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0^+(GS) → 2^+(4.44 MeV) transition density in ^{12}C

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Abstract. A model is proposed for the 2^+ first excited state in ^{12}C in which the dominant configurations outside the p shell are assumed to come from mixing with an 'idealised' isoscalar giant quadrupole resonance. A small admixture of this resonance is capable of reproducing the high collectivity of the first excited state and gives a reasonably good fit to inelastic electron scattering data. The model gives a possible explanation of the lack of energy-weighted E2 strength up to about 30 MeV in this nucleus, and a simple extension of the model gives an overall improvement in theoretical estimates of experimental quadrupole moments in the ^{12}C region. The folding model is used to discuss the properties of the corresponding transition and reorientation potentials for coupled-channels calculations of $^{12}C + ^{12}C$ scattering.

1. Introduction

The scattering of ^{12}C by ^{12}C has recently been studied extensively both experimentally and theoretically. This system is of particular interest because of the large cross sections for single and mutual excitation of the 2^+(4.44 MeV) first excited state (Stokstad *et al* 1979, Fulton *et al* 1980, Cormier *et al* 1978) and because of the presence of gross and intermediate structures (possibly 'molecular resonances') in the excitation functions of several reaction channels (see, e.g., Bromley 1978).

In the presence of large inelastic cross sections and the appearance of similar structures in different channels it is clear that the problem must be tackled theoretically in a coupled-channels formalism. Several such calculations have been performed for this system with various prescriptions for the necessary transition potentials (e.g., Kondō *et al* 1979, Tanimura 1980, Cugnon *et al* 1979).

Even in a calculation including only single and mutual excitation of the first excited state many couplings are necessary, though it may be shown (Satchler and Love 1979) that in the folding model all the radial form factors for the couplings $A + B \leftrightarrow A' + B'$ are given (for transitions of multipolarity L_1 and L_2 in the two nuclei, coupled to a total multipolarity L) by the expressions

$$U^L_{AA';BB'}(R) = i^{L_1-L_2-L}(\hat{L}_1\hat{L}_2/\hat{L})\langle L0|L_1 0 L_2 0\rangle \int_0^\infty dq\, j_L(qR) H^{L_1L_2}_{AA';BB'}(q) \tag{1.1}$$

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where $\hat{L}=(2L+1)^{1/2}$, etc, and

$$H^{L_1L_2}_{\mathrm{AA}';\mathrm{BB}'}(q)=\frac{1}{(4\pi)^{1/2}}\,\frac{1}{2\pi^2}\,q^2\tilde{v}(q)\tilde{\rho}^{L_1}_{\mathrm{AA}'}(q)\tilde{\rho}^{L_2}_{\mathrm{BB}'}(q). \tag{1.2}$$

In this latter expression the form factors of the nuclear densities are given by

$$\tilde{\rho}^{L}_{\mathrm{AA}'}(q)=4\pi\int_0^\infty j_L(qr)\rho^{L}_{\mathrm{AA}'}(r)r^2\,\mathrm{d}r \tag{1.3}$$

with

$$\rho^{L}_{\mathrm{AA}'}(r)=\left\langle \mathrm{A}'\left\|\sum_i r_i^{-2}\delta(r-r_i)Y_L(\hat{\boldsymbol{r}}_i)\right\|\mathrm{A}\right\rangle \tag{1.4}$$

where the sum runs over all nucleons i and we use the reduced matrix element of Edmonds (1957). (This means that our densities are $\hat{J}_{\mathrm{A}'}$ times those of Satchler and Love.) Also, $\tilde{v}(q)$ in equation (1.2) is the transform

$$\tilde{v}(q)=4\pi\int_0^\infty j_0(qr)v(r)r^2\,\mathrm{d}r \tag{1.5}$$

of the nucleon–nucleon potential $v(r)$, which is assumed to be a scalar in coordinate space.

Thus in the above problem all the channel couplings may be expressed in terms of the densities ρ^0_{00}, ρ^0_{22}, ρ^2_{02}, ρ^2_{22}, ρ^4_{22} and an effective nucleon–nucleon interaction v. In the above densities and henceforth we simply denote the state labels A, A′, etc, by the spins of the ground and first excited states (0 and 2 respectively).

In addition to the above total point-nucleon densities we may also introduce the point-proton (π) and point-neutron (ν) densities by restricting the sum over i in equation (1.4) appropriately, giving

$$\rho^{L}_{\mathrm{AA}'}(r)=\pi^{L}_{\mathrm{AA}'}(r)+\nu^{L}_{\mathrm{AA}'}(r) \tag{1.6}$$

(with corresponding transforms defined as in equation (1.3)) and we note that since both our states have isospin $T=0$ we have

$$\Delta\rho^{L}_{\mathrm{AA}'}(r)\equiv\pi^{L}_{\mathrm{AA}'}(r)-\nu^{L}_{\mathrm{AA}'}(r)\approx 0. \tag{1.7}$$

We also define the charge densities

$$\hat{\rho}^{L}_{\mathrm{AA}'}(r)=\int(\pi^{L}_{\mathrm{AA}'}(r')\rho_\pi(|\boldsymbol{r}-\boldsymbol{r}'|)+\nu^{L}_{\mathrm{AA}'}(r')\rho_\nu(|\boldsymbol{r}-\boldsymbol{r}'|))\mathrm{d}^3\boldsymbol{r}' \tag{1.8}$$

where ρ_α are the charge densities of a single nucleon ($\alpha=\pi$, ν throughout this paper). Henceforth we shall drop the superscript and subscripts on all quantities relating to the density $\rho^0_{00}(r)$, e.g., $\tilde{\rho}\equiv\tilde{\rho}^0_{00}$. Note also that the properly normalised ground-state density is given by $\rho_{\mathrm{GS}}=(4\pi)^{-1/2}\rho$.

