NUCLEAR STRUCTURE ASPECTS OF THE DOUBLE BETA DECAY

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OUTLINE

- Basics.
- The $0\nu$ operators.
- The nuclear wave functions; ISM or QRPA?
- ISM results for the $2\nu$ decays
- “State of the Art” ISM and QRPA nuclear matrix elements.
- The role of correlations; pairing vs deformation
- Occupation numbers.
- Digression I: The stability of the ISM predictions.
- Conclusions.
Double beta decay

Some nuclei, otherwise nearly stable, can decay emitting two electrons and two neutrinos \( (2\nu \beta\beta) \) by a second order process mediated by the weak interaction. This decay has been experimentally measured in a few cases.

This process can be observed due to the nuclear pairing interaction that favors energetically the even-even nuclei over the odd-odd ones.
Double beta decays

When the single beta decay to the intermediate odd-odd nucleus is forbidden, the only decay channel open is the $(2\nu \beta\beta)$. For instance, $^{76}\text{Ge}$ decays to $^{76}\text{Se}$ because the decay to $^{76}\text{As}$ is forbidden. The decay probability contains a phase space factor and the square of a nuclear matrix element

$$[T_{1/2}^{2\nu}]^{-1} = G_{2\nu} |M_{GT}^{2\nu}|^2$$

For $^{76}\text{Ge}$, $T_{1/2}^{2\nu} = (1.5 \pm 0.1) \times 10^{21}$ years
The $\beta\beta$ emitters

$\beta\beta$ emitters with $Q_{\beta\beta} > 2$ Mev

<table>
<thead>
<tr>
<th>Transition</th>
<th>$Q_{\beta\beta}$ (keV)</th>
<th>Abundance (%)</th>
<th>$(^{232}Th = 100)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{110}Pd \rightarrow ^{110}Cd$</td>
<td>2013</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>$^{76}Ge \rightarrow ^{76}Se$</td>
<td>2040</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>$^{124}Sn \rightarrow ^{124}Te$</td>
<td>2288</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>$^{136}Xe \rightarrow ^{136}Ba$</td>
<td>2479</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>$^{130}Te \rightarrow ^{130}Xe$</td>
<td>2533</td>
<td>34</td>
<td></td>
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<tr>
<td>$^{116}Cd \rightarrow ^{116}Sn$</td>
<td>2802</td>
<td>7</td>
<td></td>
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<tr>
<td>$^{82}Se \rightarrow ^{82}Kr$</td>
<td>2995</td>
<td>9</td>
<td></td>
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<tr>
<td>$^{100}Mo \rightarrow ^{100}Ru$</td>
<td>3034</td>
<td>10</td>
<td></td>
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<td>$^{96}Zr \rightarrow ^{96}Mo$</td>
<td>3350</td>
<td>3</td>
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<td>$^{150}Nd \rightarrow ^{150}Sm$</td>
<td>3667</td>
<td>6</td>
<td></td>
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<tr>
<td>$^{48}Ca \rightarrow ^{48}Ti$</td>
<td>4271</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>
If the neutrinos are massive Majorana particles, the double beta decay can also take place without emission of neutrinos ($0\nu \beta\beta$).
NUCLEAR STRUCTURE ASPECTS OF THE DOUBLE BETA DECAY
Has the neutrinoless double beta decay been observed?

There is an unconfirmed claim of discovery by (part of) the Heidelberg-Moscow collaboration (Klapdor 2001, 2004) of the $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ neutrinoless decay with a half-life of $(0.7-1.5) \times 10^{25}$ years
The neutrinoless double beta decay

