Clusters and fragments formed in expanding nuclear matter in heavy-ion collisions

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- Relevance of equilibrium in fragmentation
  Furuta and Ono, arXiv:0811.0428 [nucl-th]

- Cluster correlations in the AMD approach
  started at NSCL in 2005
**EOS and Collision Dynamics**

Energy of nuclear matter

\[ E(\rho, \delta)/A = E(\rho, 0)/A + E_{\text{sym}}(\rho)\delta^2 \]
\[ \delta = (\rho_n - \rho_p)/\rho \]

- \( E(\rho, 0) \) (Symmetric matter \( \rho_n = \rho_p \))
- \( E_{\text{sym}}(\rho) \): Symmetry energy
- Depends on temperature \( T \)
- Free energy rather than energy
- LG phase transition (two components)
- Effective masses \( m_n^*(\rho, \delta), m_p^*(\rho, \delta) \)
- NN cross sections \( \sigma_{NN}(\rho, \delta) \)
Clusters as bulk properties

- Many experimental observables (to probe high and low densities) are related to clusters and fragments. \((t/\bar{3}\text{He},\ \text{isoscaling etc})\)

- Clusters and fragments are the main part of the total system.

Consider four nucleons in the gas at \(T = 10\ \text{MeV}\), for example.

- Uncorrelated: \(\langle E \rangle = \frac{3}{2}T \times 4 = 60\ \text{MeV}\)

- \(\alpha\) cluster: \(\langle E \rangle = -28.3\ \text{MeV} + \frac{3}{2}T \times 1 = -13.3\ \text{MeV}\)

Clusters are important as “Bulk Nuclear Properties”.

\[
\begin{align*}
\text{Clusters}\% &
\end{align*}
\]
Excited low-density system

\[ E^* = 28A \text{ MeV} \]

\[ E^* = 10A \text{ MeV} \]

\[ E^* = 4A \text{ MeV} \]

\[ W(E) \approx e^{2\sqrt{aE^*}} \]

Volume \( V = \frac{4}{3} \pi (9 \text{ fm})^3 \)
Antisymmetrized Molecular Dynamics

**AMD wave function**

\[ |\Phi(Z)\rangle = \text{det}_{ij} \left\{ \exp\left\{ -\nu\left( \frac{Z_i}{\sqrt{\nu}} \right)^2 \right\} \chi_{\alpha_i}(j) \right\} \]

\[ Z_i = \sqrt{\nu} D_i + \frac{i}{2\hbar \sqrt{\nu}} K_i \]

\( \nu \): Width parameter = \((2.5 \text{ fm})^{-2} \)

\( \chi_{\alpha_i} \): Spin-isospin states = \( p \uparrow, p \downarrow, n \uparrow, n \downarrow \)

**Stochastic equation of motion for the wave packet centroids** \( Z \):

\[ \frac{d}{dt} Z_i = \{ Z_i, H \}_\text{PB} + \Delta Z_i(t) + (\text{NN collisions}) \]

- Mean field (Time evolution of single-particle wave functions)
- Nucleon-nucleon collisions (as the residual interaction)

Energy is conserved. No temperature in the equation. Quantum effects are included.
Mean field + Quantum branching

At each time $t_0$, for each wave packet $k$, . . .

Mean field propagation $t_0 \rightarrow t_0 + \tau$ + Branching at $t_0 + \tau$

$\tau$: Coherence time

$t = t_0$

$t = t_0 + \tau$

$|Z_k\rangle\langle Z_k| \rightarrow |\psi_k\rangle\langle \psi_k|$ (Mean field)

Branching

$\int |z\rangle\langle z| w_k(z) dz$ for $k = 1, \ldots, A$

$i\hbar \frac{d}{dt} |\psi_k(t)\rangle = h^{HF} |\psi_k(t)\rangle$ or $\frac{\partial f_k}{\partial t} = -\frac{\partial h^{HF}}{\partial p} \cdot \frac{\partial f_k}{\partial r} + \frac{\partial h^{HF}}{\partial r} \cdot \frac{\partial f_k}{\partial p}$

$\tau \rightarrow 0$ (Strongest branching)

$\tau = \tau(\rho)$ (Density-dependent)

$\tau = \tau_{NN-coll}$ (Decoherence at NN collisions)
Langevin-like equation of motion

Equation of motion for the wave packet centroids

\[
\frac{d}{dt} Z_i = \{Z_i, \mathcal{H}\}_{PB} \quad \text{Mean field}
\]
\[
+ \Delta Z_i(t) \quad \text{Mean field & Branching}
\]
\[
+ \mu(Z_i, \mathcal{H}') \quad \text{Dissipation}
\]
\[
+ \text{NN-Collision}
\]

If \( Z_i \) were canonical variables for simplicity,

\[
\{Z_i, \mathcal{H}\}_{PB} = \frac{1}{i\hbar} \frac{\partial \mathcal{H}}{\partial Z_i^*}
\]

\[
\Delta Z_{ia}(t) = 0, \quad \Delta Z_{ia}(t)\Delta Z_{jb}(t) = D_{iab}(t)\delta_{ij}\delta(t - t')
\]

\[
(Z_i, \mathcal{H}') = \frac{1}{\hbar} \frac{\partial \mathcal{H}'}{\partial Z_i^*}, \quad \mathcal{H}' = \mathcal{H} + \sum_m \beta_m Q_m
\]

\( \mu \) is determined by the total energy conservation.