The effective interaction of Bertsch *et al* (1977) known as the M3Y potential has had much success in generating folded potentials for elastic heavy-ion scattering (with a few notable exceptions (Satchler and Love 1979) which seem to be explained either in terms of break-up (Thompson and Nagarajan 1981) or coupled-channel effects (Hnizdo *et al* 1981)). This suggests that such an approach for the coupled-channels problem is worth pursuing. Stokstad *et al* (1979) have obtained such terms for use in DWBA calculations by

using the Tassie (1956) hydrodynamical model for the transition density ρ^2_{02}, i.e.

$$\rho^2_{02}(r) = Cr\, d\rho/dr \tag{1.9}$$

where C was chosen to fit the experimental $0^+ \rightarrow 2^+$ B(E2) value. The above result may also be shown to be good for a collective 2^+ state which exhausts the $\Delta T = 0$, E2 energy-weighted sum rule (EWSR) (Deal and Fallieros 1973). One finds, however, that the 4.44 MeV level in ^{12}C accounts for only 13% of the EWSR (see § 5).

One might, therefore, look to the Cohen and Kurath (1965) shell model for the description of the ground and first excited states of ^{12}C. One finds, however, that in this model the B(E2) value for the transition between these states is underpredicted by a factor of about 2.4 (see § 5), suggesting a degree of collectivity not present in the restricted basis of this model.

A rotational model might also be invoked for the description of these states, particularly since the 0^+(GS), 2^+(4.44 MeV) and 4^+(14.08 MeV) levels possess a rather accurate $J(J+1)$ spacing. Such a model has been considered by Nakada *et al* (1971). The fits to electron scattering data obtained by these authors are surprisingly good in view of the fact that ^{12}C is such a light nucleus.

The above discussion suggests, however, that the properties of the 2^+ level are intermediate between the restricted shell-model results and those of the collective model, and the purpose of this paper is, therefore, to investigate a microscopic mixed model for this 2^+ state.

2. The mixed model

The major inadequacy of the Cohen and Kurath (CK) model is the neglect of mixing with configurations outside the p shell. Clearly, however, an appreciable extension of the shell-model basis requires knowledge of many two-body matrix elements and even if these were known the size of such a calculation would rapidly become prohibitive. Although the CK shell model fits the B(E2) value of the above transition rather poorly, it is quite successful in many other respects, suggesting that the model wavefunction is missing a small component of a highly collective nature (cf Brown and Green 1966).

Our simplifying assumption is, therefore, that the 4.44 MeV level should contain a small admixture of some 'idealised' isoscalar giant quadrupole resonance (GQR) based on the ground state of ^{12}C. We take this resonance to be a coherent superposition of 1p–1h excitations of energy $2\hbar\omega$ based on the ground state. Since such a state will essentially exhaust the $\Delta T = 0$, E2 EWSR we will be able to use the Tassie model for the contribution of this component to the required transition density. Before investigating this model further let us first make some observations on the form factors $\tilde{\rho}^L_{AA'}(q)$.

3. $U(R)$ from electron scattering data

The transition $A \rightarrow A'$ may, of course, be induced by electron scattering and the resulting longitudinal form factor $F^L_{AA'}(q)$ for such an excitation is given in the plane-wave Born approximation (PWBA) by the expression

$$|F^L_{AA'}(q)|^2 = \frac{4\pi}{Z^2(2J_A+1)} \left(\int_0^\infty j_L(qr) \hat{\rho}^L_{AA'}(r) r^2\, dr \right)^2 \tag{3.1}$$

where $\hat{\rho}^L_{AA'}$ is the charge transition density. For a folded charge density of the form of equation (1.8) it may be shown that the form factor may be written as

$$F^L_{AA'}(q)=(4\pi)^{1/2}[G_\pi(q)\tilde{\pi}^L_{AA'}(q)+G_\nu(q)\tilde{\nu}^L_{AA'}(q)]/Z\hat{J}_A \tag{3.2}$$

where $G_\alpha(q)$ are the charge form factors for a single neutron or proton. Using equations (1.6) and (1.7) the expression in square brackets in equation (3.2) may be written as

$$\tilde{\rho}^L_{AA'}(q)G_S(q)+\Delta\tilde{\rho}^L_{AA'}(q)G_V(q) \tag{3.3}$$

where $G_{S,V}=\frac{1}{2}(G_\pi \pm G_\nu)$ are the isoscalar and isovector parts of the nucleon charge form factor. From equation (1.7) we have $\Delta\tilde{\rho}^L_{AA'}\approx 0$, giving

$$F^L_{AA'}(q)\approx(4\pi)^{1/2}\tilde{\rho}^L_{AA'}(q)G_S(q)/Z\hat{J}_A. \tag{3.4}$$

Thus we may obtain $\tilde{\rho}^L_{AA'}(q)$ from the experimental amplitudes $F^L_{AA'}(q)$. This is an interesting result since inelastic electron scattering data are usually used as a sensitive test of a nuclear model but here we see that if we know the experimental $F^L_{AA'}(q)$ they may be used to bypass nuclear models by inserting them directly into equation (1.1) for the transition potential.

The question now clearly arises as to whether the PWBA of equation (3.1) is adequate for the above argument. However, numerical calculations using the code DUELS (Tuan *et al* 1968) show that the PWBA is rather good up to around 4 fm^{-1} and we shall see later (§ 7) that only momentum transfers $q \lesssim 2$ fm^{-1} are important in generating our potentials. Since elastic and inelastic electron scattering data up to sufficiently high momentum transfers have been measured for ^{12}C (Sick and McCarthy 1970, Crannell 1966) we may use the above formulae to generate many of our transition potentials directly.

