Shell-model description of pfg-shell nuclei

Perspectives on the modern shell model and related experimental topics
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1. Introduction
2. Effective interaction JUN45
3. Results
   • Magicity of N=40 --- Ge isotopes : $^{70}$Ge, $^{72}$Ge, $^{74}$Ge,
   • Towards collective region --- N=Z nuclei : $^{64}$Ge, $^{66}$As, $^{68}$Se, $^{70}$Br
   • Proton-neutron multiplets --- N=49 isotones : $^{88}$Y, $^{86}$Rb, $^{84}$Br, $^{82}$As, $^{80}$Ga
4. Towards neutron-rich nuclei (Cr, Ni)
5. Summary
pfg-shell nuclei

• Interests
  – Shape
  – Isomer
  – Neutron-rich
  – Astrophysics

• Important roles of $g_{9/2}$ orbit
  – Parity coexisting in low-lying states
  – High-$j$ intruder
  – Deformation driving
  – Spin alignment

• Shell-model with $f_{5pg9}$ model space
  – $^{56}$Ni inert core
  – Valence orbits: $p_{3/2}$, $f_{5/2}$, $p_{1/2}$, $g_{9/2}$
  – No spurious center-of-mass motion
Global fit for f5pg9 model space

- **JUN45 interaction**  
  - Modify microscopic interaction G-f5pg9  
    M. Hjorth-Jensen, unpublished  
    Bonn-C potential  
    3rd order Q-box and folded diagram  
  - Keep isospin symmetry  
  - Vary 45 LC’s of 133 TBME and 4 SPE  
  - Fit to 400 energy data out of 87 nuclei of A=63~96  
    - Include low-lying states of  
      - even-Z nuclei  
      - odd-A nuclei  
  - Exclude  
    - N<46 for Z>33… large quadrupole collectivity (needs for d5/2)  
    - Ni, Cu isotopes … large effects of f7/2 core-excitations  
  - Assume \( A^{-0.3} \) mass dependence  
  - Rms error of 185 keV
G vs. FIT(JUN45)

- **TBME**
  
  \[ V(abcd;JT) \Rightarrow abcd;JT \]
  
  \[
  \begin{align*}
  3 &= p_{3/2} \\
  5 &= f_{5/2} \\
  1 &= p_{1/2} \\
  9 &= g_{9/2}
  \end{align*}
  \]

- **Modification by the fit**
  
  – **T=0** … attractive
  – **T=1** … repulsive
  – Large in 
    \[ V(abab;JT) \] diagonal, large \( J \)
    
    \[ V(aabb;01) \] monopole pairing

- **Multipole part \((V_M)\)**
  
  – \( V_M(\text{FIT}) \) is close to \( V_M(G-f_5pg_9) \)
  – Corrections mainly to \( T=1 \)
Monopole centroids

\[ V(ab; T) = \frac{\sum_j (2J + 1) V(abab; JT)}{\sum_j (2J + 1)} \]

- Similarity between
  - G-f5pg9 and S3V (realistic)
  - JUN45 and Lisetskiy (realistic + fitted)
- T=1
  - Similar to Lisetskiy except for f5-p1 and p1-p1
- T=0
  - Modifications in p1-p1 and f5-p1 due to missing d5/2 ?

Second order correction to the Sussex matrix elements
Empirical corrections to g9p3 and g9p1 diagonal T=0 tbme
Modify G-matrix interaction (T=1 part) by least squares fit

\[ V(ab; T) = \sum_j (2J + 1) V(abab; JT) \]

\[ V(ab; T) = \sum_j (2J + 1) \]

\[ N=Z=34\sim38 \text{ good test} \]
Magnetic moments

- Effective spin g-factors
  - Free-nucleon values are already good (also for sd- and pf-shell)
  - Improvements by effective spin g-factor
    \[ \frac{g_s(\text{eff})}{g_s(\text{free})} \approx 0.7 \text{ by fit to exp. data} \]
    c.f.: \( g_s^{(\text{eff})} \approx 0.85 \) … GXPF1 for pf-shell (LS-closed core)
Quadrupole moments

