Clusters and fragments formed in expanding nuclear matter in heavy-ion collisions

Akira Ono (Tohoku University)

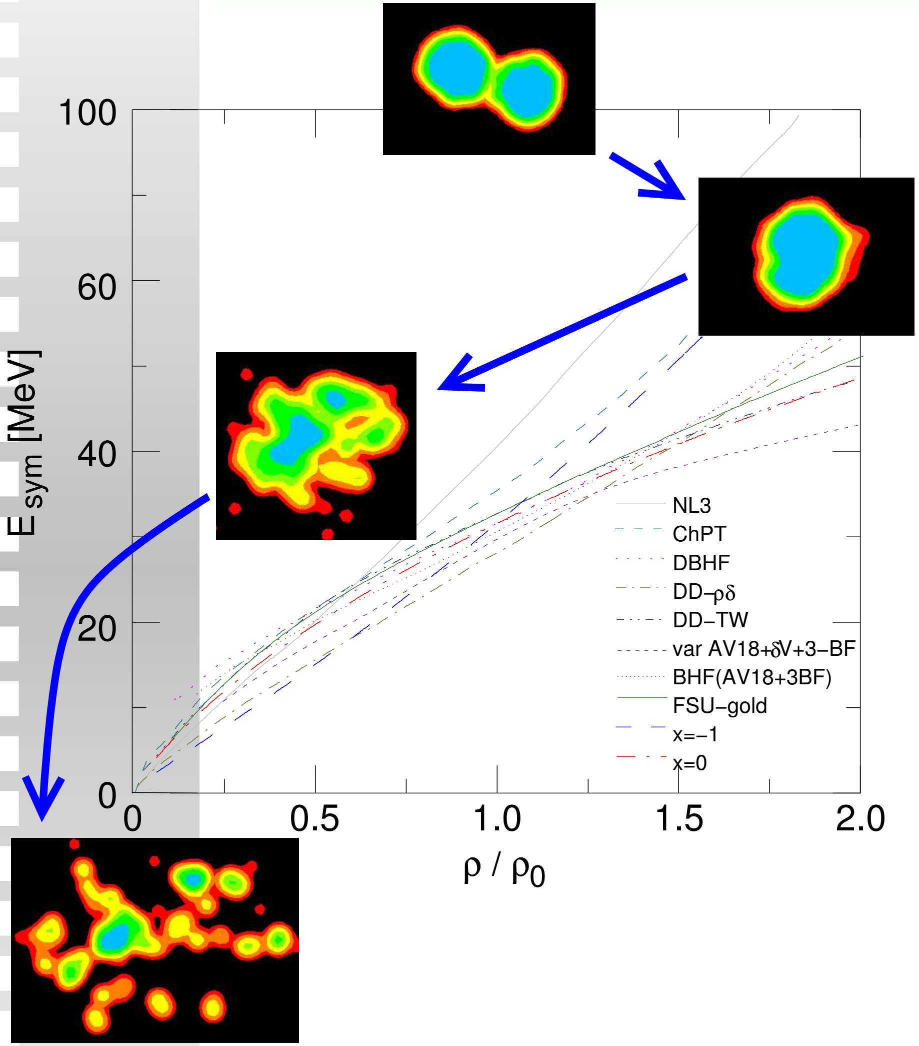
- Relevance of equilibrium in fragmentation

  Furuta and Ono, arXiv:0811.0428 [nucl-th]

- Cluster correlations in the AMD approach

  started at NSCL in 2005

EOS and Collision Dynamics



Energy of nuclear matter

$$E(\rho,\delta)/A = E(\rho,0)/A + E_{\text{sym}}(\rho)\delta^2$$

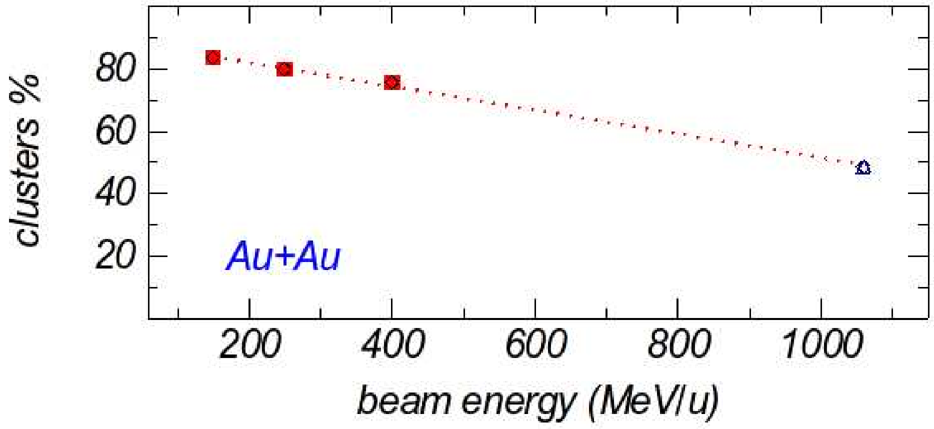
$$\delta = (\rho_n - \rho_p)/\rho$$

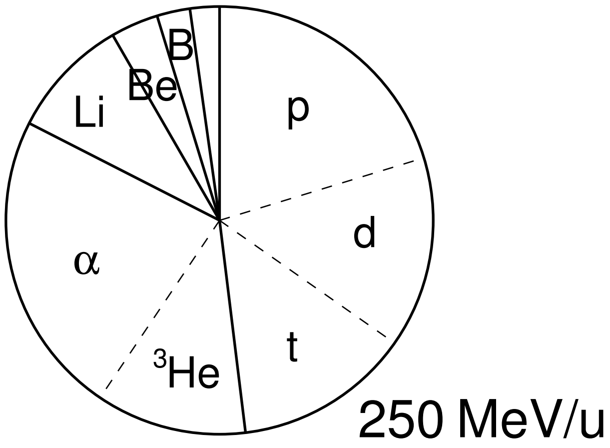
- $E(\rho, 0)$ (Symmetric matter $\rho_n = \rho_p$)
- $E_{\text{sym}}(\rho)$: Symmetry energy
- Depends on temperature T

  free energy rather than energy
- LG phase transition (two components)
- Effective masses $m_n^*(\rho,\delta)$, $m_p^*(\rho,\delta)$
- NN cross sections $\sigma_{NN}(\rho,\delta)$

Clusters as bulk properties

- Many experimental observables (to probe high and low densities) are related to clusters and fragments. ($t/^3$He, isoscaling etc)
- Clusters and fragments are the main part of the total system.

