Predicted features of the beta decay of neutron-rich *sd*-shell nuclei

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The Gamow-Teller beta-decay transitions of *sd*-shell nuclei with five or more excess neutrons are calculated from complete ($0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$)-space shell-model wave functions. These wave functions are obtained from diagonalizations of a model Hamiltonian formulation which reproduces observed energy-level structures throughout the *sd* shell. The calculations are carried out with both the "free-nucleon" normalization for the Gamow-Teller single-nucleon matrix elements and one based on the empirical values obtained for these quantities from a comparison of corresponding theoretical and experimental Gamow-Teller magnitudes near the line of stability. The phase-space factors f which connect the reduced Gamow-Teller strengths to the total half-lives and the individual decay probabilities are calculated both from the energies obtained in the shell-model calculations and, alternatively, from hybrid energy spectra in which available experimental energies are substituted for the corresponding calculated values wherever possible. Comparisons of the beta-decay predictions to existing experimental results are presented and discussed.

RADIOACTIVITY Predictions of total half-lives and decay probabilities to individual daughter states of the isotopes of O, F, Ne, Na, Mg, Al, Si, and P which have neutron excesses of five and greater; shell-model calculations, complete $0d_{5/2}$-$1s_{1/2}$-$0d_{3/2}$ basis space, empirical *sd*-shell Hamiltonian.

I. INTRODUCTION

The spectroscopy of very-neutron-rich light nuclei adds a new dimension to our understanding of nuclear structure since these systems are constructed from combinations of neutrons and protons which necessarily have symmetries different from those studied in the stable and near-stable systems whose features are the foundation of our present knowledge. The last decade has witnessed significant advances in the experimental capabilities for both the production of far-from-stability nuclei and the measurement of their properties.[1–6] The continuing development of heavy-ion accelerators and associated particle-analysis systems presage still further progress in this field of study,[7–10] and it is reasonable to hope that by these developments the domain of nuclear spectroscopy can be significantly expanded. We present in this study predictions of the Gamow-Teller (GT) beta-decay properties of the *sd*-shell nuclei which have neutron excesses of five and greater. The creation of such nuclei and the measurements of their decays are at the frontier of current activities in this field.

Simply as a guide to the selection of experiments with which to study beta-unstable isotopes of unknown properties and as an aid to the design of the requisite experimental techniques, it is valuable to have realistic estimates of half-lives and decay patterns. This practical need provides one rationale for the present calculations of these quantities. The more fundamental role of these predictions will develop, however, as data become available against which they can be thoroughly compared. The shell-model wave functions from which they derive[11] are generated in a scheme which requires that the degrees of freedom of the complete $0d_{5/2}$-$1s_{1/2}$-$0d_{3/2}$ shell-model space yield a least-squares fit of Hamiltonian eigenvalues to the experimental energies of low-lying levels of known spin and isospin. The vast preponderance of these levels are taken from nuclei which have nearly equal numbers of neutrons and protons.

Beyond the limitation to the *sd*-shell model space, the theory used to obtain the present predictions has imposed upon it the constraint of a Hamiltonian which conserves isospin, which is limited to one- and two-body interactions, and which has, more specifically, a fixed one-body spectrum and a single set of two-body matrix elements which are scaled for application to a given A value by the factor $(18/A)^{0.3}$. Comparisons between predictions and experiment for systems lying outside the domain of data from which the Hamiltonian was determined provide crucial tests of the ultimate utility of this type of theoretical formulation. These tests concern first the adequacy of the model-space assumption for systems with large neutron excesses and then the appropriateness of the Hamiltonian specification when it is used for systems quite different from those with whose energy levels it was fixed to agree.

A final goal of this study is to advance our knowledge of effective properties of nucleons in nuclei. Analysis of Gamow-Teller beta-decay data from near-stable nuclei with the corresponding members of the present family of wave functions suggests that the experimentally observed strengths are only 60% as large, on the average, as are the strengths generated from the model wave functions by the $\vec{\sigma}\,\vec{\tau}$ operator normalized to agree with the free neutron half-life and 0^+ to 0^+ Fermi decay rates.[12] This result is consistent with a sequence of related earlier investigations.[13–16] Incorporation of data from the decay of more neutron-rich systems provides not just an expansion of the data base for such studies but, more importantly, opportunities to isolate in relative purity specific one-body transition terms which either do not occur with significant intensity in the existing near-to-stability data or are masked by mixing with other terms. Of course, progress towards this last goal is attendant upon confirmation that the predicted structural properties of the systems are in good enough general agreement with experiment to make this final stage of analysis meaningful.

II. DISCUSSION OF THE CALCULATIONS

A. Model space and Hamiltonian

The foundation of the present work is a new set[11] of shell-model wave functions obtained in an attempt to reproduce all features of *sd*-shell spectroscopy from a unified formulation of the model Hamiltonian. The wave functions span the complete spaces of $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$ configurations. This is a critical aspect in calculating matrix elements of operators, such as the spin operator, for which the transitions between the spin-orbit partners are crucial.[17]

This calculation yields a family of wave functions $\psi^{NTJ\nu}$, where $N=A-16$, T and J designate the total isobaric and angular momentum spin values, and ν the counting index which identifies a particular eigenstate of the NTJ set. In this family are wave functions which are presumed to correspond to the initial and final states of the neutron-rich beta decays we wish to calculate. From these wave functions we calculate the density matrix elements $D_{jj'}$ of the one-body operators $[a^\dagger(j)\otimes a(j')]$ coupled to rank $\Delta J=1$ and $\Delta T=1$. The operators $a^\dagger(j)$ and $a(j')$ create and annihilate, respectively, nucleons in the shell-model orbits "j" of our model space, where j designates the single-nucleon state $\rho(n,l,j)$. We have the equation

$$D_{jj'}=(2\Delta J+1)^{-1/2}(2\Delta T+1)^{-1/2}\langle \psi^{N_fT_fJ_f\nu_F}|||[a^\dagger(j)\otimes a(j')]^{\Delta J=1,\Delta T=1}|||\psi^{N_iT_iJ_i\nu_i}\rangle \ , \qquad (1)$$

where the triple bar indicates reduction in both ordinary and isobaric spin. There are seven such matrix elements in the *sd*-shell space, corresponding to the j-j' transitions $d_{5/2}$-$d_{5/2}$, $d_{5/2}$-$d_{3/2}$, $s_{1/2}$-$s_{1/2}$, $s_{1/2}$-$d_{3/2}$, $d_{3/2}$-$d_{5/2}$, $d_{3/2}$-$s_{1/2}$, and $d_{3/2}$-$d_{3/2}$. These $D_{jj'}$ elements embody the entire predictive content of the shell-model calculations insofar as reduced Gamow-Teller strengths are concerned. They are uniquely and completely determined by the specification of the model Hamiltonian.

B. The Gamow-Teller operator

In order to produce a theoretical reduced matrix element for a Gamow-Teller transition the $D_{jj'}$ elements from the shell-model calculation must be combined with the single-nucleon matrix elements $S_{jj'}^{\rm GT}$ which give the amplitudes by which the Gamow-Teller process converts the model nucleons from one single-nucleon state to another. We have

$$S_{jj'}^{\rm GT}=\langle \rho(nlj)|||O_{\rm GT}|||\rho(n'l'j')\rangle \ , \qquad (2)$$

where the Gamow-Teller operator is defined as

$$O_{\rm GT}=|g_A/g_V|\,\vec{\sigma}\,\vec{\tau} \ . \qquad (3)$$

The total Gamow-Teller matrix element is then defined as

$$B({\rm GT})^{1/2}=[2(2J_i+1)]^{-1/2}\begin{pmatrix} T_f & 1 & T_i \\ -T_{Z_f} & \Delta T_z & T_{Z_i}\end{pmatrix} \times \sum_{j,j'} D_{jj'}S_{jj'}^{\rm GT} \ . \qquad (4)$$

In terms of B(GT) for a beta decay from $NT_iJ_i\nu_i$ to $NT_fJ_f\nu_f$, the partial half-life for an individual daughter state is given by

$$t_{1/2}=C[(f_v+f_{\rm EC})B({\rm F})+(f_A+f_{\rm EC})B({\rm GT})]^{-1} \ . \qquad (5)$$

The term B(F) is the Fermi matrix element

$$B({\rm F})=[T(T+1)-T_{Z_f}T_{Z_i}]\delta_{if}(1-\epsilon) \ . \qquad (6)$$

The factor $(1-\epsilon)$ accounts for the reduction in the overlap between the initial and final state nuclear wave functions which arises from isospin mixing. The calculation of the phase-space factors f_V and f_A, which depend upon the transition Q values, is discussed in Sec. II C. The term $f_{\rm EC}$ is the phase-space factor for electron capture. It is zero in the beta-minus decays which are our subject here. The values of the constants in Eqs. (5) and (6), $C=6170\pm4$ and $1-\epsilon=0.997\pm0.003$, are determined empirically from the (pure) Fermi $0+$ to $0+$ beta decays.[18] For the beta-minus decays of the neutron-rich nuclei considered here, Eq. (5) reduces to

$$t_{1/2}=(6170\pm4)f_A^{-1}/B({\rm GT}) \ . \qquad (7)$$

The predicted value for B(GT) depends, as we have seen in Eq. (4), upon the values of the $S_{jj'}^{\rm GT}$ of the Gamow-Teller operator. The conventional values for these matrix elements are obtained here by taking the value of $|g_A/g_V|=1.251\pm0.009$ in Eq. (3). (This value is consistent with the value 1.2606 ± 0.0075 recently published in Ref. 19 for the same quantity.) This normalization of the $S_{jj'}^{\rm GT}$ is consistent with the half-life of the free neutron and we refer to it, and the matrix elements obtained by using it, with the term "free-nucleon." The values of these

TABLE I. Values of the single-nucleon matrix elements of the Gamow-Teller operator in the free-nucleon normalization.

j	j'	$\langle\rho(nlj)\|\|\|(g_A/g_V)\vec{\sigma}\,\vec{\tau}\|\|\|\rho(n'l'j')\rangle$
$\frac{5}{2}$	$\frac{5}{2}$	8.88
$\frac{5}{2}$	$\frac{3}{2}$	−9.50
$\frac{1}{2}$	$\frac{1}{2}$	7.51
$\frac{1}{2}$	$\frac{3}{2}$	0.00
$\frac{3}{2}$	$\frac{5}{2}$	9.50
$\frac{3}{2}$	$\frac{1}{2}$	0.00
$\frac{3}{2}$	$\frac{3}{2}$	−4.75

free-nucleon $S_{jj'}^{GT}$ of the Gamow-Teller operator for the *sd*-shell orbits are given in Table I.

The same Hamiltonian which has yielded the wave functions of the neutron-rich nuclei presently under study has also been used to generate wave functions of the states involved in Gamow-Teller beta decay nearer the valley of stability. The $D_{jj'}$ elements from these wave functions have been used with the free-nucleon $S_{jj'}^{GT}$ to calculate values of B(GT) for these near-stability transitions, and the results compared to the corresponding experimental values.[12] As was the case with a similar analysis with wave functions of the Chung-Wildenthal Hamiltonian,[20,16] but with better quantitative accuracy, these calculations succeed in describing the experimentally observed details of the relative variations in magnitude from transition to transition quite well. The calculated matrix elements are, however, systematically larger in magnitude than the corresponding experimental values. The accuracy with which the calculations reproduce the relative features of the data is such as to suggest that the systematic discrepancy in the absolute magnitude scales of the measured values and calculated values based on the free-nucleon normalization is a consequence of limitations in the general underlying assumptions of the shell model rather than of a failure to optimally utilize in these specific calculations the degrees of freedom which these general assumptions allow.

