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A measurement of the g factor of the 0·74 MeV $\frac{5}{2}^+$ state of ^{15}C

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Abstract. The g factor of the 0·74 MeV $\frac{5}{2}^+$ state of ^{15}C has been measured by observing the hyperfine modulation of the γ-ray anisotropy in one-electron ions recoiling into vacuum following the reaction ^{3}H(^{13}C, p)^{15}C*. From the spatial frequency determined as a function of recoil distance the g factor was deduced to be $|g| = 0{\cdot}703(12)$. The shell-model interpretation of this result is discussed.

NUCLEAR REACTIONS ^{3}H(^{13}C, pγ)^{15}C, $E_{^{13}\mathrm{C}} = 24$ MeV; measured p$\gamma(\theta)$, E_γ, Doppler shift, p$\gamma(\theta, t)$. ^{15}C, 0·74 MeV level; deduced $|g|$, E_x. Ge(Li) detector, radioactive target.

1. Introduction

It is widely accepted that the single-particle shell model is successful in describing the properties of low-lying states in odd-A nuclei close to shell closures. The predictions of such calculations can be improved by relatively simple modifications to the wavefunctions using perturbation theory. The nucleus ^{15}C provides an example. The only particle-bound states of ^{15}C, the ground state ($\frac{1}{2}^+$) and the first excited state ($\frac{5}{2}^+$), are predicted to arise respectively from a neutron in the $s_{1/2}$ and in the $d_{5/2}$ orbits outside a closed ^{14}C core in the limit of j–j coupling (see, for example, Philpott 1973, Reehal and Wildenthal 1973). This view is supported by values of the spectroscopic factors close to unity for ^{14}C(d, p) populating these states (Cecil *et al* 1975, Fortune 1978, private communication and to be published).

In such nuclei, the g value is sensitive to the amplitudes of small components of the wavefunction connected by the M1 operator to the dominant component. A recent measurement of the g factor of the $\frac{5}{2}^+$ state of ^{15}C was made by Hass *et al* (1975), in which the precession of the nucleus in an external field was observed. Their result, $g = -0{\cdot}77(6)$, is consistent with the single-particle (Schmidt) value of $-0{\cdot}76$. The measured precession angle was small because of the relatively short (3·76 ns) lifetime of the state and significant corrections were required for beam-bending effects.

The present measurement was performed to obtain a more accurate g value by observing the nuclear precession in the very large hyperfine field of one-electron carbon ions. The ground-state field in such ions (3·6 kT) is sufficiently large that many periods of hyperfine oscillation occur within the nuclear lifetime. The improved precision in the g value makes it possible to observe a significant deviation from the single-particle value.

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Such a deviation can be understood within the framework of a relatively simple calculation, which can be applied also to other states arising from $d_{5/2}$ neutrons in this region whose g values have been measured. This is discussed in § 4.

2. Method and experimental details

2.1. Method

The method applied in this work has been described, for example, by Rowe *et al* (1978). In this measurement, excited $^{15}C^*$ nuclei produced by the reaction $^3H(^{13}C, p)^{15}C^*$ recoiled into vacuum from a thin target (figure 1). About 40% of the ^{15}C ions recoiling from the target were hydrogen-like. The very large hyperfine interaction in the electronic ground state of these ions produces an oscillation in the anisotropy of emitted γ radiation; this was observed time-differentially by varying the time of flight in vacuum between the target and a stopper foil. The lifetime τ of the 0·74 MeV state (3·76 ns) is so long compared with typical flight times (up to 100 ps) that essentially all the $^{15}C^*$ ions decay in the stopper. The γ-ray yield as a function of flight time, t, at a fixed angle then simplifies to the form

$$I(t) = I_0(1 + a \cos \omega t \, e^{-t/\tau}) \quad (1)$$

where ω is the hyperfine frequency and the depth of modulation, a, depends on the one-electron ground-state population and the unperturbed γ-ray angular distribution. (For further detail see, for example, Asher *et al* (1976).) The effects of the small fraction of γ rays emitted in flight can be neglected here.