Unfortunately it is not possible to obtain data on electron scattering from ^{12}C in its first excited state and thus ρ^2_{22} and ρ^4_{22} are not obtainable in this way. Indeed, even the quadrupole moment of the first excited state, which depends simply on $\tilde{\pi}^2_{22}(q=0)$,

$$\begin{aligned} Q&=(\tfrac{16}{5}\pi)^{1/2}\langle J, M=J|r^2 Y_{20}(\hat{r})|J, M=J\rangle \\ &\propto \lim_{q\to 0}\frac{1}{q^2}\tilde{\pi}^2_{22}(q), \end{aligned} \tag{3.5}$$

does not appear to be measured. We must, therefore, resort to model calculations for these terms.

These remaining densities might not be as important as ρ^2_{02} since, being diagonal in the state of ^{12}C to which they refer, they only give rise to re-orientation effects. Such a conclusion might, however, be a little suspect; in view of the enormous complexity of the elastic and inelastic excitation functions it may ultimately be necessary to perform coupled-channels calculations including all such terms.

4. The ground state of ^{12}C

Let us now return to our simple mixed model of the 2^+ state and write

$$|2^+\rangle=\cos\theta|2^+;\text{CK}\rangle+\sin\theta|2^+;\text{GQR}\rangle \tag{4.1}$$

where the mixing angle θ will be determined by fitting the experimental $B(E2)$ values and if our model is reasonable we expect to find $\sin^2\theta \ll \cos^2\theta$. Equation (4.1) leads simply to the

following expression for the transition density

$$\rho_{02}^2(r) = \cos\theta\,\rho_{02}^2(r;\text{ CK}) + \sin\theta\,\rho_{02}^2(r;\text{ GQR}) \tag{4.2}$$

and since the GQR will essentially exhaust the EWSR we may write

$$\rho_{02}^2(r;\text{ GQR}) \approx \rho_{02}^2(r;\text{ TAS}) = Cr\,\mathrm{d}\rho(r)/\mathrm{d}r, \tag{4.3}$$

where C is now chosen to fit the B(E2) of our GQR. Since the evaluation of both $\rho_{02}^2(r;\text{ CK})$ and $\rho_{02}^2(r;\text{ TAS})$ requires knowledge of the ground state of ^{12}C let us first discuss our model for this state.

We assume that in the ground state of ^{12}C the $1s_{1/2}$ orbit is filled and that the remaining nucleons occupy the $1p_{3/2}$ and $1p_{1/2}$ orbitals with occupation probabilities obtained from $(p)^8$ shell-model calculations using the Cohen and Kurath (1965) interaction POT. The average ground-state proton and neutron occupations obtained are $n_\pi(p_{3/2}) = n_\nu(p_{3/2}) = 3.24$ and $n_\pi(p_{1/2}) = n_\nu(p_{1/2}) = 0.76$. The radial wavefunctions for the $1s_{1/2}$, $1p_{3/2}$ and $1p_{1/2}$ orbits are taken to be the bound-state eigenfunctions of a central Woods–Saxon potential plus a derivative Woods–Saxon spin–orbit potential, given for example by Brown *et al* (1979). The spin–orbit parameters assume their conventional values of $V_{ls} = -6$ MeV, $R_{ls} = 1.1(A-1)^{1/3}$ fm and $a_{ls} = 0.65$ fm, and the Coulomb potential is approximated by that due to a uniformly charged sphere of radius 3.09 fm. The parameters of the central potential are then obtained by fitting the experimental $A = 12 \rightarrow A = 11$ separation energies ΔE for each orbit. The relevant experimental values are

$$\Delta E_\nu(1p_{3/2}) = 18.72 \text{ MeV} \qquad \Delta E_\nu(1p_{1/2}) = 20.70 \text{ MeV}$$

$$\Delta E_\pi(1p_{3/2}) = 15.96 \text{ MeV} \qquad \Delta E_\pi(1p_{1/2}) = 18.00 \text{ MeV}.$$

Note that the single-particle energy in a spherical potential corresponds to the average energy

$$\overline{\Delta E}(n, l, j) = \sum_{\mathrm{f}} M^2(n, l, j, n_{\mathrm{f}}) \Delta E(n, l, j, n_{\mathrm{f}})/(2j+1) \tag{4.4}$$

for one-nucleon transfer from the ^{12}C ground state to various levels n_{f} in both $A-1$ ($M^2 = C^2S$) and $A+1$ ($M^2 = (2j+1)C^2S$) nuclei. The spectroscopic factor $C^2S(1p_{1/2})$ is largest for the ground state of the $A+1$ nucleus ^{13}C and hence $\Delta E(1p_{1/2}, 2.0 \text{ MeV}, {}^{11}\text{C}) = 20.7$ MeV is significantly different from $\overline{\Delta E}(1p_{1/2}) \approx 10$ MeV. It is, therefore, important to use the above separation energies rather than centroid energies in order to have the correct experimental behaviour in the tail of the ^{12}C ground-state density.

The total ground-state densities for point neutrons and protons were obtained by summing the squares of the radial wavefunctions $\mathscr{R}_{j\alpha}(r)$ with the occupation numbers given above. The charge density was then obtained using equation (1.8) and also included the relativistic spin–orbit and Darwin–Foldy corrections as described by Brown *et al* (1979). All densities were corrected for spurious centre-of-mass motion in the harmonic-oscillator approximation using $b = 1.625$ fm; for example,

$$\tilde{\pi}^L_{\mathrm{AA}'}(q, \text{corrected}) = \tilde{\pi}^L_{\mathrm{AA}'}(q, \text{no correction}) \exp(b^2q^2/4A). \tag{4.5}$$

The radius and diffuseness parameters of the central Woods–Saxon potential were obtained by reproducing the moments $\langle r^2\rangle^{1/2} = 2.47$ fm and $\langle r^4\rangle^{1/4} = 2.78$ fm of the charge density (Sick 1974, Cardman *et al* 1980). The resulting values were $R = 3.00$ fm and $a = 0.55$ fm respectively.