The expression for the neutrinoless beta decay half-life, in the mass mode, for the $0^+ \rightarrow 0^+$ decay, can be brought to the following form:

\[
[T_{1/2}^{(0\nu)}(0^+ - > 0^+)]^{-1} = G_{0\nu} \left( M^{(0\nu)} \left( \frac{\langle m_\nu \rangle}{m_e} \right) \right)^2
\]

\[
M^{(0\nu)} = \left( \frac{g_A}{1.25} \right)^2 \left( M_{GT}^{(0\nu)} - \frac{M_F^{(0\nu)}}{g_A^2} - M_T^{(0\nu)} \right)
\]

where $\langle m_\nu \rangle$ is the effective neutrino mass

\[
\langle m_\nu \rangle = \sum_k U_{ek}^2 m_k
\]

The U’s are the matrix elements of the weak mixing matrix. $G_{0\nu}$ is the kinematic phase space factor, and $M^{0\nu}$ the nuclear matrix element (NME) that has Fermi, Gamow-Teller and Tensor contributions.
The matrix elements $M_{GT,F,T}^{(0\nu)}$ can be written as,

$$M_{K}^{(0\nu)} = \langle 0_f^+ | H_K(|\vec{r}_1 - \vec{r}_2|)(t_1^- t_2^-)\Omega_K | 0_i^+ \rangle$$

with $\Omega_F = 1$, $\Omega_{GT} = \vec{\sigma}_1 \cdot \vec{\sigma}_2$, $\Omega_T = S_{12}$

$H_K(|\vec{r}_1 - \vec{r}_2|)$ are the neutrino potentials ($\sim 1/r$) obtained from the neutrino propagator.
The neutrino potentials have the following form:

\[ H^m_K(r_{12}) = \frac{2}{\pi g_A^2 R} \int_0^\infty f_K(qr_{12}) h_K(q^2) dq \frac{h_K(q^2)q dq}{q + E_m - (E_i + E_f)/2} \]

\[ h_F(q^2) = g_V(q^2), \text{ and, neglecting higher order terms in the nuclear current, (what we shall not do),} \]
\[ h_{GT}(q^2) = g_A(q^2) \text{ and } h_T(q^2) = 0. \]
The neutrino potentials depend explicitly on the excitation energy of the states of the intermediate nucleus $E_m$. However, due to the large average energy of the virtual neutrino ($\sim 100$ MeV), they can as well be calculated in the closure approximation, that is good to better than 90%.
The Nuclear Matrix Elements

On top of that, other elements have to be taken into account before undertaking the purely nuclear calculations.

- The nucleon finite size is included by means of a dipole form factor
- The short range correlations, were most often taken into account by means of the Jastrow ansatz of Miller and Spencer, but softer prescriptions, as the UCOM (Unitary Correlation Operator Method) have been implemented as well. Actually, recent microscopic calculations (Brueckner-type) by Engel, Hagen and Mutter of the effect of the short range correlations favor the softer option
The Gamow-Teller operator needs to be quenched in the $2\nu$ mode and in the single beta decays. However, this cannot be translated naively to the $0\nu$ decay, because:

- The $1^+$ channel is just one among many others, never dominant, and often comes with opposite phase to others.
- The $1^+$ channel contains two components; the pure Gamow-Teller one and Gamow-Teller-Quadrupole term, whose behavior is completely unknown (at least to me).
- This issue should be addressed and clarified urgently.
How do the $0\nu$ operators act?

The two body transition operators can be written generically as:

$$\hat{M}^{(0\nu)} = \sum_J \left( \sum_{i,j,k,l} M_{i,j,k,l}^J \left( (a_i^\dagger a_j^\dagger)^J (a_k a_l)^J \right)^0 \right),$$

where the indices $i, j, k, l$, run over the single particle orbits of the spherical nuclear mean field. These operators can be factorized as follows:
How do the $0\nu$ operators act?

\[ \hat{M}^{(0\nu)} = \sum_{J^\pi} \hat{P}_{J^\pi}^\dagger \hat{P}_{J^\pi} \]

