Physics 231  Topic 2: One dimensional motion

Alex Brown  September 9, 2015
1) Homework set 00 due Tuesday Sept 8\textsuperscript{th} 10 pm
2) Concept survey due Thursday Sept 10th 10 pm (follow email instructions)
3) Attitude survey due Thursday Sept 10th 10 pm
4) Homework set 01 due Tuesday Sept 15\textsuperscript{th} 10 pm
5) LRC (1248 BPS) (schedule next week)
6) Will start clicker questions that count on Monday Sept 14
Key Concepts: 1D Motion

Position and coordinate systems

Displacement vs. Distance

Velocity
  × Distinguish from speed
  × Average vs instantaneous velocity

Acceleration
  × Relationship to velocity
  × Impact on position in one-dimensional (1D) motion

Constant velocity and constant acceleration equations
  × How to set up & solve problems
  × Understanding graphical representations of distance, velocity, acceleration

Covers chapter 2 in Rex & Wolfson
What mathematical forces govern accelerated motion?

A wooden and stone sphere are dropped from the tower of Pisa. Which one reaches the earth first?

Answer: they hit the ground at the same time!
The Inclined Plane Experiment

Distance traveled goes with the square of time: \( x(t) \approx t^2 \)
Coordinate Systems

We use coordinate systems as a graphical representation of the relationships of measurements:

- Distance vs time
- Espressos sold per shift
- Cell phone minutes used per day

(a) Two-dimensional coordinate system—could be used to represent motion in two dimensions.

(b) Three-dimensional coordinate system—could be used for motion in three dimensions.
Point Particle Treatment

Choosing an $x$-axis in the direction of motion makes the $y$-axis unnecessary.

The skier is represented by a simple, point-like object.
1. Walk from origin (your house) to friend’s house. Displacement:
\[ \Delta x = x_2 - x_1 = 60 \text{ m} - 0 = 60 \text{ m} \]
Position & Displacement

1. Walk from origin (your house) to friend’s house. Displacement: \( \Delta x = x_2 - x_1 = 60 \text{ m} - 0 = 60 \text{ m} \)

2. Walk from friend’s house to video store. Displacement: \( \Delta x = x_3 - x_2 = 260 \text{ m} - 60 \text{ m} = 200 \text{ m} \)
Position & Displacement

1. Walk from origin (your house) to friend’s house. Displacement: \( \Delta x = x_2 - x_1 = 60 \text{ m} - 0 = 60 \text{ m} \)

2. Walk from friend’s house to video store. Displacement: \( \Delta x = x_3 - x_2 = 260 \text{ m} - 60 \text{ m} = 200 \text{ m} \)

3. Return from video store to friend’s house. Displacement: \( \Delta x = x_2 - x_3 = 60 \text{ m} - 260 \text{ m} = -200 \text{ m} \)
Position & Displacement

1. Walk from origin (your house) to friend’s house. Displacement: \( \Delta x = x_2 - x_1 = 60 \text{ m} - 0 = 60 \text{ m} \)

2. Walk from friend’s house to video store. Displacement: \( \Delta x = x_3 - x_2 = 260 \text{ m} - 60 \text{ m} = 200 \text{ m} \)

3. Return from video store to friend’s house. Displacement: \( \Delta x = x_2 - x_3 = 60 \text{ m} - 260 \text{ m} = -200 \text{ m} \)

4. Net displacement for entire trip: \( \Delta x = x_2 - x_1 = 60 \text{ m} - 0 = 60 \text{ m} \)

Net displacement is a vector and has direction (sign)
Net displacement for the entire trip is \( 60 + 200 - 200 = 60 \text{ m} \)
Total distance is the sum of the magnitude for each part \( 60 + 200 + 200 = 460 \)
Displacement is not equal to Distance
Equally-spaced photographs of a runner reveal his position as a function of time.
Position vs. Time

Sprinter covers more distance in second 2.0-s interval than in first one.

Equally-spaced photographs of a runner reveal his position as a function of time.
Position vs Time Graph

These measurements can be translated to a “Position vs Time” plot.
Average Velocity

Average velocity = \frac{\text{Displacement}}{\text{Time interval}}

\[
\vec{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}
\]

Sprinter covers more distance in second 2.0-s interval than in first one.

\[
\Delta x = 13.6\text{m} \quad \Delta x = 20.4\text{m}
\]

\[
\Delta t = 2.0\text{s} \quad \Delta t = 2.0\text{s}
\]
Average Velocity

Average velocity = \frac{\text{Displacement}}{\text{Time interval}}

\[ v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \]

Average velocity for 2nd 2.0-s interval:

\[ v_x = \frac{\Delta x}{\Delta t} = \frac{34.0 \text{ m} - 13.6 \text{ m}}{4.0 \text{ s} - 2.0 \text{ s}} = 10.2 \text{ m/s} \]

Average velocity for 1st 2.0-s interval:

\[ v_x = \frac{\Delta x}{\Delta t} = \frac{13.6 \text{ m} - 0.0 \text{ m}}{2.0 \text{ s} - 0.0 \text{ s}} = 6.8 \text{ m/s} \]
Vectors vs Scalars

Average velocity: vector

\[ \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{0 - 30}{1 - 0} = -30 \]

Average speed: scalar

\[ \text{speed} = \frac{\text{distance}}{\text{time}} = \frac{|\Delta x|}{\Delta t} = \frac{|x_f - x_i|}{t_f - t_i} = +30 \]
Speed vs Velocity

A driver commutes from home in the suburbs to work in the city (each way = 60 km). The morning trip takes 60 minutes (1 hour). The drive home takes 90 minutes (1.5 hours).

