Spin-tensor decomposition of the effective interaction for the shell model

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Tensor force and shell structure

- Tensor force has opposite sign between $j_>$ and $j_<$.
- Change of single particle energies can be written as $(j \neq j')$,
  \[ \Delta \epsilon_p(j) = \frac{1}{2} \left( V_{jj'}^{T=0} + V_{jj'}^{T=1} \right) n_n(j') \]
  $n_n$: occupation number of neutron in orbit $j'$

Simple modeling: taking tensor force as $\pi + \rho$ meson exchange

Same potential in all nuclei, with no fit.

**Question**: Can we consider tensor force in medium so simply?

→ to answer this question, examination based on realistic nuclear force and microscopic theory is needed.
Tensor force and shell structure

- Tensor force has opposite sign between $j_{>}$ and $j_{<}$.
- Change of single particle energies can be written as ($j \neq j'$),
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Realistic nuclear force

Argonne potentials: written in operator form

For example, Argonne $V_{14}$, with operators $O_p$

$$V_{14}(r) = \sum_{p=1}^{14} v^p(r) O_p, \quad v^p(r) = v^p_\pi(r) + v^p_I(r) + v^p_S(r)$$

- Long range $v^p_\pi(r)$: One Pion Exchange Potential (OPEP)
- Intermediate range $v^p_I(r)$: phenomenological fit
- Short range $v^p_S(r)$: phenomenological fit

Operators included in Argonne $v_{18}$, $v_{14}$ and $v_{8'}$ potential

<table>
<thead>
<tr>
<th></th>
<th>CIB</th>
<th>CSB</th>
<th>operators</th>
</tr>
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<tbody>
<tr>
<td>$v_{14}$</td>
<td>No</td>
<td>No</td>
<td>${1, (\sigma_1 \cdot \sigma_2), S_{12}, L \cdot S, L^2, L^2 \sigma_1 \cdot \sigma_2, L \cdot S} \otimes {1, (\tau_1 \cdot \tau_2)}$</td>
</tr>
<tr>
<td>$v_{18}$</td>
<td>Yes</td>
<td>Yes</td>
<td>${1, (\sigma_1 \cdot \sigma_2), S_{12}, L \cdot S, L^2, L^2 \sigma_1 \cdot \sigma_2, L \cdot S} \otimes {1, (\tau_1 \cdot \tau_2)}$, $T_{12}, (\sigma_1 \cdot \sigma_2) T_{12}, S_{12} T_{12}, (\tau_{z1} + \tau_{z2})$</td>
</tr>
<tr>
<td>$v_{8'}$</td>
<td>No</td>
<td>No</td>
<td>${1, (\sigma_1 \cdot \sigma_2), S_{12}, L \cdot S} \otimes {1, (\tau_1 \cdot \tau_2)}$</td>
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- $v_{8'}$ potential is a reduction of $v_{18}$ potential.
Central: \( V_C(r) = V_1(r) + (\sigma_1 \cdot \sigma_2) V_2(r) + (\tau_1 \cdot \tau_2) V_3(r) + (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) V_4(r) \)

Tensor: \( V_T(r) = V_1(r) + (\tau_1 \cdot \tau_2) V_2(r) \)

- Central force: strong short range repulsion
- Tensor force: long range nature
Review on theories of effective interaction
Original eigenvalue problem: \( H |\Psi_i\rangle = E_i |\Psi_i\rangle \)

Effective interaction in \( P \)-space is defined as

\[
P \tilde{H} P |\phi_i\rangle = E_i |\phi_i\rangle, \quad |\phi_i\rangle = P |\Psi_i\rangle.
\]

This is accomplished by the following similarity transformation with decoupling property.

\[
\tilde{H} = e^{-\omega} H e^{\omega}, \quad Q \omega P = \omega, \quad P \tilde{H} Q = 0
\]

Formal solution is

\[
\omega = \sum_{i=1}^{d} Q |\Psi_i\rangle \langle \tilde{\phi}_i | P, \quad |\Psi_i\rangle = |\phi_i\rangle + \omega |\phi_i\rangle
\]

