$\boxed{\text{Solutions}}$

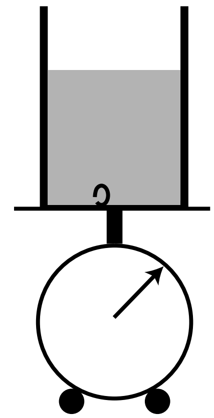
PHY321 Homework Set 4

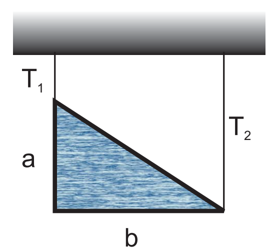
1. [~~5 pts~~ 10 pts] A beaker filled with water is placed on a scale, see figure. With the beaker and water, the dial of the scale is adjusted so that the scale reads zero. No adjustments to the dial are made during the subsequent actions. The dial is in grams.

   2 (a) A cork of mass 20 g and density of $0.25\,\mathrm{g/cm^3}$ is dropped into the beaker so that it floats there, not touching the walls of the beaker. What is the reading of the scale now?

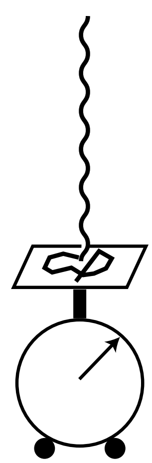
   4 (b) Subsequently a rod of negligible diameter is used to push the cork entirely underwater, without touching the walls. What is the reading of the scale now? Explain your reasoning.

   4 (c) In the subsequent step, in place of the rod a string is used to tie the cork to a hook at the bottom of the beaker, keeping the cork submerged. With negligible weight and volume of the string, what is the reading for the scale? What is the tension in the string?





2. [5 pts] A sign in the shape of a right-angle triangle, with legs of $a = 50\,\mathrm{cm}$ and $b = 110\,\mathrm{cm}$, is made out of steel sheet with aerial density of $\sigma = 1.20\,\mathrm{g/cm^2}$. The sign is suspended with two vertical wires from its corners, so that the triangle's legs are oriented in the horizontal and vertical directions, respectively, as shown. Determine the tensions T_1 and T_2 in the two wires.



3. [5 pts] A chain of mass M and length L is suspended vertically with its lower end touching the indicated scale. The chain is released and falls onto the scale. Determine the force read by the scale when the length x of the chain has fallen. Neglect the size of individual links.

4. [10 pts] N people, each of mass m_p, stand on a railway flatcar of mass m_C. They jump off one end of the flatcar with velocity u relative to the car. The car rolls in the opposite direction without friction.

4 (a) What is the final velocity of the car if all the people jump at the same time?

4 (b) What is the final velocity of the car if the people jump off one at a time? Leave the answer as a sum of terms.

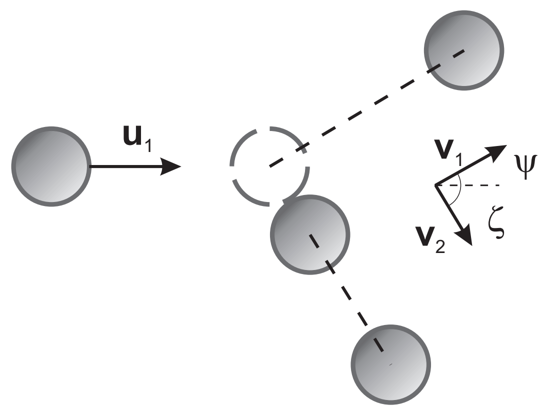
2 (c) Does case 4a or 4b yield the largest final velocity of the flat car?

5. [5 pts] A puck of mass of 0.200 kg moving at $u_1 = 3.0\,\text{m/s}$ approaches an identical puck that is stationary on frictionless ice. After the collision, the first puck leaves with a speed v_1 at $\psi = 30°$ relative to the original line of motion; the second puck leaves with speed v_2 at $\zeta = 60°$.

3 (a) Determine v_1 and v_2.

3 (b) What are the relative speeds of the pucks before and after the collision? Is the collision elastic or inelastic?