From this perception it follows that it would be appropriate to introduce state-independent corrections to the original shell-model calculations in order to compensate for the characteristic limitations to neutron and proton coordinates and a single major oscillator shell. The proportionality of the experimental and calculated matrix elements suggests that these corrections can be made in terms of renormalized transition operators, or, more specifically, renormalized or "effective" values of their single-nucleon matrix elements. In principle, these renormalizations should be calculable.[21–25] Alternatively, empirical estimates for their values can be obtained by treating them as parameters in a fit[16] of the theoretical expressions of B(GT) [see Eq. (4)] to the corresponding experimentally determined magnitudes of B(GT).

Such a fit has been carried out with this new family of wave functions.[12] The values of the Gamow-Teller $S_{jj'}^{GT}$ thus empirically determined are smaller than the free-nucleon values by factors of 0.77 ± 0.02. This near constancy of the quenching of the individual $S_{jj'}^{GT}$ terms is a simpler result than the more orbit-dependent results obtained with the Chung-Wildenthal wave functions.[16] This makes it possible to simply scale down the total $B(\mathrm{GT})^{1/2}$ values obtained for the present neutron-rich decays with the free-nucleon $S_{jj'}^{GT}$ by factors of 0.77 in order to obtain the predictions based on this empirical estimate of the net sum of the renormalizations of $S_{jj'}^{GT}$.

The approach of extracting empirical estimates of such renormalization corrections from a comparison of the predictions of nuclear structure theory with experimental data follows the path laid out by Wilkinson[13] who, from a comparison of data in the *p* shell and lower *sd* shell ($A=17$—22) with the calculations of Cohen and Kurath[26] and ancestral versions of the present wave functions,[27,15] extracted a reduction factor (assumed orbit independent) of 0.897 ± 0.035 for g_A/g_V. The empirical renormalizations of Refs. 16 and 12 are not inconsistent with this pioneering result inasmuch as the data from the *p* shell and the $A=17$—22 region do not overlap strongly with the full *sd* shell data set, and the evidence is that the *p* orbits and the $d_{5/2}$ orbit near the ^{16}O shell closure do not require as much quenching as is required overall for the entire *sd* shell.

C. The phase-space factors

The phase-space factors f_V and f_A are calculated in the present study as prescribed by Wilkinson.[12,28,18] The analytic result for a nucleus of $Z=0$ is

$$f_{Z=0}=(\tfrac{1}{60})(2W_0^4-9W_0^2-8)p_0 + (\tfrac{1}{4})W_0\ln(W_0+p_0)\ . \tag{8}$$

In this expression W_0 is the total electron (positron) end point energy and $p_0=(W_0^2-1)^{1/2}$. Several correction factors are applied to Eq. (8) to obtain f_A, the relation being expressed as

$$f_A=\delta_D\delta_R\delta_{\mathrm{WM}}f_{Z=0}\ . \tag{9}$$

The most important of these correction factors is δ_{WM}, the values of which are tabulated[28] by Wilkinson and Macefield in the form

$$\delta_{\mathrm{WM}}=\exp\left[\sum_{n=0,3}a_n(\ln E_0)^n\right]\ , \tag{10}$$

where E_0 is the end point energy of the electron in units of MeV. The calculation of δ_{WM} uses electron wave functions generated from a uniform spherical charge distribution whose radius is adjusted to fit electron scattering and muonic x-ray data, and which is corrected for the screening of the atomic electrons. Also included in the values of δ_{WM} are the effects of the energy-dependent "outer" radiative correction to order α and of the finite mass of the nucleus. The parametrization of the values of δ_{WM} by Eq. (10) is accurate to better than 0.1% throughout.[28]

The factor δ_R incorporates the effects of the outer radiative correction to orders $Z\alpha^2$ and $Z^2\alpha^3$ and is given[13,18] by

$$\delta_R=1+3.67\times10^{-4}\,|\,Z\,|+3.60\times10^{-6}Z^2\ , \tag{11}$$

where Z is the proton number of the daughter nuclei. The factor δ_D incorporates the effects of the diffuseness of the actual nuclear charge distribution[18]

$$\delta_D=1+1.8\times10^{-5}\,|\,Z\,|^{1/36}-1.2\times10^{-6}\,|\,Z\,|\,W_0\ . \tag{12}$$

The Fermi phase-space factor f_V is related[28] to the factor f_A by

$$f_V=\delta_Vf_A\ , \tag{13}$$

where

$$\delta_V=1\pm(\tfrac{2}{15})W_0R\alpha Z-(\tfrac{4}{105})(W_0R)^2 \tag{14}$$

for beta plus/minus decay, where $\alpha=\frac{1}{137}$ and we have used

$$R=1.35(A)^{1/3}\ \mathrm{fm}\ . \tag{15}$$

TABLE II. Values for the Coulomb energy correction constants $\mathscr{E}_C(Z)$ used in calculating the Q values for the beta decays from isotopes of charge Z to charge $Z+1$ [see Eq. (16)].

Z	$\mathscr{E}_C$ (keV)
8	3509
9	3999
10	4300
11	4787
12	5020
13	5495
14	5695
15	6086

We have generated two different sets of f values for the transitions treated here, corresponding to two different procedures for choosing the Q value of a transition. One procedure uses the shell-model predictions for the energies of the initial and final states combined with a Coulomb correction $\Delta E_C(A,Z)$ for a given A,Z to $A,Z+1$ decay. These Coulomb corrections are formulated according to

$$\Delta E_C(A,Z)=\mathscr{E}_C(Z)[2(Z+1)/A]^{1/3} . \tag{16}$$

The values of $\mathscr{E}_C(Z)$, obtained independently of the shell-model calculations by combining the binding-energy differences of analog states with the neutron-proton mass difference, are listed in Table II. We refer to these Q values and the f factors calculated from them, together with the resulting partial half-lives, branching fractions, and total half-lives, as the "shell-model" values.

The other of our procedures for obtaining Q values uses the shell-model energies wherever experimental information is unavailable. Where experimental values for excitation energies or for ground-state mass differences are available, these values are substituted for the corresponding shell-model values and a composite "empirical" spectrum of ground and excited state Q values is constructed. Sets of empirical f factors and resulting partial half-lives, branching fractions, and total half-lives are calculated from this alternate set of Q values.

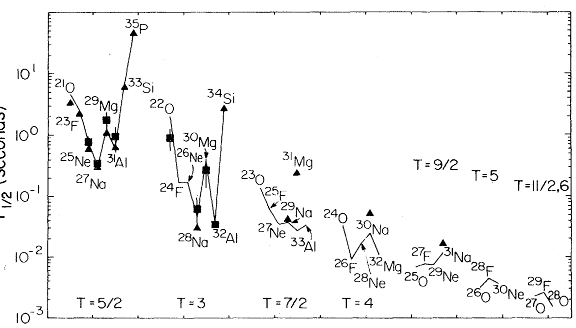

FIG. 1. Comparison of half-lives calculated for the Gamow-Teller beta decay of the sd-shell nuclei with five or more excess neutrons with experimental measurements. The measured values, taken from Table III, are shown as squares (Ref. 9) and triangles (all other experimental references), while the calculated values, also taken from Table III, are connected by the solid lines.

III. RESULTS AND DISCUSSION

A. Total half-lives

The total half-lives of the sd-shell neutron-rich nuclei which are predicted by our calculations are presented in Table III and Fig. 1, along with available experimental data. The total half-lives of Table III are calculated according to the formula

$$(T_{1/2})^{-1}=\sum_i (t^i_{1/2})^{-1} , \tag{17}$$

where the partial half-lives are calculated from the $B(\mathrm{GT})^{1/2}$ and the f factors as presented in Table IV according to Eq. (4). The results in Table III are grouped according to atomic number, those in Fig. 1 according to isospin, or neutron excess. In Table III, we list the ground-state Q values for each decay as determined by the shell-model calculation ("SM") and, if known, as experimentally measured ("emp"), followed by the predicted half-lives as calculated with the shell-model ground-state Q values and excitation energies ("pure SM") and with the hybrid combination of shell-model and experimental ground-state Q values and excitation energies ("SM + emp"). Both sets of half-life predictions incorporate the empirically determined $S^{\mathrm{GT}}_{jj'}$. Use of the free-nucleon instead of the empirical $S^{\mathrm{GT}}_{jj'}$ would yield half-lives 0.6 as long as those listed. The asterisks by the entries for ^{25,27,28}O, ^{28}F, and ^{29}Ne in Table III indicate that the present calculations for the binding energies of these systems predict them to be unbound to emission of either one or two neutrons. In the cases for which this is true, their beta decays would, presumably, not be observed. The shell-model predictions occasionally yield the incorrect ordering of closely-spaced ground-state multiplets. In these cases we have calculated the decays from the experimentally correct ground state. In the case of ^{24} F, for which the theoretical energies of the first two states are almost identical and the experimental answer is unavailable, we have listed predictions for both possibilities.

We see from Table III and Fig. 1 that for each of the isotope groups the least-neutron rich, $T_Z=-\frac{5}{2}$, member has a predicted half-life on the order of a second or longer. The shortest-lived isotopes, the most neutron-rich

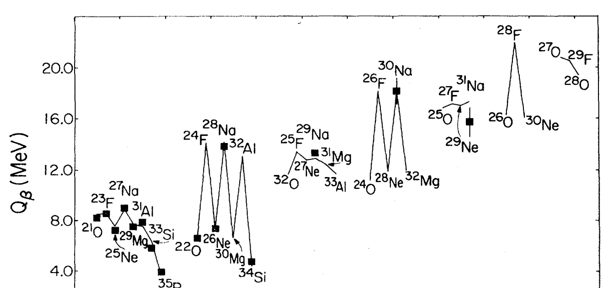

FIG. 2. Comparison of beta Q values calculated for the decays of the sd-shell nuclei with five or more excess neutrons with experimental values. The measured values, taken from Table III, are shown as squares, while the calculated values, also taken from Table III, are connected by the solid lines.

TABLE III. q values and half-lives of sd-shell nuclei which have five or more excess neutrons. Asterisks mark cases predicted to be unstable to neutron emission.