The operators $\hat{P}_{J^\pi}$ annihilate pairs of neutrons coupled to $J^\pi$ in the parent nucleus and the operators $\hat{P}_{J^\pi}^\dagger$ substitute them by pairs of protons coupled to the same $J^\pi$. The overlap of the resulting state with the ground state of the grand daughter nucleus gives the $J^\pi$-contribution to the NME. The –a priori complicated– internal structure of these exchanged pairs is dictated by the double beta decay operators.
Two main approaches have been traditionally used for the description of the nuclei involved in the transition. The Shell Model with configuration mixing in large valence spaces and the Quasi-particle RPA. More recently, the IBM suite has been also put at work. The PHFB approximation, using schematic interactions in ISM-like valence spaces is being applied as well. To assess the validity of the wave functions, quality indicators are needed such as:

- Good spectroscopy for parent, daughter and grand-daughter, even better if extended to a full mass region
- Occupancies
- GT-strengths and strength functions, $2\nu$ matrix elements, etc.

As we shall surmise later, a better understanding of the structure of the $0\nu$ transition operators in terms of the main nuclear correlators, pairing and quadrupole, can be of great help in assessing the accuracy of the nuclear descriptions.
Interacting Shell Model calculations (ISM) vs QRPA

- **Interaction**
  - ISM: Monopole corrected G-matrices
  - QRPA: Realistic or schematic interactions tuned with the $g_{ph}$ and $g_{pp}$ strengths

- **Valence space**
  - ISM: A limited number of orbits, but all the possible ways of distributing the valence particles among the valence orbits are taken into account.
  - QRPA: A larger number of orbits, but only 1p-1h and 2p-2h excitations from the normal filling are considered (and not all of them)
ISM vs QRPA calculations

- **Pairing Correlations**
  - **ISM**: Are treated exactly in the valence space. Proton and neutron numbers are exactly conserved. Proton-proton, neutron-neutron, and proton-neutron (isovector and isoscalar) pairing is included.
  - **QRPA**: Only proton-proton and neutron-neutron pairing are considered. They are treated in the BCS approximation. Proton and neutron numbers are not exactly conserved.

- **Multipole Correlations and Deformation**
  - **ISM**: Are described properly in the laboratory frame. Angular momentum conservation preserved.
  - **QRPA**: The correlations are treated at the RPA level. Permanent deformation is not incorporated.
The Valence Spaces

Miscellanea of computationally accessible valence spaces relevant for the description of double beta decay emitters:

(note: in a major HO shell of principal quantum number \( p \) the orbit \( j=p+1/2 \) is called \textit{intruder} and the remaining ones are denoted by \( r_p \))

- The \textit{pf} shell; \(^{48}\text{Ca}\)
- \( r_3\text{-g}_9/2 \): \(^{76}\text{Ge}, \, ^{82}\text{Se}, \)
- \( r_3\text{-g}_9/2 \) for protons and \( r_4\text{-h}_{11/2} \) for neutrons; \(^{96}\text{Zr}, \, ^{100}\text{Mo} \)
- \( r_4\text{-h}_{11/2} \) for neutrons and \( p_{1/2}\text{-g}_9/2\text{-r}_4 \) for protons: \(^{110}\text{Pd}, \, ^{116}\text{Cd} \)
- \( r_4\text{-h}_{11/2} \) for neutrons and protons: \(^{124}\text{Sn}, \, ^{128-130}\text{Te}, \, ^{136}\text{Xe} \)

The Strasbourg-Madrid codes can deal with problems involving basis of \( 10^{10} \) Slater determinants, using “relatively” modest computational resources
Update of the ISM $2\nu$ results

In the valence spaces $r_3-g_{9/2}$ ($^{76}\text{Ge}$, $^{82}\text{Se}$) and $r_4-h_{11/2}$ ($^{124}\text{Sn}$, $^{128-130}\text{Te}$, $^{136}\text{Xe}$) we have obtained high quality effective interactions by carrying out multi-parametrical fits whose starting point is given by realistic G-matrices. In the valence spaces proposed for $^{96}\text{Zr}$, $^{100}\text{Mo}$, $^{110}\text{Pd}$ and $^{116}\text{Cd}$, the results are still preliminary.
Update of the ISM $2\nu$ results

$q_0$ means the standard quenching used in full $0\hbar\omega$ calculations (0.75 in the $A=48$ case): $q_f$ incorporates an extra factor due to the truncations at $0\hbar\omega$ level. It is fitted to the GT single beta decays of each region: $q_f=0.6-0.57$.