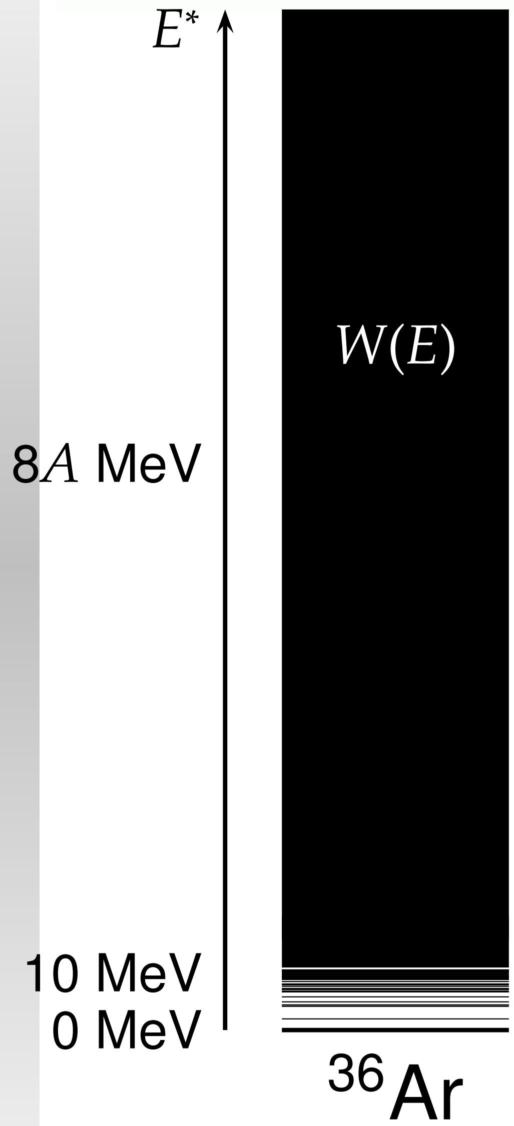




- Consider four nucleons in the gas at $T = 10$ MeV, for example.
  - Uncorrelated: $\langle E \rangle = \frac{3}{2}T \times 4 = 60$ MeV
  - α cluster: $\langle E \rangle = -28.3\,\text{MeV} + \frac{3}{2}T \times 1 = -13.3$ MeV

Clusters are important as “Bulk Nuclear Properties”.

Excited low-density system



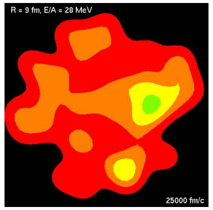
Gas

(nucleons + clusters)

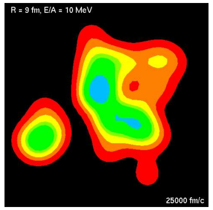
Liquid-gas

phase transition

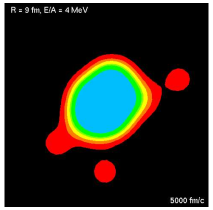
$W(E) \approx e^{2\sqrt{aE^*}}$



$E^* = 28A$ MeV



$E^* = 10A$ MeV

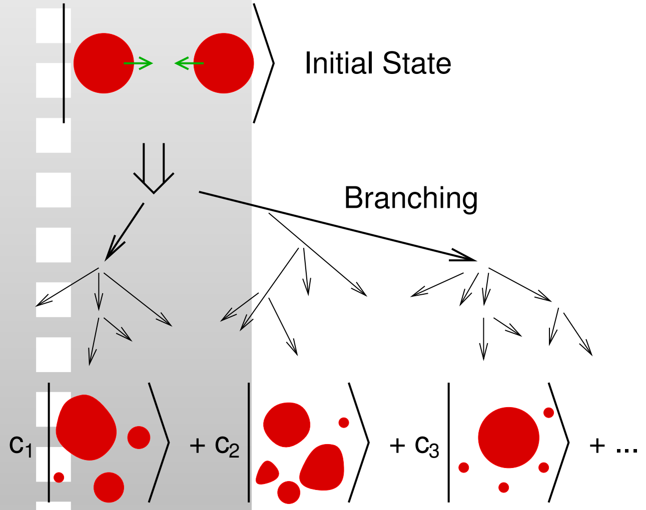


$E^* = 4A$ MeV

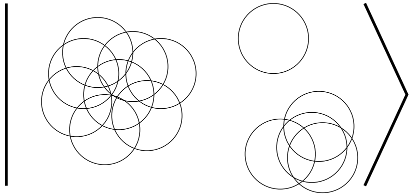
Volume $V = \frac{4}{3}\pi(9\text{ fm})^3$

Antisymmetrized Molecular Dynamics

AMD wave function



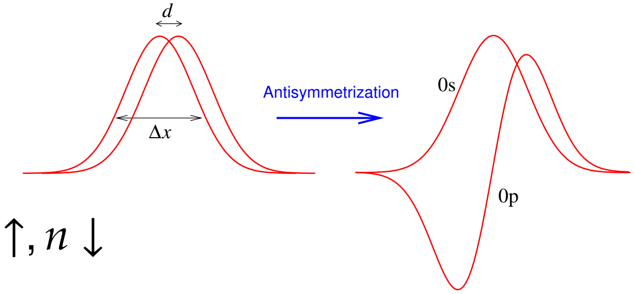
$$|\Phi(Z)\rangle = \det_{ij}\Big[\exp\Big\{-\nu\Big(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\Big)^2\Big\}\chi_{\alpha_i}(j)\Big]$$



$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = (2.5 fm)$^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$



Stochastic equation of motion for the wave packet centroids Z:

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} + \Delta\mathbf{Z}_i(t) + (\text{NN collisions})$$

- Mean field (Time evolution of single-particle wave functions)
- Nucleon-nucleon collisions (as the residual interaction)

Energy is conserved. No temperature in the equation.

Quantum effects are included.