${}^{A}Z_i(J)$	${}^{A}Z_f$	Q value (keV) (SM)	(emp)	$t(\frac{1}{2})$ (sec) (pure SM)	(SM + emp)	exp.	
^{21}O ($\frac{5}{2}$)	^{21}F	8487	8170	3.70	4.70	3.42 ±0.10	Ref. 29
^{22}O (0)	^{22}F	6449	6617	2.50	2.06	0.91 ±0.35	Ref. 9
^{23}O ($\frac{1}{2}$)	^{23}F	11 628	(11 628)	0.141	(0.141)		
^{24}O (0)	^{24}F	11 107	(11 107)	0.034	(0.034)		
^{25}O ($\frac{3}{2}$)	^{25}F	16 772	(16 772)	0.0069*	(0.0069)*		
^{26}O (0)	^{26}F	16 249	(16 249)	0.0032	(0.0032)		
^{27}O ($\frac{3}{2}$)	^{27}F	20 714	(20 714)	0.0023*	(0.0023)*		
^{28}O (0)	^{28}F	19 342	(19 342)	0.0015*	(0.0015)*		
^{23}F ($\frac{5}{2}$)	^{23}Ne	8556	8510	2.32	2.52	2.23 ±0.14	Ref. 30
^{24}F (3)	^{24}Ne	14 121	(14 121)	0.171	0.171		
^{24}F (2)	^{24}Ne	14 281	(14 218)	0.317	0.321		
^{25}F ($\frac{5}{2}$)	^{25}Ne	13 413	(13 413)	0.061	(0.061)		
^{26}F (1)	^{26}Ne	18 128	(18 128)	0.0092	(0.0092)		
^{27}F ($\frac{5}{2}$)	^{27}Ne	17 105	(17 105)	0.0078	(0.0078)		
^{28}F (3)	^{28}Ne	21 926	(21 926)	0.0045*	(0.0045)*		
^{29}F ($\frac{5}{2}$)	^{29}Ne	20 486	(20 486)	0.0027	(0.0027)		
^{25}Ne ($\frac{1}{2}$)	^{25}Na	7525	7203	0.627	0.759	0.602 ±0.008	Ref. 31
						0.780 ±0.180	Ref. 9
^{26}Ne (0)	^{26}Na	7135	7345	0.162	0.170		
^{27}Ne ($\frac{3}{2}$)	^{27}Na	12 766	(12 766)	0.035	(0.035)		
^{28}Ne (0)	^{28}Na	11 829	(11 829)	0.0169	0.0169		
^{29}Ne ($\frac{3}{2}$)	^{29}Na	16 974	(16 974)	0.0074*	(0.0074)*		
^{30}Ne (0)	^{30}Na	16 008	(16 008)	0.0037	(0.0037)		
^{27}Na ($\frac{5}{2}$)	^{27}Mg	8797	8941	0.312	0.300	0.280 ±0.020	Ref. 32
						0.304 ±0.007	Ref. 1
						0.326 ±0.062	Ref. 9
^{28}Na (1)	^{28}Mg	14 186	13 869	0.0398	0.0460	0.0305±0.0004	Ref. 1
						0.062 ±0.033	Ref. 9
^{29}Na ($\frac{3}{2}$)	^{29}Mg	12 981	13 330	0.0456	0.0374	0.0429±0.0015	Ref. 1
^{30}Na (2)	^{30}Mg	18 200	18 097	0.0238	0.0247	0.053 ±0.003	Ref. 1
^{31}Na ($\frac{3}{2}$)	^{31}Mg	17 280	15 699	0.0066	0.0118	0.0169±0.0007	Ref. 1
^{29}Mg ($\frac{3}{2}$)	^{29}Al	7715	7514	0.940	1.210	1.20 ±0.13	Ref. 33
						1.09 ±0.12	Ref. 4
						1.790 ±0.565	Ref. 9
^{30}Mg (0)	^{30}Al	6575	6575	0.340	0.389	0.325 ±0.030	Ref. 4
						0.270 ±0.135	Ref. 9
^{31}Mg ($\frac{3}{2}$)	^{31}Al	12 496	(12 496)	0.0274	0.0274	0.250 ±0.030	Ref. 4
^{32}Mg (0)	^{32}Al	11 892	(11 892)	0.0110	(0.0110)		
^{31}Al ($\frac{5}{2}$)	^{31}Si	7581	7856	0.734	0.593	0.644 ±0.025	Ref. 34
						0.945 ±0.425	Ref. 9
^{32}Al (1)	^{32}Si	13 076	(13 076)	0.0358	0.0363	0.035 ±0.005	Ref. 9
^{33}Al ($\frac{5}{2}$)	^{33}Si	11 711	(11 711)	0.0341	(0.0341)		
^{33}Si ($\frac{3}{2}$)	^{33}P	6316	5768	4.64	7.41	6.18 ±0.18	Ref. 35
^{34}Si (0)	^{34}P	4545	4700	3.07	2.90	2.77 ±0.20	Ref. 36
^{35}P ($\frac{1}{2}$)	^{35}S	3692	3909	71.1	48.3	48.1 ±1.4	Ref. 37

TABLE IV. Excitation energies of allowed final states in the Gamow-Teller beta decay of *sd*-shell nuclei with five or more excess neutrons, the values of their phase-space factors *f*, and their calculated and, where available, measured branching percentages. The asterisk denotes that all levels in the system are unstable to neutron emission.

J_f	Exc. energy (keV) (SM)	(emp)	log(*f*)	$B(\mathrm{GT})^{1/2}$	Branching (%) th (SM)	th (emp)	exp
^{21}O ($\frac{5}{2}^+$)→ ^{21}F (a)							
$\frac{3}{2}$	1853	1730	4.279	0.200	30.75	34.21	37.0
	3512	(3460)	3.653	0.166	5.65	5.63	(12.3)
	4097	(4572)	3.366	0.366	15.41	14.04	(10.5)
	4660		3.076	0.095	0.56	0.49	
	4892		2.944	0.197	1.81	1.53	
	6534		1.646	0.009	0.00	0.00	
	7660		−0.328	0.141	0.00	0.00	
	8182			0.410	0.00	0.00	
$\frac{5}{2}$	0	0	4.763	0.000	0.00	0.00	
	3681	3518	3.628	0.389	26.38	29.08	29.5
	4269		3.282	0.144	1.98	1.78	
	5122		2.804	0.045	0.07	0.06	
	5956		2.201	0.145	0.21	0.15	
	6284		1.905	0.639	2.23	1.48	
	6562		1.615	0.004	0.00	0.00	
	6790		1.342	0.327	0.19	0.11	
	7313		0.521	0.099	0.00	0.00	
	7435		0.265	0.205	0.01	0.00	
$\frac{7}{2}$	3611	3639	3.576	0.057	0.60	0.55	1.0
	4427		3.124	0.166	2.23	1.66	
	5283		2.700	0.393	4.31	3.49	
	5602		2.478	0.503	4.46	3.44	
	5720		2.390	0.444	2.89	2.18	
	6956		1.117	0.461	0.25	0.13	
	7459		0.210	0.054	0.00	0.00	
	7945		−1.581	0.186	0.00	0.00	
	8245			0.240	0.00	0.00	
B(*n*)	8168	8102(8)					
^{22}O (0^+)→ ^{22}F							
1	1629	1570	3.788	0.019	0.04	0.04	
	2327		3.466	1.193	83.30	82.48	
	3250		2.993	0.889	14.85	15.40	
	3966		2.535	0.069	0.03	0.03	
	4221		2.344	0.656	1.67	1.88	
	5118		1.484	0.482	0.10	0.14	
	5794		0.442	0.640	0.01	0.02	
	6135		−0.434	0.059	0.00	0.00	
B(*n*)	5060	5197(31)					
^{23}O ($\frac{1}{2}^+$)→ ^{23}F							
$\frac{1}{2}$	1776		5.146	0.536	54.58	54.58	
	5333		4.232	0.380	3.34	3.34	
	6379		3.867	0.775	5.99	5.99	
	7790		3.248	0.308	0.23	0.23	
	8116		3.075	0.634	0.65	0.65	
	8970		2.540	0.340	0.05	0.05	
	9105		2.441	0.627	0.15	0.15	
	9970		1.665	0.374	0.01	0.01	
	10 345		1.207	0.179	0.00	0.00	
	10 726		0.597	0.321	0.00	0.00	
$\frac{3}{2}$	3497		4.752	0.517	20.48	20.48	
	4404		4.511	0.389	6.66	6.66	
	5653		4.126	0.526	5.03	5.03	

TABLE IV. (*Continued.*)

J_f	Exc. energy (keV) (SM)	(emp)	log(f)	$B(\mathrm{GT})^{1/2}$	Branching (%) th(SM)	th (emp)	exp
^{23}O ($\frac{1}{2}^+$)→ ^{23}F							
$\frac{3}{2}$	6425		3.849	0.079	0.06	0.06	
	6767		3.714	0.529	1.96	1.96	
	7612		3.336	0.473	0.66	0.66	
	8082		3.093	0.099	0.02	0.02	
	8294		2.974	0.034	0.00	0.00	
	8450		2.881	0.221	0.05	0.05	
	8575		2.804	0.328	0.09	0.09	
$B(n)$	7581	7543(173)					
^{23}F ($\frac{5}{2}^+$)→ ^{23}Ne (b)							
$\frac{3}{2}$	1768	1822	4.365	0.027	0.41	0.42	10.0
	3237	3432	3.812	0.350	21.94	19.12	15.3
	3778	3988	3.582	0.032	0.11	0.09	
	4920	5000	3.085	0.104	0.34	0.31	
	5434		2.831	0.102	0.17	0.17	
	6003		2.442	0.129	0.11	0.11	
	6444		2.081	0.004	0.00	0.00	
	6737		1.800	0.115	0.02	0.02	
$\frac{5}{2}$	0	0	4.856	0.140	31.95	33.60	(33.6)[c]
	2184	2315	4.211	0.000	0.00	0.00	6.7
	3826	3831	3.649	0.552	31.78	32.62	24.3
	5057		3.054	0.105	0.29	0.30	
	5433		2.831	0.107	0.18	0.19	
	5955		2.477	0.134	0.13	0.13	
	6366		2.149	0.032	0.00	0.00	
	6755		1.782	0.286	0.12	0.12	
$\frac{7}{2}$	1759	1701	4.402	0.046	1.16	1.26	
	3605		3.743	0.145	2.69	2.78	
	4395	(4436)	3.396	0.344	6.91	7.09	10.1
	5535		2.767	0.258	0.93	0.93	
	5773		2.608	0.188	0.34	0.34	
	6194		2.293	0.054	0.01	0.01	
	6513		2.018	0.344	0.30	0.30	
	6976		1.539	0.313	0.09	0.08	
$B(n)$	5134	5200(3)					
^{24}O (0$^+$)→ ^{24}F							
1	861		5.227	1.231	83.46	83.46	
	2996		4.747	0.261	1.24	1.24	
	4099		4.449	1.046	10.03	10.03	
	4790		4.239	0.246	0.34	0.34	
	5230		4.093	1.070	4.63	4.63	
	6691		3.523	0.137	0.02	0.02	
	7069		3.347	0.509	0.19	0.19	
	7458		3.149	0.394	0.07	0.07	
	8091		2.781	0.253	0.01	0.01	
	8335		2.620	0.232	0.01	0.01	
$B(n)$	3410						
^{24}F (3$^+$)→ ^{24}Ne							
2	2145	1981	5.589	0.193	22.36	23.66	
	3742	3867	5.239	0.198	11.87	11.11	
	5339		4.920	0.203	5.65	5.60	

TABLE IV. (*Continued.*)