<table>
<thead>
<tr>
<th>$M^{(2\nu)}$</th>
<th>exp</th>
<th>$q_0$</th>
<th>$q_f$</th>
<th>INT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}\text{Ca} \to ^{48}\text{Ti}$</td>
<td>0.05±0.01</td>
<td>0.047</td>
<td></td>
<td>KB3</td>
</tr>
<tr>
<td>$^{48}\text{Ca} \to ^{48}\text{Ti}$</td>
<td>0.05±0.01</td>
<td>0.048</td>
<td></td>
<td>KB3G</td>
</tr>
<tr>
<td>$^{48}\text{Ca} \to ^{48}\text{Ti}$</td>
<td>0.05±0.01</td>
<td>0.065</td>
<td></td>
<td>GXPF1</td>
</tr>
<tr>
<td>$^{76}\text{Ge} \to ^{76}\text{Se}$</td>
<td>0.13±0.01</td>
<td>0.168</td>
<td>0.107</td>
<td>gcn28:50</td>
</tr>
<tr>
<td>$^{76}\text{Ge} \to ^{76}\text{Se}$</td>
<td>0.13±0.01</td>
<td>0.163</td>
<td>0.105</td>
<td>HOMHJ</td>
</tr>
<tr>
<td>$^{82}\text{Se} \to ^{82}\text{Kr}$</td>
<td>0.10±0.01</td>
<td>0.187</td>
<td>0.120</td>
<td>gcn28:50</td>
</tr>
<tr>
<td>$^{82}\text{Se} \to ^{82}\text{Kr}$</td>
<td>0.10±0.01</td>
<td>0.169</td>
<td>0.108</td>
<td>HOMHJ</td>
</tr>
<tr>
<td>$^{128}\text{Te} \to ^{128}\text{Xe}$</td>
<td>0.05±0.005</td>
<td>0.092</td>
<td>0.059</td>
<td>gcn50:82</td>
</tr>
<tr>
<td>$^{130}\text{Te} \to ^{130}\text{Xe}$</td>
<td>0.032±0.003</td>
<td>0.068</td>
<td>0.043</td>
<td>gcn50:82</td>
</tr>
<tr>
<td>$^{136}\text{Xe} \to ^{136}\text{Ba}$</td>
<td>$&lt;0.01$±0.01</td>
<td>0.064</td>
<td>0.041</td>
<td>gcn50:82</td>
</tr>
</tbody>
</table>
Running sum of the $2\nu$ matrix element: $A=48$

![Graph showing the running sum of the $2\nu$ matrix element relative to the first $1^+$ state.](image-url)
Running sum of the $2\nu$ matrix element: $A=76$ and 82

Energy relative to the first $1^+$ state

$M^{2\nu} \times 10$
In the present ISM calculations, the Form Factors (FS), the Higher Order Contributions to the Nuclear Current (HOC) and the Short Range Correlations (SRC) are treated in the same way than in the QRPA calculations of the Tubingen and Jyvaskyla groups, either with the Jastrow or the UCOM ansatzs. The effects of these terms in the NME’s, relative to the bare NME’s, are very similar in both approaches. This agreement and the fact of having a common treatment of the corrections may contribute to reduce substantially the uncertainties of the NME’s
Update of the ISM $0\nu$ results: UCOM SRC

