Practice Midterm Exam #1

Total points = 25. Show all of your work!

1. [6 points]   If **A** = 5**i** and **B** = 3**i** + 4**j** find

(a) [2] **A.B** $5\hat{i} \cdot (3\hat{i} + 4\hat{j}) = 15$

(b) [2] **A$_x$B** $5\hat{i} \times (3\hat{i} + 4\hat{j}) = 20\hat{k}$

(c) [2] The angle θ_{AB} between **A** and **B**.

$$\cos\theta_{AB} = \frac{\bar{A}\cdot\bar{B}}{A\,B} = \frac{15}{5 \cdot \sqrt{3^2+4^2}} = \frac{15}{25} = \frac{3}{5}$$

$$\theta_{AB} = \cos^{-1} 0.6 = 53^0$$

2. [7 points]   Suppose that the frictional force on an object of mass m traveling through a fluid is proportional to the cube of the velocity: F = -mkv^3 where k is a constant (and m is included to make the math a bit easier).

(a) [4] Find the velocity as a function of time, assuming that the initial velocity is v_0 at time t = 0. Neglect gravity.

$$F = ma = m\frac{dv}{dt} \qquad m\frac{dv}{dt} = -mkv^3 \qquad \frac{dv}{dt} = -kv^3$$

$$\frac{dv}{v^3} = -k\,dt \qquad \int_{v_0}^{v}\frac{dv}{v^3} = -k\int_0^t dt = -\frac{1}{2v^2}\Big|_{v_0}^{v} = -kt$$

(b) [3] After what time has the velocity slowed to half the initial velocity?

$$-\frac{1}{2v^2} + \frac{1}{2v_0^2} = -kt \qquad \frac{1}{v^2} - \frac{1}{v_0^2} = 2kt$$

$$\frac{1}{v^2} = \frac{1}{v_0^2} + 2kt \Rightarrow v = \frac{1}{\sqrt{\frac{1}{v_0^2}+2kt}}$$

$$v = \frac{v_0}{2} \qquad \frac{1}{\sqrt{\frac{1}{v_0^2}+2kt}} = \frac{v_0}{2}$$

$$\frac{1}{v_0^2} + 2kt = \frac{4}{v_0^2}$$

$$2kt = \frac{3}{v_0^2} \qquad t = \frac{3}{2kv_0^2}$$

Note: There is another question on the next page!

3. [12 points] An object of mass m_0 = 30 kg. is launched at time t = 0 with a horizontal velocity of 40.0 m/s. (There is initially no vertical component to the velocity.)

(a) [2] What is the kinetic energy of the object, K_i (in Joules)?

$$K = \frac{1}{2} m v_0^2 = \frac{1}{2} 30\,\text{kg} \left(40 \frac{m}{s}\right)^2 = 30 \times 40 \times 20$$
$$= 24000\ J$$

(b) [4] If the initial height of the object is h = 1000 m, what is the expected range, R (in meters), before it hits the ground? (Use the x origin as the point of launch, and use g = 9.81 m.s^{-2}.)

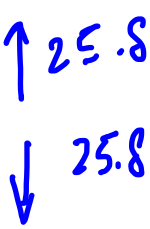
$$R = v_0 t \qquad h = \frac{1}{2} g t^2 \qquad t = \sqrt{\frac{2h}{g}}$$

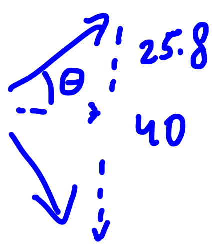
$$R = v_0 \sqrt{\frac{2h}{g}} = 40 \frac{m}{s} \sqrt{\frac{2 \times 1000\,m}{9.81\,m/s^2}} = 571\ m$$

Unfortunately, immediately after the launch, the object explodes into two fragments (each of mass equal to one-half of the original object ($m_1 = m_2 = m_0/2$ = 15 kg) i.e. we are neglecting the mass of the explosive material). The explosion contributes an additional energy of E_{ex} = 10.0 kJ (10000 Joules). The two fragments are ejected at right angles to the original line of flight of the initial object i.e. vertically in the CM frame, fragment m_1 straight up and fragment m_2 straight down.

(c) [6] Immediately after the explosion, what is the velocity (magnitude and angle relative to the horizontal) of fragment m_1 relative to an observer on the ground?

$$\frac{1}{2} m_1 v'^2 = \frac{1}{2} E_x \qquad m_1 v'^2 = E_x \qquad v' = \sqrt{\frac{E_x}{m_1}}$$





$$= \sqrt{\frac{10000\,J}{15\,kg}} = 25.8 \frac{m}{s}$$

$$v^y = v' = 25.8$$

$$\vec{v} = \vec{V} + \vec{v}'$$

$$v^x = V = 40$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{25.8^2 + 40^2} = 47.6$$

$$\tan\theta = \frac{v^y}{v^x} = \frac{25.8}{40}$$

$$\theta = \tan^{-1}(25.8/40) = 32.8°$$