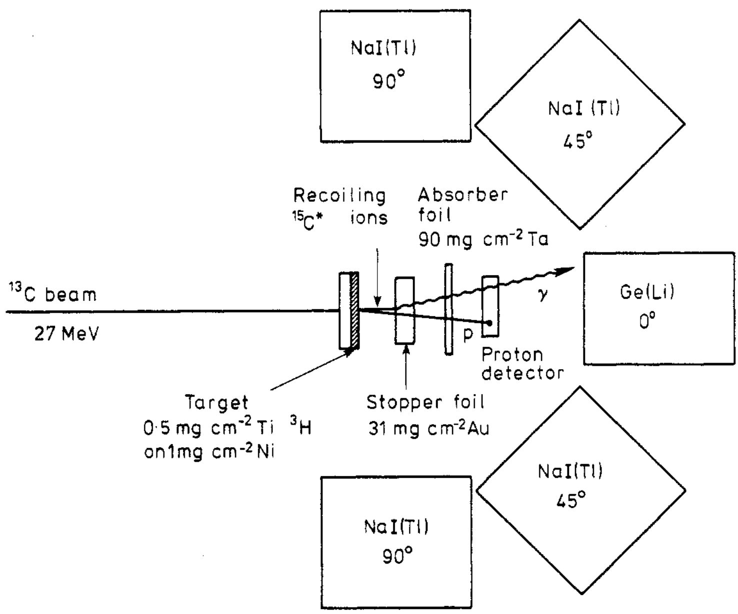

Figure 1. Diagram showing the experimental arrangement for the recoil in vacuum measurement (not to scale).

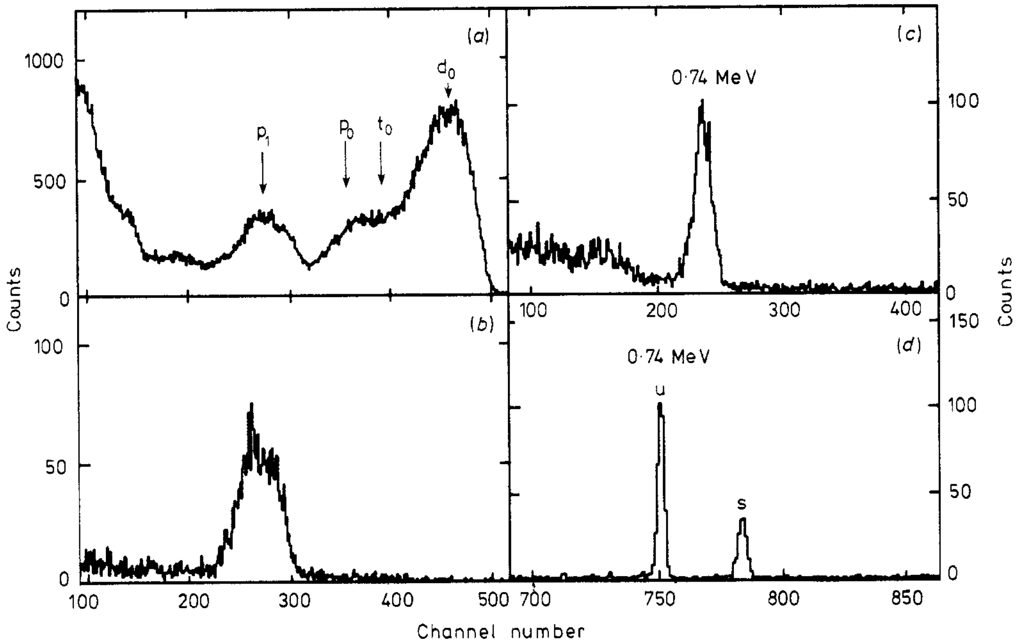

Figure 2. (*a*) Singles particle spectrum showing the proton peaks populating the ground (p_0) and first excited (p_1) states of ^{15}C, together with peaks attributed to outgoing deuterons (d_0) and tritons (t_0); (*b*) particles coincident with the 0·74 MeV γ ray; (*c*) NaI(Tl) spectrum at 90° in coincidence with p_1 protons; (*d*) Ge(Li) spectrum at 0° in coincidence with p_1 protons for a target–stopper separation of 15 mm; the Doppler shifted peak (s) and the unshifted peak (u) of the 0·74 MeV γ ray are indicated.