<table>
<thead>
<tr>
<th></th>
<th>$M^{(0\nu)}$</th>
<th>$\langle m_\nu \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$</td>
<td>0.85</td>
<td>0.63</td>
</tr>
<tr>
<td>$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$</td>
<td>2.81</td>
<td>0.72</td>
</tr>
<tr>
<td>$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$</td>
<td>2.64</td>
<td>0.37</td>
</tr>
<tr>
<td>$^{124}\text{Sn} \rightarrow ^{124}\text{Te}$</td>
<td>2.62</td>
<td>0.37</td>
</tr>
<tr>
<td>$^{128}\text{Te} \rightarrow ^{128}\text{Xe}$</td>
<td>2.88</td>
<td>1.32</td>
</tr>
<tr>
<td>$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$</td>
<td>2.65</td>
<td>0.28</td>
</tr>
<tr>
<td>$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$</td>
<td>2.19</td>
<td>0.38</td>
</tr>
</tbody>
</table>

The effective neutrino mass (in eV) corresponds to $T^{1/2} = 10^{25}$ y. Notice that the Heidelberg-Moscow claim, together with our NME leads to an effective neutrino mass of 0.7 eV.
TU07; QRPA results from Rodin, Simkovic, Faessler, and Vogel 07.
JY07; QRPA results from Kortelainen and Suhonen 07
TU07; QRPA results from Rodin, Simkovic, Faessler, and Vogel 07.
JY07; QRPA results from Kortelainen and Suhonen 07
The range of QRPA values shown in the figures derives from the different variants of the QRPA and the different choices of $g_{pp}$ and $g_A$. The larger values correspond to $g_A=1.25$ and the smaller ones to $g_A=1.0$. The ISM numbers are obtained with $g_A=1.25$.

The NME’s of the two main QRPA groups are now compatible in most decays, which was not the case not so long ago. These are good news.

However, except for $^{136}\text{Xe}$, the ISM NME’s are systematically smaller than the QRPA ones. (Preliminary calculations by the Tubingen group of the $^{124}\text{Sn}$ NME give a value 2.8, not very far from the ISM prediction. Are these bad news?)
How do the $0\nu$ operators act

Trying to unveil the physics hidden in these results we come back to the factorized operators

$$\hat{M}^{(0\nu)} = \sum_{J^\pi} \hat{P}^\dagger_{J^\pi} \hat{P}_{J^\pi}$$

and decompose the NME in the $^{82}$Se $\rightarrow$ $^{82}$Kr decay as a sum of the contributions of the pairs with different values of $J^\pi$
The contributions to the NME as a function of the $J^\pi$ of the decaying pair: $^{82}$Se $\rightarrow$ $^{82}$Kr
The contributions to the $0\nu$ matrix element as a function of the $J$ of the decaying pair: $A=130$
These results are very suggestive, because the leading contribution corresponds to the decay of $J=0$ pairs, whereas the contributions of the pairs with $J>0$ are either negligible or have opposite sign to the dominant one.

If we went to the limit of pure pairing correlations, i.e. when the initial and final states have generalized seniority zero, there will be no canceling contributions and therefore the matrix element will be maximal.

That the $J=0$ contribution be large can only be understood if the $J=0$ pairs created or destroyed by the double beta decay operators resemble to the nucleon pairs produced by the nuclear pairing interaction.

This behavior is common to all the cases that we have studied. It also occurs in the QRPA calculations, in whose context it has been previously discussed by Engel, Vogel et al.
The role of pairing

Intriguingly, this reveals that the NME’s of the neutrinoless double beta decay depend on the pair content of the nuclear wave functions of parent and grand daughter nuclei. As we shall see, if we force the nuclear wave function to be fully paired, the NME’s become very large. The nuclear correlations of multipole type (mainly quadrupole) break pairs and reduce the NME’s.
Pay special attention to the $s \leq 4$ results because, at leading order, this is the level of ground state correlations in the QRPA calculations based upon a spherical BCS solution.
Only when the decaying nucleus is a good superfluid ($^{124}\text{Sn}$), or semi-magic ($^{136}\text{Xe}$), or doubly magic ($^{48}\text{Ca}$), the $s \leq 4$ results are converged.

