

## Nucleon orbit radii in tin isotopes

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**Abstract.** The RMS radii of valence orbits in the tin isotopes are calculated using the single-particle potential model. Comparison with experimental data yields conclusions concerning the distribution of spectroscopic strength in these nuclei.

Several recent analyses of the cross sections of nucleon transfer reactions (Chapman *et al* 1979, Warwick *et al* 1979a, b) have given values of the RMS radii of the probability distributions of the nucleons in particular quantum states. These RMS radii can also be calculated from a single-particle model, so it is interesting to see how well such values agree with those obtained experimentally and what conclusions can be drawn from the comparison.

To do this, we use the global single-particle potential  $V_H$  of Bear and Hodgson (1978), whose parameters were chosen to give a good overall fit to the centroid binding energies of many single-particle states in nuclei from  $^{40}\text{Ca}$  to  $^{208}\text{Pb}$ . This potential has the standard form

$$V(r) = V_C(r) + Vf(r) + \left(\frac{\hbar}{m_\pi c}\right)^2 V_s \frac{1}{r} \frac{df}{dr} \mathbf{L} \cdot \boldsymbol{\sigma} \quad (1)$$

where  $V_C(r)$  is the Coulomb potential (for protons only) and the form factor  $f(r) = (1 + \exp[(r-R)/a])^{-1}$  with  $R = r_0 A^{1/3}$ .

The potential depth for surface states is

$$V = V_0^{p,n} \quad \text{for } -15 < E < 0 \quad (2)$$

and for deep states

$$V = V_0^{p,n} - \beta(E + 15) \quad \text{for } E < -15 \text{ MeV} \quad (3)$$

where the superscripts indicate neutron or proton potentials. These depend on the asymmetry potential  $V_1$  and can be expressed in the forms

$$V_0^p = V_0 + [(N-Z)/A]V_1 \quad V_0^n = V_0 - [(N-Z)/A]V_1. \quad (4)$$

The form factor parameters were fixed to the values  $r_0 = 1.236$  fm,  $a = 0.62$  fm for the central potential and  $V_s = 7$  MeV,  $r_s = 1.1$  fm,  $a_s = 0.65$  fm for the spin-orbit potential. The fit to a range of nuclei gave  $V_0 = 55.7$  MeV,  $V_1 = 39.3$  MeV and  $\beta = 0.51$ . The Perey non-locality correction was applied to all states with range parameter 0.87 fm and the wavefunctions were orthogonalised as described by Malaguti *et al* (1978).

This potential was used to calculate the RMS radii of some single-particle states in the tin isotopes, and the results are compared in table 1 with those found experimentally. The corresponding binding energies of the single-particle states are given in table 2. Also included in the tables are the results of Hartree–Fock calculations made by Campi *et al* (1974) using the single-particle mean field potential (quoted by Warwick *et al* 1979a). The calculated RMS radii in column A agree fortuitously well with the experimental values: the average difference between theory and experiment is only 0.03 fm, about a quarter of the quoted experimental uncertainties. This is not inconsistent, as these are systematic rather than statistical. The Hartree–Fock results are appreciably larger, around the limits of the quoted experimental uncertainties.

This comparison between theory and experiment can be extended to include the accurately known RMS total charge radii, and table 3 shows that the calculated values are significantly lower than the experimental ones. Further calculations of the radii of the individual orbits were therefore made, adjusting the radius parameter in each case to give the measured value of the charge radii, and the results are given in columns B of tables 1 and 2. The RMS radii are about 0.08 fm higher than before, on average, but still within the quoted uncertainties and much closer to the Hartree–Fock values. This comparison suggests that the experimental orbital radii are somewhat low by the standard of the very precise total RMS charge radii.