2.2. *Experimental details*

In order to achieve sufficient recoil velocity to produce a large fraction of hydrogen-like ions, the reaction $^3H(^{13}C, p)^{15}C^*$ was used to excite the $\frac{5}{2}^+$ state, at a ^{13}C energy of 24 MeV. The main experimental difficulty lay in obtaining thin and sufficiently flat tritium targets. No previous g-factor measurement using tritium targets had been carried out at this laboratory, although such targets have been used elsewhere (see, for example, Alexander *et al* (1974) and Berant *et al* (1975)). The two targets† used consisted of titanium of nominal thickness 500 μg cm^{-2} containing adsorbed tritium on the down-stream side of 1 mg cm^{-2} nickel foil. To compensate for energy loss in the nickel, the incident beam energy was increased to 27 MeV. The first of these targets had surface wrinkling comparable in depth to the spatial period of the hyperfine oscillation; the second target had a much more uniform surface. The uniformity of the tritium content was tested by scanning measurements of the tritium β-ray bremsstrahlung and was found to be better than ±5% over the area of the beam spot.

The arrangement of γ-ray detectors is shown in figure 1 and a singles particle spectrum is shown in figure 2(*a*). A 90 mg cm^{-2} tantalum foil was placed in front of the particle detector to absorb background α particles. The spectrum of γ rays from one NaI(Tl) detector in coincidence with protons (figure 2 (*b*)) populating the $\frac{5}{2}^+$ state is shown in figure 2(*c*).

† Made by NUKEM GmbH, Hanau, West Germany.

The unperturbed (p–γ) angular correlation was measured separately using a titanium–tritium target on a 35 mg cm^{-2} gold foil sufficiently thick to stop the ^{15}C ions. The resulting angular correlation coefficients, corrected for γ-detector solid angle, are

$$A_2 = 0{\cdot}44(3) \qquad \text{and} \qquad A_4 = -0{\cdot}19(5).$$

These values are consistent with a pure E2 transition and magnetic substate populations of the $\frac{5}{2}^+$ state of 0·36 ($\frac{1}{2}$), 0·13 ($\frac{3}{2}$) and 0·006 ($\frac{5}{2}$).

The recoil velocity of the ^{15}C* ions in vacuum was also measured separately by observing the Doppler shift of the 0·74 MeV γ ray in a Ge(Li) detector at 0°; figure 2(*d*) shows the Ge(Li) spectrum, coincident with the p_1 proton peak (figure 2(*a*)), in the vicinity of the shifted and unshifted peaks for a target–stopper separation of 15 mm. The velocity in this run, deduced from the measured shift, was $v = 0{\cdot}0425(3)c$. At this velocity, the equilibrium charge-state distribution (Marion and Young 1968) is

$$\phi(3^+) = 0{\cdot}01 \qquad \phi(4^+) = 0{\cdot}45 \qquad \phi(5^+) = 0{\cdot}44 \qquad \phi(6^+) = 0{\cdot}10.$$

From the measured angular correlation coefficients, it was estimated that the hyperfine oscillation would be seen with maximum statistical precision at ±90°; the angles ±45° were also chosen as being close to the next most favourable angle.

In the course of the determination of the ^{15}C recoil velocity a value for the energy of the γ ray from the $\frac{5}{2}^+$ state was obtained. The spectrum recorded with the Ge(Li) detector was calibrated internally using annihilation radiation and the ^{46}Ti 0·88925(3) MeV gamma ray; this calibration was subsequently confirmed using ^{137}Cs, ^{88}Y and ^{60}Co sources. This led to a value of 0·7424(6) MeV, which is in accord with 0·744(2) of Goss *et al* (1973) but somewhat greater than 0·739(1) of Hass *et al* (1975).