What was his average speed and average velocity over The whole trip?

a) Average speed: 60 km/h  Average velocity: 48 km/h
b) Average speed: 48 km/h  Average velocity: 0 km/h
c) Average speed: 60 km/h  Average velocity: 0 km/h
d) Average speed: 0 km/h  Average velocity: 60 km/h
e) No way to tell.

Average speed: \( \frac{120 \text{ km}}{(1 + 1.5) \text{ hrs}} = 48 \text{ km/h} \)
Average velocity: \( \frac{0 \text{ km}}{(1 + 1.5) \text{ hrs}} = 0 \text{ km/h} \)
Instantaneous Velocity

“But officer, my average speed was only 25 miles per hour…”

Sometimes we want to know the speed at one particular point in time.
Instantaneous Velocity

The velocity at a single point in time is given by the slope of the line tangent to the x-t curve at that time.

Conceptualize this condition as a distance measurement in a very tiny time interval

\[ v = \lim_{{\Delta t \to 0}} \frac{\Delta x}{\Delta t} \]
Draw a tangent line, then put a triangle with sides $\Delta x$ and $\Delta t$ underneath it (ratio does not depend on the size).

**What is the velocity at point C?**

$v = \Delta x / \Delta t = (105-85)/(3-1)
= 20/2 = 10 \text{ m/s}$

**What is the velocity at point E?**

$v = \Delta x / \Delta t = (35-75)/(4.5-3.5)
= -40/1 = -40 \text{ m/s}$

*Be careful with the sign: velocity is a vector and can be negative!*
A change in slope in the $x$-$t$ graph... 

...means a change in velocity: \textit{Acceleration}
Average acceleration: average change in velocity between B and F

\[ \bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t} \]

\[ \bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{80 - 40}{5 - 1} = 10 \]
Instantaneous Acceleration

acceleration at one point in time:

\[ a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \]

Slope of the tangent to the v-t curve at that point in time at D

\[ a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{57 - 17}{5 - 1} = 10 \]
What is the average acceleration between $t=0$ and $t=1$?

A) 15  
B) -15  
C) 0  
D) infinity

Diagram:

- At $t=0$, $v=20$
- At $t=1$, $v=5$
Relationship of Acceleration, Velocity and Displacement
Acceleration of a Car

60 mph = 27 m/s

D (m)

V (m/s)

A (m/s²)
Which $v$-$t$ diagram matches the $x$-$t$ diagram on the right?
Question

Which $v$-$t$ diagram matches which $a$-$t$ diagram?
Overview up to this point

VECTORS
- Displacement
- Average Velocity
- Instantaneous velocity
- Average acceleration
- Instantaneous acceleration

SCALARS
- Distance
- Average speed
- Instantaneous speed

MOTION DIAGRAMS

Next – equations of motion in 1d
\[ v(t) = v_0 + at \]

Velocity at time \( t \) equals...

Velocity at \( t=t_0=0 \)

Plus the gain in velocity per second multiplied by the time elapsed (every second, the velocity increases with \( a \) m/s)

\[ \bar{v} = \frac{v_0 + v(t)}{2} \]
Constant Acceleration

\[ x(t) = x_o + \bar{v}t = x_o + \left(\frac{v_o + v(t)}{2}\right)t = x_o + v_o t + \frac{1}{2} at^2 \]

Start position plus average velocity multiplied by time

Substitute \( \bar{v} = \frac{v_o + v(t)}{2} \)

Substitute \( v(t) = v_o + at \)
Constant Acceleration

\begin{align*}
v(t) &= v_0 + at \\
x(t) &= x_o + v_o t + \frac{1}{2} at^2 \\
t_o &= 0 \\
x_o &= x(t_o) \\
v_o &= v(t_o) \\
x_f &= x(t) \\
v_f &= v(t) \\
\Delta x &= x_f - x_o
\end{align*}
(A) \[ v_f = v_0 + at \]

(B) \[ \Delta x = v_o t + \frac{1}{2} at^2 \]

(C) \[ \Delta x = v_f t - \frac{1}{2} at^2 \]

(D) \[ \Delta x = \frac{1}{2} (v_o + v_f) t = \bar{v} t \]

(E) \[ \Delta x = \frac{1}{2a} (v_f^2 - v_0^2) \]

