Aim of these lectures:
To give an overview of contemporary nuclear structure theory, i.e. effective interactions, methods that solve the quantum many-body problem, and results of such calculations.
Content of lectures

**The nucleon-nucleon force (1 Lecture):**
- Determination of the Nucleon-Nucleon force from scattering.
- Chiral EFT and other approaches. Renormalization and effective interactions for the nuclear many-body problem.

**Many-body nuclear structure approaches (2 Lectures):**
- Phenomenological models ➔ shell model with a core
- Ab-initio nuclear structure ➔ building nuclei from the ground up.
- Overview of many-body methods. Coupled-cluster theory.

**Physics of nuclei at the drip lines (2 Lectures):**
- Open quantum systems and merging of experiment and theory
- Gamow-Shell Model and Coupled Cluster approaches to weakly bound and unbound nuclear systems.
Overview and challenges of low-energy nuclear structure. What are the relevant degrees of freedom, and how can we learn about the forces between interacting nucleons? Can we link low-energy nuclear structure to QCD?
What are the relevant degrees of freedom?

- **Physics of Hadrons**
  - Degrees of Freedom: quarks, gluons
  - Energy (MeV): 940 (neutron mass)
  - Degrees of Freedom: constituent quarks
  - Energy (MeV): 140 (pion mass)

- **Physics of Nuclei**
  - Degrees of Freedom: protons, neutrons
  - Energy (MeV): 8 (proton separation energy in lead)
  - Degrees of Freedom: nucleonic densities and currents
  - Energy (MeV): 1.32 (vibrational state in tin)
  - Degrees of Freedom: collective coordinates
  - Energy (MeV): 0.043 (rotational state in uranium)

Nucleonic matter
Changing shell gaps: one of the challenges

1949

Nuclear Shell Structure

N/Z

around the valley of nuclear stability
N/Z ~ 1 - 1.6

neutron-rich nuclei
N/Z ~ 3

Number of Protons

Number of Neutrons

shell gap larger than expected
shell gap less than expected
Physics of neutron-rich nuclei requires:

i. Accurate treatment of many-body correlations,

ii. Proper inclusion of coupling with continuum degrees of freedom and open channels

iii. Inclusion of both two- and three-nucleon forces.
The challenge for nuclei

“The first, the basic approach, is to study the elementary particles, their properties and mutual interaction. Thus one hopes to obtain knowledge of the nuclear forces. If the forces are known, one should, in principle, be able to calculate deductively the properties of individual nuclei. Only after this has been accomplished can one say that one completely understands nuclear structure….The other approach is that of the experimentalist and consists in obtaining by direct experimentation as many data as possible for individual nuclei. One hopes in this way to find regularities and correlations which give a clue to the structure of the nucleus….The shell model, although proposed by theoreticians, really corresponds to the experimentalist’s approach.”

–M. Goeppert-Mayer, Nobel Lecture

Two ways of doing business (I will focus primarily on the first):

QCD \(\rightarrow\) NN (and NNN) forces \(\rightarrow\) effective H \(\rightarrow\) calculate \(\rightarrow\) predict \(\rightarrow\) experiment

• Experiment \(\rightarrow\) effective forces \(\rightarrow\) calculate \(\rightarrow\) predict
• Progress involves feedback…
Aim: Bottom up approach to nuclear structure

Figure from: A. Richter, INPC 2004
The effective NN interaction

- Nuclei are made of protons and neutrons. These are composite particles.

**Q:** How do we determine the interaction between two nucleons?

**A:** Study two-nucleon scattering!
Recapitulation: Scattering theory

Phase shift is a function of relative momentum $k$; Figure shows s-wave.

Scattering length:

\[ k \cot \delta(k) \approx \frac{-1}{a}; \quad \sigma_{\text{tot}} \approx 4\pi a^2 \quad \text{for} \quad k \to 0 \]
Scattering from a spherical well

http://people.ccmr.cornell.edu/~emueller/tutorials.html

System has no bound state

Increase depth of well:
First bound state is about to enter
Scattering from a spherical well

Further increase of depth:
System has one shallow bound state

Further increase of depth:
System has one deep bound state
Scattering from a spherical well

Second bound state about to enter

System has two bound state
Deuteron is a very weakly bound system!
System has one bound state.
Steep decrease from 180 degrees due to large scattering length $a = 5.5$ fm.
Acts repulsive due to large (positive) scattering length.

System (barely) fails to exhibit bound state.
Steep rise at 0 due to large scattering length $a = -18$ fm.
Monotonous decrease due to hard core.
1990s: High precision NN potential models

- Phenomenological models based on meson exchange.
- Contain about 40 parameters; determined by fit to phase shifts/deuteron.
- Reproduce NN phase shifts with a $c^2$/datum very close to 1.0.
- “Nearly perfect” two-body physics.

Different two-body potential models disagree on structure of triton and alpha particle.

With additional three-nucleon forces, agreement with experiment is possible.

Four-body forces seem to be very small.

Q: Is there a unique three-body force?
Three nucleon forces: Why?

