

Magnetic moments of $T=0$ states in $N=Z$ nuclei

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Abstract. Theoretical calculations for magnetic moments of $T=0$ collective states in $N=Z$ nuclei are compared with recent experimental results. The $J^\pi=2^+$ and 3^- states considered all have $0.49 < g_{\text{theor}} < 0.51$ if the wavefunctions have pure $T=0$ isospin. Isospin mixing increases the calculated $^{16}\text{O } 3^-$ g factor by 9%, consistent with experiment, but has a much smaller effect on the 2^+ states. A reported negative experimental g factor for the 4^+ state in ^{20}Ne is in complete disagreement with theory.

1. Introduction

It is well known (Kurath 1961, Sugimoto 1969, van Hienen and Glaudemans 1972, Zalm *et al* 1978, Raman *et al* 1978) that for nuclei where the states have good isospin the isoscalar magnetic moment $\mu_0(J) = \frac{1}{2}(\mu(J, T, T_z = T) + \mu(J, T, T_z = -T))$ can be directly related to the expectation value of the spin density

$$g_0(J) = \mu_0(J)/J = \frac{1}{2} + (\mu_p + \mu_n - \frac{1}{2})\langle S_3 \rangle / J \quad (1)$$

where

$$\langle S_3 \rangle = \left\langle J, M=J \left| \sum_i s_{3i} \right| J, M=J \right\rangle$$

and where μ_p and μ_n are the free-nucleon moments, $\mu_p + \mu_n = 0.880$. For collective excitations built on $J^\pi=0^+$ ground states the angular momentum comes only from vibrational and rotational degrees of freedom and hence $\langle J_3 \rangle = \langle L_3 \rangle$ or $\langle S_3 \rangle = 0$, which immediately implies that $g_0 = \frac{1}{2}$. The subject of this work concerns some interesting deviations from $g_0 = \frac{1}{2}$ obtained from theoretical calculations of the effects due to other nuclear degrees of freedom and compared with recent experimental results. The experimental g factors given in table 1 have been obtained only recently due to new techniques which have been developed (Randolf *et al* 1973, van Middelkoop 1978) for measuring the moments of states with lifetimes of the order of 10^{-12} s.

For states with good isospin the contributions to the moments from the single-particle degrees of freedom can be easily understood in terms of the $\langle S_3 \rangle$ expectation values. The single-particle values for $T = \frac{1}{2}$ states in odd-even nuclei are $\langle s_3 \rangle = \frac{1}{2}$ for $j = l + \frac{1}{2}$ and $\langle s_3 \rangle = -\frac{1}{2} + 1/(2l + 1)$ for $j = l - \frac{1}{2}$. The stretched two-particle configurations for $T=0$ states in odd-odd nuclei have $\langle S_3 \rangle = 2\langle s_3 \rangle$. A good example of the latter case is the deuteron $J^\pi=1^+$ state which has $g_{\text{exp}} = 0.8574$ or, from equation (1), $\langle S_3 \rangle_{\text{exp}} = 0.941$ which is quenched from the pure s-state value of $\langle S_3 \rangle = 1$ due to the d-state admixture. In fact, all one- and two-particle configurations have $\langle S_3 \rangle_{\text{exp}}$ values which are systematically

Table 1. Experimental g factors and the $\langle S_3 \rangle_{\text{exp}}$ matrix elements deduced from equation (1) compared with $\Delta N=0$ shell-model predictions.

		g_{exp}	$g_{\Delta N=0}$	$\langle S_3 \rangle_{\text{exp}}$	$\langle S_3 \rangle_{\Delta N=0}$
^{12}C	2^+		0.510		0.054
^{16}O	3^-	0.556(4) ^a	0.511	(0.44(3)) ^h	0.085
^{20}Ne	2^+	0.54(4) ^b	0.510	0.21(21)	0.050
	4^+	-0.10(19) ^c	0.511	-6.3(20)	0.111
	6^+		0.522		0.355
	8^+		0.534		0.722
^{24}Mg	2^+	0.51(2) ^b	0.512	0.05(10)	0.063
	4^+		0.515		0.158
^{28}Si	2^+	0.56(9) ^d	0.513	0.3(5)	0.071
^{32}S	2^+	0.47(9) ^e	0.495	-0.2(5)	-0.024
^{36}Ar	2^+		0.491		-0.047
^{40}Ca	3^-	0.56(13) ^f		0.5(10)	
	5^-	0.54(10) ^g		0.5(13)	

^a Bennett (1980).