The quality of our fit to the ground-state charge density was checked by considering

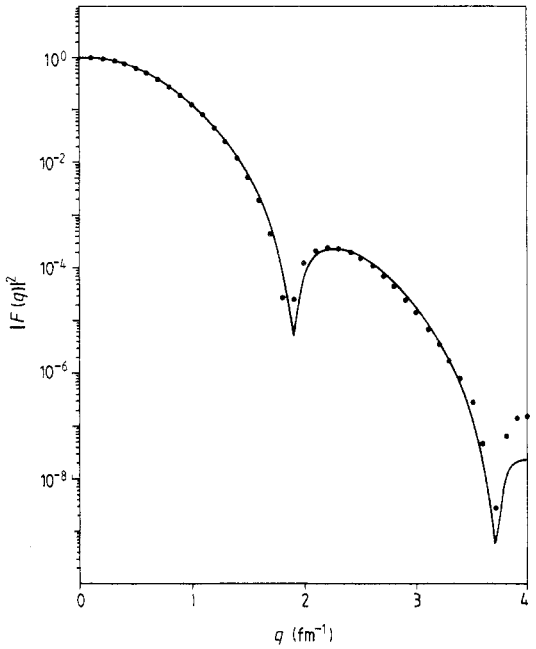

Figure 1. Squares of the experimental and theoretical elastic charge form factors for the ground state of ^{12}C.

elastic electron scattering from ^{12}C. The square of the experimental charge form factor recently obtained by Cardman *et al* (1980) is shown in figure 1 and compared with the same quantity obtained from equation (3.4) with $J_A = 0$ and with $\rho(r)$ calculated as described above, i.e.

$$|F(q)|^2 = 4\pi(\tilde{\rho}(q)G_S(q)/Z)^2. \tag{4.6}$$

We use the isoscalar form factor of Chandra and Sauer (1976) in this equation. The present model calculation is seen to reproduce well the experimental results up to the highest measured momentum transfers of $q \approx 4$ fm^{-1}.

5. Determination of the mixing angle

We may now return to equation (4.2) and calculate the Cohen and Kurath transition density $\rho^2_{02}(r;\,\text{CK})$. This may be expressed in terms of the amplitudes of the one-body transition density

$$\text{OBTD}(j_1,\, j_2)_p = \langle 2^+;\,\text{CK}\|[a^\dagger_{j_1} \otimes a_{j_2}]^{\Delta J=L}_p\|0^+;\,\text{CK}\rangle/(2\Delta J+1)^{1/2} \tag{5.1}$$

(since $\Delta T = 0$, $\text{OBTD}(j_1, j_2)_n = \text{OBTD}(j_1, j_2)_p$) which give, for $\Delta J = 2$, OBTD $= 0.537$, -0.354 and -0.219 for $(p_{1/2},\, p_{3/2})$, $(p_{3/2},\, p_{1/2})$ and $(p_{3/2},\, p_{3/2})$ respectively. The radial transition density is then obtained by combining these with the radial wavefunctions $\mathscr{R}_{j\alpha}(r)$

obtained from the ground-state calculation described above, i.e. (where again $\alpha = \pi, \nu$)

$$\alpha_{02}^2(r;\text{CK}) = \frac{1}{r^2} \sum_{j_1, j_2} \text{OBTD}(j_1, j_2)_\alpha \mathscr{R}_{j_1\alpha}(r) \mathscr{R}_{j_2\alpha}(r) \langle j_1 \| Y_L \| j_2 \rangle. \tag{5.2}$$

The $B(EL, A \rightarrow A')$ value is given by

$$B(EL, A \rightarrow A') = \frac{e^2}{2J_A + 1} \left(\int_0^\infty \pi_{AA'}^L(r) r^{L+2} \, dr \right)^2. \tag{5.3}$$

We obtain the shell-model value $B(E2, 0^+ \rightarrow 2^+) = 17.5\, e^2\ \text{fm}^4$ compared with the experimental value of $42 \pm 2\, e^2\ \text{fm}^4$.

In the case where a single state of energy ΔE exhausts the $\Delta T = 0$ model-independent EWSR we may write the sum in the form (Bohr and Mottelson 1975)

$$S(EL) \equiv \frac{L(2L+1)^2}{16\pi} \left(\frac{\hbar^2}{2m} \right) e^2 A \langle r^{2L-2} \rangle_\pi = \Delta E\, B(EL, 0^+ \rightarrow L^+) \tag{5.4}$$

where the expectation value is taken over the ground-state point-proton configuration. The corresponding transition density is given by the Tassie model:

$$\rho_{0L}^L(r;\text{TAS}) = C r^{L-1}\, d\rho(r)/dr \tag{5.5}$$

where C is a normalisation constant. In the present case $\langle r^2 \rangle_\pi = 5.72\ \text{fm}^2$, giving $S(E2) = 1412\, e^2\ \text{fm}^4\ \text{MeV}$ compared with a contribution of $(4.44\ \text{MeV}) \times (42\, e^2\ \text{fm}^4) = 186\, e^2\ \text{fm}^4\ \text{MeV}$ from the 2^+ first excited state (i.e., the 4.44 MeV level contributes only 13% of the EWSR as mentioned in § 1). We can, therefore, assume that the EWSR is essentially exhausted by our idealised GQR and use equation (5.5) for the corresponding transition density. The normalisation C in this equation must then be chosen to fit the $B(E2)$ of the GQR and this could be obtained from equation (5.4) if the energy of this state was known.