4 (c) What are the angles and magnitudes the final puck velocities in the CM system of this collision?



6. [~~5~~ 10 pts] A block of mass $m_1 = 1.00\,\text{kg}$, moving at a speed $u_1 = 4.00\,\text{m/s}$, collides with another block of mass $m_2 = 10.0\,\text{kg}$ at rest. The lighter block comes to rest after the collision.

3 (a) What is the speed of the heavier block after the collision?

3 (b) What is the coefficient of restitution for the collision?

2 (c) What is the reduced mass for the system?

2 (d) How much of the relative energy has been dissipated for this collision, in absolute terms and as percentage of original CM energy?

1) Beaker filled with H_2O

a) System mass increased by cork mass $= 20g$. The reading increases by $20g$.

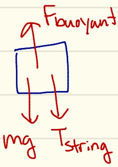
b) We have 

$$\Rightarrow \underbrace{F_{rod} + mg}_{\text{This is the net force on the beaker.}} = F_{buoyant} = \rho_{H_2O} V_{cork}\, g \quad \text{(Archimedes)}$$

This is the net force on the beaker.

which means the scale will read

$$\boxed{\frac{F_{beaker}}{g} = V_{cork}\,\rho_{H_2O} = \frac{M_{cork}}{\rho_{cork}}\rho_{H_2O} = \left(\frac{20g}{.25\, g/cm^3}\right) \times .1 \frac{kg}{cm^3} = 80\, g}$$

c) We have 

$$\Rightarrow T_{string} + mg = \rho_{H_2O} V_{cork}\, g$$

$$\Rightarrow T_{string} = \rho_{H_2O} V_{cork}\, g - mg$$

$$= \left(.1 \frac{kg}{cm^3}\right)\left(\frac{20g}{.25\, g/cm^3}\right)\left(9.8 \frac{m}{s^2}\right) - (20g)\left(9.8 \frac{m}{s^2}\right)$$

$$= 78.4\, N - .196\, N$$

$$\boxed{T_{string} = 78.2\, N}$$

* Even though T_{string} the same as F_{rod} in part b, here it makes no impact on the scale reading. (F_{rod} was an external force, whereas T_{string} is an internal force.) Thinking in term of the action-reaction of the forces in the present case, it's clear that the ~~[illegible]~~ scale just reads 20 g.

2) Balancing Sign

First find the CM of the sign

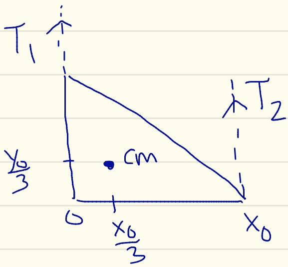
$$\bar X = \frac{\int \sigma x\,dx\,dy}{\int \sigma\,dx\,dy} = \frac{\int x\,dx\,dy}{A}$$

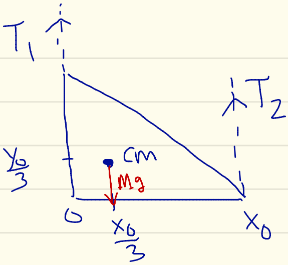
$$\bar Y = \frac{\int y\,dx\,dy}{A} \qquad A = \tfrac12 x_0 y_0$$

$$\therefore \bar X = \frac{\int_0^{x_0} x\,dx \int_0^{\frac{y_0}{x_0}x} dy}{\frac12 x_0 y_0} = \frac{2}{x_0 y_0}\,\frac{y_0}{x_0}\int_0^{x_0} dx\, x^2 = \frac23 x_0$$

$$\bar Y = \frac{\int_0^{x_0} dx \int_0^{\frac{y_0}{x_0}x} y\,dy}{\frac12 x_0 y_0} = \frac{\int_0^{x_0} dx \left(\frac{y_0}{x_0}\right)^2 \frac{x^2}{2}}{\frac12 x_0 y_0} = \frac{\left(\frac{y_0}{x_0}\right)^2 \frac{x_0^3}{6}}{\frac12 x_0 y_0} = \frac{\frac{y_0^2 \cancel{x_0}}{6}}{\frac12 \cancel{x_0} \cancel{y_0}} = \frac{y_0}{3}$$