^b Horstman *et al* (1975).

^c Speidel *et al* (1980).

^d Eberhard *et al* (1975).

^e Zalm *et al* (1979).

^f Jain *et al* (1976).

^g Hensler *et al* (1974).

^h $\langle S_3 \rangle = 0.09(3)$ from equation (4) using the calculated $\delta g = 0.044$ discussed in the text.

quenched relative to the single-particle estimate due to configuration mixing within major oscillator shells ($\Delta N=0$) (Brown and Wildenthal 1981 (unpublished), Wildenthal and Chung 1979) as well as higher-order mixing involving $2p-2h$, $4p-4h$, ... ($\Delta N \geq 2$) configurations (Shimizu *et al* 1974, Arima and Hyuga 1979). Thus for $T=0$ states, $\langle S_3 \rangle$ is determined by the competition between the collective component which has $\langle S_3 \rangle = 0$ and the quenched two-particle component which has $-1 < \langle S_3 \rangle < +1$.

It is important to remember that equation (1) is not valid if the wavefunctions do not have good isospin or if extra-nucleon degrees of freedom are important. It will be shown below that isospin mixing effects are important for the ^{16}O $J^\pi = 3^-$ state. The calculated meson-exchange corrections are small for the isoscalar moments (the one-pion-exchange current contributes only to the isovector magnetic moment operator) and will be ignored here (see Hyuga *et al* (1980) and table II of Raman *et al* (1978)).

Assuming that the wavefunctions have pure $T=0$ isospin, equation (1) has been used to extract $\langle S_3 \rangle_{\text{exp}}$ from the experimental g factors (see table 1). These are compared with shell-model calculations involving full configuration mixing within major oscillator shells $\langle S_3 \rangle_{\Delta N=0}$: $(1p)^8$ with the Cohen-Kurath (1965) interaction for ^{12}C , $(1p)^{-1}(1d, 2s)^1$ with the Millener-Kurath (1970) interaction for ^{16}O and $(1d, 2s)^n$ with the Chung-Wildenthal interactions (Chung 1976, Wildenthal 1977) for the sd -shell nuclei. For the 2^+ states considered $\langle S_3 \rangle_{\Delta N=0} \leq 0.1$, and this is in agreement with experiment. However, except for ^{24}Mg , the experimental error bars are too large for any discriminating test of the theory and new measurements of the g factors with about 1% precision are needed.

The $\Delta N=0$ predictions for the ^{20}Ne ground-state band are interesting. In contrast to $\langle S_3 \rangle_{\text{SU3}} = 0$ for all J values obtained with SU3 cluster wavefunctions (see, e.g., Strottman 1972), the $\Delta N=0$ shell-model values increase as J becomes larger and reach almost the

stretched two-particle value for the 8^+ state. The experimental value (Speidel *et al* 1980) for the 4^+ state is completely inconsistent with these predictions and in fact has an $\langle S_3 \rangle$ value an order of magnitude larger than even a two-particle configuration. Confirmation of this experimental result is essential.

The value of $\langle S_3 \rangle_{\text{exp}} = 0.44 \pm 0.03$ from the ^{16}O 3^- g -factor measurement at Oxford (Bennett 1980) is in fair agreement with that expected for the simplest shell-model configuration $\langle S_3 \rangle[(p_{1/2})^{-1}(d_{5/2})] = 0.33$. However, it is well known from the large $0^+ \rightarrow 3^- B(E3)$ value that this state is collective. Part of this collectivity comes out of the $1p-1h$ $(p)^{-1}(sd)^1$ calculation which gives $\langle S_3 \rangle_{\Delta N=0} = 0.085$ and additional $\Delta N=3$, $1p-1h$ and $3p-3h$ mixing which is needed to reproduce the $B(E3)$ value might be expected to further reduce $\langle S_3 \rangle$. Thus $\langle S_3 \rangle_{\text{exp}}$ for the ^{16}O 3^- state is in disagreement with theoretical expectations.