- Effective charges
  - Fit to exp. data $\Rightarrow e_p \sim 1.5, e_n \sim 1.1$
  - Isoscalar/isovector polarization charges $e^0/e^1$: $e_p = 1 + e^0 - e^1$, $e_n = e^0 + e^1$
  - $e^0 \sim 0.8$, $e^1 \sim 0.3$
  - pf-shell ... $e_p \sim 1.15, e_n \sim 0.8$ near $^{56}\text{Ni}$ ($^{51}\text{Fe}, ^{51}\text{Mn} B(E2; 27/2^- \rightarrow 23/2^-)$)
  - $e^0 \sim 0.5$, $e^1 \sim 0.3$

Ge around N=40

- Structure change as a function of N
- Irregular behavior of $0^+_{2}$
- $g_{9/2}$ orbit plays a crucial role

Symbols ... Exp.
Lines ..........Cal.

E. Padilla-Rondal et al., PRL94 (2005) 122501
K. Starosta et al., PRL99 (2007) 042503
$^{70}\text{Ge}$, $^{74}\text{Ge}$ \ldots $N=40 \pm 2$

- Reasonable agreement in both sides
- $N \leq 40 \Rightarrow \text{Ex(exp.)} \leq \text{Ex(cal.)}$

Insufficient description of the increase of collectivity $\Rightarrow$ small model space?
<table>
<thead>
<tr>
<th>initial $J^\pi(E_z)$</th>
<th>final $J^\pi(E_z)$</th>
<th>multipole</th>
<th>exp. (W.u.)</th>
<th>th. (W.u.)</th>
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<tr>
<td>2$^+$ (834)</td>
<td>0$^+$ (691)</td>
<td>$E2$</td>
<td>17.8 (3)</td>
<td>6.8</td>
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<td>$E2$</td>
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<td>2$^+$ (1464)</td>
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<td>3$^+$ (2515)</td>
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<td>29 ()</td>
<td>5.0</td>
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<td>—</td>
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<td>6$^+$ (2772)</td>
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<td>4 (3) $\times 10^4$</td>
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<td>$E2$</td>
<td>26 (6)</td>
<td>41.1</td>
</tr>
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</table>

![Energy level diagram for 72Ge]
0\textsuperscript{+2} states in Ge

- 0\textsuperscript{+1}, 2\textsuperscript{+1} ... no dominating configuration (< 8%) ⇒ collective character
- 0\textsuperscript{+2} \textsuperscript{72}Ge 37% filling config. ⇒ shell-model like
  \textsuperscript{70}Ge 17% (f\textsubscript{5/2})\textsuperscript{-2}
  \textsuperscript{74}Ge 27% (g\textsubscript{9/2})\textsuperscript{2}

\textbf{• Occupation number of g\textsubscript{9/2}}
  - No N=40 subshell effect in ground-state band
  - 0\textsuperscript{+2} look consistent with N=40 closed-shell configuration with fluctuations

N=40 subshell feature survives in the 0\textsuperscript{+2} states
Towards deformed region

- N=Z nuclei
  - Proton & neutron in the same orbit  
    \[ \Rightarrow \] strong proton-neutron interaction
  - Complicated shell structure  
    \[ \Rightarrow \] deformation change and shape coexistence
  - Alignment and band structure
  - Excluded from data for the fit except for $^{64}\text{Ge}$

To what extent can we explore the deformed region by shell model?
Triaxiality in $^{64}$Ge

- $E_x(4^+)/E_x(2^+) = 2.3$ (exp.) vs. 2.5 (cal.)
  $\Rightarrow \gamma$-soft (Wilets-Jean)

- $B(E2;2^+_2 \rightarrow 2^+_1)/B(E2;2^+_2 \rightarrow 0^+_1) = 14$ (cal.)
  $\Rightarrow \gamma \sim 23^\circ$ (c.f. $27^\circ$ exp.)
  triaxial (Davydov-Filippov)

- No shell-model counterpart for (3-)
  $\Rightarrow$ collective octupole state?