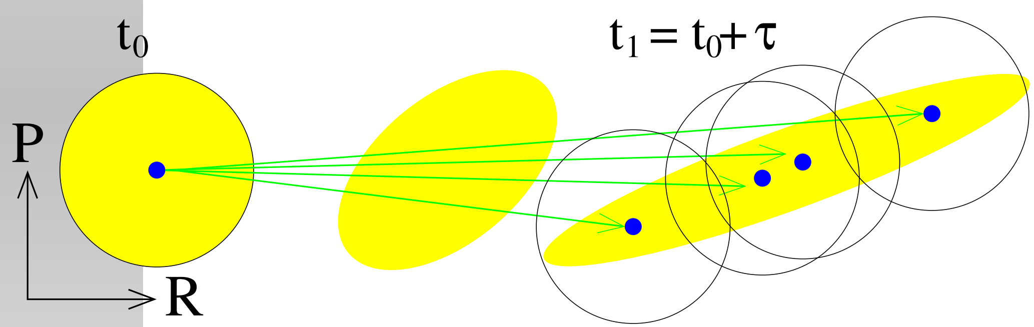
Mean field + Qauntum branching

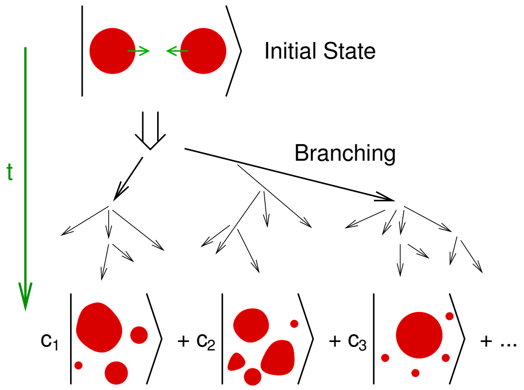
At each time t_0, for each wave packet k, ...

Mean field propagation $t_0 \to t_0 + \tau$ + **Branching at $t_0 + \tau$** τ: Coherence time

$$t = t_0 \qquad\qquad t = t_0 + \tau$$

$$|\mathbf{Z}_k\rangle\langle\mathbf{Z}_k| \xrightarrow[\text{Mean field}]{} |\psi_k\rangle\langle\psi_k| \xrightarrow[\text{Branching}]{} \int |\mathbf{z}\rangle\langle\mathbf{z}|\, w_k(\mathbf{z}) d\mathbf{z} \qquad \text{for } k = 1, \ldots, A$$





$$i\hbar\frac{d}{dt}|\psi_k(t)\rangle = h^{\mathrm{HF}}|\psi_k(t)\rangle \quad \text{or} \quad \frac{\partial f_k}{\partial t} = -\frac{\partial h^{\mathrm{HF}}}{\partial \mathbf{p}} \cdot \frac{\partial f_k}{\partial \mathbf{r}} + \frac{\partial h^{\mathrm{HF}}}{\partial \mathbf{r}} \cdot \frac{\partial f_k}{\partial \mathbf{p}}$$

- $\tau \to 0$ (Strongest branching)
- $\tau = \tau(\rho)$ (Density-dependent)
- $\tau = \tau_{\text{NN-coll}}$ (Decoherence at NN collisions)

Langevin-like equation of motion

Equation of motion for the wave packet centroids

$$
\begin{aligned}
\frac{d}{dt}\mathbf{Z}_i &= \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} && \text{Mean field} \\
&+ \Delta\mathbf{Z}_i(t) && \text{Mean field \& Branching} \\
&+ \mu(\mathbf{Z}_i, \mathcal{H}') && \text{Dissipation} \\
&+ \text{NN-Collision}
\end{aligned}
$$

If $\mathbf{Z}_i$ were canonical variables for simplicity,

$$\{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} = \frac{1}{i\hbar}\frac{\partial\mathcal{H}}{\partial\mathbf{Z}_i^*}$$

$$\overline{\Delta Z_{ia}(t)} = 0, \qquad \overline{\Delta Z_{ia}(t)\Delta Z_{jb}(t)} = D_{iab}(t)\delta_{ij}\delta(t-t')$$

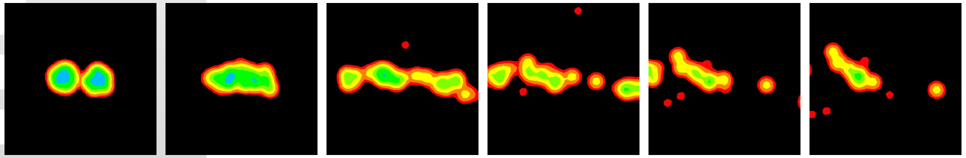
$$(\mathbf{Z}_i, \mathcal{H}') = \frac{1}{\hbar}\frac{\partial\mathcal{H}'}{\partial\mathbf{Z}_i^*}, \qquad \mathcal{H}' = \mathcal{H} + \sum_m \beta_m Q_m$$

- μ is determined by the total energy conservation.
- Lagrange multipliers β_m are determined so that Q_m are not changed by the $(\mathbf{Z}_i, \mathcal{H}')$ term.

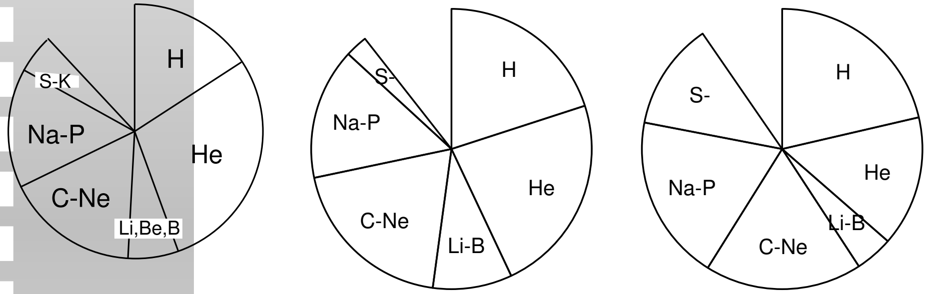
$$\{Q_m\} = \left\{\left\langle\sum_i \mathbf{r}_i\right\rangle, \left\langle\sum_i \mathbf{p}_i\right\rangle, \left\langle\sum_i \mathbf{r}_i \times \mathbf{p}_i\right\rangle, \left\langle\sum_i r_{i\sigma}r_{i\tau}\right\rangle, \left\langle\sum_i p_{i\sigma}p_{i\tau}\right\rangle\right\} \qquad \sigma, \tau = x, y, z$$