Unfortunately the E2 strength in ^{12}C appears to be appreciably fragmented (D'Erasmo *et al* 1981) and up to about 30 MeV only 20% of the EWSR has been observed experimentally. We resort, therefore, to the Tamm–Dancoff (TDA) approximation to obtain an estimate of the $B(E2)$ value:

$$B(E2;\text{TDA}) = \tfrac{1}{2} e^2 \sum_{j_p, j_h} |\langle j_p \| r^2 Y_2(\boldsymbol{r}) \| j_h \rangle|^2. \tag{5.6}$$

For a closed ^{12}C core we have $j_h = 1s_{1/2}$, $1p_{3/2}$ and $j_p = 1d_{5/2}$, $1d_{3/2}$, $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$. Using oscillator wavefunctions with $\hbar\omega = 15.7$ MeV (based on $\langle r^2 \rangle_\pi = 5.72\ \text{fm}^2$) we thus obtain $B(E2;\text{TDA}) = 45\, e^2\ \text{fm}^4$. From equation (4.2) we obtain simply

$$(B(E2;\text{exp}))^{1/2} = \cos\theta (B(E2;\text{CK}))^{1/2} + \sin\theta (B(E2;\text{TDA}))^{1/2} \tag{5.7}$$

where we have inserted the TDA estimate for the $B(E2)$ of our model GQR. The above equation may be written

$$\sin(\theta + \varphi) = \gamma \tag{5.8}$$

where $\gamma^2 = B(E2;\text{exp})/(B(E2;\text{CK}) + B(E2;\text{TDA})) = 0.67$ and $\varphi = \tan^{-1}(B(E2;\text{CK})/B(E2,\text{TDA}))^{1/2} = 32°$. Since $\gamma = 0.82 < 1$ then $\sin^{-1}\gamma = \psi = 55°$ exists and this equation clearly has the two solutions $\theta_1 = \psi - \varphi = 23°$ and $\theta_2 = \pi - (\psi + \varphi) = 93°$. The latter solution is principally giant quadrupole with a small incoherent CK amplitude and is not, therefore, the desired solution. The former solution $\theta \approx 23°$ is principally the

Cohen–Kurath state with a small coherent GQR amplitude and is, therefore, the state we are interested in. We may now insert this solution into equation (4.2) to obtain our mixed transition density.

In figure 2 we show experimental data on inelastic ($0^+ \rightarrow 2^+$) electron scattering from ^{12}C (Crannell 1966, Nakada *et al* 1971). We also show the theoretical fits to these data given by the Cohen–Kurath transition density (normalised to the CK estimate of the $B(E2)$), the Tassie model transition density (normalised to the TDA estimate of the $B(E2)$) and the mixed-model transition density (fitted to the correct experimental $B(E2)$ as described above). All the theoretical curves are corrected for centre-of-mass motion as described in § 4. We see immediately that our mixed model gives an improved fit to the shape of the experimental form factor up to around 2.5 fm^{-1}, strongly suggesting the qualitative correctness of our mixed model. The calculation does not, however, correctly reproduce the experimental bump at around 3 fm^{-1} but, at these momentum transfers, the form factor is probably sensitive to components not contained in our model wavefunction and the harmonic-oscillator correction to the centre-of-mass motion is probably not adequate for such a light nucleus (Ciofi degli Atti 1980). Meson-exchange effects should, however, be small (Sick 1982, private communication). In any case as far as our folded potentials are concerned we shall see (§ 7) that only momentum transfers up to $q \approx 2$ fm^{-1} are important.

Let us now turn for a moment to the implications of our model on the 'physical' giant quadrupole state. The energy-weighted E2 strength not possessed by the 4.44 MeV state is

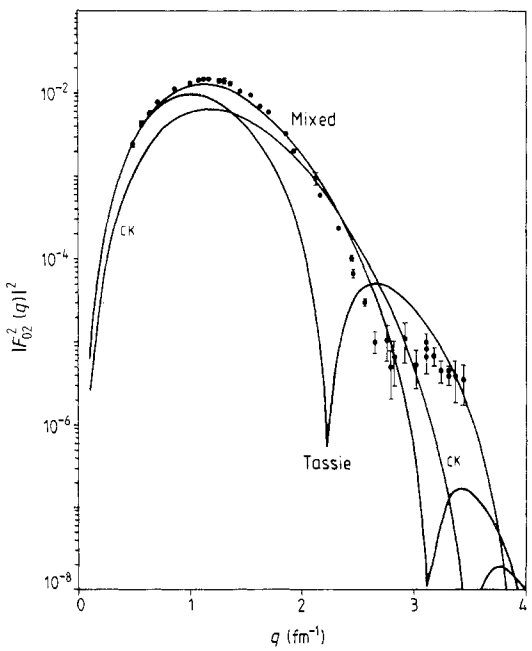

Figure 2. The square of the experimental inelastic charge form factor ($0^+ \rightarrow 2^+$) is compared with the Cohen and Kurath model (normalised to the shell-model $B(E2)$), the Tassie model (normalised to the TDA estimate of the $B(E2)$) and the mixed model (fitted to the experimental $B(E2)$) (see text).