P.J. Ennis et al., NPA535 (1991) 392
E. Farnea et al., PLB551 (2003) 56
K. Starosta et al., PRL99 (2007) 042503

$B(E2) = 27(4)W.u.$

$Q(2^+_2) = +0.29eb$

$Q(2^+_1) = -0.30eb$

$B(E2) = 25W.u.$
Isomeric states in $^{66}$As

- $(5^+)$ isomer ... $T_1/2 = 1.1(1)\mu s$
  \[E_x = 1357 \text{ keV (exp.) vs. } 407 \text{ keV (cal.)}\]
  \[B(E2) \sim 9.3 e^2\text{fm}^4 \text{ (exp.) vs. } 16 e^2\text{fm}^4 \text{ (cal. to } 3^+\)]

- $(9^+)$ isomer ... $T_1/2 = 8.2(5)\mu s$
  \[E_x = 3024 \text{ keV (exp.) vs. } 2506 \text{ keV (cal.)}\]
  \[B(E2) \sim 1.2 e^2\text{fm}^4 \text{ (exp.) vs. } 0.22 e^2\text{fm}^4 \text{ (cal. to } 7^+_2)\]

\[\pi(f_{5/2})\nu(f_{5/2})]^{J=5} \text{ config.}\]
67% in $5^+_{1}$ and 17% in $5^+_{2}$

\[\pi(g_{9/2})\nu(g_{9/2})]^{J=9} \text{ config.}\]
90% in $9^+_{1}$

([Image of a level scheme for $^{66}$As])

\[\pi(g_{9/2})\nu(g_{9/2})]^{J=9} \text{ config.}\]
Shape coexistence in $^{68}\text{Se}$

Successive excitation to $g_{9/2}$ orbit to generate higher angular momentum

JUN45 interaction gives correct shell gap between pf and $g_{9/2}$ orbits

$\nu(g_{9/2}) \sim 4.1$

$\nu(g_{9/2}) \sim 2.3$

$\nu(g_{9/2}) \sim 1.0$

$B(E2) = 26(4)\text{W.u.}$

$A.\text{Obertelli et al., PRC80 (2009) 031304(R)}$

$Q(2^+_{1}) = -41\text{efm}^2$

$Q(2^+_{2}) = +44\text{efm}^2$

$B(E2) = 32\text{W.u.}$

$A.\text{Obertelli et al., PRC80 (2009) 031304(R)}$

Exp.

Shell model
Isomeric states in $^{70}\text{Br}$

- $9^+$ isomer ... $T_{1/2}=2.2(2)$s  
  Ex= 2293 keV (exp) vs. 544 keV (cal.)  
  $(pf_{5/2})^{12}[\pi(g_{9/2})\nu(g_{9/2})]_{J=9}$ config. 87% in $9^+_1$ and 70% in $9^+_2$ (2290keV)
- High-spin states ... successive excitation to $g_{9/2}$ orbits

G. De Angelis et al., EPJA12 (2001) 51
D. G. Jenkins et al., PRC65 (2002) 064307
M. Karny et al., PRC70 (2004) 014310

Reasonable description for the lowest member of each $J$-range characterized by $n(g_{9/2})$

Correct location of $g_{9/2}$, but insufficient collectivity
Proton-neutron multiplets

- N=49 odd-odd nuclei
  - $j_{\pi}j_{\nu}$ multiplet by last proton and neutron
  - Z=39 ($^{88}$Y) to Z=31 ($^{80}$Ga)
  - Not included in the fit
- Test of effective Hamiltonian
  - proton-neutron interaction between lower ($p_{3/2}$, $f_{5/2}$) and upper ($p_{1/2}$, $g_{9/2}$) orbits

\[ \langle j_{\pi}j_{\nu} | V | j_{\pi}j_{\nu} \rangle_J \]