2. Effects of isospin mixing

It will now be shown that the discrepancy in ^{16}O mentioned above can be understood as an effect of isospin mixing. First the two-level mixing of the $T=0$ and 1 $J^\pi=3^-$ configurations $[(p_{1/2})^{-1}(d_{5/2})]$ will be considered. In perturbation theory the magnetic moment of the lowest 3^- states is given by

$$\langle 3^-|\mu|3^- \rangle = \langle 3^-T=0|\mu|3^-T=0 \rangle + \delta\mu \quad (2)$$

where

$$\delta\mu = -2 \frac{\langle 3^-T=0|V_C|3^-T=1 \rangle}{|\Delta E|} \langle 3^-T=1|\mu|3^-T=0 \rangle \quad (3)$$

and equation (1) must be modified to

$$g_0(J) = g(J) - \delta g = \frac{1}{2} + (\mu_p + \mu_n - 1)\langle S_3 \rangle/J \quad (4)$$

where $\delta g = \delta\mu/J$. For the derivation of the following formulae it is convenient to write the $1p-1h$ wavefunctions in proton-neutron formalism:

$$\begin{aligned} |j_h^{-1}j_p, J, T=0, ^{16}\text{O} \rangle &= -(|(\pi j_h)^{-1}(\pi j_p)J \rangle - |(\nu j_h)^{-1}(\nu j_p)J \rangle)/\sqrt{2} \\ |j_h^{-1}j_p, J, T=1, ^{16}\text{F} \rangle &= |(\nu j_h)^{-1}(\pi j_p)J \rangle \\ |j_h^{-1}j_p, J, T=1, ^{16}\text{O} \rangle &= (|(\pi j_h)^{-1}(\pi j_p)J \rangle + |(\nu j_h)^{-1}(\nu j_p)J \rangle)/\sqrt{2} \\ |j_h^{-1}j_p, J, T=1, ^{16}\text{N} \rangle &= |(\pi j_h)^{-1}(\nu j_p)J \rangle. \end{aligned} \quad (5)$$

One can show that

$$\begin{aligned} \langle JT=1|\mu|JT=0 \rangle &= \left(\frac{4\pi}{3} \frac{J}{(J+1)(2J+1)} \right)^{1/2} \langle JT=1||M1||JT=0 \rangle \\ &= \frac{-1}{2(J+1)} \{ [J(J+1) + j_p(j_p+1) - j_h(j_h+1)]g_1(j_p) \\ &\quad + [J(J+1) + j_h(j_h+1) - j_p(j_p+1)]g_1(j_h) \} \mu_N \end{aligned} \quad (6)$$

where $g_1(j) = \frac{1}{2}(g(\pi j) - g(\nu j))$ are the single-particle isovector g factors. The Schmidt

values are $g_1(1d_{5/2})=1.342$ and $g_1(1p_{1/2})=-0.899$ and hence $\langle 3^-T=1|\mu|3^-T=0\rangle = -2.90\mu_N$. Alternatively, this off-diagonal matrix element can be related to the $B(M1)$ between these two states:

$$|\langle J T=1|\mu|J T=0\rangle| = \left(\frac{4\pi}{3} \frac{J}{J+1}\right)^{1/2} (B(M1))^{1/2}. \quad (7)$$

The experimental value for the transition between the 13.26 MeV $T=1$ and 6.13 MeV $T=0$ 3^- states is $B(M1)=(2.16 \pm 0.35)\mu_N^2$ (Ajzenberg-Selove 1977, Gorodetzky *et al* 1968), which gives $|\langle 3^-T=1|\mu|3^-T=0\rangle|=(2.6 \pm 0.2)\mu_N$ in fair agreement with the $(1p_{1/2})^{-1}(1d_{5/2})$ calculation given above.