AMD results for fragmentation

^{40}Ca + ^{40}Ca at 35 MeV/u, $b = 0$



Experiment AMD AMD

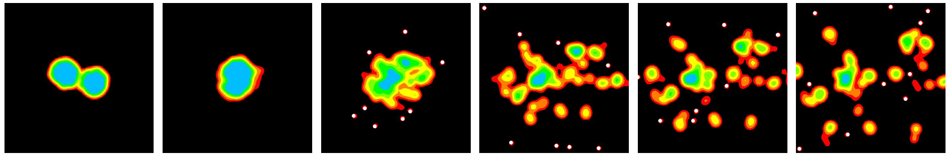


Hagel et al.
PRC50(1994)2017

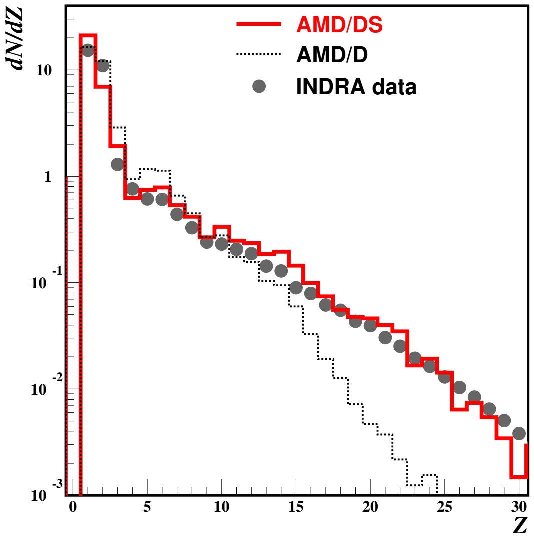
$\tau(\rho)$

$\tau_{\text{NN-coll}}$

Xe + Sn at 50 MeV/u, $0 \leq b \leq 4$ fm



Charge distribution

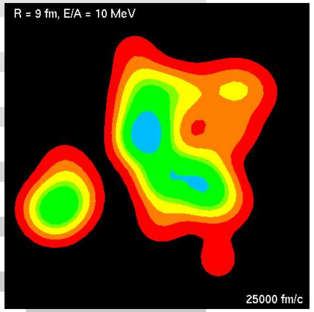


Can we reproduce different data with the same model of branching?
(Cluster correlations?)

- AMD ($\tau \to 0$)
- AMD ($\tau_{\text{NN-coll}}$)

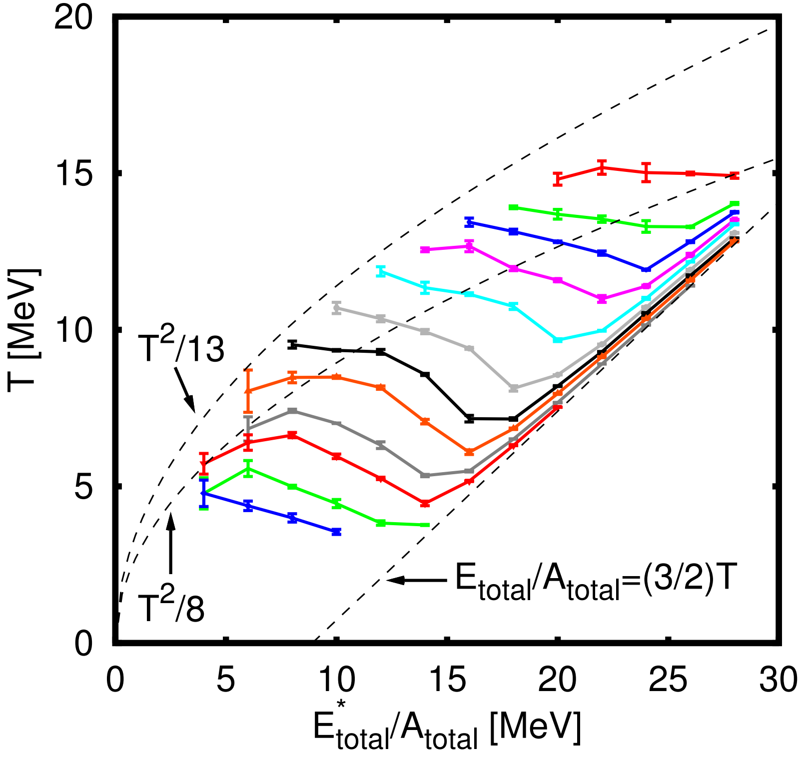
Equilibrium ensembles and caloric curves

Microcanonical ensemble ⇐ Simply solve the time evolution for a long time



- Total energy: E
- Volume: $V = \frac{4}{3}\pi R^3$ (reflections at the wall of container)
- Neutron and proton numbers: $N = 18$, $Z = 18$

⇒ Temperature $T(E, V)$ and Pressure $P(E, V)$

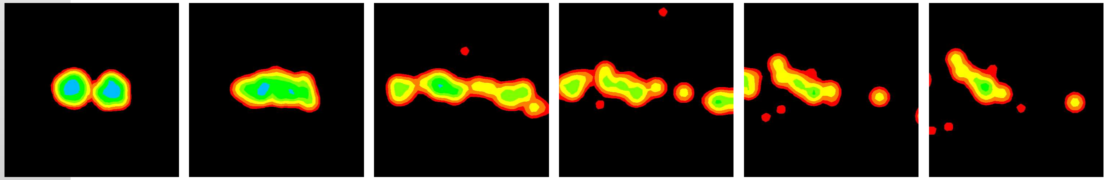


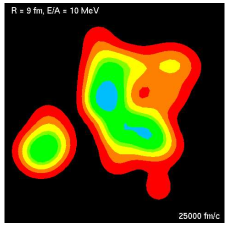
Furuta and Ono,
arXiv:0811.0428 [nucl-th];
PRC74 (2006) 014612.

Comparison of reaction and equilibrium

Furuta and Ono, arXiv:0811.0428 [nucl-th].