A possible reason for this discrepancy is the potential assumed for the deep states, which is not known experimentally for tin. To test this, calculations were made with the same potential for all states, and this was found to increase the total radii by about 0.03 fm. This is in the right direction, but only by about a fifth of the amount needed to

**Table 1.** Comparison of calculated RMS point charge radii of proton states with experimental and Hartree–Fock values (Warwick *et al* 1979a). The calculated values are obtained (A) from the BH potential, (B) after adjustment of the proton radius parameter to give the experimental total RMS charge radius and (C) after adjustment of the potential depth to give the measured separation energies. (Radii in fm  $\equiv 10^{-13}$  cm.)

Nucleus	Orbit	Experimental	Calculated			Hartree–Fock
			A	B	C	
$^{112}\text{Sn}$	1g <sub>9/2</sub>	4.95 $^{+0.17}_{-0.12}$	4.98	5.06	5.10	5.16 $\pm$ 0.04
	2p <sub>1/2</sub>	4.51 $^{+0.17}_{-0.12}$	4.55	4.62	4.74	4.59 $\pm$ 0.04
	2p <sub>3/2</sub>	4.54 $^{+0.15}_{-0.11}$	4.53	4.58	4.75	4.57 $\pm$ 0.04
$^{116}\text{Sn}$	1g <sub>9/2</sub>	5.01 $^{+0.16}_{-0.11}$	4.99	5.10	5.15	5.18 $\pm$ 0.04
	2p <sub>1/2</sub>	4.46 $^{+0.15}_{-0.11}$	4.55	4.64	4.77	4.60 $\pm$ 0.04
	2p <sub>3/2</sub>	4.55 $^{+0.14}_{-0.10}$	4.53	4.58	4.78	4.57 $\pm$ 0.04
$^{118}\text{Sn}$	1g <sub>9/2</sub>	5.00 $^{+0.15}_{-0.14}$	5.00	5.12	5.17	5.21 $\pm$ 0.04
	2p <sub>1/2</sub>	4.52 $^{+0.14}_{-0.11}$	4.54	4.63	4.78	4.60 $\pm$ 0.04
	2p <sub>3/2</sub>	4.46 $^{+0.14}_{-0.12}$	4.53	4.57	4.79	4.58 $\pm$ 0.04
$^{120}\text{Sn}$	1g <sub>9/2</sub>	4.99 $^{+0.16}_{-0.13}$	5.01	5.13	5.18	5.22 $\pm$ 0.04
	2p <sub>1/2</sub>	4.49 $^{+0.14}_{-0.11}$	4.54	4.62	4.79	4.63 $\pm$ 0.04
	2p <sub>3/2</sub>	4.57 $^{+0.14}_{-0.11}$	4.51	4.56	4.80	4.59 $\pm$ 0.04
$^{124}\text{Sn}$	1g <sub>9/2</sub>	5.02 $^{+0.14}_{-0.11}$	5.03	5.15	5.22	5.22 $\pm$ 0.04
	2p <sub>1/2</sub>	4.58 $^{+0.14}_{-0.10}$	4.53	4.61	4.80	4.61 $\pm$ 0.04
	2p <sub>3/2</sub>	4.50 $^{+0.13}_{-0.11}$	4.49	4.55	4.81	4.60 $\pm$ 0.04

**Table 2.** Comparison of experimental separation energies and calculated binding energies for protons (in MeV). The columns A and B are as in table 1. (Energies in MeV.)

Nucleus	Orbit	Experimental <sup>†</sup>	Calculated		
			A	B	Hartree-Fock
<sup>112</sup> Sn	1g <sub>9/2</sub>	7.51	9.69	11.84	8.87
	2p <sub>1/2</sub>	8.05	10.76	12.45	12.29
	2p <sub>3/2</sub>	8.32	12.22	13.88	14.11
<sup>116</sup> Sn	1g <sub>9/2</sub>	9.27	11.49	14.23	10.51
	2p <sub>1/2</sub>	9.61	12.52	14.67	13.86
	2p <sub>3/2</sub>	9.87	13.96	16.90	15.74
<sup>118</sup> Sn	1g <sub>9/2</sub>	10.01	12.36	15.69	11.13
	2p <sub>1/2</sub>	10.33	13.37	16.33	14.58
	2p <sub>3/2</sub>	10.60	14.79	18.78	16.46
<sup>120</sup> Sn	1g <sub>9/2</sub>	10.67	13.21	17.46	11.84
	2p <sub>1/2</sub>	10.98	14.20	18.03	15.39
	2p <sub>3/2</sub>	11.27	16.08	20.46	17.17
<sup>124</sup> Sn	1g <sub>9/2</sub>	12.10	14.85	21.27	13.27
	2p <sub>1/2</sub>	12.42	16.39	21.66	16.63
	2p <sub>3/2</sub>	12.76	18.83	24.09	18.53