J_f	Exc. energy (keV) (SM)	(emp)	log(f)	$B(GT)^{1/2}$	Branching (%) th(SM)	th (emp)	exp
^{24}F $(3^+)\rightarrow$ ^{24}Ne							
2	5957		4.771	0.124	1.49	1.48	
	6610		4.601	0.061	0.24	0.24	
	7265		4.415	0.281	3.38	3.36	
	7618		4.308	0.032	0.03	0.03	
	8322		4.078	0.185	0.67	0.67	
	8582		3.986	0.190	0.57	0.57	
	8811		3.901	0.194	0.49	0.49	
3	4568		5.093	0.077	1.20	1.19	
	5472		4.889	0.025	0.08	0.08	
	6927		4.513	0.394	8.34	8.27	
	7240		4.423	0.204	1.80	1.79	
	7842		4.238	0.284	2.30	2.28	
	8683		3.949	0.507	3.76	3.73	
	8880		3.875	0.151	0.28	0.28	
	9187		3.755	0.110	0.11	0.11	
	9875		3.458	0.161	0.12	0.12	
	10023		3.388	0.174	0.12	0.12	
4	3996		5.213	0.327	28.70	28.46	
	5645		4.848	0.023	0.06	0.06	
	5941		4.775	0.167	2.73	2.71	
	7394		4.377	0.130	0.67	0.66	
	7701		4.283	0.252	2.01	1.99	
	7744		4.269	0.035	0.04	0.04	
	7983		4.192	0.052	0.07	0.07	
	8560		3.994	0.084	0.11	0.11	
	8840		3.890	0.033	0.01	0.01	
	9110		3.786	0.268	0.72	0.72	
$B(n)$	8921	8865(11)					
^{24}F $(2^+)\rightarrow$ ^{24}Ne							
1	7391		4.407	0.373	10.82	10.94	
	7976		4.226	0.249	3.19	3.22	
	8555		4.030	0.038	0.05	0.05	
	8857		3.920	0.139	0.49	0.49	
	9296		3.750	0.096	0.16	0.16	
	9467		3.680	0.015	0.00	0.00	
	9716		3.573	0.009	0.00	0.00	
	10095		3.400	0.222	0.38	0.38	
2	2145	1981	5.606	0.114	14.96	16.13	
	3742	3867	5.259	0.253	37.34	35.65	
	5339		4.943	0.082	1.81	1.83	
	5957		4.795	0.012	0.03	0.03	
	6610		4.627	0.217	6.09	6.16	
	7265		4.444	0.142	1.71	1.73	
	7618		4.338	0.151	1.52	1.53	
	8322		4.111	0.465	8.52	8.61	
3	4568		5.114	0.016	0.10	0.10	
	5472		4.912	0.114	3.26	3.30	
	6927		4.540	0.231	5.65	5.71	
	7240		4.451	0.071	0.44	0.44	
	7842		4.269	0.048	0.13	0.13	
	8683		3.984	0.051	0.08	0.08	
	8880		3.912	0.271	1.82	1.84	
	9187			0.279	1.47	1.49	
$B(n)$	8921	8865(11)					

TABLE IV. (*Continued.*)

J_f	Exc. energy (keV) (SM)	(emp)	log(f)	$B(\mathrm{GT})^{1/2}$	Branching (%) th(SM)	th (emp)	exp
$^{25}O\ (\frac{3}{2}^+) \rightarrow {}^{25}F$							
$\frac{1}{2}$	911		6.136	0.194	3.38	3.38	
	4069		5.673	0.118	0.43	0.43	
	6017		5.327	0.435	2.64	2.64	
	7083		5.112	0.255	0.55	0.55	
$\frac{3}{2}$	3073		5.830	0.047	0.10	0.10	
	3987		5.686	0.155	0.77	0.77	
	5049		5.506	0.075	0.12	0.12	
	5461		5.432	0.323	1.86	1.86	
$\frac{5}{2}$	0		6.252	0.799	75.05	75.05	
	3756		5.723	0.462	7.43	7.43	
	4799		5.550	0.526	6.47	6.47	
	5631		5.400	0.270	1.21	1.21	
$B(n)$	4852						
$^{25}F\ (\frac{5}{2}^+) \rightarrow {}^{25}Ne$							
$\frac{3}{2}$	1687		5.517	0.350	23.40	23.40	
	2967		5.278	0.055	0.33	0.33	
	4595		4.929	0.223	2.46	2.46	
	5676		4.661	0.138	0.51	0.51	
	6174		4.526	0.342	2.28	2.28	
	6409		4.459	0.230	0.88	0.88	
	6895		4.313	0.105	0.13	0.13	
	7341		4.170	0.225	0.43	0.43	
$\frac{5}{2}$	1779		5.501	0.224	9.24	9.24	
	2971		5.277	0.303	10.09	10.09	
	4226		5.013	0.443	11.73	11.73	
	4648		4.916	0.052	0.13	0.13	
	5574		4.688	0.498	7.03	7.03	
	5741		4.644	0.205	1.07	1.07	
	6182		4.524	0.334	2.16	2.16	
	6561		4.414	0.274	1.13	1.13	
	7027		4.272	0.598	3.88	3.88	
	7409		4.147	0.050	0.02	0.02	
$\frac{7}{2}$	3639		5.141	0.484	18.79	18.79	
	4249		5.008	0.029	0.05	0.05	
	4692		4.906	0.028	0.04	0.04	
	6175		4.526	0.177	0.61	0.61	
	6205		4.517	0.188	0.68	0.68	
	6775		4.350	0.082	0.09	0.09	
	7080		4.255	0.519	2.82	2.82	
	7324		4.176	0.015	0.00	0.00	
$B(n)$	4093	4278(96)					
$^{25}Ne\ (\frac{1}{2}^+) \rightarrow {}^{25}Na$ (d)							
$\frac{1}{2}$	1159	1069	4.202	0.439	21.94	22.38	19.2
	4048		2.891	0.443	1.41	1.11	
	5134		2.096	0.317	0.14	0.09	
	5725		1.486	0.370	0.06	0.03	
$\frac{3}{2}$	132	90	4.501	0.556	70.57	71.38	76.8
	2130	2202	3.793	0.297	4.67	4.00	2.0

TABLE IV. (*Continued.*)

J_f	Exc. energy (keV) (SM)	(emp)	log(f)	$B(\mathrm{GT})^{1/2}$	Branching (%) th(SM)	th (emp)	exp
^{25}Ne ($\frac{1}{2}^+$)$\rightarrow$ ^{25}Na (d)							
$\frac{3}{2}$	3304		3.302	0.232	0.93	0.79	
	3623		3.136	0.149	0.27	0.22	
$B(n)$	9047	9011(7)					
^{26}O (0$^+$)$\rightarrow$ ^{26}F							
1	0		6.186	1.326	82.70	82.70	
	2054		5.904	0.268	1.76	1.76	
	4633		5.487	0.595	3.33	3.33	
	5105		5.401	0.907	6.35	6.35	
	5874		5.253	0.175	0.17	0.17	
	6418		5.142	0.381	0.62	0.62	
	7316		4.945	1.354	4.95	4.95	
	7797		4.831	0.239	0.12	0.12	
$B(n)$	1201						
^{26}F (1$^+$)$\rightarrow$ ^{26}Ne							
0	0		6.425	0.353	29.24	29.24	
	3812		5.932	0.347	9.08	9.08	
	5539		5.665	0.186	1.40	1.40	
	7257		5.360	0.125	0.31	0.31	
	8468		5.116	0.012	0.00	0.00	
	9442		4.898	0.208	0.30	0.30	
1	4356		5.581	0.077	0.37	0.37	
	6210		5.551	0.153	0.74	0.74	
	7285		5.355	0.161	0.52	0.52	
	8399		5.131	0.135	0.22	0.22	
	8910		5.020	0.278	0.71	0.71	
	9154		4.965	0.115	0.11	0.11	
2	2011		6.179	0.471	29.57	29.57	
	3448		5.984	0.389	12.87	12.87	
	4717		5.796	0.008	0.00	0.00	
	5818		5.618	0.511	9.56	9.56	
	6087		5.572	0.193	1.22	1.22	
	6560		5.489	0.360	3.51	3.51	
	6914		5.425	0.104	0.26	0.26	
	7428		5.327	0.019	0.01	0.01	
$B(n)$	5868	5475(119)					
^{26}Ne (0$^+$)$\rightarrow$ ^{26}Na							
1	−181	(88)	4.542	1.266	90.50	90.92	
	1349	(1996)	3.927	0.458	3.97	2.89	
	2090	(2290)	3.814	0.609	3.74	3.94	
	2538	(2697)	3.648	0.340	0.76	0.84	
	3348		3.351	0.024	0.00	0.00	
	3871		3.077	0.850	1.02	1.41	
	4174		2.901	0.054	0.00	0.00	
	4567		2.648	0.089	0.00	0.01	
$B(n)$	5606	5618(18)					
^{27}O ($\frac{3}{2}^+$)$\rightarrow$ ^{27}F							
$\frac{1}{2}$	1997		6.482	0.120	0.97	0.97	
	3299		6.331	0.375	6.70	6.70	
	7479		5.758	0.075	0.07	0.07	

TABLE IV. (*Continued.*)

J_f	Exc. energy (keV) (SM)	(emp)	log(f)	$B(\mathrm{GT})^{1/2}$	Branching (%) th(SM)	th (emp)	exp
$^{27}\mathrm{O}$ $(\frac{3}{2}^+)\rightarrow$ $^{27}\mathrm{F}$							
$\frac{1}{2}$	8527		5.587	0.183	0.29	0.29	
	9114		5.484	0.332	0.75	0.75	
	10300		5.261	0.336	0.46	0.46	
	11087		5.099	0.377	0.40	0.40	
	11308		5.051	0.000	0.00	0.00	
$\frac{3}{2}$	2861		6.383	0.785	33.13	33.13	
	4436		6.190	0.150	0.77	0.77	
	5772		6.011	0.194	0.86	0.86	
	6228		5.946	0.318	1.99	1.99	
	7031		5.828	0.050	0.04	0.04	
	7584		5.742	0.440	2.38	2.38	
	7726		5.719	0.141	0.23	0.23	
	9388		5.435	0.179	0.19	0.19	
$\frac{5}{2}$	0		6.694	0.116	1.48	1.48	
	3385		6.321	0.762	27.05	27.05	
	4445		6.189	0.697	16.69	16.69	
	5676		6.024	0.162	0.62	0.62	
	6736		5.872	0.379	2.39	2.39	
	7432		5.766	0.441	2.53	2.53	
	8496		5.592	0.003	0.00	0.00	
	9120		5.483	0.038	0.01	0.01	
$B(n)$	2674						
$^{27}\mathrm{F}$ $(\frac{5}{2}^+)\rightarrow$ $^{27}\mathrm{Ne}$							
$\frac{3}{2}$	0		6.303	0.419	26.38	26.38	
	2539		5.968	0.036	0.09	0.09	
	3371		5.846	0.413	8.91	8.91	
	4678		5.638	0.150	0.73	0.73	
	5041		5.576	0.015	0.01	0.01	
	5697		5.460	0.232	1.16	1.16	
	5773		5.446	0.178	0.66	0.66	
	6389		5.331	0.238	0.91	0.91	
$\frac{5}{2}$	2171		6.020	0.539	22.72	22.72	
	2582		5.962	0.330	7.46	7.46	
	3265		5.862	0.355	6.86	6.86	
	4579		5.654	0.160	0.86	0.86	
	4896		5.601	0.363	3.93	3.93	
	5184		5.551	0.045	0.05	0.05	
$\frac{7}{2}$	2017		6.041	0.381	11.96	11.96	
	4253		5.707	0.248	2.34	2.34	
	4662		5.640	0.063	0.13	0.13	
	5092		5.567	0.371	3.79	3.79	
	5791		5.443	0.211	0.92	0.92	
	6215		5.364	0.097	0.16	0.16	
$B(n)$	1605						
$^{27}\mathrm{Ne}$ $(\frac{3}{2}^+)\rightarrow$ $^{27}\mathrm{Na}$							
$\frac{1}{2}$	1630		5.420	0.074	0.48	0.48	
	3741		4.987	0.118	0.45	0.45	
	4406		4.830	0.354	2.81	2.81	
	4759		4.742	0.045	0.04	0.04	
	5342		4.588	0.147	0.28	0.28	

