CORE EXCITATIONS AND M1 STRENGTHS IN THE Ca ISOTOPES

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The existence of strong M1 transitions from $T = 1$ and $T = 2$ states in ^{40}Ca provides an interesting challenge to nuclear structure models. We show that the recent M1 data can be reconciled with a shell model study of ^{40}Ca and that the fragmentation and disappearance of strengths in ^{42}Ca and ^{44}Ca is consistent with a weak-coupling extension of the ^{40}Ca system in which the M1 strength is divided among T_z and T_{z+1} components.

Recent results of electron scattering on the calcium isotopes have provided an interesting challenge to nuclear structure theorists. These results indicate:

(i) The occurrence of strong M1 excitations in the L–S closed shell ^{40}Ca, [1–3].

(ii) A pronounced fragmentation of M1 strength in ^{42}Ca and to a greater degree in ^{44}Ca [4,5].

The importance of core excited configurations in ^{40}Ca is now well established and the strong M1 transitions from $T = 1$ and $T = 2$ states [1–3] indicate as expected that the independent particle model proposed by McGrory and Wildenthal [6] in their study of the heavier Ca isotopes has to be considerably enlarged.

The aim of our paper is indeed to show that the M1 data can be reconciled with a shell model study of ^{40}Ca and that the fragmentation and disappearance of strength in ^{42}Ca and ^{44}Ca is consistent with a weak-coupling extension of the ^{40}Ca system in which the M1 strength is divided among components populating $T = T_z$ and T_{z+1} states, where $T_z = (N-Z)/2$.

We first concentrate on ^{40}Ca where the recent finding by (ee′) of an unusually strong M1 transition to a 1^+ state at 10.219 MeV ($B{:}\mathrm{M1}\uparrow = 1.12 \pm 0.07\,\mu_k^2$) [1] has been confirmed by the backward angle high resolution data of Steffen et al. and Burt et al. [3,4]. Then there is the more recent ^{36}Ar(α, γ) ^{40}Ca experiment [2] indicating that the $T = 2 \rightarrow T = 1$ summed M1 decay of 5.9 WU is stronger than any other measured M1 decay from $T = 2$ states. In table 1, we present the results of a shell model calculation for $J = 0^+$ and 1^+ ($T = 0$, $T = 1$ and $T = 2$) states including up to 4 holes in the $d_{3/2}$ shell and 4 particles in the $f_{7/2}$ shell. Calculations within a larger space, although highly desirable are computationally more difficult for now. Two interactions were used: one (column A), found successful in a previous calculation of core excited 0^+ states in ^{40}Ca [7], the other column (column B), similar to

Table 1
Excitation energies of $T = 0^+$ and 1^+ states in ^{40}Ca.

J	T	ν	E (MeV)			
			calc. (A)	calc. (B)	exp.	ref.
0^+	0	1	0.00	0.00	0.0	4
0^+	0	2	5.34	3.18	3.36	4
0^+	0	3	7.47	5.08	5.21	4
0^+	0	4	9.57	7.79	7.30	4
0^+	0	5	10.93	8.17	7.70	4
1^+	1	1	9.88	8.62	9.87	2
1^+	1	2	10.86	9.16	10.32	2
1^+	1	3	11.13	9.62		
1^+	1	4	11.59	10.01		
1^+	1	5	12.05	10.28		
0^+	2	1	12.56	11.23	11.97	2

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Table 2
$B(\mathrm{M1}; 0^+ \to 1^+, T = 1)$ (in nm) in ^{40}Ca.

	Calc. (A), $J_i(v_i)\ T_i$			calc. (B), $J_i(v_i)\ T_i$			Exp. (ref. [2]), $J_i(v_i)\ T_i$	
v_f	0(1), 0	0(2), 0	0(1), 2)	0(1), 0	0(2), 0	0(1), 2	0(1), 0	0(1), 2
1	0.38	0.40	2.61	0.23	0.20	2.63	0.24(6)	4.1(6)
2	1.45	2.52	6.65	0.48	2.67	3.72	1.17(6)	6.4(9)
3	0.66	0.05	1.54	0.52	0.06	1.80		
4	0.51	0.33	2.62	0.47	0.39	3.32		
5	0.00	0.01	0.02	0.28	0.15	2.41		

A but with the diagonal matrix elements $\langle \mathrm{dd}|v|\mathrm{ff}\rangle$ multiplied by 0.6 in order to obtain better agreement within our truncation scheme with the energies of the 0^+ states in ^{40}Ca. For that reason, interaction B is probably similar to the one adopted by Federman et al. [8] and Seth et al. [9] in their study of f–p shell nuclei. The main effect of the reduction in size of the diagonal matrix element is to spread out the 2p–2h component over many 4p–4h 1^+ states. The effect of this 2p–2h fragmentation is clearly seen in table 2 where the use of interaction B distributes the M1 strength over more states with the result that agreement with experiment is less satisfactory. It is therefore difficult to adopt an interaction which conciliates quantitative agreement with both experimental energy levels and transition rates although the qualitative agreement is generally acceptable. (There are of course many variations in the interaction which we have not explored.)