$(1412-186)=1226\ e^2\ \text{fm}^4$ MeV and so the energy of the GQR predicted by the EWSR is

$$E_{\text{GQR}} = 1226/B(\text{E2; GQR}). \tag{5.9}$$

Inserting the TDA estimate of the B(E2) into this equation we find $E_{\text{GQR}} \approx 28$ MeV, which is surprisingly close to the value given by the empirical formula $E_{\text{GQR}} \approx 63\ \text{A}^{-1/3}$ MeV (Bertrand 1980) obtained for heavier nuclei. However, as stated previously, very little energy-weighted E2 strength has been found up to around 30 MeV. We see, though, that in our mixed model the 'physical' GQR must remain orthogonal to the 4.44 MeV state and becomes, therefore,

$$|2^+;\ \text{physical GQR}\rangle = \cos\theta|2^+;\ \text{GQR}\rangle - \sin\theta|2^+;\ \text{CK}\rangle \tag{5.10}$$

which, using an equation analogous to equation (5.7), yields B(E2; physical) $\approx 22\ e^2\ \text{fm}^4$. Inserting this value into equation (5.9) gives the estimate $E_{\text{GQR}} \approx 56$ MeV, which is certainly consistent with the noted *lack* of energy-weighted E2 strength at lower energies.

The above number could be somewhat large since a different model estimate might give a larger B(E2) for our idealised state. For example, in the random-phase approximation, Kirson (1980) suggests $B(\text{E2; RPA}) \approx 1.5\ B(\text{E2; TDA})$. This gives a smaller mixing angle $\theta_1 = 18°$, less reduction of strength predicted by equation (5.10) and consequently E_{GQR}(physical) comes back down to around 28 MeV. The important point is that our model does give a possible explanation for the E2 strength in ^{12}C lying higher than expected.

It is common in shell-model calculations to define an effective charge in order that the model wavefunctions fit experimental electromagnetic quantities, and in figure 3 we show the CK fit to the inelastic scattering data above with $e_\pi + e_\nu = 1.55e$ chosen to reproduce the experimental B(E2). The overall shape of the theoretical curve is not as good as the mixed model. It is important here to realise the significance of our calculation: in a restricted shell-model calculation the effective charge is chosen to fit the B(E2) and thus mock-up configurations which are not included, whereas in our calculations *no* effective charge is used but the B(E2) is fitted by choosing an appropriate value of a physically meaningful parameter (the mixing angle) which determines the mixing with what we believe to be the *dominant* configuration not included in the shell-model calculations.

Before leaving this section we note that an alternative collective model for the transition density ρ^2_{02} is furnished by the Bohr and Mottelson (1975) result

$$\rho^2_{02}(r;\ \text{BM}) = C\ \text{d}\rho(r)/\text{d}r \tag{5.11}$$

which is based on a model in which the surface thickness of the deformed nucleus is independent of angle and deformation. The fit to inelastic scattering data obtained from this density is also shown in figure 3, where C has again been chosen to fit B(E2; exp). The overall shape of this fit is seen to be quite good.

6. Extension of mixed model to other densities

In order to extend our model for ρ^2_{02} to the general density $\rho^L_{\text{AA}'}$ it is convenient to go back to a microscopic first-order perturbation theory approach where

$$\rho^L_{\text{AA}'} = \rho^L_{\text{AA}'}(\text{CK}) + \delta\rho^L_{\text{AA}'} \tag{6.1}$$

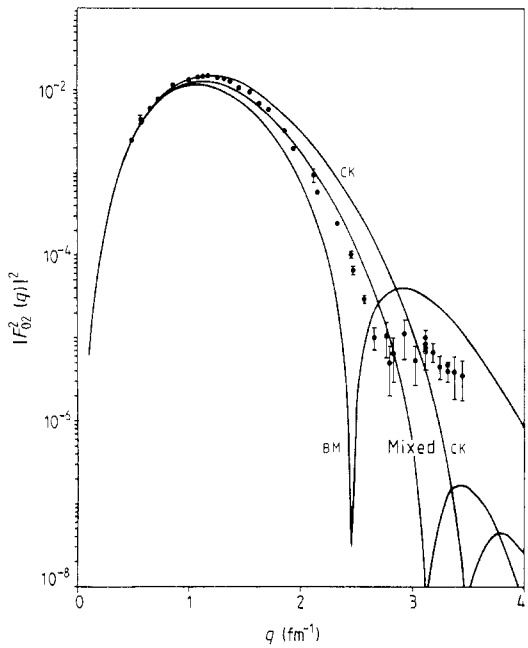

Figure 3. The same as figure 2, but we now show the Bohr and Mottelson result, the mixed model result and the Cohen and Kurath result all normalised to the experimental B(E2).

and

$$I^L_{AA'} = 4\pi \int \rho^L_{AA'} r^2 \, dr = I^L_{AA'}(\text{CK}) + \delta I^L_{AA'}. \tag{6.2}$$

The correction $\delta\rho$ is given by the diagrams of figure 4, where the three arrows in the initial and final states symbolise the $(p)^8$ wavefunction and the energy difference in the intermediate state is $\Delta N\hbar\omega$, where $\Delta N \geqslant 2$ and $N = 2n + l$; the broken lines indicate an antisymmetric two-body residual interaction and the wavy lines represent the one-body operator of rank L. Diagrams (a) and (b) are coherent whereas diagrams (c) and (d) tend to be about equal but opposite in sign. Therefore, the most important terms are (a) and (b).

In the limit where the residual interaction can be represented by a quadrupole–quadrupole interaction, the first-order corrections (a) and (b) to the transition density are proportional to the Tassie-model expression if only $\Delta N = 2$ is allowed, as we have assumed above. Furthermore the ratio $\delta I^L_{AA'}/I^L_{AA'}$ is independent of J_A and $J_{A'}$ and is equal to the isoscalar effective charge e_{eff}. Thus we are led to the general expression

$$\rho^L_{AA'}(r) = \rho^L_{AA'}(r; \text{CK}) + e_{\text{eff}} \frac{I^L_{AA'}(\text{CK})}{T^L_{AA'}} r \frac{d}{dr} \rho(r) \tag{6.3}$$

where

$$T^L_{AA'} = \int r \frac{d\rho}{dr} \, dr \tag{6.4}$$

normalises the Tassie transition density. The isoscalar effective charge obtained from the $0^+ \rightarrow 2^+$ transition is $e_{\text{eff}} = 0.55e$.