^{40}Ca + ^{40}Ca, $E/A = 35$ MeV, $b = 0$



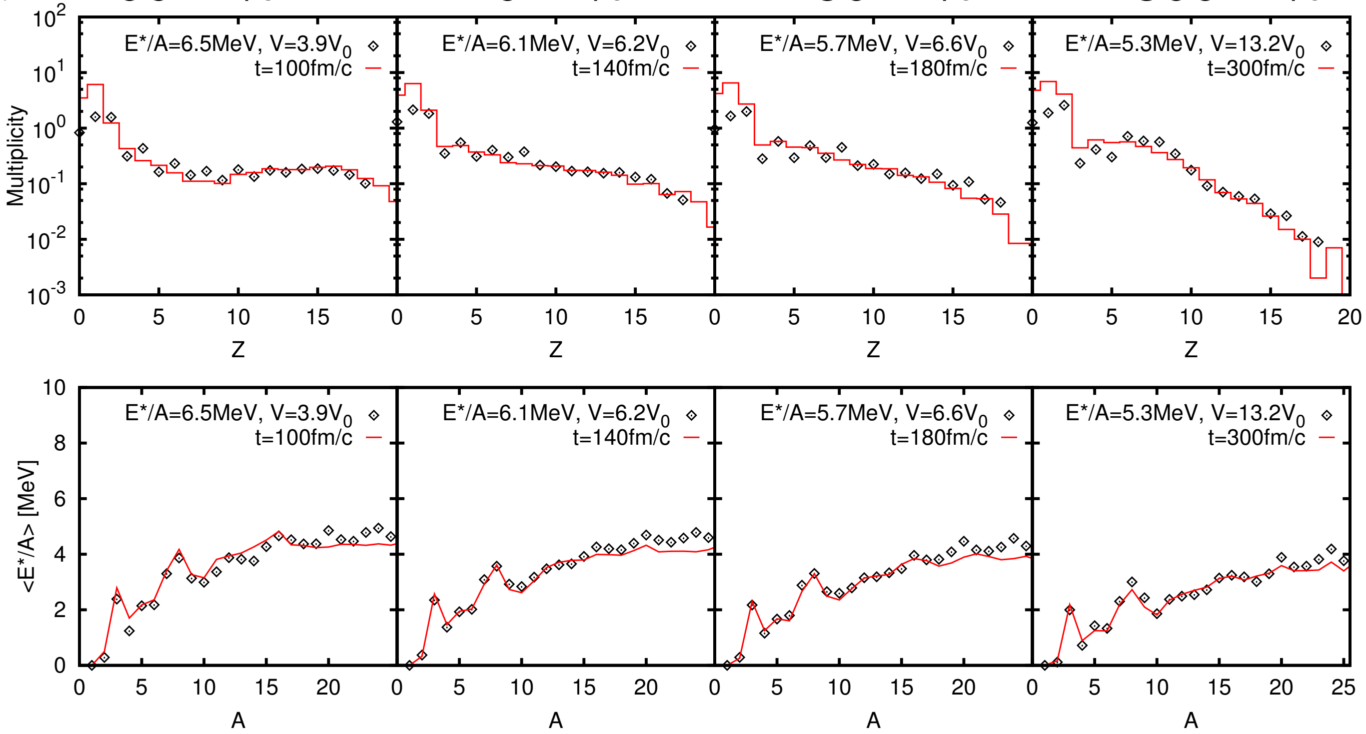


$$\{\text{States at the reaction time } t\} \quad = \overset{?}{=} = \quad \text{Equilibrium ensemble}(E, V, A)$$

t = 100 fm/c 140 fm/c 180 fm/c 300 fm/c

$Y(Z)$

$\langle E^*/A \rangle$

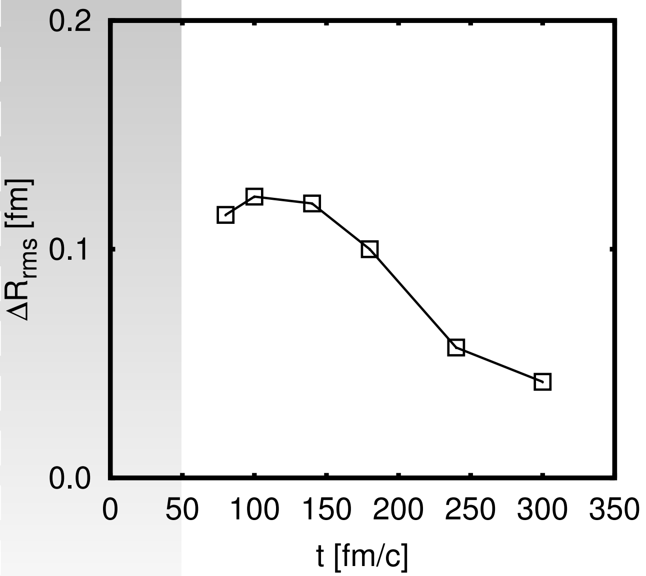


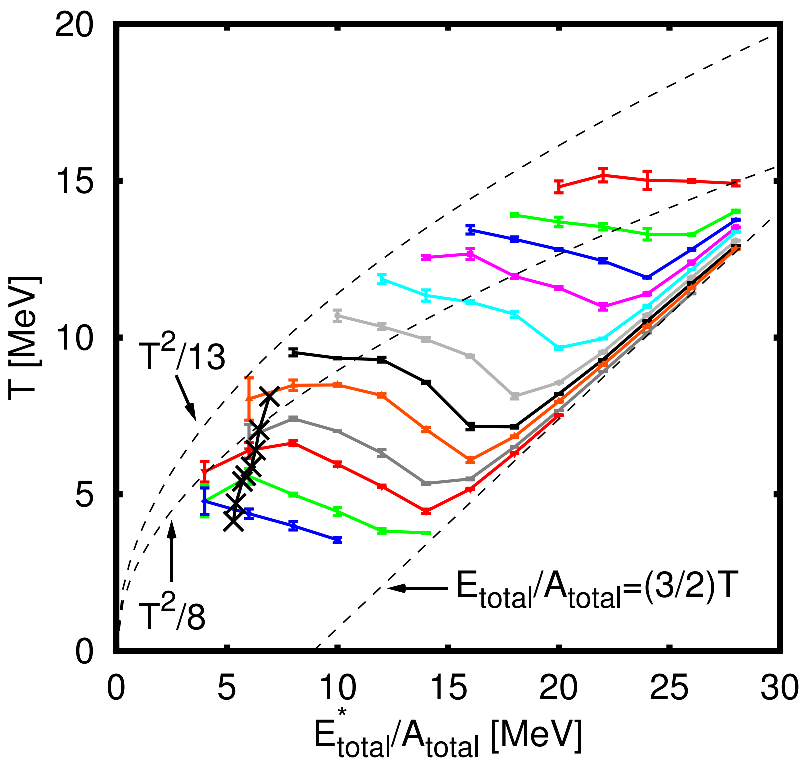
Result of comparison

Fragment observables during the reaction ($80 \le t \le (300+)$ fm/c) are well explained as eqilibrium properties of nuclear many-body system.

