Magnetic Dipole and Quadrupole Transitions in Nuclei – Open Problems

• Qualitative nature of the M1 response
• Orbital M1 scissors mode: low and high
• Spin M1 resonance in heavy deformed nuclei
• Quenching of spin-magnetic strength
• Orbital M2 strength: the twist mode
• Fine structure and scales: spin M1 resonance in fp-shell nuclei
• Forbidden M1 transitions

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Structure of the M1 Operator

\[ T(M1) = \sum_{i=1}^{A} \{ g_l(i) \vec{l}_i + g_s(i) \vec{s}_i \} = T(M1)_{IV} + T(M1)_{IS} \quad [\mu_N] \]

\[ T(M1)_{IS} = \frac{1}{2} \vec{J} + \frac{1}{2} (g_p + g_n) \vec{S} = \frac{1}{2} \vec{J} + 0.38 \vec{S} \quad [\mu_N] \]

small “spin“
Structure of the M1 Operator

\[ T(M1)_{IV} = \sum_{i} t_{x}(i) \vec{l}_{i} + (g_{p} - g_{n}) \sum_{i} t_{x}(i) \vec{s}_{i} \quad [\mu_{N}] \]

\[ = \frac{1}{2} (\vec{L}_{p} - \vec{L}_{n}) + \frac{1}{2} (g_{p} - g_{n}) T(M1)_{\Delta T_{z}=0} \quad [\mu_{N}] \]

\[ = \frac{1}{2} (\vec{L}_{p} - \vec{L}_{n}) + 4.71 T(M1)_{\Delta T_{z}=0} \quad [\mu_{N}] \]

- rotation generator
- enhanced “spin-flip”
- “scissors motion”
- (Gamow-Teller)

• Similar relation for M2 and \( J^{\pi} = 2^- \) component of the spin-dipole resonance
Isospin Components

Isospin Symmetry

\[ T_0 \]

\[ T_0+1 \]

\[ T_0-1 \]

\[ T_0-1 \text{ G.S.} \]

\[ T_z = T_0-1 \]

\[ N-1,Z+1 \]

\( \beta^- \) decay

\( (p,n) \)

\( (p,p') \)

\( (e,e') \)

\[ N,Z \]

\[ T_z = T_0 \]

\[ T_0 \text{ G.S.} \]

\[ T_0+1 \text{ G.S.} \]

\[ T_z = T_0+1 \]

\[ T_0+1 \]

\[ N+1,Z-1 \]

\( \beta^+ \) decay

\( (n,p) \)
Example: $A = 58$

\[
\begin{align*}
\text{GT}^- & \quad \text{GT}_0 \quad \text{GT}^+ \\
\hline
\text{Counts / 10 keV} & \text{Counts / mC Channel} & \text{Counts / 10}^3 \\
\text{Excitation Energy (MeV)} & \text{Excitation Energy (MeV)} & \text{Excitation Energy (MeV)} \\
\end{align*}
\]

H. Fujita \textit{et al}.,
PRC 75 (2007) 034310

W. Mettner \textit{et al}.,
NPA 473 (1987) 160

M. Hagemann \textit{et al}.,
PLB 579 (2004) 251

- Benchmark tests of modern microscopic nuclear theory
Schematic M1 Response in Heavy Deformed Nuclei

HEAVY NUCLEI

STRENGTH

$\ell_j \rightarrow \ell_j$  
$(j) \ell + 1/2 \rightarrow (j) \ell - 1/2$

REPULSIVE

$\sigma \cdot \tau$

$2\Delta$

2QP

ORBITAL

IV

SPIN

$\sigma \cdot \tau$

$G \cdot T$

GIANT QUADRUPOLE

IV

$K^\pi = 1^+$

SCISSORS

$2\hbar \omega$

$\hbar \omega$

ORBITAL

COLL

$j = \ell - 1/2$

$j = \ell + 1/2$
Orbital M1 Strength: the Scissors Mode

D. Bohle et al., PLB 137 (1984) 27
Deformation Dependence of the Scissors Mode: Data

W. Ziegler et al., PRL 65 (1990) 2515
Deformation Dependence of the Scissors Mode: Models

- E. Garrido et al., PRC 44 (1991) R1250
- R.R. Hilton et al., PRC 47 (1993) 602
- N. Lo Iudice and A. Richter, PLB 304 (1993) 193
- K. Heyde et al., PRC 49 (1994) 156
- P. Sarriguren et al., JPG 20 (1994) 315
- N. Shimizu et al., PRL 86 (2001) 1171
Energy and Strength of the Scissors Mode