- Nucleons are not point particles (i.e. elementary).
- We neglected some internal degrees of freedom (e.g. D-resonance, “polarization effects”, …), and unconstrained high-momentum modes.

Example from celestial mechanics:  
Some tidal effects are included in the two-body interaction

Other tidal effects cannot be included in the two-body interaction! Three-body force unavoidable.

Renormalization group transformation:  
Removal of “stiff” degrees of freedom at expense of additional forces.
Three-body forces cont’d

Figure 23: Eliminating degrees of freedom leads to three-body forces.
(from Bogner, Furnstahl, Schwenk, arXiv:0912.3688)

Leading three-nucleon force
1. Long-ranged two-pion term (Fujita & Miza …)
2. Intermediate-ranged one-pion term
3. Short-ranged three-nucleon contact

The question is not: Do three-body forces enter the description? **The (only) question is: How large are three-body forces?**
Role of TNF in light nuclei: overall binding & level ordering

Argonne v_{18}
With Illinois-2
GFMC Calculations
6 November 2002
Three-nucleon forces

- Different NN forces must be associated with different three-nucleon forces.
- Modern understanding that there are no 'TRUE' NN and 3N forces.
- **Question:** Is there a consistent and systematic way of relating 3NF's to a given nucleon-nucleon force?
- **Question:** Is there a systematic way of linking low-energy nuclear structure with QCD?
- **Answer:** YES! Chiral effective field theory.
Effective Field Theory

Its pretty complicated inside a nucleon!!

Starting point is an effective Lagrangian.

\[ L_{\pi N} = L_{\pi N}^{(1)} + L_{\pi N}^{(2)} + L_{\pi N}^{(3)} + \cdots \]

- Obeys QCD symmetries (spin, isospin, chiral symmetry)

\[ L_{\pi N}^{(1)} = \bar{N} \left( iD_0 - \frac{g_A}{2} \vec{\sigma} \cdot \vec{u} \right) N \approx \bar{N} \left[ i\vec{\sigma} \cdot \left( \frac{1}{4} f_\pi^2 \vec{\tau} \cdot (\vec{\pi} \times \vec{\partial}_0 \vec{\pi}) - \frac{g_A}{2 f_\pi} \vec{\tau} \cdot (\vec{\sigma} \cdot \vec{\nabla}) \vec{\pi} \right] N + \cdots \]
“If you want more accuracy, you have to use more theory (more orders)"

Effective Lagrangian $\rightarrow$ obeys QCD symmetries (spin, isospin, chiral symmetry breaking)

Lagrangian $\rightarrow$ infinite sum of Feynman diagrams.

Expand in $O(Q/\Lambda_{QCD})$

Weinberg, Ordonez, Ray, van Kolck

NN amplitude uniquely determined by two classes of contributions: contact terms and pion exchange diagrams.

24 parameters (rather than 40 from meson theory) to describe 2400 data points with
At which order do we get sufficient accuracy?

Nucleon-Nucleon phase shifts up to 300 MeV for different orders in Chiral Perturbation theory.

Red line from Entem & Machleidt
( Phys. Rev. C 68, 041001 (2003) and green lines from Epelbaum et al.

Green dotted line:
Next to leading order NLO

Green dashed line:
Next-to-next-to leading order NNLO

Red Line: N3LO

Phase shifts reproduced to $c^2/datum=1$
About 40+ parameters

Three-nucleon force at NNLO

Challenge: Deliver the best NN and NNN interactions with their roots in QCD.

dashed -> NLO
dot -> NNLO
Solid -> N3LO
Chiral interactions and role 3NF in light nuclei.

FIG. 4: States dominated by p-shell configurations for $^{10}$B, $^{11}$B, $^{12}$C, and $^{13}$C calculated at $N_{\text{max}} = 6$ using $\hbar \Omega = 15$ MeV (14 MeV for $^{10}$B). Most of the eigenstates are isospin $T=0$ or $1/2$, the isospin label is explicitly shown only for states with $T=1$ or $3/2$. The excitation energy scales are in MeV.

Chiral interactions at N$^2$LO
$^{12}$C from lattice effective field theory

Ground-state energy as a function of imaginary time evolution.
LO from lattice Monte Carlo. All others from perturbation theory.
(IB = isospin breaking, EM = electro magnetic, 4N contact includes effects from N$^3$LO.
Epelbaum, Krebs, Dean Lee, Meißner,
Chiral interactions in medium sized nuclei

G. Hagen et al,
Summary of Lecture 1

- High precision two-body potential models available
  - Fit two-nucleon scattering data with $\chi^2$/datum close to 1.
  - Yield different results for triton and alpha particle.
- Calculations in light systems require three-nucleon forces
  - No “best” potential.
  - Different TNF for each two-nucleon potential.
- Chiral effective field theory
  - Based on QCD symmetries.
  - Permits systematic construction of nuclear forces (parametrization of our ignorance).
  - TNF arise naturally.

Discussion:
What does the Tjon-line for the A=3, 4 system teach us about the role of three-nucleon forces in medium mass nuclei?
Is there a recipe to minimize the effect of many-body forces?