Some dynamical effects

- Finite flow
- Fragment radius (the figure below)
- Actual volume

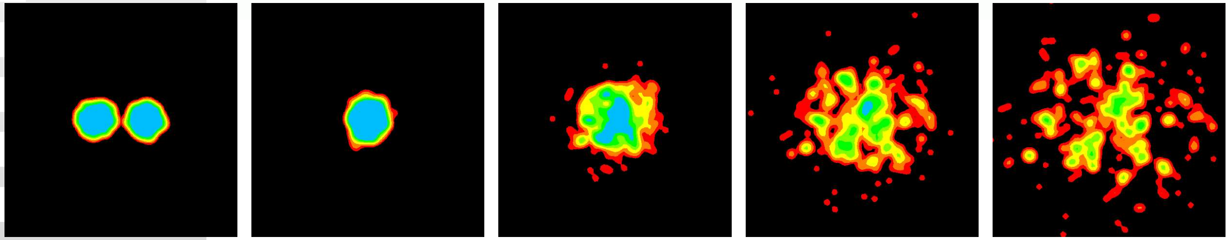




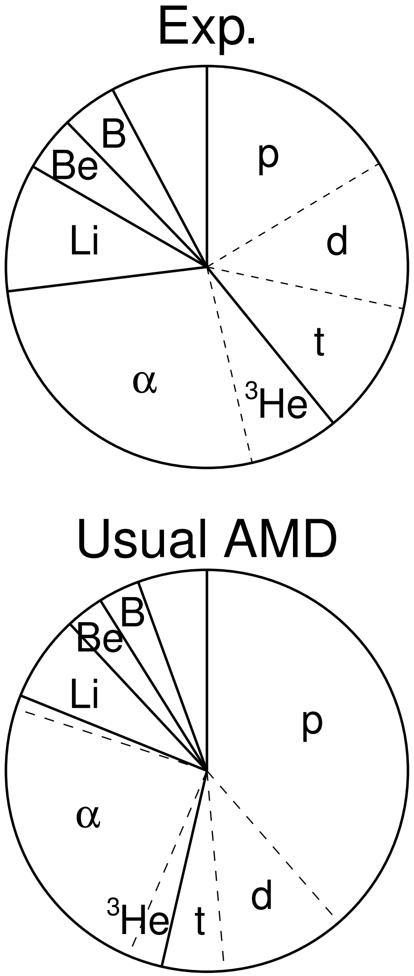
^{40}Ca + ^{40}Ca, $E/A = 35$ MeV, $b = 0$

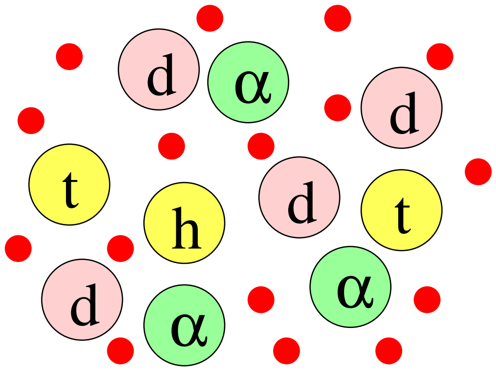
arXiv:0811.0428 [nucl-th]

Cluster correlations



^{197}Au + ^{197}Au at 150 MeV/u

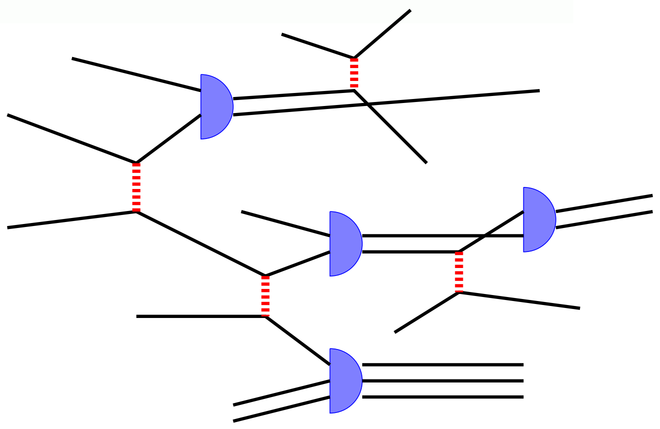




Cluster formation

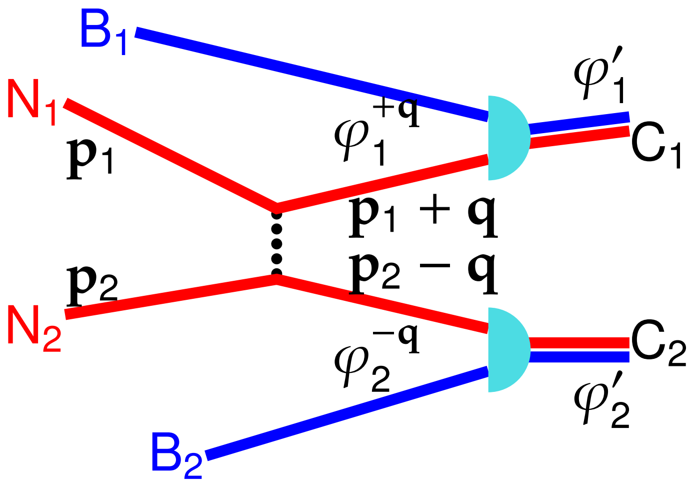
During the time evolution of AMD,

- Cluster formation
- Propagation
- Breakup



$$N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2$$

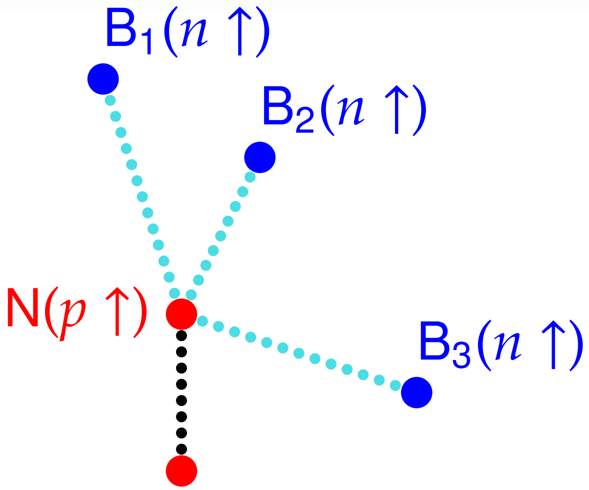
- N_1, N_2 : Colliding nucleons
- B_1, B_2 : Spectator nucleons/clusters
- C_1, C_2 : N, $(2N)$, $(3N)$, $(4N)$



$$\frac{d\sigma}{d\Omega} = F_{\rm kin}\, |\langle\varphi'_1|\varphi_1^{+\mathbf{q}}\rangle|^2\, |\langle\varphi'_2|\varphi_2^{-\mathbf{q}}\rangle|^2 \left(\frac{d\sigma}{d\Omega}\right)_{\rm NN}$$

c.f. Danielewicz et al., NPA533 (1991) 712.