So we have





* at equilibrium, there must be no net torques about the 2 lower corners:

$$0 = (Mg)\left(\tfrac{2}{3}x_0\right) - T_1 x_0 \qquad \text{(right corner)}$$

$$0 = (Mg)\left(\tfrac{1}{3}x_0\right) - T_2 x_0 \qquad \text{(left corner)}$$

1st eq => $\boxed{T_1 = \frac{2}{3}Mg = \frac{2}{3}(3.3\,\text{kg})(9.81\,\frac{\text{m}}{\text{s}^2}) = 21.6\ \text{N}}$

(used $M = A\sigma = \frac{1}{2} \times 50\,\text{cm} \times 110\,\text{cm} \times 1.20\,\text{g}/\text{cm}^2 = 3.3\ \text{kg}$)

2nd eq => $\boxed{T_2 = \frac{1}{3}Mg = \frac{1}{2}T_1 = 10.8\,\text{N}}$

3) Falling Chain

total force on scale = Normal force + Impulse force

$$F = mg + \frac{dp}{dt}$$

let $\lambda = \frac{\text{mass}}{\text{length}}$ $\Rightarrow m = \lambda x$ for the length x that has fallen

What is $\frac{dp}{dt}$? In time dt, a mass of $\lambda(v\,dt)$ drops to the scale.

$\therefore$ this imparts a momentum $dp = \lambda(v\,dt)v = \lambda v^2 dt$ to the scale

$$\therefore \frac{dp}{dt} = \lambda v^2$$

$$\Rightarrow F = mg + \frac{dp}{dt} = \lambda x g + \lambda v^2$$

finally, use energy conservation to find what v is. If we consider a piece δm that has fallen x

$$\cancel{\delta m}\, g x = \frac{1}{2}\cancel{\delta m}\, v^2 \Rightarrow v^2 = 2gx$$

$$\boxed{\therefore F = \lambda g x + 2\lambda g x = 3\lambda x g \text{ on the scale}}$$

4.) N people on a railway car

a.) Jump off all at once: Use momentum conservation

$P_i = P_f$

M_T = mass of train car

m_p = mass of 1 person

u = speed of person w.r.t. train

v_T = speed of train w.r.t. ground

$u - v_T$ = speed of person w.r.t. ground

$$0 = N m_p (u - v_T) - M_T v_T$$

$$\Rightarrow 0 = N m_p u - v_T (N m_p + M_T)$$

$$\boxed{\Rightarrow v_T = \frac{N m_p}{N m_p + M_T} u}$$

b.) Jump 1-by-1:

1st jumper:

$$P_i = P_f$$

$$0 = m_p (u - v_T^{(1)}) - (M_T + (N-1) m_p) v_T^{(1)}$$

$$0 = m_p u - v_T^{(1)} (\cancel{m_p} + M_T + (N - \cancel{1}) m_p)$$

$$\therefore v_T^{(1)} = \frac{m_p u}{N m_p + M_T}$$

2nd jumper:

$$P_i = P_f$$

$$-(M_T + (N-1) m_p) v_T^{(1)} = m_p (u - v_T^{(2)}) - (M_T + (N-2) m_p) v_T^{(2)}$$

$$= m_p u - v_T^{(2)} (M_T + (N-1) m_p)$$

$$\Rightarrow v_T^{(2)} = v_T^{(1)} + \frac{m_p u}{M_T + (N-1) m_p}$$

$$\Rightarrow v_T^{(2)} = v_T^{(1)} + \frac{m_p u}{M_T + (N-1)m_p} = \frac{m_p u}{M_T + N m_p} + \frac{m_p u}{M_T + (N-1)m_p}$$