<sup>†</sup> These figures are the separation energies of Gove and Wapstra (1972) plus the excitation energy. The centroid energies of known fragments are approximately 0.5 MeV higher, a difference that is not significant for the present purpose.

give the experimental values. The calculated radii are thus insensitive to the potentials chosen for the deep states.

The discrepancy could also be resolved by altering the assumed value of the spectroscopic factors of the transitions used to determine the orbit radii. Warwick *et al* (1979a, b) found that a 10% change in the spectroscopic factor corresponds to a 1.0% change in the orbit radius. A radius increase of 0.08 fm is about 1.6%, requiring a 16% decrease of the spectroscopic factor. Since the spectroscopic factors used by Warwick *et al* are normalised to the sum-rule limit obtained from the observed fragments of the single-particle strength this would imply that about 14% of the spectroscopic strength lies in unobserved states not included in their analysis.

It should be noted that the experimental uncertainties quoted by Warwick *et al* include many effects, of which the most important are those due to ambiguities in the distorting

**Table 3.** Comparison between the calculated (A) charge RMS radii and the experimental results of Ficenc *et al* (1972). (Radii in fm  $\equiv 10^{-13}$  cm.)

Nucleus	Calculated <sup>†</sup>	Experimental
<sup>112</sup> Sn	4.499	4.586 $\pm$ 0.005
<sup>116</sup> Sn	4.506	4.619 $\pm$ 0.005
<sup>118</sup> Sn	4.511	4.634 $\pm$ 0.005
<sup>120</sup> Sn	4.514	4.646 $\pm$ 0.005
<sup>124</sup> Sn	4.520	4.670 $\pm$ 0.005

<sup>†</sup> The calculated values are found from the point proton RMS radii by folding in the proton radius.

**Table 4.** Comparison of neutron orbit radii. The columns A, B and C are as in table 1, keeping the neutron and proton radius parameters equal. (Radii in fm  $\equiv 10^{-13}$  cm.)

Nucleus	Orbit	Experimental <sup>†</sup>	Calculated		
			A	B	C
<sup>113</sup> Sn	3s <sub>1/2</sub>	5.31 ± 0.12	5.30	5.28	5.28
	1g <sub>7/2</sub>	5.00 ± 0.12	4.99	5.09	5.07
<sup>115</sup> Sn	3s <sub>1/2</sub>	5.34 ± 0.12	5.33	5.30	5.36
	2d <sub>3/2</sub>	5.29 ± 0.12	5.25	5.27	5.31
	1h <sub>11/2</sub>	5.27 ± 0.12	5.34	5.40	5.39
<sup>117</sup> Sn	3s <sub>1/2</sub>	5.37 ± 0.12	5.35	5.32	5.43
	2d <sub>3/2</sub>	5.38 ± 0.12	5.27	5.30	5.36
	1h <sub>11/2</sub>	5.29 ± 0.12	5.36	5.44	5.43
<sup>119</sup> Sn	3s <sub>1/2</sub>	5.52 ± 0.12	5.37	5.34	5.47
	2d <sub>3/2</sub>	5.42 ± 0.12	5.29	5.32	5.39
	1h <sub>11/2</sub>	5.44 ± 0.12	5.39	5.48	5.47

<sup>†</sup> Chapman *et al* (1979).

potentials. It would therefore be possible to account for the discrepancy in this way as well. Without embarking on a complete re-analysis of the data, it is however preferable to discuss the uncertainties with reference to the spectroscopic factors alone.