Non-orthogonality of final states



Non-orthogonality of final states:

$N_{BB'} \equiv \langle \Phi_B | \Phi_{B'} \rangle \neq \delta_{BB'}$

The probability that N forms a cluster with one of B's:

$$P = \langle \Phi^q | \hat{X} | \Phi^q \rangle \qquad \hat{X} = \sum_{BB'} |\Phi'_B\rangle N^{-1}_{BB'} \langle \Phi'_{B'}|$$

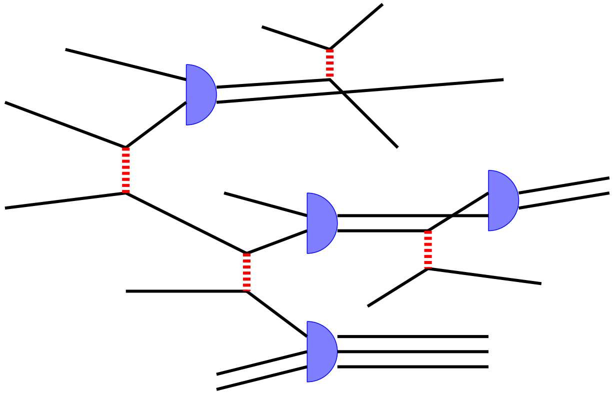
$$= \sum_B |\langle \tilde{\Phi}'_B | \Phi^q \rangle|^2 \qquad |\tilde{\Phi}'_B\rangle = (N^{-1/2})_{BB'} |\Phi_{B'}\rangle$$

$|\langle \tilde{\Phi}'_B | \Phi^q \rangle|^2$ is regarded as the probability that N forms a cluster with B.

The details of cluster correlations

- **Formation**
  - $(d\sigma/d\Omega)_{\text{NN}} \Rightarrow$ Cluster formation cross section
  - Clusters: N, $2N$, $3N$, $4N = (0\text{s})^n$
  - Pauli-blocking factor: $\prod_{i\in\text{C}}(1 - f_i)$
  - Avoid double countings of final states
  - Take care of the non-orthogonality of final states
- **Propagation**
  Nucleons i in a cluster C are propagated as usual, except that the internal fluctuations are turned off:

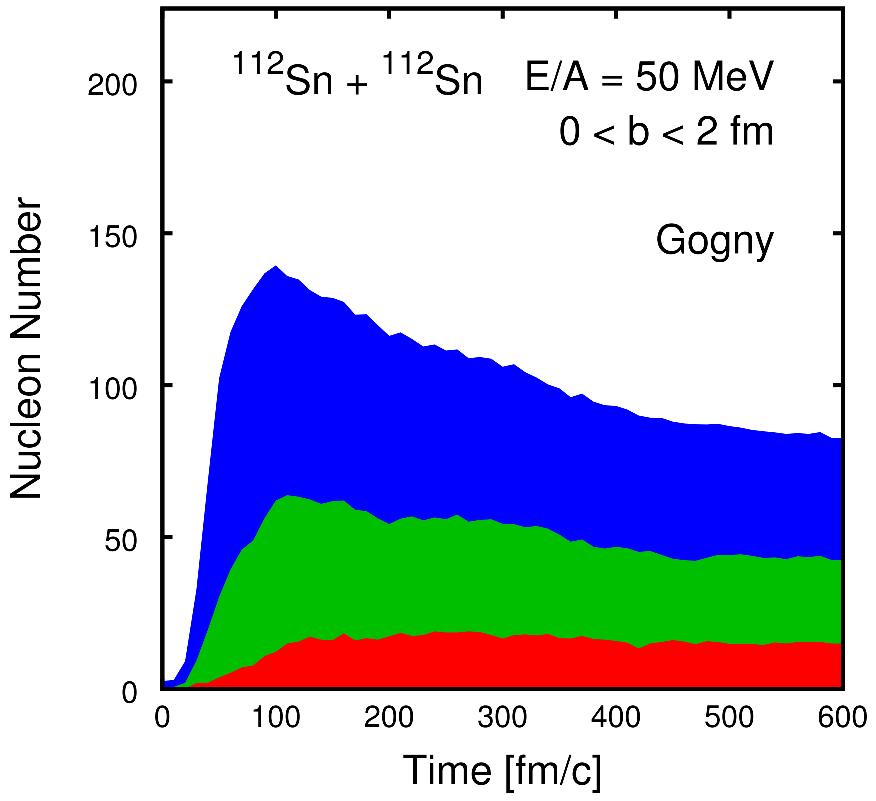
$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} + \Delta\mathbf{Z}_i(t), \quad \Delta\mathbf{Z}_i(t) := \frac{1}{\text{C}}\sum_{j\in\text{C}}\Delta\mathbf{Z}_j(t)$$



- **Breakup**
  A cluster C is broken when a nucleon in C collides with another nucleon.

Time evolution of number of clusters

Number of nucleons in correlated clusters



Non-clustered

(2N)