We will assume that the isospin mixing is due to the Coulomb interaction V_C between two protons. In the one-particle-one-hole model the off-diagonal matrix element $\langle T=0|V_C|T=1\rangle$ can be related to $A=15, 16$ and 17 binding energies ($E=-BE$):

$$E(^{16}\text{F}J, T=1) = \varepsilon(\pi j_p) - \varepsilon(\nu j_h) + \langle V_s \rangle + E(^{16}\text{O GS}) \quad (8a)$$

$$E(^{16}\text{N}J, T=1) = \varepsilon(\nu j_p) - \varepsilon(\pi j_h) + \langle V_s \rangle + E(^{16}\text{O GS}) \quad (8b)$$

$$E(^{16}\text{O}J, T=1) = \frac{1}{2}(\varepsilon(\pi j_p) - \varepsilon(\nu j_h) + \varepsilon(\nu j_p) - \varepsilon(\pi j_h) + \langle V_s \rangle) + \frac{1}{2}\langle V_C \rangle + E(^{16}\text{O GS}) \quad (8c)$$

$$\langle J, T=0|V_C|J, T=1\rangle = -\frac{1}{2}(\Delta\varepsilon_1 + \langle V_C \rangle) \quad (8d)$$

where

$$\Delta\varepsilon_1 = (\varepsilon(\pi j_p) - \varepsilon(\nu j_p)) - (\varepsilon(\pi j_h) - \varepsilon(\nu j_h))$$

$$\varepsilon(j_p) = E(A=17, T=\frac{1}{2}) - E(^{16}\text{O GS}) \quad (8e)$$

and

$$\varepsilon(j_h) = E(^{16}\text{O GS}) - E(A=15, T=\frac{1}{2}). \quad (8f)$$

$\langle V_s \rangle$ is the strong isospin-conserving (T_z -independent) particle-hole matrix element and $\langle V_C \rangle$ is the Coulomb particle-hole matrix element between two protons. First equations (8e) and (8f) can be used to obtain $\varepsilon(j)$, then these are put into equations (8a) or (8b) to obtain $\langle V_s \rangle$ and finally ε and $\langle V_s \rangle$ are put into equation (8c) to obtain $\langle V_C \rangle$. The numerical values obtained from the experimental binding energies (Wapstra and Bos 1977, Ajzenberg-Selove 1976, 1977) of $A=15$ $\frac{1}{2}^-$, $A=17$ $\frac{1}{2}^+$ and $A=16$ 3^- states are $\varepsilon(\pi d_{5/2}) - \varepsilon(\nu d_{5/2}) = 3.54$ MeV, $\varepsilon(\pi p_{1/2}) - \varepsilon(\nu p_{1/2}) = 3.54$ MeV, $\langle V_s \rangle(^{16}\text{F}) = 1.87$ MeV, $\langle V_s \rangle(^{16}\text{N}) = 1.95$ MeV and $\langle V_s \rangle(^{16}\text{O}) + \frac{1}{2}\langle V_C \rangle = 1.73$ MeV. The values of $\langle V_s \rangle$ obtained from ^{16}F and ^{16}N are inconsistent due to an effect which will be discussed below. The average value will be used to obtain $\langle V_C \rangle = -0.36$ MeV. Notice that empirically $\Delta\varepsilon_1 \approx 0$ in equation (8d), leaving only $\langle V_C \rangle$ to contribute to the isospin-mixing matrix element $\langle 3^-T=0|V_C|3^-T=1\rangle = +0.18$ MeV.

Thus in the two-level mixing approximation the matrix elements required for $\delta\mu$ can be obtained from experimental quantities and the sign can be deduced from the $(p_{1/2})^{-1}(d_{5/2})$ calculation. From equation (3) the result for mixing of the 13.26 MeV $T=1$ and 6.13 MeV $T=0$ 3^- states is ($|\Delta E| = 7.1$ MeV)

$$\delta\mu = \frac{-2(+0.18)(-2.6)}{7.1} \mu_N = 0.13\mu_N. \quad (9)$$

This, together with the $\Delta N=0$ value for the $T=0$ component $\langle 3^-T=0|\mu|3^-T=0\rangle = 1.53$,

gives $\langle 3^- | \mu | 3^- \rangle = 1.66 \mu_N$ or $g(3^-) = 0.555$, which is in remarkably good agreement with experiment.

Relationships between masses and isospin mixing were first used by Braithwaite *et al* (1972) to estimate isospin mixing for the ^{12}C 1^+ states and the relation given in their paper is equivalent to using only the matrix element $\langle V_s \rangle$ obtained from the neutron-rich nucleus (^{16}N in this case) and ignoring information about the proton-rich nucleus (^{16}F in this case). Similar relationships have been used since then (see, e.g., Sato and Zamick 1977, Shlomo and Wagner 1978).