And, only in these cases the QRPA NME’s are close to the ISM ones!!
ISM vs QRPA NME’s; Jastrow SRC

\[ M^{0v\beta\beta} \]

\[ A = 48, 76, 82, 124, 128, 130, 136 \]

- ISM \( s_m = 4 \)
- JY07
- TU07
- ISM

NUCLEAR STRUCTURE ASPECTS OF THE DOUBLE BETA DECAY
NUCLEAR STRUCTURE ASPECTS OF THE DOUBLE BETA DECAY

ISM vs QRPA NME's ; UCOM SRC

(ISM $s_m=4$

JY07

TU07

ISM)

$M^{0\nu\beta\beta}$

$A$

48 76 82 124 128 130 136
ISM vs QRPA NME’s

- The QRPA results are reasonably close to the ISM ones at $s \leq 4$.
- The ISM values at $s \leq 4$ are far from converged, except in the $A=48$, $A=124$ and $A=136$ decays.
- Thus, we surmise that, except in these cases, the QRPA overestimates the values of the NME’s.
Decomposition of the $0\nu\beta\beta$ NME as a function of the seniority components of the initial, $s_i$, and final, $s_f$, wave functions. Results for the $A=82$ decay. The coefficients in parenthesis indicate the percentage of the wave function that belongs to each particular seniority.

<table>
<thead>
<tr>
<th>$s_f$</th>
<th>$s_f = 0$</th>
<th>$s_f = 4$</th>
<th>$s_f = 6$</th>
<th>$s_f = 8$</th>
<th>$s_f = 10$</th>
<th>$s_f = 12$</th>
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</thead>
<tbody>
<tr>
<td>(44)</td>
<td>(41)</td>
<td>(6)</td>
<td>(8)</td>
<td>(1)</td>
<td>(0.1)</td>
<td></td>
</tr>
<tr>
<td>$s_i = 0$ (50)</td>
<td>8.8</td>
<td>-5.6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$s_i = 4$ (39)</td>
<td>-0.3</td>
<td>4.9</td>
<td>-1.2</td>
<td>-6.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$s_i = 6$ (10)</td>
<td>-</td>
<td>-0.2</td>
<td>2.2</td>
<td>-0.3</td>
<td>-3.0</td>
<td>-</td>
</tr>
<tr>
<td>$s_i = 8$ (1)</td>
<td>-</td>
<td>-0.02</td>
<td>-0.07</td>
<td>0.6</td>
<td>-0.08</td>
<td>-4.3</td>
</tr>
</tbody>
</table>

Notice that the $\Delta s=4$ matrix elements have the same size but opposite sign to the diagonal ones, being responsible for the cancellations that lead to small final values of the NME’s.
The nuclear wave functions in the seniority basis

<table>
<thead>
<tr>
<th>Element</th>
<th>$s = 0$</th>
<th>$s = 4$</th>
<th>$s = 6$</th>
<th>$s = 8$</th>
<th>$s = 10$</th>
<th>$s = 12$</th>
<th>$s = 14$</th>
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<tbody>
<tr>
<td>$^{48}$Ca</td>
<td>97</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{48}$Ti</td>
<td>59</td>
<td>36</td>
<td>4</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{76}$Ge</td>
<td>43</td>
<td>41</td>
<td>7</td>
<td>8</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{76}$Se</td>
<td>26</td>
<td>41</td>
<td>11</td>
<td>16</td>
<td>4</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$^{82}$Se</td>
<td>50</td>
<td>39</td>
<td>10</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{82}$Kr</td>
<td>44</td>
<td>41</td>
<td>6</td>
<td>8</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{124}$Sn</td>
<td>95</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{124}$Te</td>
<td>60</td>
<td>33</td>
<td>6</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{128}$Te</td>
<td>70</td>
<td>26</td>
<td>3</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{128}$Xe</td>
<td>37</td>
<td>41</td>
<td>9</td>
<td>10</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{130}$Te</td>
<td>79</td>
<td>20</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{130}$Xe</td>
<td>46</td>
<td>39</td>
<td>7</td>
<td>7</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{136}$Xe</td>
<td>97</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{136}$Ba</td>
<td>72</td>
<td>25</td>
<td>2</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Recently, the spectroscopic factors of the nuclei $^{76}\text{Ge}$ and $^{76}\text{Se}$ have been measured by a team led by J. P. Schiffer. It turns out that the ISM occupancies are much closer to the experiment than those produced by the QRPA. New QRPA calculations have been performed, modifying the single particle energies as to reproduce the experimental occupancies. The new QRPA NME’s are much closer to the ISM ones.
Benchmarking with the occupation numbers