If the unperturbed model space is degenerate \( H_0 |\phi_i\rangle = E_0 |\phi_i\rangle \),

\[
V^{(n)}_{\text{eff}} = \hat{Q}(E_0) + \sum_m \hat{Q}_m(E_0) \{ V^{(n-1)}_{\text{eff}} \}^m
\]

\[
\hat{Q}(E_0) \equiv P H_1 P + P H_1 \frac{1}{E_0 - Q H Q} Q H_1 P, \quad \hat{Q}_m(E_0) \equiv \frac{1}{m!} \frac{d^m \hat{Q}(E_0)}{dE_0^m}
\]
Solve decoupling equation in momentum space

\[ \omega = \sum_{i=1}^{d} Q |\psi_i\rangle \langle \phi_i | P \]

- Conserve all the low-momentum observables and wave functions.
- Coupling between high momentum and low momentum decreases.
- Low momentum attraction increases.

\( ^1S_0 \) channel

\( \Lambda : \) boundary between low momentum and high momentum.

\( \rightarrow \) cutoff parameter
Another method to obtain low-momentum effective interaction

Flow equation

\[ H(s) = U(s) H U^{\dagger}(s) \equiv T_{\text{rel}} + V(s) \]

\[ \frac{dH(s)}{ds} = [\eta, H(s)], \quad \eta(s) = \frac{dU(s)}{ds} U^{\dagger}(s) = -\eta^{\dagger}(s) \]

If we choose \( \eta(s) = [T_{\text{rel}}, H(s)] \)

\[ \frac{dV(k, k')}{ds} = -(k^2 - k'^2)^2 V(k, k') + \int q^2 dq (k^2 + k'^2 - 2q^2) V(k, q) V(q, k') \]

Figures are taken from http://www.physics.ohio-state.edu/ntg/srg/

Channel: \( ^3S_1 - ^3S_1 \) \( \lambda = s^{-4} \approx \Lambda \) in \( V_{\text{low}k} \)
Tensor force in effective interaction
Transform jj-coupled matrix elements to LS-coupled matrix elements

\[
\langle ABLSJT | V | CDL'S'JT \rangle = [(1 + \delta_{AB})(1 + \delta_{CD})]^{1/2} \sum_{j_a,j_b,j_c,j_d} \left[ \begin{array}{cccc} l_a & 1/2 & j_a \\ l_b & 1/2 & j_b \\ L & S & J \\ \end{array} \right] \left[ \begin{array}{cccc} l_c & 1/2 & j_c \\ l_d & 1/2 & j_d \\ L' & S' & J' \\ \end{array} \right]
\times [(1 + \delta_{ab})(1 + \delta_{cd})]^{1/2} \langle abJT | V | cdJT \rangle
\]

Two-body interaction:

\[
V = \sum_p V_p = \sum_p U_p \cdot X_p
\]

\(U_p\): rank \(p\) operator in coordinate space

\(X_p\): rank \(p\) operator in spin space

**Spin-tensor decomposition**

\[
\langle ABLS | V_p | CDL'S' \rangle_{JT} = (-1)^{J'} \hat{p} \left\{ \begin{array}{ccc} L & S & J' \\ S' & L' & p \\ \end{array} \right\} \times \sum_J (-1)^J \hat{J} \left\{ \begin{array}{ccc} L & S & J \\ S' & L' & p \\ \end{array} \right\} \langle ABLS | V | CDL'S' \rangle_{JT}
\]

Uniquely decomposed into central, spin-orbit and tensor component
Tensor-force monopole is not affected much even in the low-momentum interaction $V_{\text{low}k}$, whereas the central force change much by renormalization.
Tensor force in low-momentum interaction ($pf$-shell)