3. Results

Figure 3 shows the data accumulated during a run with the better of the two titanium tritide targets where the amplitude of the surface wrinkling was less than 10 μm over the beam spot. The full curves are derived from simultaneous least-squares fits of functions of the form of equation (1) to the 45° and 90° data: the relative depths of modulation were fixed on the basis of the measured angular correlation coefficients. The value of χ^2 for these fits is 49·7 for 43 degrees of freedom. The linear slopes apparent in the data are the effects of differential γ-ray absorption in the moving plunger, and were separately measured using a γ-ray source in the same geometry.

The best-fit values of the parameters from two targets are shown in table 1. The errors shown include statistical errors, correlation effects and uncertainties in recoil velocity. The final value for the hyperfine frequency, averaging the results of the two runs, is

$$\omega = 0{\cdot}731(13)\ \text{rad ps}^{-1}.$$

The nuclear g factor is related to the hyperfine frequency ω by the equation

$$\hbar\omega = (2I + 1)g\mu_N B(0)$$

where I is the nuclear spin, μ_N is the nuclear magneton and $B(0)$ is the 1s; $^2S_{1/2}$ hyperfine field; for ^{15}C, this field is 3·62 kT (Grotch and Yennie 1969). From the hyperfine

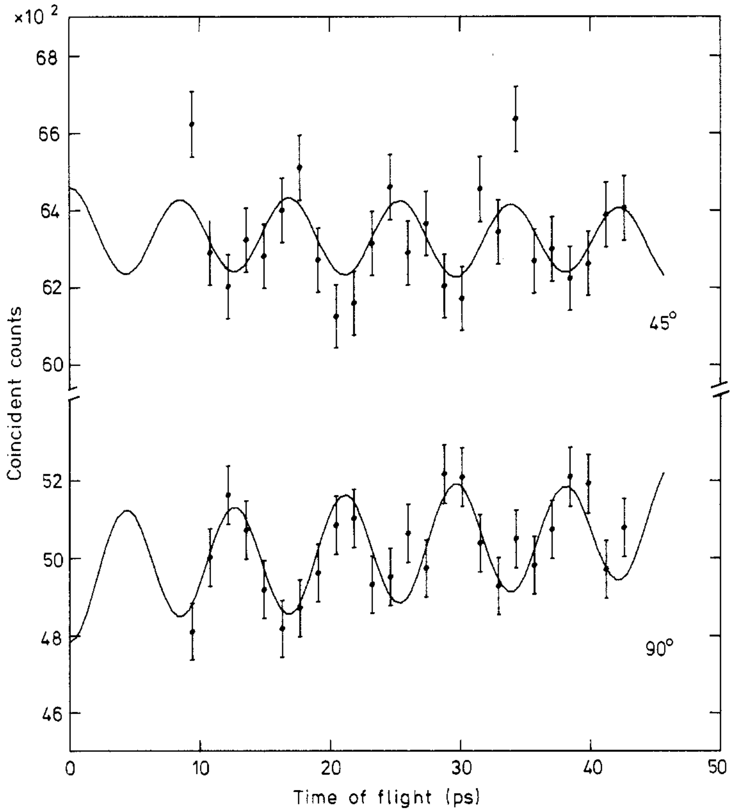

Figure 3. Oscillations in the γ-ray yields at 45° and 90° to the beam axis as a function of target–stopper separation expressed as time of flight.

frequency, the nuclear g factor was deduced to be

$$|g| = 0{\cdot}703(12)$$

where the principal source of error is in counting statistics.

The effect of the unevenness in the earlier target can be seen in the small and apparently inconsistent values of the ground-state hydrogen-like population shown in table 1. Due to the uncertainties in the target surface, it was not possible to make reliable deductions regarding the true atomic population. It should be noted that the result of such a distortion of the target in this type of measurement is simply to reduce the depth of modulation; the value of the hyperfine frequency is not directly affected. The effects of other perturbing atomic states, such as the triplet states of helium-like ions, were included in the data analysis, but no significant conclusions could be drawn regarding their population. The value of the g factor was insensitive to the inclusion of these states.