- Excitation energy approximately constant, independent of deformation
- Strength depends strongly on deformation
- Midshell saturation

![Graph showing excitation energy and strength as functions of mass number A.](image-url)
Sum-Rule Approach

E. Lipparini and S. Stringari, PLB 130 (1983) 139

- Sum rules
  \[ S_j(M) = \sum_i B_i(M) E_{2i}^j \]

- \[ S_{+1}(M1) = \frac{3}{5\pi} r_0^2 A^{5/3} \delta^2 E_{GDR}^2 m_N g_{IV}^2 \]
  \[ E_{x} = \sqrt{S_{+1}/S_{-1}} \]

- \[ S_{-1}(M1) = \frac{3}{16\pi} \Theta_{M1} g_{IV}^2 \]
  \[ B(M1) = \sqrt{S_{+1} \cdot S_{-1}} \]

- Sum rules depend on two parameters:
  \[ g_{IV} \approx g_{IS} = g(2_1^+) \]
  \[ \Theta_{IV} = \Theta_{IS} = 3\hbar^2/E_{2_1^+} \]
Parameter-Free Sum Rule Description

\[ E_x \sim \sqrt{E_{21}} \cdot \delta \approx \text{const} \]

Contributions from deformation and moment of inertia cancel each other.

\[ B(M1) \sim \frac{\delta}{\sqrt{E_{21}}} \sim \delta^2 \]

“\( \delta^2 \) law” results from an interplay of deformation and the moment of inertia.

J. Enders et al., PRC 71 (2005) 014306
Scissors Mode Coupled to a Neutron Skin

- Theoretical studies needed
- M1 strength at threshold in nuclei along the r-process path
Mixed-Symmetry States in Vibrational Nuclei: Signature

\[ F = F_{\text{max}} \quad (\text{sym. states}) \]
\[ F = F_{\text{max}} - 1 \quad (\text{ms states}) \]

- **Strong E2 transitions** for decay of symmetric Q-phonon
- **Weak E2 transitions** for decay of ms Q-phonon
- **Strong M1 transitions** for decay of ms states to symmetric states

- Experimentally demonstrated for $^{94}\text{Mo}$ (N. Pietralla et al.)
Phase-Shape Transitions and the Scissors Mode

**Vibrator**

- $1^+_n$
- $2^+_m$
- $2^+_s$
- $4^+$
- $2^+$
- $0^+$

**Rotor**

- $2^+_r$
- $2^+_T$
- $0^+_g$
- $6^+$
- $4^+$
- $2^+$
- $0^+$

- $K=1$
- $K=2$
- $K=0$

- Properties of the Scissors Mode: signature of phase-shape transitions?
Spin M1 Resonance in $^{154}$Sm

D. Frekers et al., PLB 244 (1990) 178
M1 Strength in Deformed Rare-Earth Nuclei

\[ B_{\text{orb}}(\text{M1}) (\mu_N^2) \]

\[ B^q(\text{M1}) (\mu_N^2/80 \text{ keV}) \]

\[ E_x \ (\text{MeV}) \]
Spin M1 Strength in $^{154}$Sm: Experiment vs. Models

- Nature of the double-hump structure: $p/n$ or $IS/IV$?