The reason for the difference in $\langle V_s \rangle$ between ^{16}N and ^{16}F can be understood as an implicit effect of the Coulomb interaction in the model space (Lawson 1978). For ^{16}F the $1d_{5/2}$ proton single-particle wavefunction is bound by only 0.6 MeV while for ^{16}N the $1d_{5/2}$ neutron single-particle wavefunction is bound by 4.14 MeV. For a delta-function residual interaction the residual particle-hole interaction is proportional to the integral

$$\int R_{1p_{1/2}}^2(r) R_{1d_{5/2}}^2(r) r^2 dr$$

and the large spatial extent of the proton orbit in ^{16}F due to its small binding energy reduces the value of this integral compared with that for ^{16}N . The ^{16}F to ^{16}N ratio for this integral using Woods-Saxon wavefunctions is 0.90 compared with the empirical value of $1.87/1.95 = 0.96$ for the 3^- level (and $1.45/1.65 = 0.95$ for the 2^- level). (For the $(1p_{1/2})^{-1}(2s_{1/2})$ $J^\pi = 0^-$ and 1^- levels the calculation gives 0.76 compared with the empirical values of $0.65/0.90 = 0.72$ and $0.85/1.18 = 0.72$, respectively.) The discrepancies between theory and experiment may be due to the finite range of the residual interaction, but the effect is understood qualitatively. In this model $\langle V_s \rangle$ for the middle nucleus ^{16}O should be about the average of the values for ^{16}N and ^{16}F as we have assumed above.

In the limit of an infinitely long-range Coulomb interaction it is easy to see that $\Delta \varepsilon_1 = \langle V_c \rangle$ (≈ 0.36 MeV) and there would be no isospin mixing. $\Delta \varepsilon_1$ is nearly vanishing in this case partly because of the small binding energy of the $d_{5/2}$ orbit. In fact, for this reason the isospin matrix elements of the $(1p_{1/2})^{-1}(2s_{1/2})$ $J^\pi = 0^-$ and 1^- states should even be larger (about 0.40 MeV) since $\Delta \varepsilon_1 = -0.37$ MeV for the difference between the $1p_{1/2}$ and $2s_{1/2}$ displacement energies.

The effects due to more complicated structures for the lowest 3^- states as well as the effects due to isospin mixing with more highly excited $T=1$ 3^- states have been considered using the relation

$$\delta\mu = -2 \sum_i \frac{\langle 3_1^- T=0 | V_c | 3_i^- T=1 \rangle}{|\Delta E_i|} \langle 3_i^- T=1 | \mu | 3_1^- T=0 \rangle.$$

The wavefunctions were obtained by allowing complete configuration mixing within the model space $(1p_{1/2}, 1d_{5/2}, 2s_{1/2})^4$ (ZBM) with the Reehal-Wildenthal (1973) interaction and a separate calculation within the model space $(1p)^{-1}(1d, 2s)^1$ (PHSD) with the Millener-Kurath (1970) interaction.

In both model spaces the sum was found to be dominated by more than 90% from the contribution from the lowest $T=1$ 3^- state. The theoretical $B(\text{M}1)$ values between the lowest $T=0$ and $T=1$ states are $1.25 \mu_N^2$ for the PHSD model space and $2.30 \mu_N^2$ for the ZBM model space compared with the $(1p_{1/2})^{-1}(1d_{5/2})$ value of $2.68 \mu_N^2$ from equations (6) and (7) and the experimental value of $(2.16 \pm 0.35) \mu_N^2$. In the PHSD model space the M1 matrix element is small due to destructive interference between the large $(p_{1/2})^{-1}(d_{5/2})$ component and the relatively small $(p_{3/2})^{-1}(d_{5/2})$ and $(p_{3/2})^{-1}(d_{3/2})$ components. These

results indicate that the Millener–Kurath interaction induces somewhat too large an admixture of the $1p_{3/2}$ and $1d_{3/2}$ orbits into the 3^- wavefunctions.