Table 1. Best-fit values of the fitted parameters.

Run	Hyperfine frequency ω(rad ps^{-1})	Apparent 1s: $^2S_{1/2}$ fraction
1	0·715(20)	0·20(3)
2	0·743(18)	0·36(5)

4. Discussion

The present g value, 0·703(12), of the 0·74 MeV state of ^{15}C is consistent with the result of −0·77(6) published by Hass *et al* (1975). However, the greater precision of the present result, which lies below the Schmidt value (−0·764) by 8(3)%, allows for a more sensitive test of the wavefunction.

In order to help understand this deviation and to put it into perspective, we consider the measured g factors in other nuclei, listed in table 2, which should also have exactly the $d_{5/2}$ neutron Schmidt value in the j–j coupling limit. These include the cases of a $d_{5/2}$ neutron outside ^{12}C, ^{14}C and ^{16}O cores as well as the $|(\nu d_{5/2})^n J\rangle$ configurations of the oxygen isotopes for which we recall that $g[(\nu d_{5/2})^n J] = g(\nu d_{5/2})$, independent of n and J ($J \neq 0$).

The experimental g factors are plotted in figure 4. The trend is seen immediately; the ^{17}O $\frac{5}{2}^+$ g factor is very close to the Schmidt value. Relative to $A = 16$, all the others deviate monotonically. The ^{15}C g factor is the closest to the Schmidt value apart from ^{17}O.

These trends can be understood quantitatively using first-order core-polarisation theory as was demonstrated by the extensive calculations of Noya *et al* (1958). This theory is appropriate to cases where the deviations are caused by small admixtures in the wavefunction, whose amplitudes can be calculated by perturbation theory. The g factors are most sensitive to those additional configurations which are directly connected to the zeroth-order configuration by the magnetic moment operator and hence contribute to the g factors linearly in their amplitude.

Since the magnetic moment operator connects only single-particle states with $\Delta n = 0$ and $\Delta l = 0$, there are no first-order core-polarisation corrections for ^{17}O. However, in ^{13}C and ^{15}C configurations in which a particle is excited from the $p_{3/2}$ to the $p_{1/2}$ orbital are important. For the oxygen isotopes with two or more (sd) shell particles, the $d_{5/2}$ to $d_{3/2}$ excitation is important. These configurations are illustrated schematically in figure 5.

We have calculated these first-order core-polarisation corrections by the standard techniques of perturbation theory (Brussard and Glaudemans 1977) using harmonic-

Table 2. Experimental and calculated g factors.

Nucleus	J^π	Experimental g value	Experimental Reference	Core-polarisation theory	Shell model $(sd)^n$†	Shell model $(sd)^2 + (sd)^4p^{-2}$‡
^{13}C	$\frac{5}{2}^+$	±0·59(5)	Beene *et al* (1974)	−0·610		
^{15}C	$\frac{5}{2}^+$	0·703(12)	Present work	−0·714		
^{17}O	$\frac{5}{2}^+$	−0·757	Alder and Yu (1951)	−0·764	−0·764	
^{18}O	2^+	−0·287(15)	Asher *et al* (1976)	−0·609	−0·425	−0·315
	4^+	−0·62(10)	Berant *et al* (1974)	−0·635	−0·498	−0·614
^{19}O	$\frac{3}{2}^+$	−0·48(6)	Goldring *et al* (1976)	−0·545	−0·607	
	$\frac{5}{2}^+$			−0·545	−0·600	
	$\frac{9}{2}^+$			−0·545		
^{20}O	2^+	−0·39(4)	Berant *et al* (1975) Gerber *et al* (1976)	−0·485	−0·335	
	4^+			−0·459		
^{21}O	$\frac{5}{2}^+$			−0·330		

†Wildenthal and Chung (1979) and Chung (1976).
‡Lawson *et al* (1976).