TABLE IV. (*Continued.*)

J_f	Exc. energy (keV) (SM)	(emp)	log(*f*)	$B(GT)^{1/2}$	Branching (%) th(SM)	th (emp)	exp
^{27}Ne ($\frac{3}{2}^+$)→ ^{27}Na							
$\frac{1}{2}$	5811		4.456	0.315	0.94	0.94	
	6441		4.263	0.265	0.43	0.43	
	7138		4.029	0.151	0.08	0.08	
$\frac{3}{2}$	14		5.701	0.162	4.40	4.40	
	3121		5.124	0.517	11.81	11.81	
	3643		5.009	0.314	3.34	3.34	
	3749		4.985	0.042	0.06	0.06	
	4431		4.824	0.157	0.55	0.55	
	4794		4.733	0.000	0.00	0.00	
$\frac{5}{2}$	0		5.704	0.494	41.00	41.00	
	2671		5.218	0.456	11.41	11.41	
	3571		5.025	0.362	4.60	4.60	
	3922		4.946	0.538	8.47	8.47	
	4337		4.847	0.466	5.07	5.07	
	4902		4.705	0.186	0.58	0.58	
	5123		4.647	0.117	0.20	0.20	
	5655		4.501	0.535	3.01	3.01	
B(*n*)	7003	6812(47)					
^{27}Na ($\frac{5}{2}^+$)→ ^{27}Mg (e,f)							
$\frac{3}{2}$	895	985	4.740	0.734	85.89	85.29	86.0, 84
	3162	3490	3.865	0.056	0.10	0.07	
	3632		3.719	0.189	0.79	0.54	
	5226		3.220	0.023	0.00	0.00	
	5404		3.124	0.150	0.07	0.09	
	5656		2.981	0.165	0.06	0.08	
	5887		2.841	0.193	0.06	0.07	
	6428		2.471	0.037	0.00	0.00	
	6648		2.300	0.039	0.00	0.00	
	7230		1.762	0.365	0.02	0.02	
$\frac{5}{2}$	1667	1698	4.549	0.278	7.64	7.91	14.0, 13
	1978	1940	4.480	0.121	1.17	1.28	<3.5,1
	4032		3.768	0.011	0.00	0.00	
	4201		3.698	0.201	0.53	0.58	
	4719		3.470	0.196	0.29	0.33	
	5101		3.284	0.324	0.51	0.58	
	5454		3.097	0.357	0.39	0.46	
	6086		2.713	0.040	0.00	0.00	
	6483		2.430	0.066	0.00	0.00	
	6759		2.208	0.236	0.02	0.03	
$\frac{7}{2}$	3149		4.098	0.082	0.22	0.24	
	3300		4.045	0.218	1.41	1.52	
	4616		3.517	0.193	0.31	0.35	
	5206		3.230	0.279	0.33	0.38	
	5481		3.081	0.119	0.04	0.05	
	5813		2.887	0.251	0.12	0.14	
B(*n*)	6312	6444(<2)					
^{28}O (0^+)→ ^{28}F							
1	1847		6.341	1.474	70.25	70.25	
	2774		6.227	0.767	14.63	14.63	
	6904		5.629	0.773	3.75	3.75	

TABLE IV. (*Continued.*)

J_f	Exc. energy (keV) (SM)	(emp)	log(*f*)	$B(GT)^{1/2}$	Branching (%) th(SM)	th (emp)	exp
$^{28}O\ (0^+)\rightarrow {}^{28}F$							
1	8080		5.423	0.075	0.02	0.02	
	9549		5.134	2.246	10.14	10.14	
	11418		4.699	1.275	1.20	1.20	
B(*n*)	*						
$^{28}F\ (3^+)\rightarrow {}^{28}Ne$							
2	1785		6.645	0.002	0.00	0.00	
	3678		6.439	0.337	13.48	13.48	
	4300		6.366	0.286	8.26	8.26	
	5343		6.239	0.141	1.49	1.49	
	6203		6.127	0.153	1.36	1.36	
	6888		6.035	0.267	3.34	3.34	
	7892		5.891	0.066	0.15	0.15	
	8514		5.796	0.056	0.09	0.09	
3	5106		6.268	0.340	9.28	9.28	
	5838		6.175	0.127	1.05	1.05	
	7039		6.014	0.493	10.89	10.89	
	7535		5.943	0.538	11.03	11.03	
	8400		5.814	0.062	0.11	0.11	
	8782		5.754	0.149	0.55	0.55	
	9724		5.600	0.033	0.02	0.02	
	9895		5.570	0.006	0.00	0.00	
4	3301		6.481	0.353	16.35	16.35	
	5182		6.259	0.412	13.35	13.35	
	6868		6.037	0.300	4.25	4.25	
	6979		6.022	0.231	2.43	2.43	
	7821		5.901	0.043	0.06	0.06	
	8261		5.835	0.069	0.14	0.14	
	8619		5.780	0.261	1.78	1.78	
	8898		5.736	0.153	0.55	0.55	
B(*n*)	4336						
$^{28}Ne\ (0^+)\rightarrow {}^{28}Na$							
1	0		5.545	0.912	47.33	47.33	
	1567		5.251	1.051	31.99	31.99	
	2167		5.127	0.450	4.41	4.41	
	2704		5.010	0.760	9.58	9.58	
	3803		4.747	0.836	6.33	6.33	
	3900		4.722	0.011	0.00	0.00	
	4510		4.559	0.250	0.37	0.37	
	4939		4.436	0.027	0.00	0.00	
B(*n*)	3442	3576(141)					
$^{28}Na\ (1^+)\rightarrow {}^{28}Mg$ (f)							
0	0	0	5.886	0.367	44.09	45.77	63.0
	3802	3863	5.210	0.632	29.55	28.67	21.0
	6187		4.669	0.107	0.25	0.24	
	6855		4.484	0.201	0.58	0.55	
	7867		4.169	0.178	0.23	0.21	
	8900		3.792	0.110	0.04	0.03	
1	4396	4561	5.061	0.355	7.04	6.41	(5.0)[g]
	5412	5193	4.917	0.401	5.36	5.88	
	6863		4.482	0.213	0.65	0.61	
	7381		4.326	0.205	0.42	0.39	

TABLE IV. (*Continued.*)

J_f	Exc. energy (keV) (SM)	(emp)	log(*f*)	$B(\mathrm{GT})^{1/2}$	Branching (%) th(SM)	th (emp)	exp
^{28}Na $(1^+)\to$ ^{28}Mg (f)							
0	8229		4.045	0.397	0.86	0.77	
2	1543	1473	5.653	0.126	3.00	3.16	11.0
	4264	4557	5.062	0.193	2.23	1.91	(5.0)[g]
	4773	4878	4.990	0.024	0.03	0.03	
	5402		4.867	0.067	0.15	0.14	
	6013		4.714	0.322	2.47	2.37	
	6757		4.512	0.327	1.63	1.54	
	7330		4.342	0.171	0.30	0.28	
	7420		4.314	0.338	1.12	1.04	
B(*n*)	8684	8503(3)					
^{29}F $(\frac{5}{2}^+)\to$ ^{29}Ne							
$\frac{3}{2}$	0		6.679	0.034	0.14	0.14	
	3006		6.347	0.583	19.43	19.43	
	4658		6.139	0.144	0.73	0.73	
	5940		5.963	0.346	2.83	2.83	
	7152		5.781	0.287	1.28	1.28	
	7672		5.698	0.208	0.55	0.55	
$\frac{5}{2}$	2087		6.454	0.007	0.00	0.00	
	4155		6.204	0.394	6.40	6.40	
	4811		6.119	0.739	18.50	18.50	
	5372		6.043	0.585	9.72	9.72	
	6436		5.890	0.057	0.06	0.06	
	7113		5.787	0.464	3.39	3.39	
$\frac{7}{2}$	2023		6.461	0.173	2.23	2.23	
	3647		6.268	0.818	31.96	31.96	
	5198		6.066	0.092	0.25	0.25	
	5946		5.962	0.326	2.50	2.50	
B(*n*)	*						
^{29}Ne $(\frac{3}{2}^+)\to$ ^{29}Na							
$\frac{1}{2}$	1825		6.060	0.023	0.04	0.04	
	3269		5.851	0.115	0.67	0.67	
	3611		5.799	0.299	4.01	4.01	
	5771		5.433	0.332	2.13	2.13	
	5960		5.398	0.317	1.79	1.79	
	7121		5.168	0.013	0.00	0.00	
$\frac{3}{2}$	0		6.297	0.353	17.64	17.64	
	2669		5.940	0.216	2.91	2.91	
	4037		5.731	0.009	0.00	0.00	
	4379		5.676	0.170	0.98	0.98	
	4968		5.576	0.511	7.04	7.04	
	5521		5.479	0.122	0.32	0.32	
$\frac{5}{2}$	−137		6.314	0.432	27.41	27.41	
	3085		5.879	0.419	9.49	9.49	
	4122		5.717	0.244	2.22	2.22	
	4454		5.663	0.509	8.51	8.51	
	4676		5.626	0.667	13.43	13.43	
	5409		5.499	0.250	1.40	1.40	
B(*n*)	5017	4292(199)					

TABLE IV. (*Continued.*)

J_f	Exc. energy (keV) (SM)	(emp)	log(f)	$B(\mathrm{GT})^{1/2}$	Branching (%) th(SM)	th (emp)	exp
^{29}Na $(\frac{3}{2}^+)\rightarrow$ ^{29}Mg							
$\frac{1}{2}$	−39		5.810	0.107	2.78	2.68	
	2398		5.393	0.611	33.16	33.09	
	5221		4.779	0.522	5.52	5.88	
	5803		4.627	0.178	0.44	0.48	
	6621		4.394	0.150	0.18	0.20	
	6752		4.354	0.068	0.03	0.04	
	7219		4.206	0.015	0.00	0.00	
	7873		3.979	0.178	0.09	0.11	
$\frac{3}{2}$	0		5.804	0.333	26.35	25.40	
	2153		5.439	0.073	0.53	0.53	
	3187		5.238	0.511	16.03	16.24	
	4796		4.883	0.028	0.02	0.02	
$\frac{5}{2}$	1503		5.556	0.055	0.40	0.39	
	2999		5.276	0.272	4.99	5.03	
	3492		5.175	0.188	1.87	1.91	15.0
	3934		5.081	0.180	1.37	1.41	
	4895		4.860	0.375	3.47	3.66	
	4944		4.848	0.340	2.77	2.93	
$B(n)$	3622	3753(51)					
^{29}Mg $(\frac{3}{2}^+)\rightarrow$ ^{29}Al (f)							
$\frac{1}{2}$	1214	1398	4.218	0.119	2.80	2.72	
	3330	3433	3.415	0.393	5.01	4.65	
	4229		2.993	0.016	0.00	0.00	21.0
	4650		2.731	0.158	0.16	0.16	6.0
	5470		2.100	0.155	0.04	0.04	
	5803		1.776	0.141	0.02	0.01	
	6214		1.287	0.031	0.00	0.00	
	6482		0.887	0.161	0.00	0.00	
	7499		−5.251	0.163	0.00	0.00	
$\frac{3}{2}$	1959	2224	3.928	0.327	12.06	10.52	
	2735	2866	3.671	0.227	2.99	2.81	26.0
	3578		3.344	0.325	2.64	2.72	5.0
	3976		3.137	0.436	3.01	3.02	27.0
	4459		2.854	0.008	0.00	0.00	
	5064		2.436	0.212	0.16	0.14	
	5591		1.988	0.047	0.00	0.00	
	6113		1.419	0.154	0.01	0.01	
$\frac{5}{2}$	0	0	4.635	0.313	43.10	49.00	
	2801	3062	3.586	0.434	10.29	8.45	
	3017	3184	3.531	0.622	17.19	15.27	
	3816		3.223	0.136	0.35	0.36	
	4128		3.052	0.016	0.00	0.00	
	4501		2.828	0.042	0.01	0.01	
	5281		2.263	0.135	0.04	0.04	
	5648		1.933	0.259	0.08	0.07	
$B(n)$	9461	9436(7)					
^{30}Ne $(0^+)\rightarrow$ ^{30}Na							
1	66		6.166	1.065	59.36	59.36	
	2511		5.819	0.531	6.65	6.65	
	3251		5.702	1.124	22.72	22.72	