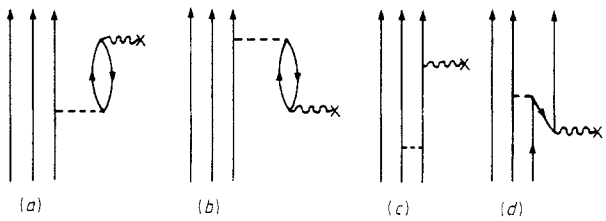

Figure 4. Perturbation diagrams relevant to the calculation of $\delta\rho$ (see § 6). The three arrows in the initial and final states represent the $(p)^8$ wavefunction, the broken lines indicate an antisymmetric two-body residual interaction and the wavy lines represent a one-body operator of rank L.

The quadrupole moment of the 2^+ state in ^{12}C as well as those for the ground states of several neighbouring nuclei have been calculated with equation (6.3) and are shown in table 1 together with the Cohen–Kurath results and where possible compared with experiment. The overall improvement in the agreement over the CK quadrupole moments is encouraging.

7. Transition potentials

We are now in a position to compare the transition potentials obtained from the above models. We start by displaying in figure 5 for the mixed model the quantities $H^{L_1L_2}_{AA';BB'}(q)$ which appear in the integrand in equation (1.1) for $U^{L}_{AA';BB'}(R)$. Essentially all combinations of L_1, L_2, AA′ and BB′ are shown since it is reasonable to assume that $\rho^0_{22}(r)\approx\rho(r)$ and since $\rho^4_{22}\equiv 0$ in the approximation (6.3). We see in this figure that the most important momentum transfers are $q\leqslant 2$ fm^{-1}, justifying the arguments of § 3.

The factor $\langle L0|L_1 0 L_2 0\rangle$ occurring in equation (1.1) expresses the fact that channel spins $L=1, 3$ are not allowed and thus all our potentials are generated by folding the above quantities with $j_L(qR)$, where $L=0, 2, 4$ (shown in figure 6) and multiplying by the appropriate normalisation. An important feature of figure 5 is that $H^{20}_{02;00}$ and $H^{20}_{22;00}$ have roughly the same form factor in q space (though they differ somewhat in magnitude). Therefore the transition potential $U^2_{02;00}$ and the re-orientation potential $U^2_{22;00}$ have similar radial form factors. The former potential is shown in figure 7(*a*) calculated using CK, Tassie and mixed-model values of $\rho^2_{02}(q)$. The CK and Tassie form factors were normalised to give the correct experimental B(E2) (though note from figure 2 that the TDA estimate already normalises the Tassie model almost exactly) and thus all three potentials are similar for large R.

Table 1. Calculated and experimental quadrupole moments (in units of e fm^2) for nuclei with $A=11, 12$.

Nucleus	J^π	T	Q_{theor}(CK)[a]	Q_{theor}(CK + δe)[b]	Q_{exp}
^{11}C	$\frac{3}{2}^-$	$\frac{1}{2}$	1.31	2.44	3.08 ± 0.06
^{11}B	$\frac{3}{2}^-$	$\frac{1}{2}$	2.88	4.02	4.07 ± 0.03
^{12}C	2^+	0	3.91	6.04	—
^{12}B	1^+	1	1.34	1.77	1.34 ± 0.14

[a] $e_p+e_n=e$; $e_p-e_n=e$.
[b] $e_p+e_n=1.55e$; $e_p-e_n=e$.

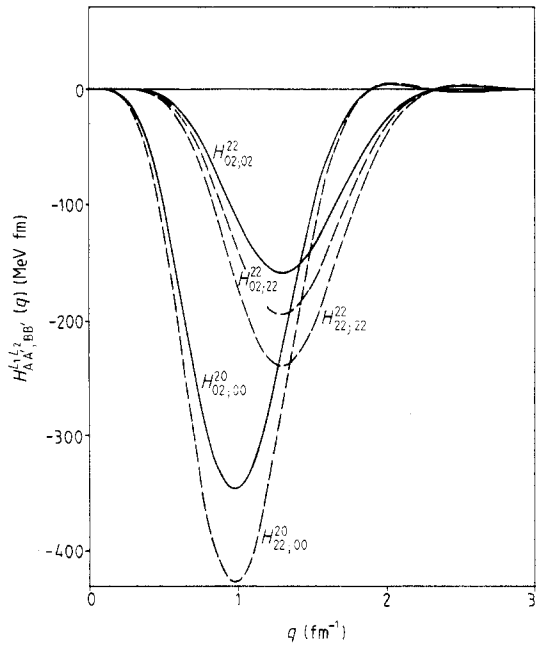

Figure 5. The mixed-model values of the quantities $H^{L_1L_2}_{AA';BB'}(q)$ are shown for essentially all combinations of indices required to reproduce the channel couplings for single and mutual excitation of the 2^+ state of ^{12}C.

The potential is seen to have its largest negative value around $R = 3$ fm and here significant differences in the three models are seen (these would have been even greater had the CK form factor not been normalised). The reason the above potential has a deep minimum around 3 fm is that for $R \simeq 3$ the first maximum of $j_2(qR)$ occurs for $q \approx 1$ fm^{-1} (see figure 6) and coincides with the deep minimum in $H^{20}_{02;00}(q)$ (see figure 5).