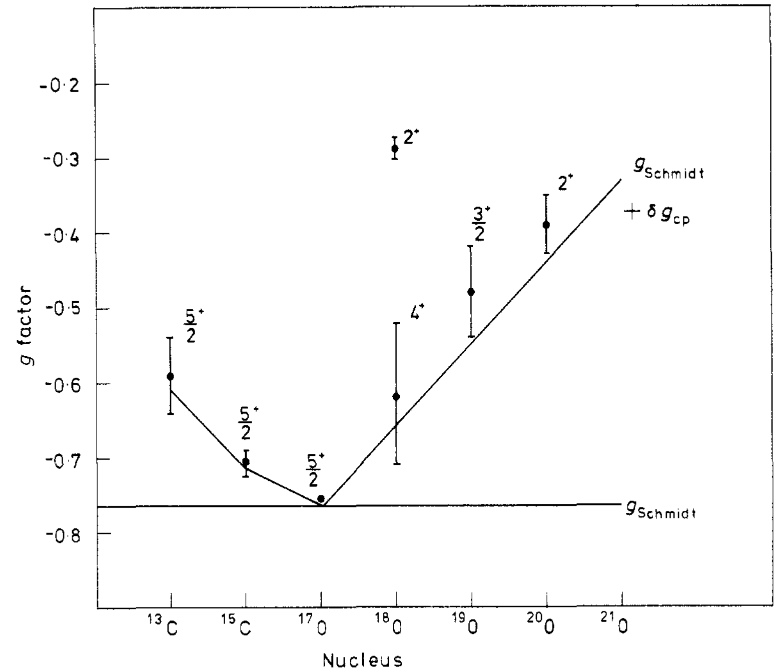

Figure 4. Measured and calculated g factors. The full lines represent theoretical values for the single-particle (g_{Schmidt}) g factors and the deviations resulting from core-polarisation (δg_{cp}).

oscillator wavefunctions with $\hbar\omega = 14\,\mathrm{MeV}$, and a delta-function interaction $(V_0 + V_1\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\delta(\boldsymbol{r}_{12})$. Interaction strengths of $V_0 = -500\,\mathrm{MeV\,fm^3}$ and $V_1 = -60$ $\mathrm{MeV\,fm^3}$ were chosen to match the average properties of empirical and realistic interactions. For the M1 operator free-nucleon g factors were used: $g_s^{\mathrm{p}} = 5{\cdot}59$, $g_s^{\mathrm{n}} = -3{\cdot}83$,

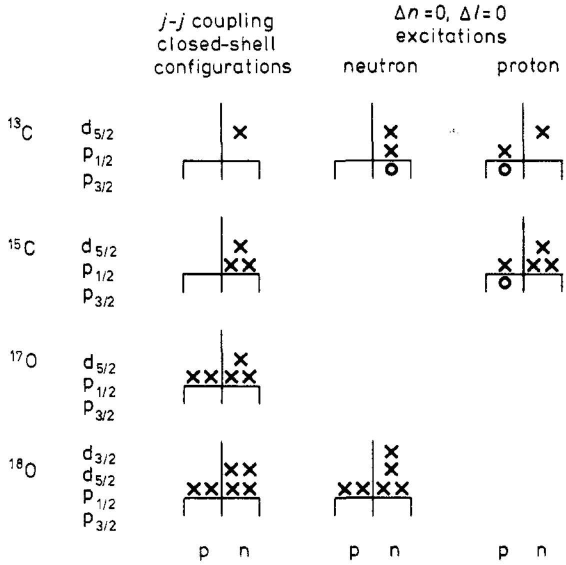

Figure 5. Diagrams representing the configurations considered in calculating core-polarisation corrections.