Proton Occupancies

Neutron Vacancies

A. Poves
NUCLEAR STRUCTURE ASPECTS OF THE DOUBLE BETA DECAY
Benchmarking with the occupation numbers

Values of $M^{0\nu\beta\beta}$ for the $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ decay

<table>
<thead>
<tr>
<th></th>
<th>$M^{0\nu\beta\beta} (J)$</th>
<th>$M^{0\nu\beta\beta} (UCOM)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISM (GCN28.50)</td>
<td>2.30</td>
<td>2.81</td>
</tr>
<tr>
<td>ISM (RG.PROLATE)</td>
<td>2.70</td>
<td>3.26</td>
</tr>
<tr>
<td>Jy (WS)</td>
<td>4.03</td>
<td>5.36</td>
</tr>
<tr>
<td>Jy (ADJ)</td>
<td>2.78</td>
<td>4.11</td>
</tr>
<tr>
<td>Tu (WS)</td>
<td>4.15-5.11</td>
<td>5.07-6.25</td>
</tr>
<tr>
<td>Tu (ADJ)</td>
<td>3.56-4.06</td>
<td>4.59-5.44</td>
</tr>
</tbody>
</table>
BUT, to have good occupancies is not enough

Occupancies and NME for the $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ decay in function of the maximum seniority in the wave functions, $s_m$. UCOM type SRC’s.

<table>
<thead>
<tr>
<th>Neutrons</th>
<th>Protons</th>
<th>NME</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{76}\text{Ge}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1p$</td>
<td>$0f_{5/2}$</td>
<td>$0g_{9/2}$</td>
</tr>
<tr>
<td>$s_m = 0$</td>
<td>4.8</td>
<td>5.2</td>
</tr>
<tr>
<td>$s_m = 4$</td>
<td>4.8</td>
<td>5.0</td>
</tr>
<tr>
<td>$s_m = 10$</td>
<td>4.8</td>
<td>4.8</td>
</tr>
</tbody>
</table>

| $^{76}\text{Se}$ | | |
| $1p$ | $0f_{5/2}$ | $0g_{9/2}$ | $1p$ | $0f_{5/2}$ | $0g_{9/2}$ |
| $s_m = 0$ | 3.9 | 4.6 | 5.5 | 1.8 | 3.3 | 0.9 | 11.85 |
| $s_m = 4$ | 4.3 | 4.4 | 5.3 | 2.1 | 2.6 | 1.3 | 7.99 |
| $s_m = 14$ | 4.1 | 4.1 | 5.9 | 2.1 | 2.8 | 1.1 | 3.26 |
To measure the amount of quadrupole correlations in the ground state of the nuclei participating in the decay, we refer to the non energy weighted sum rule:

$$\langle Q^2 \rangle = \sum_i |\langle 2^+ | Q | 0^+ \rangle|^2$$
The role of deformation: The ideal but unreal case of a mirror decay

\[ ^{50}\text{Cr} \rightarrow ^{50}\text{Fe} \]
\[ ^{66}\text{Ge} \rightarrow ^{66}\text{Se} \]
\[ ^{110}\text{Xe} \rightarrow ^{110}\text{Ba} \]

If we compute both nuclei with the same interaction, they have the same deformation. If we compute the parent with \( H_0 \) and the grand daughter with \( H_0 + \lambda Q \cdot Q \) we can evaluate the influence of the differences of deformation in the NME.
$^{66}\text{Ge} \rightarrow ^{66}\text{Se}$

