Name: Solutions **Student ID:** ____________

S14 PHY321: Midterm 1

February 7, 2014

NOTE: Show all your work in a neat and logical fashion to maximize your partial credit points. *No credit* will be given for unsupported answers.

Turn the front page only when advised by the instructor!

Total points for this exam: **25**

1. For the vectors $\vec{A} = 4\hat{i} - 5\hat{k}$ and $\vec{B} = 6\hat{j}$, find

(a) [2 pts] $\vec{C} = \vec{A} \times \vec{B}$

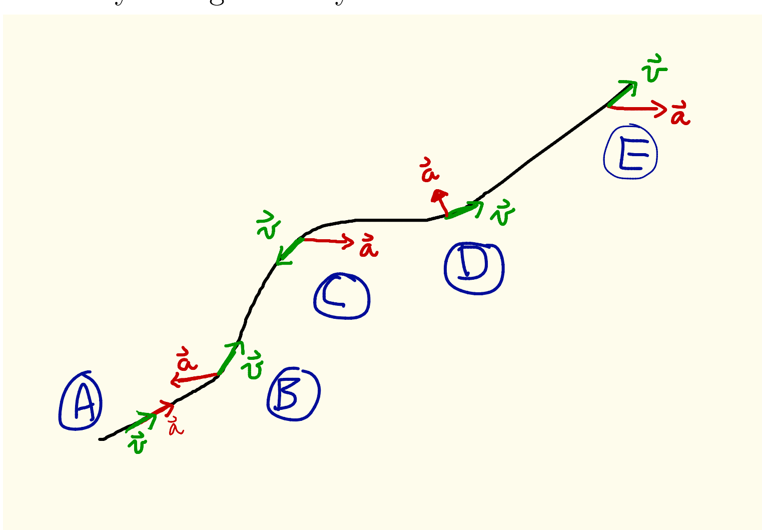
$$\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & -5 \\ 0 & 6 & 0 \end{vmatrix} = 30\hat{i} + 24\hat{k}$$

(b) [2 pts] the angle between $\vec{A}$ and positive direction of the x-axis.

$$A_x = \vec{A}\cdot\hat{i} = |\vec{A}|\cos\theta \Rightarrow \theta = \cos^{-1}\left[\frac{A_x}{|\vec{A}|}\right] = \cos^{-1}\left[\frac{A_x}{|\vec{A}|}\right] = \cos^{-1}\left[\frac{4}{\sqrt{41}}\right]$$

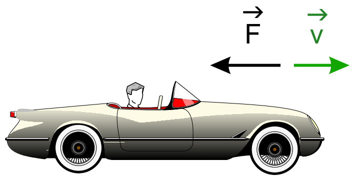
$$\Rightarrow \underline{\theta = 51.3^\circ}$$

(c) [4 pts] A student measures $\vec{a}(t)$ and $\vec{v}(t)$ for a particle moving along the trajectory shown in the figure. Which measurement is obviously wrong and why?



E is wrong. This is because $\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$, where $\vec{a}_{\perp}$ is perp. to trajectory. as discussed in class, non-zero $\vec{a}_{\perp}$ means the trajectory should curve in the direction of $\vec{a}_{\perp}$. at E, we see non-zero $\vec{a}_{\perp}$, but the trajectory is totally straight, so it's wrong.

2. A car of mass m begins to coast at time $t = 0$, while advancing at speed v_0. The car is subject simultaneously to a friction force from the wheels and from the ground, approximately independent of the car's velocity v, and to an air drag force that generally increases with v. These combine to a net force F which opposes the motion of the car. Find the dependence of v on time t, if the opposing force is given by:



(a) [2 pts] $F = -A$, where A is a positive constant,

$$m\frac{dv}{dt} = -A \Rightarrow v(t) - v(0) = -\frac{A}{m}t$$

$$\Rightarrow \boxed{v(t) = v_0 - \frac{A}{m}t}$$

(b) [5 pts] $F = -A - B\,v$, where A and B are positive constants.