An extension of the present shell model to the calculation of M1 strength in $^{40+n}$Ca isotopes is still at present outside the range of computational practicality. However a simple weak-coupling extension of the ^{40}Ca n-particle–n-hole structure allows one to isolate a possible mechanism responsible for the fragmentation and decrease in strength in the ^{42}Ca and ^{44}Ca systems. Indeed if one attributes the strong M1 decays in ^{40}Ca as a constructive interference of 2p–2h excitations [1] with ^{40}Ca represented as

$$^{40}\mathrm{Ca} \equiv (1-\beta^2)^{1/2}\,|0\rangle + \beta\,|\mathrm{d}_{3/2}^{-2}\,(\mathrm{f}_{7/2})^2\rangle ,$$

then the weak-coupling approximation suggests that the $^{40+n}$Ca ground state would be written as

$$^{40+n}\mathrm{Ca} \equiv (1-\beta^2)^{1/2}\,|\mathrm{f}_{7/2}\rangle^n + \beta\,|\mathrm{d}_{3/2}^{-2}\,(\mathrm{f}_{7/2})^{n+2}\rangle ,$$

and the $T = T_{z+1}$ states should have the following M1 strength ratios

$$^{40}\mathrm{Ca} : {}^{42}\mathrm{Ca} : {}^{44}\mathrm{Ca} = 1 : 1/2 : 1/3 .$$

It is interesting to note that the information on ground state occupation in the Ca isotopes is indeed consistent with the weak-coupling hypothesis. A plot of the number of holes in the s–d shell in the Ca isotopes ground states as a function of A (fig. 1) indicates a maximum for ^{44}Ca and a decrease thereafter i.e. *core breaking increased up to A = 44 and decreases above that* [10]. This can be understood as *competition between α correlations which favor core breaking and the Pauli Principle which prevents it.*

As we mentioned earlier, a microscopic calculation

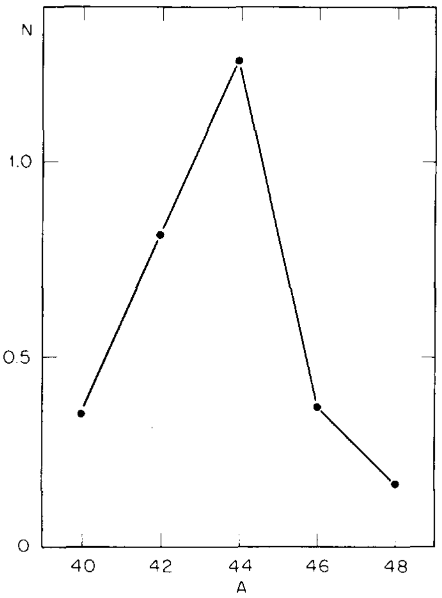

Fig. 1. Experimental plot of the number of holes in the s–d shell in the Ca isotopes ground states as a function of A. Additional theoretical estimates are given in ref. [10].

of the M1 strengths in ^{42}Ca is not yet practical. However, it is interesting to consider the oxygen isotopes for which the analogous calculations can be carried out. We note that the qualitative trend of the core breaking appears to be similar in the oxygen and calcium isotopes. For example, the experimental differences in the $\langle r^2 \rangle$ matrix elements is $(Z = 8) \times [\mathrm{ms}(^{18}\mathrm{O}) - \mathrm{ms}(^{16}\mathrm{O})] = 3.3\ \mathrm{fm}^2$ compared to $(Z = 20) \times [\mathrm{ms}(^{42}\mathrm{Ca}) - \mathrm{ms}(^{40}\mathrm{Ca})] = 4.2\ \mathrm{fm}^2$ (where "ms" is the mean square charge radius) [10]. Also M1 transitions comparable in strength to those in ^{40}Ca have also been observed in ^{16}O $T = 0 \to 1$ [11,12] and ^{18}O $T = 1 \to 2$ [13] transitions.