TABLE IV. (*Continued.*)

J_f	Exc. energy (keV) (SM)	(emp)	log(f)	$B(\mathrm{GT})^{1/2}$	Branching (%) th(SM)	th (emp)	exp
^{30}Ne $(0^+)\rightarrow$ ^{30}Na							
1	4115		5.557	0.728	6.81	6.81	
	5443		5.312	0.468	1.60	1.60	
	5922		5.216	0.312	0.57	0.57	
	6500		5.094	0.384	0.65	0.65	
	7406		4.889	0.767	1.63	1.63	
$B(n)$	1381	2333(354)					
^{30}Mg $(0^+)\rightarrow$ ^{30}Al (f)							
1	493	(685)	4.143	1.316	91.21	89.95	84.5
	2139	(2163)	3.579	0.786	7.65	8.75	3.0
	2538		3.394	0.032	0.01	0.01	
	2981		3.167	0.019	0.00	0.00	
	3185		3.054	0.161	0.10	0.11	
	3551		2.835	0.672	1.01	1.15	
	4100		2.455	0.119	0.01	0.02	
	4573		2.062	0.143	0.01	0.01	
$B(n)$	5583	5751(41)					
^{30}Na $(2^+)\rightarrow$ ^{30}Mg							
1	5242		5.728	0.053	0.36	0.36	
	5959		5.610	0.377	13.73	13.72	
	7304		5.366	0.290	4.66	4.63	
	8449		5.135	0.071	0.17	0.17	
	8837		5.051	0.117	0.37	0.37	
	9065		5.000	0.003	0.00	0.00	
	9366		4.930	0.403	3.33	3.28	
	10115		4.747	0.161	0.35	0.35	
2	1671	(1484)	6.238	0.161	10.53	10.63	
	3446		5.997	0.234	12.86	12.94	
	4802		5.798	0.229	7.79	7.81	
	5192		5.736	0.392	19.86	19.89	
	6495		5.516	0.295	6.77	6.76	
	6746		5.471	0.062	0.27	0.27	
	7315		5.364	0.288	4.59	4.56	
	7792		5.271	0.098	0.43	0.43	
3	4692		5.815	0.067	0.69	0.70	
	6499		5.515	0.188	2.76	2.75	
	7267		5.373	0.285	4.57	4.55	
	7450		5.338	0.137	0.98	0.98	
	8247		5.178	0.094	0.32	0.32	
	8667		5.088	0.318	2.97	2.94	
	8852		5.047	0.136	0.49	0.49	
	9221		4.964	0.226	1.14	1.12	
$B(n)$	6778						
^{31}Na $(\frac{3}{2}^+)\rightarrow$ ^{31}Mg							
$\frac{1}{2}$	2366		5.804	0.209	3.02	3.15	
	3745		5.578	0.721	22.56	22.24	
	5604		5.228	0.017	0.01	0.01	
	6090		5.127	0.122	0.26	0.23	
	7662		4.761	0.396	1.34	1.02	
	8619		4.503	0.465	1.12	0.78	
$\frac{3}{2}$	0		6.144	0.261	9.53	10.73	
	3206		5.669	0.445	10.37	10.48	

TABLE IV. (*Continued.*)

J_f	Exc. energy (keV) (SM)	(emp)	log(*f*)	$B(GT)^{1/2}$	Branching (%) th(SM)	th (emp)	exp
^{31}Na $(\frac{3}{2}^+)\rightarrow$ ^{31}Mg							
$\frac{3}{2}$	4649		5.415	0.503	7.88	7.43	
	5329		5.284	0.382	3.48	3.16	
	6865		4.954	0.055	0.04	0.03	
	7233		4.867	0.159	0.26	0.21	
$\frac{5}{2}$	1550		5.928	0.272	6.61	7.10	
	3232		5.665	0.711	26.17	26.40	
	4268		5.485	0.439	6.93	6.66	
	5415		5.267	0.071	0.12	0.10	
	6209		5.101	0.011	0.00	0.00	
	6422		5.055	0.144	0.32	0.27	
$B(n)$	1798						
^{31}Mg $(\frac{3}{2}^+)\rightarrow$ ^{31}Al (h)							
$\frac{1}{2}$	944	946	5.517	0.046	0.18	0.18	17
	3314		5.044	0.069	0.14	0.14	
	4982		4.635	0.030	0.01	0.01	
	5561		4.472	0.016	0.00	0.00	
	6867		4.052	0.497	0.73	0.73	
	7543		3.797	0.190	0.06	0.06	
	7955		3.625	0.238	0.06	0.06	
	8565		3.342	0.087	0.00	0.00	
$\frac{3}{2}$	1744	1636	5.369	0.165	1.67	1.67	14
	3793		4.934	0.120	0.32	0.32	
	4052	3623	4.873	0.228	1.02	1.02	8
	4752		4.696	0.159	0.33	0.33	
	5227		4.567	0.159	0.24	0.24	
	5633		4.451	0.131	0.13	0.13	
	6178		4.284	0.010	0.00	0.00	
	6349		4.229	0.454	0.92	0.92	
$\frac{5}{2}$	0	0	5.680	0.781	76.84	76.84	10
	3171	2529	5.076	0.083	0.21	0.21	0.9
	3951	3239	4.897	0.750	11.68	11.68	24
	4781	4809	4.688	0.504	3.27	3.27	5
	5245		4.562	0.184	0.33	0.33	
	5905		4.369	0.505	1.57	1.57	
	6057		4.322	0.191	0.20	0.20	
	6445		4.197	0.120	0.06	0.06	
$B(n)$	7667	7271(78)					
^{31}Al $(\frac{5}{2}^+)\rightarrow$ ^{31}Si (f,i)							
$\frac{3}{2}$	0	0	4.736	0.457	65.72	64.72	65.0, (65)
	2295	2317	4.031	0.636	23.70	24.79	26.0, 23
	3825	4260	3.181	0.216	0.57	0.40	
	5129		2.651	0.109	0.02	0.03	
	5709		2.204	0.130	0.01	0.02	
	6215		1.714	0.140	0.00	0.01	
$\frac{5}{2}$	1606	1695	4.245	0.298	9.17	8.88	7.5, 11
	2871	2789	3.854	0.079	0.21	0.25	1
	5037		2.714	0.061	0.01	0.01	

TABLE IV. (*Continued.*)

$2J_f$	Exc. energy (keV) (SM)	(emp)	$\log(f)$	$B(\mathrm{GT})^{1/2}$	Branching (%) th(SM)	th (emp)	exp
^{31}Al ($\frac{5}{2}^+$)→ ^{31}Si (f)							
$\frac{5}{2}$	5398		2.455	0.137	0.02	0.03	
	5764		2.156	0.773	0.30	0.49	
	6074		1.863	0.282	0.02	0.03	
	6640		1.185	0.038	0.00	0.00	
	6884		0.801	0.198	0.00	0.00	
	7198		0.154	0.081	0.00	0.00	
	7414		−0.478	0.092	0.00	0.00	
$\frac{7}{2}$	3697	3875	3.379	0.076	0.08	0.08	
	4711		2.922	0.149	0.08	0.11	
	5632		2.269	0.308	0.06	0.10	
	6170		1.763	0.192	0.01	0.01	
	6506		1.367	0.557	0.02	0.04	
	6555		1.303	0.270	0.00	0.01	
	7009		0.569	0.288	0.00	0.00	
	7302		−0.123	0.021	0.00	0.00	
$B(n)$	6326	6588(<2)					
^{32}Mg (0$^+$)→ ^{32}Al							
1	0		5.577	1.531	93.87	93.87	
	2547		5.080	0.313	1.25	1.25	
	5071		4.438	0.047	0.01	0.01	
	5309		4.367	1.025	2.59	2.59	
	6012		4.139	0.848	1.05	1.05	
	6738		3.876	0.758	0.46	0.46	
	7211		3.685	1.207	0.75	0.75	
	7500		3.559	0.254	0.02	0.02	
$B(n)$	3841						
^{32}Al (1$^+$)→ ^{32}Si							
0	0	0	5.784	0.608	77.38	78.43	
	4809	4983	4.797	0.785	14.67	13.45	
	5755		4.593	0.045	0.03	0.03	
	8056		3.835	0.012	0.00	0.00	
	8813		3.513	0.170	0.03	0.03	
	9877		2.955	0.177	0.01	0.01	
	10987		2.153	0.268	0.00	0.00	
	11464		1.682	0.097	0.00	0.00	
1	5582		4.640	0.253	0.96	0.97	
	7457		4.060	0.576	1.31	1.33	
	7869		3.908	0.261	0.19	0.19	
	8419		3.687	0.155	0.04	0.04	
	9031		3.410	0.229	0.05	0.05	
	9435		3.205	0.392	0.08	0.09	
2	2094	1941	5.452	0.117	1.25	1.36	
	4212	4232	4.978	0.276	2.51	2.52	
	5649		4.622	0.048	0.03	0.03	
	7076		4.191	0.229	0.28	0.28	
	7631		3.997	0.366	0.46	0.46	
	7968		3.870	0.295	0.22	0.22	

TABLE IV. (*Continued.*)