$$m\frac{dv}{dt} = -A - Bv \Rightarrow -\frac{dv}{A+Bv} = \frac{1}{m}dt$$

$$\int_{v_0}^{v}\frac{dv}{A+Bv} = \frac{1}{B}\ln A+Bv\Big|_{v_0}^{v} = \frac{1}{B}\ln\left(\frac{A+Bv}{A+Bv_0}\right) \Rightarrow \therefore \frac{1}{B}\ln\left(\frac{A+Bv}{A+Bv_0}\right) = -\frac{t}{m}$$

$$\frac{A+Bv}{A+Bv_0} = e^{-Bt/m}$$

$$\Rightarrow A+Bv = (A+Bv_0)\,e^{-Bt/m}$$

$$\Rightarrow \boxed{v(t) = \frac{1}{B}(A+Bv_0)e^{-Bt/m} - \frac{A}{B}}$$

(c) [2 pts] Complete this problem by examining whether your answer $v(t)$ from 2b reduces to that from 2a when $B \to 0$.

$$e^{-Bt/m} \approx 1 - \frac{Bt}{m} \quad \text{as } B \to 0 \quad \therefore v(t) \approx \left(\frac{A}{B} + v_0\right)\left(1 - \frac{Bt}{m}\right) - \frac{A}{B}$$

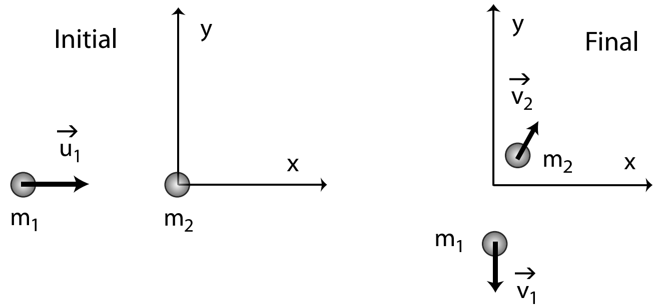
$$= \cancel{\frac{A}{B}} - \frac{A}{B}\frac{Bt}{m} + v_0 - \frac{v_0 Bt}{m} - \cancel{\frac{A}{B}}$$

taking now $B \to 0$

$$\underline{v(t) \to v_0 - \frac{A}{m}t} \quad \checkmark$$

(Over)

3. A particle of mass m_1 and velocity $\vec{u}_1$ strikes *elastically* a particle of mass $m_2 > m_1$, at rest. After the collision, the particle m_1 emerges at a right angle to its original direction, as shown in the figure.



(a) [1 pt] What is the final velocity component v_{2x} of particle m_2, in terms of the particle masses and u_1?

$m_1 u_1 = m_2 v_{2x}$ by $P_x^{in} = P_x^{fin}$

$\boxed{v_{2x} = \frac{m_1}{m_2} u_1}$

(b) [1 pt] What is the velocity component v_{2y} of particle m_2, in terms of the masses and final speed v_1?

$P_y^{in} = P_y^{fin} = 0 = m_2 v_{2y} + m_1 v_{1y} \Rightarrow \boxed{v_{2y} = -\frac{m_1}{m_2} v_{1y}}$

(c) [4 pts] Consider the energy and find v_1 in terms of the data for the initial state, i.e. the masses and u_1.

$E^{in} = E^{fin} \Rightarrow \frac{m_1}{2} u_1^2 = \frac{m_1}{2} v_1^2 + \frac{m_2}{2} v_2^2$

$v_1^2 = u_1^2 - \frac{m_2}{m_1} v_2^2$ but $v_2^2 = v_{2x}^2 + v_{2y}^2 = \left(\frac{m_1}{m_2}\right)^2 u_1^2 + \left(\frac{m_1}{m_2}\right)^2 v_1^2$

$\therefore v_1^2 = u_1^2 - \frac{m_2}{m_1}\left(\frac{m_1}{m_2}\right)^2 u_1^2 - \frac{m_2}{m_1}\left(\frac{m_1}{m_2}\right)^2 v_1^2$

$\Rightarrow v_1^2\left(1 + \frac{m_1}{m_2}\right) = u_1^2\left(1 - \frac{m_1}{m_2}\right) \Rightarrow \boxed{v_1 = u_1\sqrt{\frac{1 - m_1/m_2}{1 + m_1/m_2}}}$

(d) [2 pts] If $m_1 = 1\,\text{kg}$, $m_2 = 4\,\text{kg}$ and $u_1 = 3\,\text{m/s}$, what is the net energy (in Joules) for the final state?

$E^{in} = E^{out} = \frac{1}{2} m_1 u_1^2 = \frac{1}{2}(1\text{kg})(9\frac{m^2}{s^2}) = \frac{9}{2} = 4.5$ Joules

Scratch paper