The comparison in table 2 shows that the calculated binding energies are considerably larger than the corresponding separation energies, and this is still the case when allowance is made for the known fragmentation of strength among some of the lower states. A further set of calculations was therefore made in which the binding energy for each state was adjusted to the experimental separation energy, always re-adjusting the radius parameter to give the experimental total RMS charge radius. The results of these calculations are given in column C of table 1, and are on average about 0.22 fm greater than the experimental values, corresponding to a spreading of about 30% of the spectroscopic strength to unobserved states.

We therefore conclude that the available experimental data for proton states in the even tin isotopes, when analysed by the single-particle model, imply that the spectroscopic factors used by Warwick *et al* are about 30% too high, and that their orbital radii should be increased by an average of about 0.22 fm. This is very similar to the 2–6% discrepancy between calculated and experimental orbital radii noted for lighter nuclei by Brown *et al* (1979).

Similar calculations were made for the neutron orbits and the results are compared with the experimental data of Chapman *et al* (1979) in tables 4 and 5. As before, the calculations in column A correspond to the  $\text{BH}$  potential, and those in column B correspond to potentials with neutron radius parameters set to the values that give the measured total charge radii. Column C gives the results of calculations with the potential depths adjusted to give the measured separation energies. In this case the calculated RMS radii are on average only 0.02 fm greater than the experimental values, well within the quoted uncertainty.

In order to improve on this analysis it would be necessary to consider the quantity  $S|\psi(r \approx 11 \text{ fm})|^2$  which is directly measured in sub-Coulomb transfer reactions, where  $S$  is the spectroscopic factor and  $\psi(r)$  is the radial wavefunction. Unfortunately this quantity is

**Table 5.** Comparison of experimental separation energies and calculated binding energies for neutrons (in MeV). The columns A and B are as in table 1. (Energies in MeV.)

Nucleus	Orbit	Experimental <sup>†</sup>	Calculated	
			A	B
<sup>113</sup> Sn	3s <sub>1/2</sub>	10.80	9.37	11.06
	1g <sub>7/2</sub>	10.88	10.45	12.32
<sup>115</sup> Sn	3s <sub>1/2</sub>	10.32	9.26	11.16
	2d <sub>3/2</sub>	9.82	8.97	11.02
	1h <sub>11/2</sub>	9.60	8.19	10.75
<sup>117</sup> Sn	3s <sub>1/2</sub>	9.57	9.16	11.25
	2d <sub>3/2</sub>	9.41	8.87	11.15
	1h <sub>11/2</sub>	9.43	8.14	10.95
<sup>119</sup> Sn	3s <sub>1/2</sub>	9.33	9.05	11.32
	2d <sub>3/2</sub>	9.31	8.78	11.23
	1h <sub>11/2</sub>	9.24	8.10	11.12

<sup>†</sup> From separation energies of Gove and Wapstra (1972) plus the excitation energy.

not usually quoted in experimental papers, but instead a calculation with a potential of the form of equation (1) is carried out in order to convert this into an 'RMS' radius. This conversion introduces an unnecessary model dependence into the information extracted from experiment. Once it is realised that  $S|\psi|^2$  is the model-independent quantity it is clear that the results are extremely sensitive to the separation energy since this determines the exponential fall-off of  $|\psi|^2$  and only calculations of the type 'C' above where the separation energy is fixed are justified. The fact that the observed separation energies are not in good agreement with the 'B' or the Hartree-Fock calculations is partly connected with the fact that the spectroscopic factor  $S$  is smaller than the sum-rule value. The energies obtained in 'B' or Hartree-Fock calculations should correspond more closely with the centroid energies obtained from the energy-weighted sum of all stripping and pick-up fragments with a given  $nlj$  value (which is not easily obtained experimentally). This means that the total charge density is more closely related to centroid energy calculations whereas the tail densities of individual orbits are related to some separation-energy prescription calculation.

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