J_f	Exc. energy (keV) (SM)	(emp)	log(*f*)	$B(GT)^{1/2}$	Branching (%) th(SM)	th (emp)	exp
^{32}Al $(1^+) \rightarrow$ ^{32}Si							
2	8193		3.781	0.421	0.37	0.37	
	8485		3.659	0.289	0.13	0.13	
$B(n)$	9330	9200(<2)					
^{33}Al $(\frac{5}{2}) \rightarrow$ ^{33}Si							
$\frac{3}{2}$	0		5.556	0.872	89.53	89.53	
	4421		4.584	0.472	2.80	2.80	
	4693		4.507	0.320	1.08	1.08	
	6327		3.975	0.468	0.68	0.68	
	7290		3.584	0.052	0.00	0.00	
	7763		3.362	0.652	0.32	0.32	
	8530		2.944	0.153	0.01	0.01	
	8797		2.777	0.152	0.00	0.00	
$\frac{5}{2}$	4378		4.596	0.343	1.52	1.52	
	5677		4.203	0.501	1.31	1.31	
	5994		4.095	0.215	0.19	0.19	
	6279		3.992	0.460	0.68	0.68	
	7160		3.641	0.455	0.30	0.30	
	7959		3.263	0.194	0.02	0.02	
	8329		3.062	0.564	0.12	0.12	
	8549		2.933	0.181	0.01	0.01	
$\frac{7}{2}$	3985		4.702	0.117	0.23	0.23	
	6609		3.868	0.556	0.75	0.75	
	7106		3.665	0.390	0.23	0.23	
	7546		3.467	0.131	0.02	0.02	
	8165		3.153	0.620	0.18	0.18	
	8544		2.935	0.027	0.00	0.00	
	8850		2.742	0.088	0.00	0.00	
	9240		2.465	0.590	0.03	0.03	
$B(n)$	4328	4562(50)					
^{33}Si $(\frac{3}{2}^+) \rightarrow$ ^{33}P (j)							
$\frac{1}{2}$	0	0	4.124	0.043	1.64	1.72	(2)
	4242		1.598	0.302	0.58	0.26	
	5728		−3.882	0.105	0.00	0.00	
$\frac{3}{2}$	1531	1432	3.558	0.019	0.09	0.09	4.2
	2646	2539	2.985	0.386	11.35	10.27	8.8
	3606		2.230	0.337	2.26	1.37	
	4997		0.430	0.459	0.21	0.04	
$\frac{5}{2}$	1997	1848	3.360	0.703	78.15	80.67	85
	3787	3490	2.327	0.599	5.31	5.41	
	3888		1.974	0.150	0.28	0.15	
	5153		0.062	0.478	0.14	0.02	
$B(n)$	10371	10103(3)					
^{34}Si $(0^+) \rightarrow$ ^{34}P (k)							
1	0	0	3.717	0.621	50.77	55.91	(56)
	1408	1608	2.902	1.390	48.43	42.98	44
	2954		1.840	0.758	0.80	1.11	
	4233		−0.374	0.285	0.00	0.00	
$B(n)$	6024	6295(9)					

TABLE IV. (*Continued.*)

J_f	Exc. energy (keV) (SM)	(emp)	log(*f*)	$B(\text{GT})^{1/2}$	Branching (%) th(SM)	th (emp)	exp
$^{35}P\ (\frac{1}{2}^+)\rightarrow{}^{35}S$ (l)							
$\frac{1}{2}$	1557	1572	2.387	0.937	99.39	99.43	100.00
$\frac{3}{2}$	0	0	3.367	0.005	0.04	0.03	
	2800	2939	0.828	0.416	0.58	0.54	
$B(n)$	6824	6986(<2)					

[a]Values in the last column taken from Ref. 29.
[b]Values in the last column taken from Ref. 30.
[c]Experimental ground-state percentage set equal to theoretical value and relative excited-state percentages renormalized to make a total of 100%.
[d]Values in the last column taken from Ref. 31.
[e]Values in the last column taken from Ref. 32.
[f]Values in the last column taken from Ref. 4.
[g]Experimental levels unresolved.
[h]Values in the last column taken from Ref. 38.
[i]Reference 34.
[j]Values in the last column taken from Ref. 35.
[k]Values in the last column taken from Ref. 36.
[l]Values in the last column taken from Ref. 37.

members of the O, F, and Ne chains, have half-lives of the order of a millisecond. As a rough average, the half-life decreases by a factor of 5±2 for each increase in the neutron excess, the decrease being less rapid at the largest values. From Fig. 2, which displays the calculated Q values for each T_Z group plotted in the same arrangement as that used in Fig. 1 for the plot of the half-lives, we can see that these trends of half-lives with neutron excess, as well as a significant amount of the local variations within a T_Z group, are correlated with the phase-space factors.

Total half-lives have been measured for all of the nuclei in the $T_Z=-\frac{5}{2}$ group. We see from Table III and Fig. 1 that the predictions for these nuclei are in consistently good agreement with the experimental results. The predicted ^{21}O decay deviates from experiment[29] by the largest amount. In the $T_Z=-3$ group, the Si, Al, and Mg predictions agree well with the experimental values, the Na and O predicted half-lives are significantly longer than the experimental values, and the F and Ne decays are unmeasured. In the $T_Z=-\frac{7}{2}$, -4, $-\frac{9}{2}$, and -5 groups, measured values are reported[4] only for the three remaining Na isotopes and ^{31}Mg. The ^{29}Na prediction and experiment are in good agreement with each other, but the predictions for ^{30}Na and ^{31}Na are too short by a factor of 2 to 3. Finally, the most significant discrepancy between experiment and theory for any of the nuclei studied, indeed the only glaring disagreement, is found in ^{31}Mg, for which the predicted half-life is a factor of 10 shorter than that experimentally quoted.

B. Branching percentages to the daughter states

The detailed distributions of Gamow-Teller strength among the energetically accessible states of the daughter nuclei yield information about the nuclear structure of the systems involved which can be much more revealing than the total half-life values. This is because the total half-lives are so strongly affected by the Q values and usually result from a sum over several competing branches. The predictions and measurements for individual transitions between parent and daughter states are presented in Table IV. In this table we give the energies calculated for the states in the daughter nuclei whose population is allowed by the GT selection rule, the experimental values for these excitations where they are known, the f values calculated with the Coulomb-corrected energies from the shell-model calculation, the $B(\text{GT})^{1/2}$ values as calculated with the free-nucleon $S_{jj'}^{\text{GT}}$, and the percentages of the total decay which go to each daughter state, as calculated alternatively with the pure SM and SM + emp f factors and as quoted experimentally. We also note at the bottoms of the energy columns the calculated and, where available, experimental values of the neutron-emission thresholds in the daughter systems. States above these energies which are populated in beta decay would not show up in subsequent gamma-ray spectra.

The branching percentages of Table IV are calculated as

$$``\%"_i = 100(T_{1/2}/t_{1/2}^i)\ . \tag{18}$$

We have assumed that the states listed in Table IV comprise all those that are appreciably populated in the decays. While in the cases of the largest Q values more states than listed are energetically accessible, the phase-space factors are such as to render their contributions negligible. As alluded to above, the shell-model calculations[11] predict incorrect orderings of the ground-state doublets in ^{26,28}Na and ^{29}Mg and a several hundred keV inversion of the ground and first excited states in ^{31}Na. This is reflected in a few negative excitation energy values in the relevant columns. In calculating the Q values for these cases (listed in Table III) we have used as the shell-model values the calculated energies of the experimentally correct ground states.

In quoting the experimental results for the branching percentages we do not, in keeping with the summary presentations in the original publications, attempt to assign uncertainties. Statistical uncertainties alone range from small to quite significant among the various data, but uncertainties from correlation effects and systematic errors are often the dominant factors in limiting the accuracy of the quoted numbers. In many instances the experimental technique does not produce a value for the ground-state branch. In these instances we adopt a scale for the relative fractions of the measured branches to the excited states such that the sum of these fractions is equal to the corresponding sum predicted for the excited states.

In this and the following paragraphs we review some of the salient results presented in Table IV. The decay of ^{21}O to the ground state of ^{21}F, though allowed, is predicted to be vanishingly small. The experimental technique[29] used to acquire the excited-state branches would not have detected the ground-state branch in any case. The two dominant decay branches of ^{21}O are to the first $\frac{3}{2}^+$ state, at a (theoretical/experimental) excitation energy of (1853/1730) keV and the second $\frac{5}{2}^+$ state, at an excitation energy of (3681/3518) keV. The predicted and observed percentages of these branches are in good agreement with each other. Smaller but significant branches are observed to levels at 3460 and 4572 keV excitation energy. The predicted branch to the second $\frac{3}{2}^+$ state, at 3512 keV excitation energy, is consistent with the 3460 keV observation, but the calculated 4097 keV excitation energy of the remaining large theoretical branch, to the third $\frac{3}{2}^+$ state, is rather far from the 4572 keV value of the remaining significant observed branch.

The decay pattern of ^{23}F is complex. The unmeasured ground-state branch is predicted to be 34% of the total. In this normalization, the first and second $\frac{3}{2}^+$ states, at (1768/1822) and (3237/3432) keV excitation energies, respectively, the second and third $\frac{5}{2}^+$ states, at (2184/2315) and (3826/3831) keV excitation energies, respectively, and the third $\frac{7}{2}^+$ state, at (4395/4436) keV excitation energy, are observed[30] to have important branches. The predictions agree with the outline of these results, the most important aspect of which is the concentration of strength into the $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ triplet near 4 MeV excitation energy. The principal failing of the predictions is to concentrate too much strength into the second $\frac{3}{2}^+$ and third $\frac{5}{2}^+$ states while leaving the first $\frac{3}{2}^+$ and second $\frac{5}{2}^+$ states too weak.

The decay of ^{25}Ne to the ground state of ^{25}Na is not allowed. The observed[31] dominant branches to the lowest $\frac{3}{2}^+$ and $\frac{1}{2}^+$ states, at (132/90) and (1159/1069) keV (theoretical/ experimental) excitation energies, respectively, are accurately reproduced by the calculations. The decay of ^{27}Na to the ground state of ^{27}Mg is similarly not allowed. Experiment[32,4] and theory agree in assigning the preponderance of the ^{27}Na decay intensity to the first $\frac{3}{2}^+$ state, at (895/985) keV excitation energy, with the other transition of consequence being that to the first $\frac{5}{2}^+$ state at (1667/1690) keV excitation energy.

The decay of ^{29}Mg to the ground state of ^{29}Al is predicted to be 49% of the total. The remaining 51% of the predicted decay strength is distributed over several states, with the lowest $\frac{3}{2}^+$ state, at (1959/2224) keV, and the third $\frac{5}{2}^+$ state, at (3017/3184) keV (theoretical/ experimental) excitation energy, receiving the majority. The experimental value of the ground-state decay percentage relative to those of the excited states is obtained[4] from different data than those which establish the individual fractions[33,4] for the excited states. The experimental value quoted is appreciably smaller than the prediction. Within the relative values for the excited states, the agreement between experiment and theory is reasonably good; the most significant discrepancy is for the lowest $\frac{1}{2}^+$ state, at (1214/1318) keV excitation energy. The observed state has all of the strength seen for $\frac{1}{2}^+$ states, while in the model spectrum much of this strength is fragmented into the second $\frac{1}{2}^+$ state. As an example of $T=\frac{5}{2}$ decays, the distribution of transition strength with excitation energy and the correspondence between theory and experiment are illustrated for this decay of ^{29}Mg in Fig. 3.

The decay of ^{31}Al is predicted to proceed dominantly to the $\frac{3}{2}^+$ ground state of ^{31}Si, with the only two appreciable branches to excited states proceeding to the first $\frac{5}{2}^+$ and second $\frac{3}{2}^+$ states. The experimental results[34,4] are very similar to the predictions in their pattern of decay intensities. The decay of ^{33}Si to the ground state of ^{33}P, on the other hand, is predicted to be very weak. The dominant decay branch is predicted to proceed to the first $\frac{5}{2}^+$ state, with the other appreciable branches going to the first and second $\frac{3}{2}^+$ states. As for the case of ^{31}Al, there is good agreement between the predicted and observed[35] decay patterns.