Lagrange multipliers \( \beta_m \) are determined so that \( Q_m \) are not changed by the \((Z_i, \mathcal{H}')\) term.

\[
\{Q_m\} = \{\langle \sum_i r_i \rangle, \langle \sum_i p_i \rangle, \langle \sum_i r_i \times p_i \rangle, \langle \sum_i r_{i\sigma} r_{i\tau} \rangle, \langle \sum_i p_{i\sigma} p_{i\tau} \rangle\} \quad \sigma, \tau = x, y, z
\]
**AMD results for fragmentation**

\[ ^{40}\text{Ca} + ^{40}\text{Ca} \text{ at } 35 \text{ MeV/u, } b = 0 \]

\[ \text{Xe} + \text{Sn} \text{ at } 50 \text{ MeV/u, } 0 \leq b \leq 4 \text{ fm} \]

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**Experiment**

\[ \begin{align*}
\text{H} & \quad \text{He} \\
\text{S-K} & \quad \text{Na-P} \\
\text{C-Ne} & \quad \text{Li-B}
\end{align*} \]

**AMD**

\[ \tau(\rho) \]

**AMD**

\[ \tau_{\text{NN-coll}} \]

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Can we reproduce different data with the same model of branching? (Cluster correlations?)

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Equilibrium ensembles and caloric curves

Microcanonical ensemble $\Leftrightarrow$ Simply solve the time evolution for a long time

- Total energy: $E$
- Volume: $V = \frac{4}{3}\pi R^3$ (reflections at the wall of container)
- Neutron and proton numbers: $N = 18$, $Z = 18$

$\Rightarrow$ Temperature $T(E, V)$ and Pressure $P(E, V)$

Furuta and Ono,
arXiv:0811.0428 [nucl-th];
Comparison of reaction and equilibrium

$^{40}\text{Ca} + ^{40}\text{Ca}, \ E/A = 35 \text{ MeV}, \ b = 0$

\{States at the reaction time $t$\} = Equilibrium ensemble($E, V, A$)

\begin{align*}
t &= 100 \text{ fm/c} & 140 \text{ fm/c} & 180 \text{ fm/c} & 300 \text{ fm/c} \\
Y(Z) & \quad E^*/A=6.5\text{MeV}, \ V=3.9V_0 & E^*/A=6.1\text{MeV}, \ V=6.2V_0 & E^*/A=5.7\text{MeV}, \ V=6.6V_0 & E^*/A=5.3\text{MeV}, \ V=13.2V_0 \\
\langle E^*/A \rangle & \quad t=100\text{fm/c} & t=140\text{fm/c} & t=180\text{fm/c} & t=300\text{fm/c}
\end{align*}
Result of comparison

Fragment observables during the reaction \((80 \leq t \leq (300+) \text{ fm/c})\) are well explained as equilibrium properties of nuclear many-body system.

Some dynamical effects
- Finite flow
- Fragment radius (the figure below)
- Actual volume

\[ {^{40}\text{Ca} + ^{40}\text{Ca}, E/A = 35 \text{ MeV}, b = 0} \]

\[ E_{\text{total}}/A_{\text{total}} = (3/2)T \]

\[ T^2/8 \]

\[ T^2/13 \]

\[ \Delta R_{\text{rms}} \text{ [fm]} \]

\[ 0 \]

\[ 0.0 \]

\[ 0.1 \]

\[ 0.2 \]

\[ 0 \]

\[ 50 \]

\[ 100 \]

\[ 150 \]

\[ 200 \]

\[ 250 \]

\[ 300 \]

\[ 350 \]

\[ \text{t [fm/c]} \]

\[ \text{t [fm/c]} \]

\[ 0 \]

\[ 5 \]

\[ 10 \]

\[ 15 \]

\[ 20 \]

\[ 25 \]

\[ 30 \]

\[ 0 \]

\[ 5 \]

\[ 10 \]

\[ 15 \]

\[ 20 \]

\[ 25 \]

\[ 30 \]

\[ T \text{ [MeV]} \]

\[ E_{\text{total}}/A_{\text{total}} \text{ [MeV]} \]

arXiv:0811.0428 [nucl-th]
Cluster correlations

$^{197}\text{Au} + ^{197}\text{Au}$ at 150 MeV/u

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Cluster formation

During the time evolution of AMD,
- Cluster formation
- Propagation
- Breakup

\[ N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2 \]

