Time-Dependent Green’s Functions approach to nuclear reactions

Understanding the 1D mean-field dynamics

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Time-dependent formalism for nuclear reactions

Our "ambitious" goal:

- **Simulate** time evolution of nuclear phenomena in **3D**
- Use **Time-Dependent Green’s Functions** formalism
  
  - Good in the **static** case
  - Fully **quantal**
  - **Correlations** in initial state and in dynamics
  - Microscopic **conservation laws** are preserved

Where we are right now:

- **Mean-field** simulation of collisions of **1D** slabs
- Understanding **TDGF** formalism before...
  
  - Including **correlations** in **1D** (NN collisions)
  - Extending to **higher** dimensions (2D, 3D)
**Kadanoff-Baym equations**

Eventually:

\[ \rho(x_{11}, x_{11}') = G^<(11') = i \langle \Phi_0 | \hat{a}^\dagger(x_{11}, t_1, \tau) \hat{a}(x_{11}, t_1) | \Phi_0 \rangle \]

\[ G^>(11') = i \langle \Phi_0 | \hat{a}(x_{11}, t_1) \hat{a}^\dagger(x_{11}', t_1') | \Phi_0 \rangle \]

\[
\begin{aligned}
\left\{ i \frac{\partial}{\partial t_1} + \frac{\nabla^2_{\bar{r}}}{2m} \right\} G^\leq (11') &= \int d\bar{r}_1 \Sigma_{HF}(1\bar{I}) G^\leq (\bar{I}1') \\
&\quad + \int_{t_0}^{t_1} d\bar{I} \left[ \Sigma^> (1\bar{I}) - \Sigma^< (1\bar{I}) \right] G^\geq (\bar{I}1') - \int_{t_0}^{t_1'} d\bar{I} \Sigma^\geq (1\bar{I}) \left[ G^> (\bar{I}1') - G^< (\bar{I}1') \right]
\end{aligned}
\]

\[
\begin{aligned}
\left\{ -i \frac{\partial}{\partial t_1'} + \frac{\nabla^2_{\bar{r}}}{2m} \right\} G^\geq (11') &= \int d\bar{r}_1 G^\geq (\bar{I}1) \Sigma_{HF}(\bar{I}1') \\
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\end{aligned}
\]

- **Evolution equations for non-equilibrium systems (Keldysh)**
- Can be derived from general principles
- Include correlation and memory effects
- Complicated numerical solution!

Kadanoff-Baym equations

Right now:

\[ \rho(x_1 t_1, x_1', t_1') = \mathcal{G}^<(11') = i\langle \Phi_0 | \hat{a}^\dagger(x_1', t_1') \hat{a}(x_1 t_1) | \Phi_0 \rangle \]

\[ \mathcal{G}^>(11') = i\langle \Phi_0 | \hat{a}(x_1 t_1) \hat{a}^\dagger(x_1', t_1') | \Phi_0 \rangle \]

\[ \left\{ i \frac{\partial}{\partial t_1} + \frac{\nabla^2_{t_1}}{2m} \right\} \mathcal{G}^{\leq}(11') = \int d\vec{r}_1 \Sigma_{HF}(1\vec{1}) \mathcal{G}^{\leq}(\bar{1}\vec{1}') \]

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- Evolution equations for non-equilibrium systems (Keldysh)
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TDGF in the MF approximation vs. TDHF

- TDGF and TDHF give exactly the **same results**...
- but are expressed in **rather different terms**!

<table>
<thead>
<tr>
<th>Time Dependent Green’s Functions</th>
<th>Time Dependent Hartree-Fock</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i \frac{\partial}{\partial t} G(x, x'; t) = \left{ -\frac{1}{2m} \frac{\partial^2}{\partial x^2} + U(x) \right} G(x, x'; t) )</td>
<td>for ( \alpha = 1, \ldots, N_\alpha )</td>
</tr>
<tr>
<td>( -i \frac{\partial}{\partial t} G(x, x'; t) = \left{ -\frac{1}{2m} \frac{\partial^2}{\partial x'^2} + U(x') \right} G(x, x'; t) )</td>
<td>( i \frac{\partial}{\partial t} \phi_\alpha(x, t) = \left{ -\frac{1}{2m} \frac{\partial^2}{\partial x^2} + U(x) \right} \phi_\alpha(x, t) )</td>
</tr>
</tbody>
</table>

- 2 equations ... \( N_x \times N_x \) matrix
- **Testing** ground & new perspective
- 1 equation ... \( N_x \times N_\alpha \) matrix
- **Limited** to mean-field!