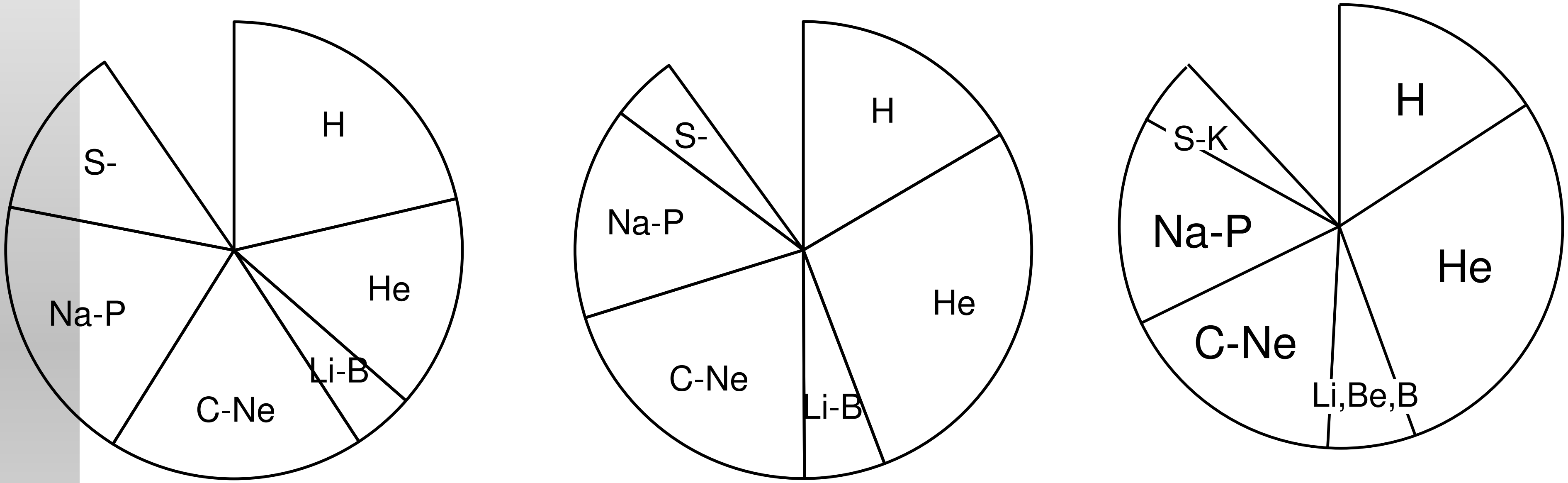
(3N)

(4N)

Effects of cluster correlations

^{40}Ca + ^{40}Ca, E/A = 35 MeV, filtered violent collisions

w/o cluster correlations — with cluster correlations — experiment



	w/o cluster correlations	with cluster correlations
p	6.7	4.4
d	1.5	1.8
t	0.3	0.5
^{3}He	0.3	0.6
α	2.7	5.0

Effects of clusters

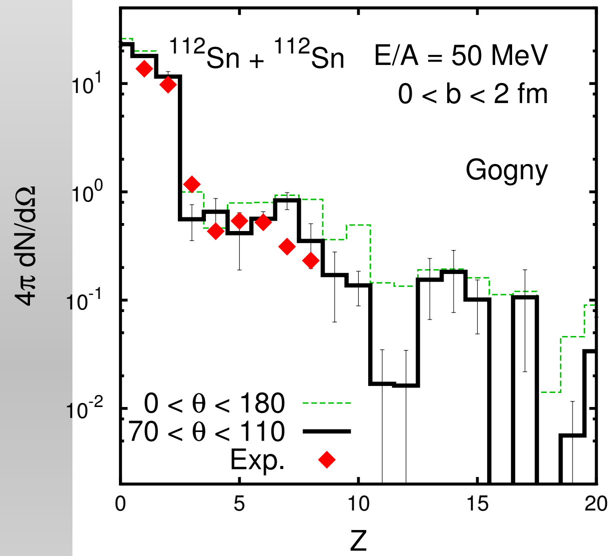
- M_p $\downarrow$
- M_α $\uparrow$
- $\sum_{\text{IMF}} Z$ $\downarrow$

Coherence time: $\tau_{\text{NN-coll}}$

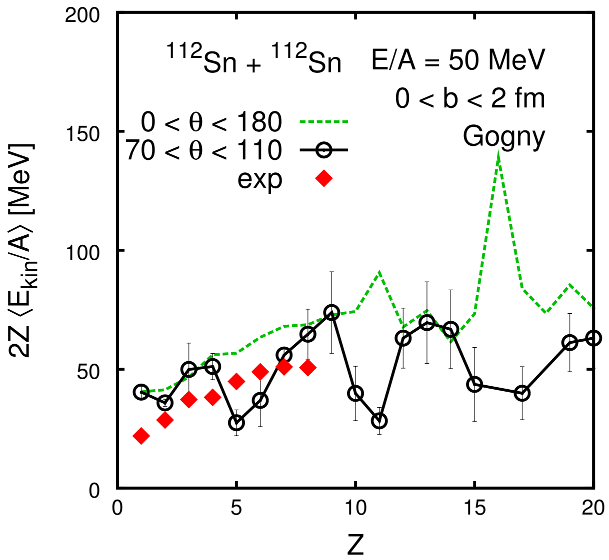
Results for Sn + Sn **system**

^{112}Sn + ^{112}Sn at $E/A = 50$ MeV/nucleon, $0 < b < 2$ fm

With cluster correlations $\Sigma Z(70^\circ < \theta < 110^\circ) = 25$



n	26.0
p	9.9
d	6.5
t	3.6
^{3}He	1.4
α	10.4



- Reasonable numbers of clusters.
- Maybe too much transverse emission (i.e. too large σ_{NN}).
- Exact calculation ($\propto A^4$) with Gogny force.

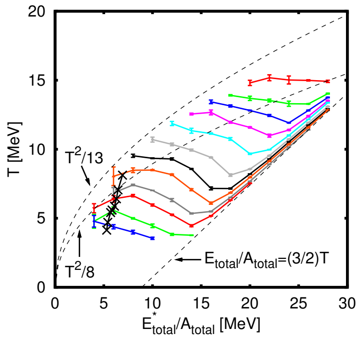
Xe+Sn, INDRA data

p	8.4
d	4.4
t	3.3
^{3}He	0.9
α	10.1

multiplicities of detected particles

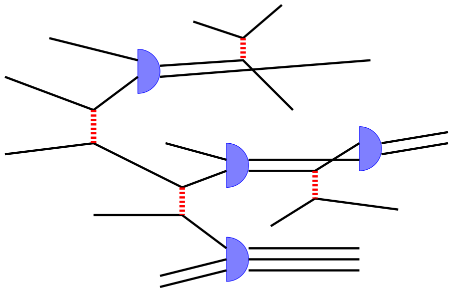
Summary

Clusters and fragments are important as an aspect of bulk properties of expanding nuclear matter.



Reaction and Equilibrium — A unified study with AMD

- Equivalence for fragment observables at each reaction time [$80 \lesssim t < (300+)$ fm/c]
- Some dynamical effects



Cluster correlations in AMD

- N$_1$ + B$_1$ + N$_2$ + B$_2$ $\to$ C$_1$ + C$_2$, based on $(d\sigma/d\Omega)_{NN}$
- Cluster correlations have systematic effects on M_p, M_α, and $\sum_{\text{IMF}} Z$.
- Consistent reproduction of various multifragmentation data may be improved.