From Stopping to Viscosity in Nuclear Reactions

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Reactions of Heavy Nuclei

Late in a reaction, matter describable in terms of a local temp \( T \) and velocity \( \vec{v} \). Dissipation is responsible for equilibration.

⇒ ?Pace of the dissipation? ?Quantitative description of the dissipative transport??

Reactions well described in terms of the Boltzmann equation

\[
\frac{\partial f}{\partial t} + \frac{\partial \epsilon_p}{\partial p} \frac{\partial f}{\partial r} - \frac{\partial \epsilon_p}{\partial r} \frac{\partial f}{\partial p} = \int \mathrm{d}p_2 \mathrm{d}\Omega' \, v_{12} \frac{d\sigma}{d\Omega'} \left( \tilde{f}_1 \tilde{f}_2 f_1' f_2' - \tilde{f}_1' \tilde{f}_2' f_1 f_2 \right)
\]

where \( f \) - nucleon Wigner function, \( \tilde{f} = 1 - f \) - blocking factor, \( \epsilon \) - single-nucleon energy related to the equation of state

Boltzmann valid at low-\( n \)/high-\( T \). At high-\( n \) - phenomenological.

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Dissipative Transport

Examples of the transport on the way towards equilibrium:

- Transport of momentum
- Transport of isospin, i.e. of neutron-proton imbalance

Transport of isospin of interest due to availability of exotic beams - with unusual neutron-proton content.
Transport Coefficients

For slow changes, the flux of a transported quantity is linear in gradients, with proportionality coefficients characteristic for the matter.

Complication: Different gradients present in a reaction - many coefficients?!

⇒ Curie (Pierre) Principle allows some sorting of relations.

Strategy: Identify nuclear reaction observables sensitive to a particular transport coefficient.

Manipulate the Boltzmann equation to reproduce data → deduce the coefficient; make sure that the sensitivity exclusive.

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Flux of z-momentum in the x-direction, $\Pi^{zx}$, proportional to collective velocity gradient:

$$\Pi^{zx} = -\eta \frac{\partial u^z}{\partial x}$$

where $\eta$ - shear viscosity coefficient.
Elementary estimate

Density $n = \text{const}$:

$$ u^z $$

Net Momentum Flux Up = Flux Up - Flux Down

$$ \Pi^{zx} = \frac{1}{6} n v_{\text{kin}} m u^z (x - \lambda) - \frac{1}{6} n v_{\text{kin}} m u^z (x + \lambda) $$

$$ \simeq - \frac{1}{3} n v_{\text{kin}} m \lambda \frac{\partial u^z}{\partial x} $$

Thus, the shear viscosity coefficient:

$$ \eta \simeq \frac{n m v_{\text{kin}}}{3} \lambda \simeq \frac{0.16 \text{ fm}^{-3} 939 \text{ MeV}/c^2 0.3 \text{ c}}{3} 2 \text{ fm} \sim 30 \text{ MeV/fm}^2 $$
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Solving Boltzmann Equation

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\frac{\partial f_j}{\partial t} + \frac{\partial \epsilon_j}{\partial \mathbf{p}} \frac{\partial f_j}{\partial \mathbf{r}} - \frac{\partial \epsilon_j}{\partial \mathbf{r}} \frac{\partial f_j}{\partial \mathbf{p}} = \mathcal{I}_j
\]

where \( j = 1, 2 \) for neutrons and protons. Collision integrals \( \mathcal{I}_j \) vanish, if the local equilibrium Wigner functions are substituted

\[
f_i^{\text{eq}} = \frac{1}{\exp \left( \frac{(p - m u)^2}{2m} \frac{T}{\mu_i} - \mu_i \right) + 1}
\]

However, the LHS does not vanish if there are gradients in the system. Thus, \( f_i^{\text{eq}} \) cannot be a precise solution and

\[
f_i = f_i^{(0)} + f_i^{(1)} + f_i^{(2)} + \ldots
\]

where \( f_i^{(0)} \equiv f^{\text{eq}} \) and \( f^{(n)} \) is of \( n \)'th order in gradients. \( f^{\text{eq}} \) produces no dissipative fluxes, while \( f^{(1)} \) yields lowest-order fluxes linear in gradients & transport coefficients. \( f^{(1)} \) obtained by substituting \( f^{(0)} \) to the LHS and expanding \( \mathcal{I} \) in \( f^{(1)} \).
Solving Boltzmann Equation

\[ \frac{\partial f_j}{\partial t} + \frac{\partial \varepsilon_j}{\partial p} \frac{\partial f_j}{\partial r} - \frac{\partial \varepsilon_j}{\partial r} \frac{\partial f_j}{\partial p} = I_j \]

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Shear Viscosity from Boltzmann Eq

From the structure of the Boltzmann equation
\[ f_i^{(1)} = \phi_i f_i^{(0)} (1 - f_i^{(0)}) \]

where \( \phi \) is a smooth function

Following the Curie principle, anisotropy of symmetric momentum-flux tensor should be driven by the anisotropy of symmetric tensor of velocity gradient:

\[ \phi_i = b_i \left( p_k p_\ell - \frac{p^2}{3} \delta_{k\ell} \right) \left( \nabla_k u_\ell + \nabla_\ell u_k - \frac{2}{3} \delta_{k\ell} \nabla_n u_n \right) \]

Upon substituting to the Boltzmann eq, the result on viscosity is

\[ \eta = \frac{5 T}{9} \frac{\left( \int dp f p^2 \right)^2}{\int dp_1 dp_2 d\Omega f_1 f_2 (1 - f'_1) (1 - f'_2) \nu_{12} \frac{d\sigma}{d\Omega} \rho_{12}^4 \sin^2 \theta} \]

Shi&PD PRC68(03)064604
**Numerical Results**

Free-space cross-sections used; density \( n \) in units of normal \( n_0 \)

At low-\( T \) and high-\( n \), divergence due to diverging mean-free-path.