![Graph showing NME, $\langle qq \rangle$, and $\langle \text{pair} \rangle$ vs. $\lambda_{qq}$](image-url)
$^{66}\text{Ge} \rightarrow ^{66}\text{Se}$

NME

$NME_{\text{nondiag}}$

$\langle qq \rangle$

$\langle \text{pair} \rangle$

$\langle \text{overlap} \rangle$
$^{66}\text{Ge} \rightarrow ^{66}\text{Se}$

$A=66, \beta_0=0.22$

NME

$\Delta \beta$

$A_{\langle \text{pair} \rangle}$

NME
ISM calculations in QRPA-like valence spaces: $^{82}\text{Se}$

The ISM valence space for the $^{76}\text{Ge}$ and $^{82}\text{Se}$ decays has been traditionally:

$$1p_{3/2}, 0f_{5/2}, 1p_{1/2}, 0g_{9/2}$$

Although only recently full calculations in this space have been possible.

In the QRPA, it is rather:

$$0f_{7/2}, 1p_{3/2}, 0f_{5/2}, 1p_{1/2}, 0g_{9/2}, 1d_{5/2}, 0g_{7/2}, 2s_{1/2}, 1d_{3/2}$$
ISM calculations in QRPA-like valence spaces: $^{82}$Se

As a first step toward a more complete benchmarking, we have evaluated the influence of the 2p-2h jumps from the 0f$_{7/2}$ orbit – $^{56}$Ni core excitations– in our results for the $^{82}$Se decay. Similar calculations for the $^{76}$Ge decay are under way.

The calculation in the full $r_3g$ space plus 2p-2h proton excitations from the 0f$_{7/2}$ orbit gives a 20% increase of $M^{0\nu}$, but probably we overestimate the amount of core excitations. Our 0f$_{7/2}$ proton occupancies, 7.71 and 7.69 in $^{82}$Se and $^{82}$Kr are smaller than the BCS occupancies of Rodin et al. 7.84 and 7.84. Therefore the above 20% must be taken as an upper bound.

The $2\nu$ matrix element remains nearly constant, even if the total Gamow-Teller strengths, (GT+) and (GT-), increase from 0.15 to 0.34 and from 20.5 to 26.9.
We have also computed the $^{136}$Xe decay in the $r_4h$ space including 2p-2h excitations from the $0g_{9/2}$ proton orbit and the matrix element increases less than 10%.

In another set of calculations, we have included 2p2h neutron excitations toward the $0h_{9/2}$ and $1f_{7/2}$ orbits. The occupancies that we obtain are relatively large (0.25 neutrons in each orbit) and the effect is to increase the matrix element by 15%. It is interesting to note that the increase with the two orbits simultaneously active is equivalent to that obtained including one or another orbit separately. Therefore there is no pile-up of the contributions of the small components of the wave function.

In summary, the ISM results seem to be robust against the inclusion of small components of the wave function.
Conclusions

- Large scale shell model calculations with high quality effective interactions are available or will be in the immediate future for all but one of the neutrinoless double beta emitters.

- We have found that the superfluid correlations in parent and grand daughter favor the neutrinoless decay.

- We have also seen that in the realistic cases, where many other correlations are present, their contributions to the matrix elements come with opposite sign to the pairing ones.

- In order to take properly into account these cancellations, it is crucial to describe correctly the pair structure of the wave functions.
Conclusions

- State of the art QRPA calculations using the same prescription for the short range correlations are now compatible. The softest possible choice, UCOM, seems to be the more realistic one.

- Low seniority truncations $s \leq 4$, similar to those present in the spherical QRPA approaches based in a BCS treatment of the pairing interaction, are shown to fall short in the capture of the proper correlations, and hence to overestimate the nuclear matrix elements in several decays.
The difference in the amount of quadrupole correlations in the ground states of parent and grand daughter hinders the transition.

When the quadrupole correlations are large, low seniority approximations fail to treat them properly, and thus, again, the nuclear matrix elements are overestimated.