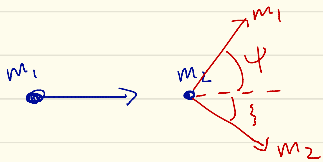
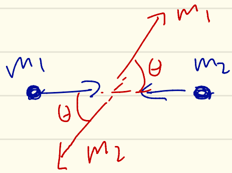
Recap: Elastic ($T_i = T_f$) Collisions



LAB frame



CM frame

$$\boxed{\tan\psi = \frac{\sin\theta}{\frac{m_1}{m_2} + \cos\theta}}$$

* Limiting cases

1.) $m_2 \to \infty \quad \left(\frac{m_1}{m_2} \to 0\right)$

$$\boxed{\Rightarrow \psi = \theta}$$ (m_2 fixed scattering center)

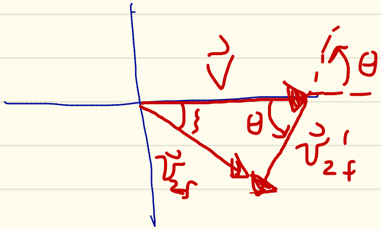
2.) $\frac{m_1}{m_2} \to \infty \qquad \boxed{\Rightarrow \psi = 0}$

3.) $\frac{m_1}{m_2} = 1 \quad \Rightarrow \tan\psi = \frac{\sin\theta}{1+\cos\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\theta/2} = \tan\frac{\theta}{2}$

$$\Rightarrow \boxed{\psi = \frac{\theta}{2}}$$

* Can follow the same steps to find relation between ξ + θ

$$\vec{v}_{2_f} = \vec{V} + \vec{v}\,'_{2f}$$



$$\left.\begin{aligned} \underline{Y}: \quad & v_{2f} \sin\xi = v'_{2f} \sin\theta \\ \underline{X}: \quad & v_{2f} \cos\xi = V - v'_{2f} \cos\theta \end{aligned}\right\} \Rightarrow \tan\xi = \frac{\sin\theta}{\left(\frac{V}{v'_{2f}}\right) - \cos\theta}$$

* but

$$v_i^{rel} = v_f^{rel}$$

$$|\vec{v}\,'_{1i} - \vec{v}\,'_{2i}| = |\vec{v}_{1i} - \cancelto{0}{\vec{v}_{2i}}| = |\vec{v}\,'_{1f} - \vec{v}\,'_{2f}|$$

* but $\vec{p}\,'_{1f} = -\vec{p}\,'_{2f} \Rightarrow \boxed{\vec{v}\,'_{1f} = -\frac{m_2}{m_1}\vec{v}\,'_{2f}}$

* from last lecture, $\boxed{v'_{1f} = \frac{m_2}{M} v_{1i} = \frac{m_2}{M} \times \frac{M}{m_1} V = \frac{m_2}{m_1} V}$

$$\Rightarrow \frac{V}{v'_{2f}} = 1 \qquad \Rightarrow \tan\xi = \frac{\sin\theta}{1 - \cos\theta} = \cot\frac{\theta}{2} = \tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$\Rightarrow \boxed{2\xi = \pi - \theta}$$

* For $m_1 = m_2$, $\psi = \frac{\theta}{2}$

$$\therefore \; \xi = \frac{\pi}{2} - \frac{\theta}{2} \Rightarrow \boxed{\xi + \psi = \frac{\pi}{2}} \; *$$

$$\Rightarrow \vec{v}_{1f} \cdot \vec{v}_{2f} = 0$$

* A few words about inelastic collisions

* Still have $\vec{P}_i = \vec{P}_f$ (always!), but no longer $T_i = T_f$

$$Q + T_i = T_f$$

$Q < 0$ Endoergic

$Q > 0$ Exoergic

Coefficient of Restitution

$$\varepsilon = \frac{v_{rel}^f}{v_{rel}^i} = \frac{|\vec{v}_1^f - \vec{v}_2^f|}{|\vec{v}_1^i - \vec{v}_2^i|}$$

$\varepsilon = 1$ elastic
$\varepsilon < 1$ endoergic
$\varepsilon > 1$ exoergic