Similarity in
\[ \langle j_{\pi}j_{\nu} | V | j_{\pi}j_{\nu} \rangle_J \]

Test of G-$f_5p_{g9}$ (multipole part)
Systematics of $j_{\pi}-j_\nu$ multiplets

- parity
  - p1g9
  - p3g9
  - f5g9

+ parity
  - g9g9
  - p1p1
  - p3p1
  - f5p1

- $j_{\pi}-j_\nu$ config. >30%
- “p1g9” $\Rightarrow j_{\pi} = p_{1/2}, j_\nu = g_{9/2}$
\[88_{\text{Y}}\]

- Reasonable description for yrast states
- Problem in 1+ and 2− states
  - Indication of insufficient model space
  - Needs for fine tuning of T=0 TBME
Comparison to results in \((p_{1/2}, g_{9/2})\) model space

- F.J.D. Serduke et al., NPA256 (1976) 45 … allow isospin mixing
- R. Gross et al., NPA267 (1976) 85 … exclude \(1^+\) data

"intruder" i.e.,
\((p1g9)\) config. < 10%
86Rb, 84Br, 82As

- 2⁻ ground state (f5g9)
- Good correspondence between exp. and th.

- 6⁻ isomer at 320 ± 100 keV
- Exp. data above 6⁻:
  A. Astier et al., EPJA30 (2006) 541
- J(g.s.)=3 is suggested (decay to $2^+$, $4^+$ in $^{80}$Ge)
  - Cal. 6$^-$ gs ($\pi f_{5/2} v_{g9/2}$)
    but $3^-$ ($\pi p_{3/2} v_{g9/2}$) and $3^+$ ($\pi f_{5/2} v_{p1/2}$) are close
- 1$^+$ data $\leftarrow$ $\beta$-decay experiment
  - $T_{1/2} = 0.54(2)$ s (exp.) vs. 0.22 s (cal.)
  - Difference in the GT distribution $\Rightarrow$ deformation effect?

J.A. Winger PRC36 (758) 1987

log ft

Ex (MeV)

0 1 2 3

th.(+) exp. $^{80}_{32}$Ga

$^{80}_{32}$Ga

0.55s
Difficulties for neutron-rich nuclei

• Model space
  – Description of shell-evolution, new feature of collective motion
  – Multi-major-shell calculation

• Effective interaction
  – Very limited data
  – Empirical correction is difficult

• Possible approach
  – Combine partial interactions determined in smaller model spaces including stable nuclei
  – How to fix unknown parts?
  – Utilize results of mean-field model (with proper tensor force effects)
Neutron-rich Cr

- Binding energy systematics and $E_x(2^+)$ suggest large deformation
- pf-shell model fails for $N \geq 36$

Even-Z

Collectivity due to $g_{9/2}$-d$_{5/2}$?
pfg9d5-shell

- $(f_7/2, p_3/2, f_5/2, p_1/2, g_9/2, d_5/2)$
- 6 SPE + 608 TBME (assume isospin symmetry)

**TBME**
- GXPF1A (pf)
- JUN45 (other f5pg9)
  - mass dependence correction
- G-pfsdg (others)
  - microscopic G-matrix int. by M. Hjorth-Jensen
  - N3LO potential, without Coulomb
  - Modification in paring matrix elements for g9/2 orbit
    - $V(J=0,T=1) + 0.3\text{MeV}, V(J=2,T=1) - 0.15\text{MeV}$

**SPE & monopole corrections**
- pfg9-part … fit to GXPF1A & JUN45 prediction
- Others … fit to WS spe of stable sem-magic nuclei
Ex(2⁺₁) and B(E2 ↓)

- pf ... GXPF1A
- pf+g9 ... PFG9B3
- pf+g9d5 ... A3DA

MCSM 5-dim. Approx.