The decay of the 1^+ ground state of ^{28}Na is predicted to proceed principally to the ground and first excited 0^+ states of ^{28}Mg. The residual strength is predicted to be

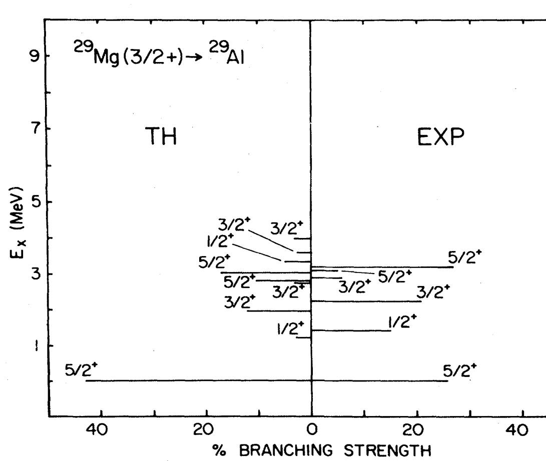

FIG. 3. Comparison of the calculated and measured branching percentages of the decay of ^{29}Mg.

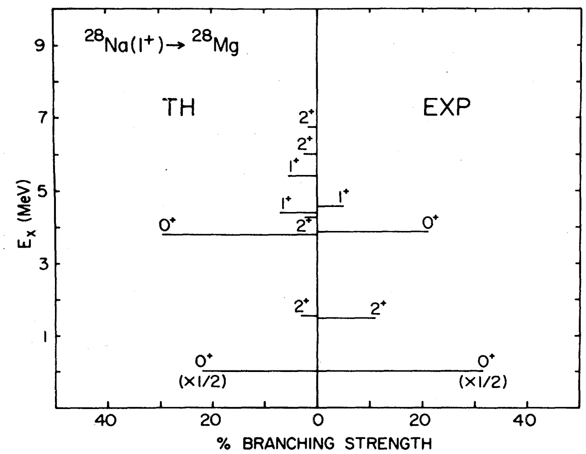

FIG. 4. Comparison of the calculated and measured branching percentages of the decay of ^{28}Na.

distributed over the first two 1^+ states and first two 2^+ states. The agreement of these predictions with the experimental results[4] is satisfactory except for the first 2^+ state, for which the predicted stength is too small. The energy distribution of the ^{28}Na decay strengths and the correspondence between theory and experiment is shown in Fig. 4. Roughly 90% of the decay of ^{30}Mg is predicted to go to the first predicted 1^+ state in ^{30}Al, at 493 keV excitation energy, and most of the remainder to the second predicted 1^+ state, at 2139 keV. These predictions are in good agreement with the reported branchings[4] of 84% and 3%, respectively, for transitions to levels observed at 685 and 2165 keV.

We recall that the total half-life of ^{31}Mg was completely anomalous in the context of *sd*-shell systematics. We can now see from Table IV and Fig. 5 that the pattern of branchings in this decay is equally anomalous. The anomaly in both the total half-life and the branching pattern resides in the ground-state decay branch. This is predicted to be 77% of the total decay, a result which is similar to that for the corresponding decay branch of ^{29}Mg. This theoretical result is easily understood as the wave functions of ground states of the $A=31$ isotopes of Al and Mg should, and do in the model calculations, resemble those of the corresponding states in the $A=29$ isotopes. The difference between the $A=29$ and 31 states lies essentially only in the presence of an additional pair of $d_{3/2}$ neutrons coupled to $J=0$, $T=1$, and this additional pair does not radically affect the calculated beta decay. The experimental result[38] of only a 10% branch to the ground state is in sharp contradiction to the shell-model prediction. In effect, this means that it is anomalous with respect to *sd*-shell systematics and suggests that some non-*sd*-shell effect is present in this decay. Since ^{31}Al is closer to the "normal" N/Z ratio and its decay is itself "well-behaved," it is reasonable to locate the source of the anomaly in ^{31}Mg.

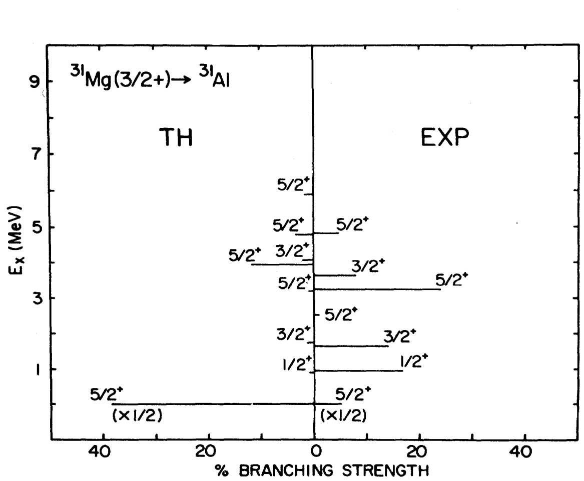

FIG. 5. Comparison of the calculated and measured branching percentages of the decay of ^{31}Mg.

Measurements of the individual decay branches of a beta-unstable nucleus conventionally involve detection of the gamma rays deexciting the daughter levels populated in the decay of the parent. In many instances, only these gamma rays are detected. It is hence important in planning experiments to estimate whether or not it is reasonable to expect strong branches to particle-bound excited states. In this context inspection of Table IV suggests that of the presently unmeasured decays, those of $^{22-24}$O, $^{24-27}$F, and ^{26,28}Ne would lend themselves to study. Of course, it is necessary to form the parent before observing its decay, and the spectrum of excited states in the parent may inhibit the formation cross section of some of these examples.[8] Nuclei which are predicted to decay predominantly to the ground state of the daughter system, and hence may be difficult to detect via gamma-ray measurements, include ^{26}O, ^{32}Mg, and ^{32,33}Al. Of course, this does not preclude measurement of their total half-lives, as is evident from the recent[9] measurement for ^{32}Al.

IV. SUMMARY AND CONCLUSIONS

The predicted features of the Gamow-Teller beta decay of $T_Z=-\frac{5}{2}$ nuclei obtained by using the empirically renormalized (quenched) single-nucleon matrix elements of the GT operator,[12] together with the complete *sd*-shell-model wave functions of the new Hamiltonian,[11] agree well with experimentally measured values of the total half-lives and the branching percentages to individual final states for these systems. This result indicates that the basic assumptions of the model space and Hamiltonian made in the calculations are valid for neutron excesses up through five. The agreement in magnitudes also confirms the insensitivity of the GT quenching factor, extracted from less neutron-rich systems, to significant excursions away from $N=Z$. Several of the present results involve strong $d_{3/2}$ neutron to $d_{5/2}$ proton one-body transitions. Direct information on this element is sparse in the beta decays closer to stability.

Experimental beta-decay results for nuclei with neutron excesses larger than five are progressively less abundant. Agreement between these data and the corresponding predictions is not significantly worse than for the $T_Z=-\frac{5}{2}$ systems, except for ^{30}Na, ^{31}Na, and, particularly, for ^{31}Mg. For these three nuclei, the predicted half-lives are too short. Information on the branching percentages is available for the case in which the discrepancy in half-life is worst, ^{31}Mg. Experiment and theory disagree in this aspect of the decay as well, in that the predicted strong ground-state branch is observed to be quite weak, a feature which is consistent with the total half-life discrepancy.

These anomalies in the decays of the $N=19$ and 20 isotopes of Na and Mg are particularly interesting in the context of anomalies of comparable magnitude which were revealed previously[1,5] in measurements of their masses. These are found to correspond to binding energies 1 or 2 MeV in excess of predictions[39,11] of shell-model calculations designed for less neutron-rich nuclei. Calculations in which orbits above the *sd* shell are added to the model space can replicate these binding energy trends.[40,41]

Further measurements[3,4,6,38] on these systems have yielded the beta decays quoted here, electromagnetic moments, and some excited-state energies. The electromagnetic-moment measurements do not yield conclusive information about the structures of the ground states, but a striking feature does emerge from the excitation-energy measurements,[4] namely a value for the first 2^+ state of ^{32}Mg which is much lower than systematics would predict. To summarize, the experimentally observed features of the very-neutron-rich nuclei which were noted previously as being qualitatively different from *sd*-shell systematics are the masses of ^{31}Na, ^{31}Mg, and ^{32}Mg and the first excited state energy of ^{32}Mg. To this list the present analysis adds the half-life of ^{31}Mg. The half-lives of ^{30}Na and ^{31}Na tend to be in the same direction but are not as dramatically in disagreement with the systematic trends.

It is plausible to hypothesize that these anomalies in the very-neutron-rich Na and Mg isotopes correspond to qualitatively different structures from those of the usual energy levels of the *sd* shell. This different structure presumably involves an inversion of the lowest *fp* orbits with the highest *sd* orbits. Phenomena analogous to this have long been known in the neutron-rich members of the upper 0*p* shell and recently were suggested for the $A=100$ region, as well as for the present cases.

The present analysis thus shows that the anomalies present in the energies of the $N=19$ and 20 isotopes of Na and Mg also appear in their beta decays. This suggests that it will be possible to pursue this issue experimentally in situations where mass measurements themselves are not feasible. The analysis also indicates that this region of anomaly is rather tightly circumscribed, in that the decays of the lighter isotopes ^{29}Na and ^{30}Mg seem well understood in terms of *sd*-shell systematics, as do those of the relevant other (higher *Z*) systems nominally within the shell for which data are available.

The properties of the $N=20$ isotopes ^{30}Ne and ^{24}F are most crucial to a better understanding of the nature of the $N=19$ and 20 behavior in the Na and Mg isotopes. However, the predictions for these decays suggest that measurements of their decays will be difficult with techniques currently at hand. The present calculations predict an even more pessimistic situation for the cases of the $N=19$ isotopes of F and Ne, namely that they are unbound. Of course, anomalies in F and Ne comparable to those found in Na and Mg might have the effect of making experimental measurements easier than our predictions suggest. In any case, our predictions for the $N=18$ isotopes of these elements indicate that measurements of their properties should be feasible. Such data, along with more detailed measurements on ^{30}Na with which to compare our predictions of the branching pattern (see Table IV and Fig. 6), should go far towards clarifying whether the currently known anomalies are to be associated with $N=19$ and 20 in particular, or with large neutron excesses in general.

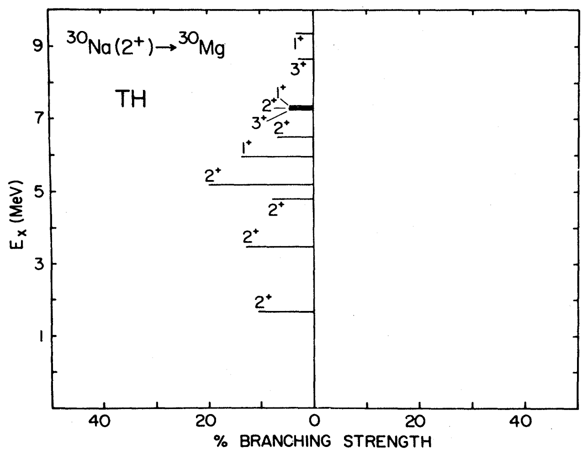

FIG. 6. The calculated branching percentages of the decay of ^{30}Na.

In summary, we have seen that the juxtaposition of the present generation of nuclear structure predictions for neutron-rich *sd*-shell nuclei with the relatively sparse body of data serves to validate, within limits, the general assumptions and specific parameters of the shell model as formulated to reproduce systems near to $N=Z$. This limited validation highlights, in turn, the few sharp discrepancies between existing data and these predictions. It is reasonable to hope that continued experimental progress against the background of realistic theoretical expectations will result in the quantitative explication of the present anomalies, if not in the unambiguous discovery of additional ones. We conclude with the observation that the currently available evidence vividly illustrates the fundamental interest of studies of very-neutron-rich nuclei in this region.

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