$g_l^p = 1$ and $g_l^n = 0$. The energy denominators are estimated by considering at what excitation energy we expect the excited states which are connected to the state under consideration by a strong Ml transition. In the $(sd)^2$ model for ^{18}O there are two 4^+ states with dominant configurations of $(d_{5/2})^2$ and $(d_{5/2}d_{3/2})$ separated by about 4·7 MeV (Chung 1976). We have used $\Delta E = 5$ MeV for all of our oxygen isotope calculations. For ^{13}C and ^{15}C we have assumed a weak coupling model in which the $d_{5/2}$ neutron is coupled to the 0^+ ground state and 1^+ excited state in ^{12}C and ^{14}C. The first 1^+ state in ^{12}C is at 12·71 MeV and we have used $\Delta E = 13$ MeV for both ^{13}C and ^{15}C.

The results are given in table 2. For ^{17}O, ^{19}O and ^{21}O the core-polarisation correction is exactly proportional to $A - 17$, the number of particles which can be excited from the $d_{5/2}$ to the $d_{3/2}$ orbital. For ^{18}O and ^{20}O, the core-polarisation effect is not exactly linear in $A - 17$, but nearly so, and thus the core-polarisation correction, δg_{cp}, for the oxygen isotopes is represented in figure 4 by a straight line.

The calculations are in good agreement with the trends of the observed deviations from the Schmidt value; the agreement could be slightly improved by increasing the interaction strengths by about 15%. The only extreme exception is the g factor of the ^{18}O 2^+ state; this is due to the large admixtures of $d_{5/2}$ $s_{1/2}$ and 4p–2h configurations in this state.

For ^{13}C the ratio of the contributions to the g factor from the proton and neutron $p_{3/2}$ excitations from the core is given by Morinaga and Yamazaki (1976) as

$$\frac{\delta g_{cp}^{\text{core protons}}}{\delta g_{cp}^{\text{core neutrons}}} = \frac{-2V_1}{V_0 - 3V_1} \frac{(g_s^p - g_l^p)}{(g_s^n - g_l^n)} = 0{\cdot}45.$$

Thus for a $d_{5/2}$ neutron the like-particle excitations are predicted to be about twice as important as the unlike-particle excitations; this is in agreement with the experimental values for ^{13}C, ^{15}C and ^{17}O. ^{15}C has a relatively small deviation from the Schmidt value since only the proton (unlike) excitations can contribute. The relative importance of the like-particle excitations over the unlike-particle excitations is a general feature of the magnetic dipole core-polarisation corrections up to the lead region. The ^{13}C, ^{15}C and ^{17}O g factors offer, theoretically, probably the most clear-cut test of this feature.

It is interesting to recall (Noya *et al* 1958) that with a delta-function interaction the first-order core-polarisation corrections for the $p_{1/2}$ ground states of ^{13}N and ^{13}C vanish because $\langle p_{1/2}^2|\delta(r_{12})|p_{1/2}p_{3/2}\rangle = 0$ ($J = 1$, $T = 0$). This is the matrix element that connects the $p_{1/2}$ state to the $p_{3/2}$ core excitation in perturbation theory. The experimental values in these cases are close to the Schmidt values.

Also in table 2 the results of the $(sd)^n$ shell-model calculations of Wildenthal and Chung (1979) and Chung (1976) for the oxygen isotopes are given. These must be considered better than the present calculation since they include all possible $d_{5/2}$, $d_{3/2}$ and $s_{1/2}$ configurations. The two-body matrix elements in this $(sd)^n$ calculation were obtained by modifying the Kuo G matrix (Kuo 1967) to the extent required to fit the binding energies of low-lying levels in nuclei from $A = 18$–24. However, the matrix elements involving the $d_{3/2}$ orbital of the type $\langle d_{5/2}^2| V(r)|d_{5/2}d_{3/2}\rangle$ cannot be well determined from fits to the energy levels and hence the reliability of the calculated g factors depends on the reliability of the Kuo G matrix.

Finally, for ^{18}O the results of calculations by Lawson *et al* (1976) (table I in this reference: 'constrained I') which include 4p–2h states are given in table 2. Only at this level is satisfactory agreement obtained for the ^{18}O 2^+ state.

Acknowledgments

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