D. Zawischa and J. Speth, PLB 252 (1990) 4

C. De Coster et al., NPA 542 (1992) 375

P. Sarriguren et al., JPG 19 (1993) 291

R.R. Hilton et al., EPJA 1 (1998) 257
Schematic M1 Response in Heavy Deformed Nuclei

**HEAVY NUCLEI**

- $\ell_j \rightarrow \ell_j$
- $(j) \ell + 1/2 \rightarrow (j) \ell - 1/2$

- REPULSIVE
- $\sigma \cdot \tau$

- $2 \Delta$

- ORBITAL IV
- SPIN
- G-T

- $\omega$
- ORBITAL COLL

- GIANT QUADRUPOLE IV
- $K^\pi = 1^+$

- SCISSORS

- $2 \hbar \omega$
Search for High-Lying Scissors Mode

- Result depends on modeling the quasifree background
- Other experimental methods?
**Quenching of Spin – Isospin Strength**

<table>
<thead>
<tr>
<th>mechanism</th>
<th>highly excited (1p-1h)</th>
<th>configuration mixing (2p-2h); tensor</th>
<th>Δ-admixture (Δ - N⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>≈0</td>
<td>≈40%</td>
<td>≈10%</td>
</tr>
<tr>
<td>GT</td>
<td>increase</td>
<td></td>
<td>decrease</td>
</tr>
<tr>
<td>M2</td>
<td></td>
<td></td>
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<tr>
<td>M3</td>
<td></td>
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<tr>
<td>Mλ</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(high spin)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- $\bar{\sigma} \cdot \bar{\tau}$ strength $\approx 50\%$ reduced
M1 Transition in $^{48}$Ca as a Prime Example of Quenching

\[ \begin{align*}
\text{(e,e')} \\
E_0 = 39 \text{ MeV} \\
\theta = 165^\circ \\
\text{(p,p')} \\
E_0 = 319 \text{ MeV} \\
\theta = 3.5^\circ \\
\text{(\pi'^-,\pi'^-)} \\
E_0 = 116 \text{ MeV} \\
\theta = 20^\circ \\
\text{(\pi'^+,\pi'^+)} \\
\end{align*} \]
M1 Transition in $^{48}$Ca as a Prime Example of Quenching

- $B(M1)_{\text{exp}} \leq B(M1)_{\text{the}}$

- Total contribution of weak transitions?
M1 Transition in $^{48}$Ca as a Prime Example of Quenching

- Still sizable discrepancies between experiment and theory
M1 Strength: Two Alternative Approaches

• $2^{\text{nd}}$ RPA (2p - 2h) + $n\hbar\omega$

• SM ($np - nh$) + $0\hbar\omega$ (one major shell)

• Example: N=28 isotones
N=28 Isotones: Experiment vs. Shell Model Predictions

• Data show considerable fine structure: sign of configuration mixing

• Global description quite good (apart from the interaction KB3→KB3G)

P. von Neumann-Cosel et al., PLB 443 (1998) 1
• Quenching factor agrees with quenching of $g_A$ in fp-shell
$^{52}$Cr: Experiment vs. “State of the Art” SM Calculations

- Still significant differences between different effective interactions
- Knowledge of these strength distributions important for astrophysics

K. Langanke et al., PRL 93 (2004) 202501
Relation of $B(M1)$ and GT Strength in Selfconjugate Nuclei

\[ B(M1) = \left[ M(\sigma) + M(l) + M_\Delta + M_{MEC} \right]^2 \frac{3(\mu_p - \mu_n)^2}{8\pi} \]

\[ B(GT) = \left[ M(\sigma) + M_\Delta + M_{MEC} \right]^2 \]

$\rightarrow$ $R(M1/GT)$ measures $M(l)$ and $M_{MEC}$
The $^{28}\text{Si}(e,e')$ Reaction

$E_0 = 62.1$ MeV

$\theta = 180^\circ$
Form Factor Examples

- Distinction of multipolarities
M1 Strength Distribution in $^{28}\text{Si}$

- Brown-Wildenthal effective operator
M1 Enhancement Factor

A. Richter et al., PRL 65 (1990) 2519
F. Hofmann et al., PRC 65 (2002) 024311

• Enhancement $R > 1 \rightarrow$ signature of meson exchange currents
M2 Strength and First-Forbidden Matrix Elements

