positive real frequency. System parameters correspond to $\tilde{\omega}_{pb}^2/2\gamma_b^2\omega_{\beta\perp}^2 = 0.07$, $v_{T||e} = v_{T||b}$, $m_b/m_e = 1836$ (protons), $(\gamma_b - 1)m_b c^2 = 800$ MeV, $r_b/r_w = 0.5$, and $f = 0.1$. Curves are shown for several values of normalized ion thermal spread $v_{T||b}/V_b$ ranging from 0 to 0.01.

velocity V_b [Eq. (10.105)], it is expected that Landau damping by parallel ion kinetic effects can have a strong stabilizing influence at modest values of $v_{T||b}/V_b$. That this is indeed the case is evident from Fig. 10.12, which shows a substantial reduction in maximum growth rate and eradication of the instability over the instability bandwidth as $v_{T||b}/V_b$ is increased from 0 to 0.01.

We now make use of the nonlinear particle perturbative simulation method [60, 61] described in Sec. 8.5 to illustrate several important properties of the two-stream instability in intense beam systems (Sec. 10.4.2).

PREVENTIVE MEASURES (from J. Weid & L. Macek, GEN electron cloud workshop 2003)

- SUPPRESS ELECTRON GENERATION

- SURFACE TREATMENT OF THE VACUUM PIPE

- KICKER MAGNETS IN GAPS

- VACUUM VOLTS SCREENED TO REDUCE E-Field

- CLEANING ELECTRODES

- HIGH VACUUM

- SOLENOIDS - TO REDUCE MULTIBUNCHING

SUMMARY OF ELECTRON, GAS, PRESSURE, & SCATTERING EFFECTS

1. COULOMB COLLISIONS WITHIN BEAM CAN TRANSFER ENERGY FROM \perp TO \parallel AND PROVIDE LOWER LIMIT ON T_{\parallel} , HIGHER THAN FLOW ACCELERATIVE COOLING.
2. COULOMB INTERACTIONS WITH RESIDUAL GAS NUCLEI PROVIDE A SOURCE OF EMITTANCE GROWTH (BUT NOT IMPORTANT FOR HIGH CURRENT AND LONG RESIDENCE TIMES).
3. PRESSURE INSTABILITY FROM DESOLATION OF RESIDUAL GAS BY STRIPPED BEAM IONS HITTING WALL OR BEAM-IONIZED RESIDUAL GAS ATOMS, FORCED TO WALL BY E-FIELD OF BEAM. LIMITS CURRENT IN RINGS OR HIGH VELOCITY LINAC.
4. ELECTRONS CAN CASCADE AND REACH A "QUASI" EQUILIBRIUM (POPULATION OF SIMILAR LINE CHARGE TO THE ION BEAM). ELECTRON-ION TWO STREAM INSTABILITY IS UNSTABLE, AND CAN LEAD TO TRANSVERSE INSTABILITY, SIMILAR TO WHAT IS OBSERVED IN SOME PROTON RINGS.