- **First calculation**: mean-field time evolution of 1D slabs
- **Understand** time evolution of **uncorrelated** density matrices
Collisions of 1D slabs

- Frozen $y,z$ coordinates, dynamics in $x$
- Simplified Skyrme mean field:

\[ U(x) = \frac{3}{4}t_0 n(x) + \frac{2 + \sigma}{16} t_3 [n(x)]^{(\sigma+1)} \]

- Initial state from adiabatic theorem or static HF
- Ground state of $G^<$ with $A = 8$ ($N_\alpha = 2$)

Two aspects:
- **Phenomenology**: dependence on bombarding energy?
- **New** perspective from Green’s functions $\Rightarrow$ off-diagonal elements
Mean-field approximation

Diagonal & off-diagonal matrix elements

- **Diagonal elements yield physical properties:**

\[
n(x) = G^<(x, x' = x) = \sum_{\alpha=0}^{N_\alpha} n_\alpha |\varphi_\alpha(x)|^2
\]

\[
K = \sum_k \frac{k^2}{2m} G^<(k, k' = k)
\]

- What happens off the diagonal?

\[
G^<(x, x') = \sum_{\alpha=0}^{N_\alpha} n_\alpha \varphi_\alpha(x) \varphi_\alpha^*(x')
\]

- What is their meaning? ⇒ \(G^<\) ~ correlation function
- Are all \(x \neq x'\) necessary for the time-evolution?
- Eliminate them at higher D’s to avoid \(N_x^D \times N_x^D \times N_t\) matrices?
Diagonal & off-diagonal matrix elements

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\[ n(x) = \mathcal{G}^<(x, x' = x) = \sum_{\alpha=0}^{N_{\alpha}} n_\alpha |\varphi_\alpha(x)|^2 \]

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- **What is their meaning?** \( \Rightarrow \mathcal{G}^< \sim \) correlation function

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- **Eliminate** them at higher D’s to avoid \( N_x^D \times N_x^D \times N_t \) matrices?
Collisions of 1D slabs: fusion (off-diagonal)

\[ \mathcal{G}^\prec (x, x', P) = e^{iPx} \mathcal{G}^\prec (x, x', P = 0)e^{-iPx} \]

\[ E_{CM}/A = 0.1 \text{ MeV} \]
Collisions of 1D slabs: break-up (off-diagonal)

\[ G^< (x, x') = \sum_\alpha n_\alpha \varphi_\alpha (x) \varphi_\alpha (x') \]

\[ E_{CM}/A = 4 \text{ MeV} \]
Collisions of 1D slabs: multifrag. (off-diagonal)

\[ E_{CM}/A = 25 \text{ MeV} \]
Erasing off-diagonal elements

- Use a cut-off imaginary potential off the diagonal $\sim$ superoperator

$$\mathcal{G}^<(x, x', t + \Delta t) \sim e^{i(\varepsilon(x) + iW(x, x'))\Delta t} \mathcal{G}^<(x, x', t)e^{-i(\varepsilon(x') - iW(x, x'))\Delta t}$$

- Properties of the field set to preserve norm, periodicity, etc
- Other choices do not yield good results!
Collisions

Erasing off-diagonal elements

Time evolution of the local density: $x_0$ dependence

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19th November, 2008
Collisions

Erasing off-diagonal elements

Time evolution of the local density: $x_0$ dependence

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Erasing off-diagonal elements

Time evolution of the local density: $x_0$ dependence

Density, $\rho$ [fm$^{-3}$]

- 90 %
- 0 %

$t=0$ fm/c
$t=60$ fm/c
$t=90$ fm/c

Density, $\rho$ [fm$^{-3}$]

$t=0$ fm/c
$t=60$ fm/c, 90 %
$t=90$ fm/c, 90 %

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Erasing off-diagonal elements

- Up to 60% (safe) off-diagonal elements can be neglected safely?
- Small effect of erasure for observables in high energy reactions!
**Wigner** transform:

\[
f(x, P) = \int \frac{dy}{2\pi} e^{-iPy} G^< \left( x + \frac{y}{2}, x - \frac{y}{2} \right)
\]

- **\( f(x, P) \)** phase space density
- **Obeys Boltzmann** equation under assumptions
- **Momentum average of** \( f(x, P) \) **∼ erasure of off-diagonal elements**
Conclusions

- KB equations offer a consistent framework for TD calculations
- Time evolution of 1D slabs in the mean-field
- New perspective is gained
- 60% off-diagonal elements are unimportant with appropriate cut-off
- Wigner functions are being analyzed

- Qualitative agreement with previous calculations*
- Beyond mean-field calculations are being implemented
- Extension to 2D and 3D ⇒ Promising approach to study correlations in time-dependent approaches!

*Bonche, Koonin and Negele, PRC 13, 1226 (1976).
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