C. Rangacharyulu et al.,
PLB 135 (1984) 29

- Orbital matrix elements zero within error bars
- Quenching of $g_A$ and $g_V$ must be similar
M2 Resonance in 180° Electron Scattering

$^{90}\text{Zr}(e,e')$

$E_0 = 42.7$ MeV

$\theta = 180°$

P. von Neumann-Cosel et al., PRL 82 (1999) 1105
M2 Resonance in 180° Electron Scattering

- Strong interference
- Quantitative description possible
Running Sums

- Quenching comparable to M1 case
M3 Strength in $^{26}$Mg

K.K. Seth et al., PRL 74 (1995) 642

- Comparison to shell model: no quenching!?
 Orbital M2 Strength: the Nuclear “Twist”

- Operator \( \hat{I} = e^{-i\alpha z \hat{l}_z} = e^{i\alpha \vec{u} \cdot \vec{\nabla}} \) with \( \vec{u} = (yz, -xz, 0) \), i.e. rotation around the body-fixed z-axis with a rotation angle proportional to z (clockwise for \( z > 0 \) and counterclockwise for \( z < 0 \)).

- Operator \( z \hat{l}_z \) has spin-parity \( J^\pi = 2^- \) (because the scalar part of the tensor product \( \vec{r} \otimes \vec{l} \)), i.e. \( \vec{r} \cdot \vec{l} \), vanishes identically).

- Although for axially symmetric nuclei there is evidently no change in the local density, the “twist” still creates a distortion of the local Fermi surface characterized by \( \alpha \).
Currents of the Twist Mode

- Clockwise respective counterclockwise flow in the two hemispheres
- Reversal of direction of flow in the interior $\rightarrow$ node of the transition current
- Semiclassical picture confirmed in microscopic calculations
Indirect Evidence for the Twist Mode

- Indirect evidence through strong interference
Direct Evidence for Orbital M2 Excitations

B. Reitz et al., PLB 532 (2002) 179
The Twist Mode: Electron Scattering Form Factors

$^{208}\text{Pb}(e,e')$

$\theta = 180^\circ$

$|F_T|^2$

$E_0$ (MeV)
Fine Structure of the Spin-Flip GTR

\(^{60}\text{Zr}^{3}\text{He},t)^{60}\text{Nb}

E_n = 140 \text{ MeV/u}

RCNP

\(\Delta E = 50 \text{ keV}\)

Y. Kalmykov et al., PRL 96 (2006) 012502

- High energy resolution
- Asymmetric fluctuations
Wavelet Analysis

\[ \int_{-\infty}^{\infty} \Psi^*(x) \, dx = 0 \quad \text{and} \quad \int_{-\infty}^{\infty} \left| \Psi^*(x) \right|^2 \, dx < \infty \]

\[ \Psi(x) = \cos(2\pi\omega x) e^{-x^2/2} \]

Wavelet coefficients:

\[ C(\delta E, E_x) = \frac{1}{\sqrt{\delta E}} \int \sigma(E) \, \Psi^* \left( \frac{E_x-E}{\delta E} \right) \, dE \]

Continuous: \( \delta E, E_x \) are varied continuously
Extraction of Scales from the Data

- scales at 80, 300, 950, 2500 keV
• 2-phonon QPM (equivalent to SRPA) yields scales at 100, 380, 950, 1600 keV very similar to experiment
Scales of the Spin-Flip GTR in $^{90}$Nb

- Similar power spectra
- Scales are a global phenomenon of giant resonances (GQR, GDR, GT, …)
  application to spin M1 resonance
Scales of the Spin M1 Resonance in fp-Shell Nuclei

I. Petermann et al., PRC (submitted)
• Number of iterations to stabilize scales depends on scale energy
Impact of Model Space

- Power spectrum stable for $t \geq 3$
Scales and Effective Interactions

- $^{52}$Cr
- KB3G
- GXPF1
- FPD6