PHY321 Homework Set 1

1. [5 pts] Expand $\sin x$ about the point $x = \pi/4$. Hint: Represent the function as $\sin x = \sin(y + \pi/4)$ and assume y to be small.

2. [5 pts] For the two vectors

$$\vec{A} = \hat{i} - \hat{j} + 2\hat{k}\,, \quad \text{and} \quad \vec{B} = -2\hat{i} + \hat{j} + 3\hat{k}\,,$$

find

   (a) $\vec{A} - \vec{B}$ and $|\vec{A} - \vec{B}|$,

   (b) component of $\vec{B}$ along $\vec{A}$,

   (c) angle between $\vec{A}$ and $\vec{B}$,

   (d) $\vec{A} \times \vec{B}$,

   (e) $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$.

3. [10 pts] For a hill the elevation in meters is given by $z = 10 + 0.5\,x + 0.25\,y + 0.5\,x\,y - 0.25\,x^2 - 0.5\,y^2$, where x is the distance east and y is the distance north of the origin.

   (a) Where is the top of the hill and how high is it?

   (b) How steep is the hill at $x = y = 1$, i.e. what is the angle between a vector perpendicular to the hill and the z axis?

   (c) In which compass direction is the slope at $x = y = 1$ steepest? Indicate whether the angle you provide is the angle measured in the standard way in the counter-clockwise direction from x-axis (east) or whether it is the compass azimuth. The compass azimuth is normally measured in the *clockwise* direction from *north*.

4. [5 pts] Consider action of two forces, $\vec{F}^A(\vec{r})$ and $\vec{F}^B(\vec{r})$, on a particle. These forces depend on particle position $\vec{r}$ and their components are given by

$$F_x^A = F_x^B = y^2\,, \quad F_y^A = 2xy\,, \quad F_y^B = xy\,, \quad F_z^A = F_z^B = 0\,,$$

   where the force components are in newtons and coordinates are in meters.

   (a) Compute the work in joules done by the two forces, $W^A = \int \vec{F}^A \cdot \mathrm{d}\vec{r}$ and $W^B = \int \vec{F}^B \cdot \mathrm{d}\vec{r}$, on the particle moving within the x-y plane along a parabolic trajectory $y = x^2$ from $(x, y) = (0, 0)$ to $(x, y) = (1, 1)$. The coordinates in the equation for the trajectory are in meters. Hint: Under the integral write $\vec{F}^A \cdot \mathrm{d}\vec{r} = F_x\,\mathrm{d}x + F_y\,\mathrm{d}y = \left(F_x + F_y\,\frac{\mathrm{d}y}{\mathrm{d}x}\right)\mathrm{d}x$ and carry out integration over x.

   (b) Compute the work in joules done by the two forces, on the particle moving along another trajectory joining $(0, 0)$ and $(1, 1)$ and consisting of two straight at right angles to each other, first at constant y to $(x, y) = (1, 0)$ and then at constant x to $(1, 1)$. Compare the results to those obtained in 4a. Hint: Sketch the trajectory.

(c) Comment on your results. Can you draw any conclusions about the nature of the two forces?

5. [5 pts] Corners of a uniform horizontal flat triangular plate are at $(x, y) = (0, 0)$, $(0, 1)$ and $(1, 2)$, see the figure. Find the coordinates (X, Y) of the center of mass of that plate, by directly carrying out integrations within the mathematical definition for location of the center, $\vec{R} = \int \mathrm{d}^2\, r\, \vec{r} / \int \mathrm{d}^2\, r$.