Simple estimate gave \( \eta \sim 30 \text{ MeV/fm}^2 \text{ c} \).

Validation in terms of data??

Calcs of in-medium cross-sections for Boltzmann eq.: Schnell PRC57(98)806 & Fuchs PRC64(01)024003 - effects of Pauli & eff mass on intermediate states; general suppression of cross-section, particularly of low-energy resonant behavior
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FOPI Measurements of Stopping

Central symmetric collisions of different nuclei from 0.09 to 1.93 GeV/nucleon. Generally, all ptcles with $Z < 10$, excluding pions, weighted with their charge.

Reisdorf et al.
PRL92(04)232301

Rapidity distributions wider in the longitudinal than transverse direction: incomplete stopping

Ratio of rapidity widths:

$$vartl = \frac{\Delta y_t}{\Delta y_l}$$
vartl Excitation Function

Stopping $\leftrightarrow$ In-Medium Cross-Sections $\leftrightarrow$ Viscosity

Boltzmann equation simulations by Brent Barker.

$vartl = \Delta y_t / \Delta y_l$. In the model $A \leq 3$, while FOPI data $Z < 10$: only $E_{lab} \gtrsim 400$ MeV/nucleon relevant; note the INDRA point.

symbols - data

free cross-sections?
**vartl Excitation Function**

**Stopping ↔ In-Medium Cross-Sections ↔ Viscosity**

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Free cross-sections yield far too much stopping.

Is there a role for microscopic cross-sections?
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symbols - data

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Rostock cross-sections yield still too much stopping; Fuchs’ similarly
Cross-Section Phenomenology

‘Microscopic’ cross-sections don’t include effects of other collisions in the vicinity. Phenomenology: size of strong-int cross-section should not exceed interparticle distance.

\[ \sigma \lesssim \sigma_0 = \nu n^{-2/3} \]

Practical realization:

\[ \sigma = \sigma_0 \tanh \left( \frac{\sigma_{\text{free}}}{\sigma_0} \right) \]

with \( \nu \) adjusted.

For \( n \to 0, \sigma \to \sigma_{\text{free}} \).

For \( n \to \infty, \sigma \to \sigma_0 \).
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\( \nu \sim 0.6 \) best
Ca+Ca $v_{artl}$ Excitation Function

FOPI central Ca+Ca events - symbols
(Reisdorf et al. PRL92(04)232301)

Again free cross-sections yield far too much stopping.

$\nu \sim 0.4$ seems favored, but there may be an issue of the quality of central-event selection.
Stopping: Linear Momentum Transfer

Linear mo transfer: Mass asymmetric reactions ($b \sim 0$) examined in lab frame

Velocity component along the beam of the most massive fragment determined and its average compared to the cm velocity.

Limits:

Little stopping: $\langle v_\parallel \rangle \sim 0$ small c.s.?

Large stopping, fusion: $\langle v_\parallel \rangle \sim v_{cm}$ large c.s.?
**In-Medium Cross-Section Reduction?**

Data: Conlin et al. PRC57(98)R1032 - symbols

Ar + Cu, Ag, Au High multiplicity events $\langle b \rangle \sim b_{\text{max}}/4$

Free cross-sections overestimate stopping.

Low $\nu \sim 0.4$ favored.
Number of collisions:
Rostock $\sigma$ yields the same $\langle v_\parallel \rangle / \nu_{cm}$ at all energies, as tempered $\sigma$ with $\nu = 1$, but very different collision No.
What do these $\sigma$s share that decides on the same $\langle v_\parallel \rangle$?

No of collisions with viscous weight $q^4 \sin^2 \theta'$ is nearly the same for the two $\sigma$s, $\sim 3/4$ of the No for free $\sigma$. Found at all the energies in question.
Net In-Medium Cross-Sections??

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Shear Viscosity Data Comparisons

**Viscosity from Transport Analysis of Reaction Data**

Shi&PD PRC68(03)064604:

\[
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\]

**Nuclear Viscosity**

Significant enhancement of the viscosity due to in-medium cross-section & effective-mass reduction.

At \( n \sim n_0 \), \( \eta \sim 75 \text{ MeV/fm}^2 \text{ c} \)
Conclusions

- **Transport theory may be used for deducing macroscopic transport coefficients of nuclear matter.**
- Free cross-sections yield more stopping in collisions than exhibited by data.
- Close correspondence results in simulations between reduced stopping and inferred shear viscosities.
- In-medium modifications appear to raise nuclear viscosity by nearly a factor of 2, compared to expectations based on free-space cross sections and dispersion relation.
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Viscosity vs EOS

QGP phase-transition search focused now on tricritical point

Empirical evidence: close to critical temperature viscosity normalized to entropy minimizes.

Compilation
Csernai et al. PRL97(06)152303

At RHIC $\eta$ limited from above by the strength of collective flow ($v_2$).

Lower limit from strong-coupling limit of gauge theories: $\eta/s \geq 1/4\pi \sim 1/12$.

⇒ Where is nuclear matter in medium-energy collisions?
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Entropy from Cluster Yields

Simple formula: \( S/A \approx 3.9 - \log \left( \frac{N_d}{N_p} \right) \)

Siemens & Kapusta PRL43(79)1486

Independent testing in transport theory

PD & Bertsch NPA533(81)712
In intermediate-energy reactions, $s/n \equiv S/A = (3-4.5)$, corresponding to $T = (40-70)$ MeV at $n = n_0$, yielding $\eta/s = (0.5-0.7)$

Results fall in the ballpark of other but follow from nuclear data.

Lacey et al PRL98(07)092301