Lowering of Ex(2⁺) and Ex(4⁺)

Data: O. Sorlin et al., EPJA16 (2003) 55
S. Zhu et al., PRC74 (2006) 064315

Large B(E2) for N ≥ 36
Common effective charge ep = 1.15, en = 0.8

(translated from δ_{pp})

Large B(E2) for N ≥ 36
Common effective charge ep = 1.15, en = 0.8

(translated from δ_{pp})
Energy surface: pfg9d5

- Constrained HF: $\langle r^2 Y_{2\mu} \rangle$, $\langle J_x \rangle$
- $q$-$\gamma$ plot: $\langle 2z^2 - x^2 - y^2 \rangle = q \cos \gamma$, $\langle x^2 - y^2 \rangle = \frac{q}{\sqrt{3}} \sin \gamma$

<table>
<thead>
<tr>
<th>N</th>
<th>$^{58}$Cr$_{34}$</th>
<th>$^{60}$Cr$_{36}$</th>
<th>$^{62}$Cr$_{38}$</th>
<th>$^{64}$Cr$_{40}$</th>
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<tr>
<td>Q-mom. (efm$^2$)</td>
<td>-12</td>
<td>-27</td>
<td>+29</td>
<td>-6</td>
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<tr>
<td>$\langle n(g9/2) \rangle + \langle n(d5/2) \rangle$</td>
<td>0.3</td>
<td>1.6</td>
<td>2.0</td>
<td>2.3</td>
</tr>
<tr>
<td>B(E2;↓) (e$^2$fm$^4$) ep=1.15, en=0.8</td>
<td>195</td>
<td>222</td>
<td>281</td>
<td>375</td>
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Ex (2+)

- **S3V, Lisetskiy**
  - f5pg9-shell

- **LSSM (Nowacki)**
  - O.Sorlin et al., PRL88 (2002) 092501
  - 48Ca core
    - $\pi$: f7/2, p3/2, f5/2, p1/2
    - $\nu$: p3/2, f5/2, p1/2, g9/2
  - TBME
    - pf: KB3G
    - f5pg9: Nowacki
    - f7-g9: KLS
  - SPE of vg9/2
    - 9MeV in 41Ca
    - Monopole shift for $V(f7-g9)$ to reproduce 9/2+ in 57Ni at 0p-0h

R.Grzywacz EPJA25,s01(2005) 89

![Graph showing energy levels versus mass number for $Z=28$.]
B(E2; 0+ → 2+)

- **LSSM (Nowacki)**
  - O.Sorlin et al., PRL88 (2002) 092501
  - ep=1.5, en=0.5

- **QRPA**
  - K.Langanke et al., PRC67 (2003) 044314
  - Quadrupole and quadrupole-pairing forces
Ni isotopes

- pf+g9d5 shell
- 1-dim/5-dim approx.

f5pg9 $ep=1.5, en=1.1$
pf+g9d5 $ep=1.15, en=0.8$

- O. Sorlin et al., PRL88 (2002) 092501
- O. Perru et al., PRL96 (2006) 232501
- O. Kenn et al., PRC63 (2000) 021302(R)
- G. Kraus et al., PRL73 (1994) 1773
- ENSDF database
Summary

• Empirical fits to experimental energy data demand sizable modifications of the microscopic effective interaction based on a realistic G-matrix, especially in the monopole part.
• N=40 subshell structure vanishes in Ge isotopes in the ground states, but can be found in the second 0+ states.
• The triaxial/γ-soft character of 64Ge, shape coexistence in 68Se, isomer states in 66As and 70Br are described simultaneously, in qualitative agreement with experimental data.
• The appearance of the jπ-jν multiplets in neutron-rich odd-odd nuclei is successfully described, suggesting the good property of effective proton-neutron interaction between pf-shell orbits and g9/2 orbit.
• Discrepancies between the shell-model results and the experimental data in collective properties can basically be understood as a result of insufficient model space.
• More detailed tuning of the proton-neutron TBME is also needed for quantitative description of the odd-odd nuclei.
• The f5pg9 model space cannot be a good framework for the unified description of pfg-shell nuclei, but can provide a good starting point for future investigations with more extended model space.