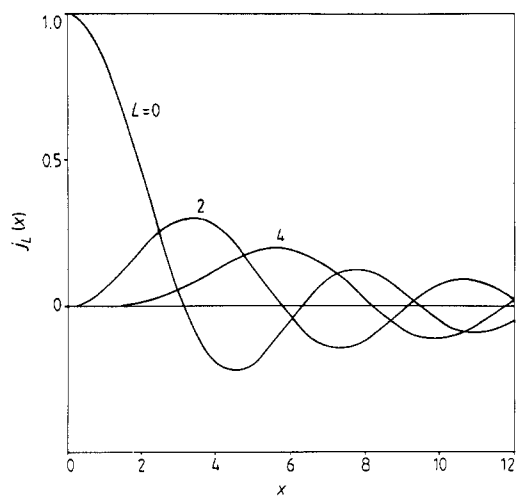

Figure 6. The functions $j_L(x)$ for $L = 0, 2, 4$.

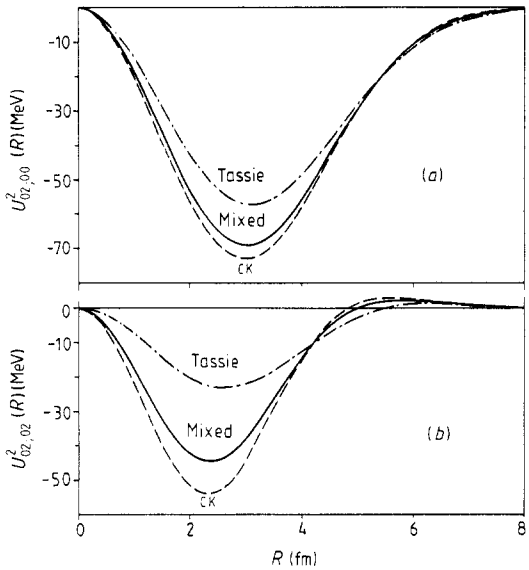

Figure 7. (*a*) The single excitation potential $U^2_{02;00}$ generated by the Cohen and Kurath, Tassie and mixed models. Note that the full curve corresponds to the full curve $H^{20}_{02,00}$ in figure 5. (*b*) The same as (*a*) for the mutual excitation term $U^2_{02;02}$. The full curve now corresponds to the full curve $H^{22}_{02;02}$ in figure 5.

We see from figure 5 that all three quantities $H^{22}_{AA';BB'}$, i.e., with $L_1 = L_2 = 2$, have roughly the same shape and, therefore, as above, we may confine our discussion to the potentials generated by any one of these terms. We choose the term $H^{22}_{02;02}$ responsible for mutual excitation of the ^{12}C nuclei (note, though, that the re-orientation terms are actually larger than these). The potential $U^2_{02;02}$ is shown in figure 7(*b*) for comparison with the single-excitation potential of the same total multipolarity, and $U^0_{02;02}$ and $U^4_{02;02}$ are shown in figures 8(*a*) and (*b*) respectively, again for all three models for ρ^2_{02}. The differences between the three models are now seen to be more important since $H^{22}_{02;02}$ peaks at larger momentum transfers than $H^{20}_{02;00}(q)$. The maxima or minima in these three potentials may again be understood by searching the values of R for which the maxima or minima of $j_L(qR)$ coincide with the deep minimum in $H^{22}_{02;02}$.

Note that for all these potentials the Tassie model greatly underpredicts the magnitude of the maximum or minimum since the Tassie form factor $F^2_{02}(q)$ falls well below the experimental value even for $q \approx 1$ fm^{-1} (see figure 2). Our mixed model, however, is quite good up to around 2.5 fm^{-1}.

While all our transition potentials could have been generated directly from the experimental form factor $F^2_{02}(q)$, this is not true of the corresponding re-orientation potentials (see § 3) though the success of our mixed model in fitting ground-state quadrupole moments gives us some confidence in the corresponding re-orientation terms it generates.

8. Conclusions

We have presented a model in which we attempt to incorporate the dominant

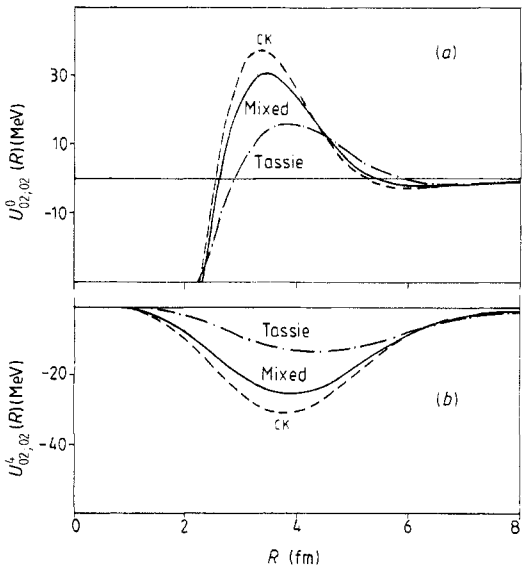

Figure 8. The mutual excitation terms of (*a*) multipolarity 0 and (*b*) multipolarity 4.

configurations outside the p shell into the shell-model wavefunctions for p-shell nuclei, in particular ^{12}C. In order to explain the high collectivity of such states these dominant configurations were assumed to be those which built up an 'idealised' GQR but, despite this enormous simplification, the model appears to be rather successful in qualitatively fitting inelastic electron scattering data.

A further success is obtained by re-calculating the quadrupole moments of the ground states of nuclei in the ^{12}C region whereupon a general improvement over the shell-model results is achieved. The model also affords a possible explanation of why little energy-weighted E2 strength has been observed experimentally in ^{12}C up to 30 MeV.

The above results of the mixed model give us some confidence in the transition and re-orientation potentials they generate and allow a simple qualitative discussion of the major features of these potentials which will be used elsewhere to perform coupled-channels calculations for the $^{12}C + {}^{12}C$ system.

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