The ingredients for the shell-model calculations of the oxygen isotopes have been comparatively rather thoroughly explored both in the $(1p_{1/2}, 1d_{5/2}, 2s_{1/2})$ model space [11–18] [which is comparable to the $(1d_{3/2}, 1f_{7/2})$ model space used above for ^{40}Ca] and in a larger space which includes in addition the $1p_{3/2}$ and $1d_{3/2}$ orbits for the excited states of ^{16}O [12,19]. As noted in ref. [12], the ^{16}O M1 transitions are very poorly described in this larger space which includes the $1p_{3/2}$ and $1d_{3/2}$ orbits, which is possibly due to problems with the Millener–Kurath interaction [19]. Thus, here we will concentrate on the results obtained within the $(1p_{1/2}, 1d_{5/2}, 2s_{1/2})$ or "ZBM" model space, and in particular on those obtained with the "Z" interaction of ref. [17]. The results of these calculations for the M1 transitions in the oxygen isotopes are shown in fig. 2. We note that these calculations do not include the $d_{5/2}$ to $d_{3/2}$ neutron "spin-flip" transition which are most important and in fact would dominate the $T = T_z$ to $T = T_z$ M1 spectrum of ^{22}O (the analog of ^{48}Ca), and thus our comments here are only concerned with that part of the M1 strength that comes from the core-breaking component. The total M1 strength should be approximately equal to the sum of this core-breaking component and the neutron "spin-flip" component such as been calculated for the calcium isotopes by McGrory and Wildenthal [6]. The results indicate:

(i) A rapid decrease of M1 strength to T_{z+1} states with increasing A (the decrease is faster than the weak-coupling estimate). Based on the calcium–oxygen analogy we should indeed expect little T_{z+1} M1 strength in ^{44}Ca.

(ii) The M1 strength to T_z states is also distributed among several states as suggested by the ^{42}Ca experiment [5].

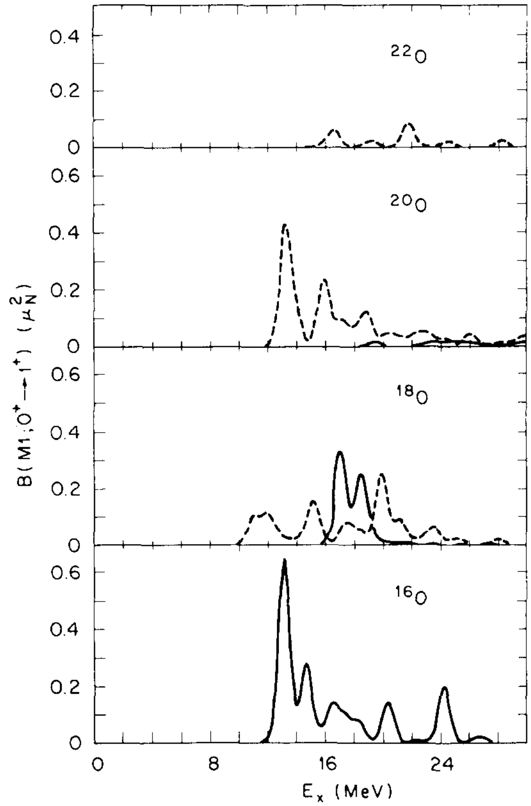

Fig. 2. Energy distribution of M1 strength to final states T_z (dotted line) and T_{z+1} (solid line) calculated with the "Z" interaction (ref. [17]) for the oxygen isotopes.

In conclusion we have shown that our present knowledge of M1 strength distribution in the light Ca isotopes is qualitatively consistent with a microscopic calculation of core excitations in ^{40}Ca and a weak-coupling extension to the ^{42}Ca and ^{44}Ca systems. We should also note that in a recent ^{42}Ca(p, n) ^{42}Sc study [20] for the analogue of the 1^+ state observed in ^{42}Ca (ee′) at 11.27 MeV was only weakly populated, if at all [4]. This could be due to a $T = 2$ character for that state which would then imply a low cross section above the ^{42}Ca(p, n) ^{42}Sc background, consistent with our own calculation of T_{z+1} strength in ^{42}Ca.

Several charge exchange reactions could be envisaged to investigate the isospin character of 1^+ states in the Ca isotopes. We hope that this present study will encourage such attempts as well as provide an incentive for more extensive and elaborate shell model calculations.

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