- \( N_1, N_2 \): Colliding nucleons
- \( B_1, B_2 \): Spectator nucleons/clusters
- \( C_1, C_2 \): \( N \), \( (2N) \), \( (3N) \), \( (4N) \)

\[ \frac{d\sigma}{d\Omega} = F_{\text{kin}} |\langle \varphi'_1 | \varphi_{1}^+ | q \rangle|^2 |\langle \varphi'_2 | \varphi_{2}^- | -q \rangle|^2 \left( \frac{d\sigma}{d\Omega} \right)_{NN} \]

Non-orthogonality of final states:

\[ N_{BB'} \equiv \langle \Phi_B | \Phi_{B'} \rangle \neq \delta_{BB'} \]

The probability that \( N \) forms a cluster with one of \( B \)'s:

\[
P = \langle \Phi^q | \hat{X} | \Phi^q \rangle = \sum_{BB'} |\Phi'_B \rangle N^{-1}_{BB'} \langle \Phi'_B | \hat{X} | \Phi'_B \rangle
\]

\[
= \sum_B |\langle \Phi'_B | \Phi^q \rangle|^2
\]

\[
|\langle \Phi'_B | \Phi^q \rangle|^2 \text{ is regarded as the probability that } N \text{ forms a cluster with } B.
\]
The details of cluster correlations

**Formation**
- $(d\sigma/d\Omega)_{\text{NN}} \Rightarrow \text{Cluster formation cross section}
- Clusters: $N, 2N, 3N, 4N = (0s)^n$
- Pauli-blocking factor: $\prod_{i\in C}(1 - f_i)$
- Avoid double countings of final states
- Take care of the non-orthogonality of final states

**Propagation**
Nucleons $i$ in a cluster $C$ are propagated as usual, except that the internal fluctuations are turned off:

$$\frac{d}{dt}Z_i = \{Z_i, \mathcal{H}\}_{\text{PB}} + \Delta Z_i(t), \quad \Delta Z_i(t) := \frac{1}{C} \sum_{j\in C} \Delta Z_j(t)$$

**Breakup**
A cluster $C$ is broken when a nucleon in $C$ collides with another nucleon.
Time evolution of number of clusters

Number of nucleons in correlated clusters

$^{112}$Sn + $^{112}$Sn E/A = 50 MeV
0 < b < 2 fm

Gogny

Non-clustered

(2N)
(3N)
(4N)
Effects of cluster correlations

$^{40}\text{Ca} + ^{40}\text{Ca}, \frac{E}{A} = 35 \text{ MeV, filtered violent collisions}$

w/o cluster correlations

with cluster correlations

experiment

Coherence time: $\tau_{\text{NN-coll}}$

<table>
<thead>
<tr>
<th>Particle</th>
<th>w/o Cluster Correlations</th>
<th>with Cluster Correlations</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>6.7</td>
<td>4.4</td>
<td>(M_p) ↓</td>
</tr>
<tr>
<td>d</td>
<td>1.5</td>
<td>1.8</td>
<td>(M_\alpha) ↑</td>
</tr>
<tr>
<td>t</td>
<td>0.3</td>
<td>0.5</td>
<td>(\sum_{\text{IMF}} Z) ↓</td>
</tr>
<tr>
<td>$^3\text{He}$</td>
<td>0.3</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2.7</td>
<td>5.0</td>
<td></td>
</tr>
</tbody>
</table>
Results for Sn + Sn system

$^{112}\text{Sn} + ^{112}\text{Sn}$ at $E/A = 50$ MeV/nucleon, $0 < b < 2$ fm

With cluster correlations $\Sigma Z(70^\circ < \theta < 110^\circ) = 25$

Reasonable numbers of clusters.

Maybe too much transverse emission
(i.e. too large $\sigma_{\text{NN}}$).

Exact calculation ($\propto A^4$) with Gogny force.

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Xe+Sn, INDRA data

\begin{align*}
\mu &\quad 8.4 \\
d &\quad 4.4 \\
t &\quad 3.3 \\
^3\text{He} &\quad 0.9 \\
\alpha &\quad 10.1
\end{align*}

multiplicities of detected particles

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Summary

Clusters and fragments are important as an aspect of bulk properties of expanding nuclear matter.

Reaction and Equilibrium — A unified study with AMD

- Equivalence for fragment observables at each reaction time \(80 \leq t < (300+) \text{ fm/c}\)
- Some dynamical effects

Cluster correlations in AMD

- \(N_1 + B_1 + N_2 + B_2 \rightarrow C_1 + C_2\), based on \((d\sigma/d\Omega)_{NN}\)
- Cluster correlations have systematic effects on \(M_p, M_\alpha,\) and \(\Sigma_{\text{IMF}} Z\).
- Consistent reproduction of various multifragmentation data may be improved.