Tensor-force monopole

Tensor force survives in $pf$-shell also

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In momentum space

$$V(k; ^3S_1, k'; ^3D_1)$$

$^3S_1 - ^3D_1$ channel: only tensor force

- Long-range nature of the tensor force
\( V_{\text{low}k} \) from Av8’ Tensor subtracted (Av8’ TS)
\( V_{\text{low}k} \) from Av8’ (Av8 full)

Central-force monopole

\[ \Lambda = 2.1 \text{fm}^{-1} \]

**Short range** part of the tensor force is renormalized to predominantly the central force.

The process shown is expected to have the largest contribution.
Schrodinger equation for deuteron:

\[-\frac{\hbar^2}{M} \frac{d^2 u(r)}{dr^2} + V_C u(r) + \sqrt{8} V_T w(r) = E_d u(r)\]

\[-\frac{\hbar^2}{M} \frac{d^2 w(r)}{dr^2} + \left( \frac{6\hbar^2}{Mr^2} + V_C - 2V_T - 3V_L S \right) w(r) + \sqrt{8} V_T u(r) = E_d w(r)\]

Effective central force

\[V_{\text{eff}}(r;^3S_1) = V_C(r;^3S_1) + \Delta V_{\text{eff}}(r;^3S_1), \quad \Delta V_{\text{eff}}(r;^3S_1) \equiv \sqrt{8} V_T(r) \frac{w(r)}{u(r)}.\]

→ This leads additional attraction to the central force

Feshbach’s formal solution of the effective interaction

\[PHP|\psi\rangle + PHQ \frac{1}{E - QHQ} QHP|\psi\rangle = E|\psi\rangle, \quad H_{\text{eff}} = PHP + PHQ \frac{1}{E - QHQ} QHP\]

\[V_{\text{low}k}\]
- \(P\)-space: low-momentum
- \(Q\)-space: high-momentum

\[\text{Effective central force}\]
- \(P\)-space: \(S\)-wave wavefunction
- \(Q\)-space: \(D\)-wave wavefunction

→ can be understood on the same footing
We calculated tensor-force monopole of $V_{\text{low}k}$ starting from another potential ($\chi$N3LO)

**sd-shell**

**pf-shell**

Similar result is obtained when the original potential is N3LO → result does not depend on specific model of the nuclear force
Effective interaction for shell model

- Starting from $V_{\text{low}k}$
- Q-box and its folded diagrams is taken into account
- Effective interaction for $sd$-shell and $pf$-shell
- Harmonic oscillator basis

Diagrams included in 2nd order Q-box:

- Core polarization
- Particle-particle ladder
- Hole-hole ladder
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Tensor-force monopole ($sd$-shell)

Tensor-force monopole ($pf$-shell)

- Tensor force **survives** even in the effective interaction for shell model, in both $sd$-shell and $pf$-shell
- complicated and specific structure of the tensor force

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Evaluation of the Q-box

- Predominantly the central force
- Tensor component is not coherent

Only exchange diagram contribute to Tensor-force monopole
→ always has the same sign
→ not predominantly the tensor force (tensor force has strong state dependence)
- Good agreement in $T = 0$ channel of SDPF-M
- USD families has weaker tensor force
- $T = 1$ channel includes effects from neutron rich nuclei including dripline
When the Tensor-force monopole is $\pi + \rho$ exchange, remaining can be well approximated by simple Gaussian central force.

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Similar observation also in $T=1$ matrix monopole.
Tensor force in $V_{SRG}$ is also similar to that of original Av8’ tensor force.
Three body force contribution to effective interaction
Three-body force

Virtual excitation to the \( \Delta(1232) \): lowest excited state of the nucleons

exchange \( \pi \) meson two times

Renormalization of single particle energies affected by the Pauli’s exclusion principle in nuclear medium

This effect is included automatically if we consider exchange diagram (Delta-hole diagram)

\[ \rightarrow \text{effective two-body force} \]

\[ \rightarrow \text{we call this effective twobody force comes from} \Delta \text{ hole diagram} \]