You can get C, D and E from A and B.
Constant position

\[ x(t) = x_0 \]

\[ v(t) = 0 \]

\[ a(t) = 0 \]
\[ x(t) = x_0 \]
\[ v(t) = 0 \]
\[ a(t) = 0 \]

\[ x(t) = x_0 + v_0 t \]
\[ v(t) = v_0 \]
\[ a(t) = 0 \]
### Constant position
- \( x(t) = x_0 \)

### Constant velocity
- \( x(t) = x_0 + v_0 t \)
- \( v(t) = v_0 \)
- \( a(t) = 0 \)

### Constant acceleration
- \( x(t) = x_0 + v_0 t + \frac{1}{2} a t^2 \)
- \( v(t) = v_0 + at \)
- \( a(t) = a_0 \)
Galileo’s Free Fall Experiment
Gravitational Free Fall

\[ \Delta x = v_o t + \frac{1}{2} at^2 \]

\[ v(t) = v_o + at \]

\( a \) is the acceleration felt due to gravitation (it is called ‘g’ and its value is 9.81 m/s\(^2\))

\( g \) does not depend on the mass of falling object
A person throws a ball straight up in the air. Which of the following is true?

a) The velocity at the highest point is 0.
b) The acceleration at the highest point is 0.
c) Both velocity and acceleration at the highest point are 0.
d) Neither velocity nor acceleration are 0 at the highest point.
Dave throws a ball straight up with a velocity of 5.0 m/s.

1) How high does it go relative to the height at which it was released?
2) After how much time does it reach that height?

\[
x(t) = x_0 + v_0 t + \frac{1}{2} a t^2
\]
\[
v(t) = v_0 + a t
\]

\[
x(t) = 0 + 5t + \frac{1}{2}(-9.8)t^2 \quad \text{it decelerates}
\]
\[
v(t_{\text{top}}) = 0 = 5 - 9.8t_{\text{top}} \quad \text{at its highest point, the velocity is zero}
\]

\[
t_{\text{top}} = \frac{5}{9.8} = 0.51 \text{ s} \quad \text{time to reach highest point}
\]

\[
x_{\text{top}} = x(t=0.51\text{s}) = 5 \times 0.51 - \frac{1}{2} \times 9.8 \times (0.51)^2 = 1.3 \text{ m}
\]
\[ \Delta x = \frac{1}{2} (v_o + v_f) t = \bar{v} t \]

The area under the v-t curve is equal to the displacement (change of position) of the object.

If the area is negative then the change of position is negative.
\[ v_f = v_0 + at \]

\[ v_f - v_0 = at \]

The area under the a-t curve is equal to the change of velocity.

If the area is negative then the change of velocity is negative (its slowing down).
The figure shows velocity versus time. 

a) What is the displacement after 4 s?
b) What is the average velocity?

a) The distance covered is the area under the v-t diagram. 
\[ \Delta x = \frac{1}{2} \times 2 \times 4 + 1 \times 4 + (2 + \frac{1}{2} \times 1 \times 2) = 11 \text{ m} \]

b) Average velocity = \[ \frac{\Delta x}{\Delta t} = \frac{11 \text{ m}}{4 \text{ s}} = 2.75 \text{ m/s} \]
A vs T Diagram Example

Given this a-t diagram, calculate:

a) Velocity after 4 s
b) Distance traveled in 4 s

The object was originally at rest.

a) \( v(t) = v_0 + at \) (area)

After 2 seconds: \( v(2) = 0 + 2 \times 1 = 2 \text{ m/s} \)
After 4 seconds: \( v(4) = v(2) + 2 \times 2 = 2 + 4 = 6 \text{ m/s} \)

b) for \( x(t) \) use area under v-t curve

After 2 seconds: \( x(2) = 0 + \frac{1}{2} \times 2 \times 2 = 2 \text{ m} \)
After 4 seconds: \( x(4) = x(2) = 2 + \frac{1}{2} (2 \times 4) + 2 \times 2 = 10 \text{ m} \)
Galileo’s Free Fall Experiment

Galileo throws a cannon ball from the tower of Pisa. Ignoring friction what is the distance covered between $t=1$ and $t=3$ seconds? The initial velocity was 0 m/s.

\[ x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \]

\[ = x_0 + v_0 t + \frac{1}{2}(-g)t^2 \]

\[ = h - \frac{1}{2}gt^2 \]

\[ x_0 = h \quad \text{(height of tower)} \]

\[ v_0 = 0 \quad \text{(started at rest)} \]

\[ a = -g \]

\[ (9.81 \text{ m/s}^2 \text{ in the } -x \text{ direction}) \]

\[ \Delta x_1 = x(1) - h = -\frac{1}{2} \times 9.8 \times 1^2 = -4.9 \text{ m} \]

\[ \Delta x_3 = x(3) - h = -\frac{1}{2} \times 9.8 \times 3^2 = -44.1 \text{ m} \]

Distance traveled:

\[ \Delta x = | [x(3)-h] - [x(1)-h] | = |-44.1 + 4.9| = 39.2 \text{ m} \]