Recently the Oxford shell-model code has been extended to calculate two-body transition densities and two-body Coulomb matrix elements (Brown *et al* 1981, unpublished). The two-body Coulomb matrix elements were calculated with harmonic-oscillator wavefunctions and the single-particle energies were taken as adjustable parameters to fit the $A = 15$ and $A = 17$ displacement energies. The isospin-mixing matrix element was calculated to be 0.15 MeV in the ZBM model space and 0.12 MeV in the PHSD space, to be compared with the $(1p_{1/2})^{-1}(1d_{5/2})$ value obtained above of 0.18 MeV. Although the calculations can be criticised because harmonic-oscillator wavefunctions were used for the two-body Coulomb matrix elements, the reductions relative to 0.18 MeV are expected because of the more complex structure of the 3^- states in these model spaces. The moment correction becomes $\delta\mu = 0.09\mu_N$ when a value of 0.12 MeV is used for the matrix element V_C in equation (3) (together with the experimental off-diagonal M1 matrix element), which is still in fair agreement with the experimental value of $\delta\mu = 0.135 \pm 0.012$.

The quantity $\delta\mu$ is rather large for the 3^- state in ^{16}O because of the strong $(T=1) \rightarrow (T=0)$ M1 strength of (1.2 ± 0.2) Wu, the relatively small gap (7.1 MeV) between the states and the large isospin-mixing matrix element. As discussed above, normally the isospin matrix element would be smaller because of a cancellation between the terms involving $\Delta\varepsilon_1$ and $\langle V_C \rangle$ in equation (8d). For example, in ^{12}C $\Delta\varepsilon_1 = 0.24$ MeV from the $A = 11$ and 13 binding energies and $\langle V_C \rangle$ should have about the same value of -0.36 MeV and thus $\langle T=0 | V_C | T=1 \rangle \approx 0.06$. Also the $(T=1) \rightarrow (T=0)$ strength is smaller ((0.34 ± 0.06) Wu for ^{12}C) and $\Delta\varepsilon$ is larger (11.7 MeV) and hence $|\delta g| \approx 0.007$ for the 2^+ state in ^{12}C . From the experimental properties of the 4^+ $T=1$ state in ^{20}Ne (Fifield *et al* 1980), equation (9) takes the form $|\delta\mu| = 2(0.10)(1.7)/(6.8) = 0.050$ or $|\delta g| = 0.012$ (an isospin-mixing matrix element of 100 keV was assumed), which is far too small to account for the experimental results reported by Speidel *et al* (1980). Isospin-mixing effects on other positive-parity states should be similar to these two examples. For the 3^- and 5^- states in ^{40}Ca the isospin-mixing effects should again be large since $\Delta\varepsilon_1 \approx 0$, but the present experimental errors are an order of magnitude too large to be sensitive to this effect.

3. Conclusions

In conclusion, shell-model calculations for the collective 2^+ and 3^- states in $N=Z$ nuclei give small values for $\langle S_3 \rangle$ consistent with the collective-model assumption that $\langle S_3 \rangle = 0$. Since the meson-exchange corrections are small for the isoscalar magnetic operator it is expected that $g = \frac{1}{2}$ for the collective states in $N=Z$ nuclei. Experimental results for 2^+ states are in agreement with this expectation, but at present the experimental errors are too large to offer a discriminating test of the theory. The g factor of the ^{16}O 3^- state at 6.13 MeV excitation has been measured an order of magnitude more precisely than any other collective state and the experimental g factor is 10% larger than $g = \frac{1}{2}$. This deviation is found to be due to isospin mixing with the $T=1$ 3^- state at 13.26 MeV excitation, which is important because of the large $(3^-, T=1) \rightarrow (3^-, T=0)$ M1 transition strength and the large isospin-mixing matrix element. For the positive-parity states the isospin mixing effects are estimated to be smaller, of the order of $|\delta g| \leq 0.01$. The calculated values for $\langle S_3 \rangle$ in the ground-state band of ^{20}Ne increase with increasing J up to $\langle S_3 \rangle = 0.72$ for the 8^+ state. The shell-model g factor is then $g_{\Delta N=0}(8^+) = 0.534$ compared with the collective value of $\frac{1}{2}$, which would be interesting to confirm experimentally. The measured value of

$g(^{20}\text{Ne}, 4^+) = -0.10 \pm 0.19$ (Speidel *et al* 1980) is completely inconsistent with existing theory and a confirmation of this result is essential.

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