FM-twobody force

we calculate the multipole of FM-twobody force in \( T = 1 \) channel
- Three-body force
- Virtual excitation to the $\Delta(1232)$: lowest excited state of the nucleons
- Exchange $\pi$ meson two times

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FM-twobody force

we calculate the multipole of FM-twobody force in $T = 1$ channel
Interaction between N and \( \Delta \) is similar in structure to OPEP

\[
V_{NN}^\pi(k) = \frac{f_{\pi NN}^2}{m_\pi} \frac{(\sigma_1 \cdot k)(\sigma_2 \cdot k)}{m_\pi^2 + k^2} \tau_1 \cdot \tau_2
\]

\[
V_{N\Delta}^\pi(k) = \frac{f_{\pi NN} f_{\pi N\Delta}}{m_\pi} \frac{(\sigma \cdot k)(S \cdot k)}{m_\pi^2 + k^2} \tau \cdot T
\]

- \( S, T \) is spin and isospin operators which convert nucleon to \( \Delta \) (spin = \( \frac{3}{2} \), isospin = \( \frac{3}{2} \))
- For simplicity, we neglect \( \rho \) meson contribution and others
Spin-tensor decomposition of FM-twobody force

- Predominantly central force
- Weak LS and tensor component (other than $p1-p1$ monopole)
- Relatively strong ALS (anti-symmetric spin-orbit) component

→ ALS component does not exist realistic nuclear force
→ consequence of renormalization in nuclear medium
Spin-tensor decomposition (revisited)

Transform \( jj \)-coupled matrix elements to \( LS \)-coupled matrix elements

\[
\langle ABLSJT | V | CDL'S'JT \rangle = [(1 + \delta_{AB})(1 + \delta_{CD})]^{1/2} \sum_{ja, jb, jc, jd} \begin{bmatrix} l_a & \frac{1}{2} & j_a \\ l_b & \frac{1}{2} & j_b \\ L & S & J \end{bmatrix} \begin{bmatrix} l_c & \frac{1}{2} & j_c \\ l_d & \frac{1}{2} & j_d \\ L' & S' & J \end{bmatrix} \times [(1 + \delta_{ab})(1 + \delta_{cd})]^{1/2} \langle abJT | V | cdJT \rangle
\]

Two-body interaction: \( V = \sum_p V_p = \sum_p U^p \cdot X^p \)

- \( U_p \): rank \( p \) operator in coordinate space
- \( X_p \): rank \( p \) operator in spin space

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\]

Uniquely decomposed into central, spin-orbit and tensor component
In LS-coupled matrix element $\langle ABLS | V_p | CDL'S' \rangle_{J' T}$, $S \neq S'$ component is called anti-symmetric spin-orbit (ALS) force.

Consider a two-body problem. If $S \neq S'$, this force is automatically the rank 1 force. Moreover, from the Pauli’s exclusion principle, orbital angular momenta which have different parity must be mixed, if isospin is conserved.

→ such a component does not exist realistic nuclear force

→ consequence of renormalization of nuclear force in nuclear medium
Let us consider diagrams included in 2nd order Q-box

In pp-ladder and hh-ladder diagrams interactions does not change total spin $S$ in each vertex
$\rightarrow$ does not induce ALS component to the effective interaction

Core-polarization diagram is different in structure
$\rightarrow$ total spin of two nucleons can be changed
(Delta-hole diagram has a similar structure)

- Similar state dependence
- Comparable in strength
Tensor force is not affected much by renormalization, at least in its monopole part.

- Tensor-force monopole in $V_{\text{low } k}$ is quite similar to that of original potential.
- Tensor-force monopole in the effective interaction for the shell model calculated by the Q-box expansion is also similar to that of original potential.
- These results do not rely on the specific model of the nuclear force.

→ Tensor force survives both of the two-step renormalization.

- Effective two-body force produced by Fujita-Miyazawa three-body force can be described by a simple picture, central force and anti-symmetric spin-orbit force (ALS). The strength of ALS is comparable to that of core-polarization diagram.
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