The pattern is obvious. We have

$$v_T^{(n)} = m_p u \times \sum_{k=1}^{n} \frac{1}{M_T + (N+1-k)m_p}$$

Therefore, after all N jump we have

$$\boxed{v_T = m_p u \times \sum_{k=1}^{N} \frac{1}{M_T + (N+1-k)m_p} = \frac{m_p u}{M_T} \sum_{k=1}^{N} \frac{1}{1 + (N+1-k)\frac{m_p}{M_T}}} \quad (*)$$

c) Which is bigger, v_T in part a or b?

$$v_T^a = \frac{N m_p}{N m_p + M_T} u = \frac{m_p u}{M_T} \frac{N}{1 + N\frac{m_p}{M_T}} = \frac{m_p u}{M_T} \sum_{k=1}^{N} \frac{1}{1 + N\frac{m_p}{M_T}}$$

* but $\underbrace{\sum_{k=1} \frac{1}{1 + N\frac{m_p}{M_T}}}_{(a)} < \underbrace{\sum_{k=1}^{N} \frac{1}{1 + (N+1-k)\frac{m_p}{M_T}}}_{(b)}$ for all $N > 1$

Therefore, v_T in part b is bigger. **

5.) Hockey Pucks

a) $\cancel{m}u_1 = \cancel{m}v_1\cos\psi + \cancel{m}v_2\cos\xi \qquad (p_x^i = p_x^f)$

$0 = \cancel{m}v_1\sin\psi - \cancel{m}v_2\sin\xi \qquad (p_y^i = p_y^f)$

$\Rightarrow v_2 = v_1\frac{\sin\psi}{\sin\xi}$ → plug into 1st eqn:

$$u_1 = v_1\cos\psi + v_1\frac{\sin\psi\cos\xi}{\sin\xi}$$

$$= v_1\frac{\sin\xi\cos\psi + \cos\xi\sin\psi}{\sin\xi} = \frac{v_1\sin(\xi+\psi)}{\sin\xi} = v_1\frac{\sin 90°}{\sin 60°}$$

$$= v_1\frac{2}{\sqrt{3}}$$

$$\boxed{\therefore v_1 = u_1\frac{\sqrt{3}}{2} = 2.6\ \frac{m}{s}} \ *$$

$$\boxed{v_2 = v_1\frac{\sin\psi}{\sin\xi} = 2.6\frac{\sin 30}{\sin 60} = 1.5\ \frac{m}{s}} \ *$$

b) $u_{rel} = |\vec{u}_1 - \cancel{\vec{u}_2}^{\,0}| = 3\ \frac{m}{s}$ (initial relative speed)

$v_{rel} = |\vec{v}_1 - \vec{v}_2| = \sqrt{v_1^2 + v_2^2 - 2\underbrace{\cancel{\vec{v}_1\cdot\vec{v}_2}}_{0}} = \sqrt{(2.6)^2 + (1.5)^2} = 3\ \frac{m}{s}$

Collision is <u>elastic</u> since coeff. of restitution $\varepsilon = \frac{v_{rel}}{u_{rel}} = 1$

c) $\theta = 2\psi = 60°$

other particle moves at $180 - \theta = 120°$.

6.) Inelastic Collision problem

a) $m_1 u_1 = m_2 v_2 \Rightarrow v_2 = \frac{m_1}{m_2} u_1 = \frac{1}{10} \cdot 4 \frac{m}{s} = .4 \frac{m}{s}$

b) $\left.\begin{array}{l} u_{rel} = u_1 = 4 \frac{m}{s} \\ v_{rel} = v_2 = .4 \frac{m}{s} \end{array}\right\} \Rightarrow \varepsilon = \frac{v_{rel}}{u_{rel}} = \frac{.4}{4} = .1$

c) $\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{10}{11} \text{ kg} = .909 \text{ kg}$

d) $\frac{\mu}{2}(u_{rel}^2 - v_{rel}^2) = \frac{.909 \text{ kg}}{2} \cdot \left(4^2 \frac{m^2}{s^2} - (.4)^2 \frac{m^2}{s^2}\right) = 7.2 \text{ Joule}$

relative $\frac{u_{rel}^2 - v_{rel}^2}{u_{rel}^2} = 1 - \varepsilon^2 = 99\%$