

Estimate of the parity-violating α -decay width of the 0^+ , $T = 1$ state of ${}^6\text{Li}$

R. G. H. Robertson

Physics Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

B. A. Brown

Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824

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A full two-body shell-model calculation has been used to estimate the pion-exchange contribution to the parity-violating α decay of the 0^+ , $T = 1$ state of ${}^6\text{Li}$. For a neutral-current-induced weak pion amplitude equal to the "best" value given by Desplanques, Donoghue, and Holstein, the α width is 1.3×10^{-8} eV. There is no contribution in lowest order from the intrinsic parity impurity in the α particle, but there is a small contribution from the deuteron.

[NUCLEAR STRUCTURE ${}^6\text{Li}$. Calculation of parity-violating α decay of 0^+ , $T = 1$ state.]

Purely hadronic parity violation is experimentally well established¹ and, as exemplified by the recent work² of Desplanques, Donoghue, and Holstein (DDH), is becoming theoretically tractable. In an isospin decomposition of the effective parity-violating force, the $\Delta T = 1$ component has received special interest because it is sensitive to the neutral current. Finding experimental evidence for this component, however, has proven to be exceptionally difficult. Recently it has been observed^{1,3-5} that measurements in ${}^{19}\text{F}$ and ${}^{21}\text{Ne}$ can be resolved into isoscalar and isovector components in good accord with the "best" values for the coupling constants given by DDH. The nuclear-model calculations for ${}^{21}\text{Ne}$, in particular, are rather difficult, and the desirability of observing $\Delta T = 1$ parity violation in isolation has long been recognized. Only three cases have appeared to satisfy the twin constraints of experimental feasibility and theoretical tractability, namely, the capture of polarized neutrons by protons,⁶ mixing⁷ between the lowest $2^-, T = 0$ and $2^+, T = 1$ states in ${}^{10}\text{B}$, and mixing⁸ between the lowest $0^-, T = 0$ and $0^+, T = 1$ states in ${}^{18}\text{F}$. In this Brief Report we present a calculation for a fourth case, the parity-forbidden α decay of the 0^+ , $T = 1$ state at 3.56 MeV in ${}^6\text{Li}$.

A decay energy of 2.08 MeV is available for the 0^+ , $T = 1$ state to break up into a deuteron (d) and an α particle, but because the deuteron spin is 1, the relative orbital angular momentum of α -d must also be 1 to conserve total angular momentum. This requires a parity impurity in the deuteron, the α , or ${}^6\text{Li}$ (3.56). The ${}^6\text{Li}$ case is unusual in that the parity mixing does not occur between two close-lying states of opposite parity—no $0^-, T = 0$ states occur at low excitation in the nucleus. Thus there is no reason to expect that impurities in the d or α -wave functions will be any less important, and it is essential to examine their contribution with regard to both magnitude and phase. Our strategy is to use the six-nucleon shell model both for ${}^6\text{Li}$ and for the separated α -d system. In the latter case, the wave function of relative motion will not, of course, be realistic, but antisymmetrization effects between the clusters are automatically included, and the unified treatment addresses the problem of relative phases. To obtain a quantitative result wherein relative motion is realistically described, we make use of a reduced width and an experimentally normalized penetrability.

The regular-parity ${}^6\text{Li}$ wave function (henceforth we omit explicit reference to the 0^+ , $T = 1$ state) is taken to be con-

tained entirely in the $0s0p$ basis. Action of the parity-violating force can promote core nucleons into the $0p$ shell or valence nucleons into the $1s0d$ shell, but we neglect $3\text{-}\hbar\omega$ excitations. The α particle is analogously described. The deuteron is taken to have an internal wave function of the form

$$|d\rangle = \alpha|00\rangle + \beta|10\rangle + \gamma|02\rangle + \delta|01\rangle,$$

where the quantum numbers are nl , the principal and orbital quantum numbers of internal motion. Since the weak nucleon-nucleon pion-exchange potential² is diagonal in total spin S , all components (including the parity-irregular part) have $S = 1$.

To transform from the α -d system to the six-nucleon system, we make use of the Moshinsky brackets.⁹ The quantum numbers of relative α -d motion are NL . As we have remarked, $L = 1$ is required by angular momentum conservation, and values of $N \geq 2$ are excluded by the requirement that the Moshinsky energy index ρ not exceed three in the space we have chosen. We introduce a mixing angle θ such that the $N = 0$ amplitude is proportional to $\sin\theta$ and the $N = 1$ amplitude is proportional to $\cos\theta$.

Parity-violating excitation of the α particle plays, at most, a minor role. That may be seen by considering the two principal components of the ${}^6\text{Li}$ wave function, $|(0s_{1/2})^4(0p_{3/2})^2\rangle$ and $|(0s_{1/2})^4(0p_{1/2})^2\rangle$. Inasmuch as the force under consideration is a scalar, the allowed core excitation is only into the $0p_{1/2}$ orbit. This is permissible for the first component, but projecting $0p_{3/2}0s_{1/2}$ nucleons onto a regular-parity deuteron leaves a spin-1 negative-parity object, which is not an irregular-parity α particle. On the other hand, one could construct a regular-parity deuteron from $0p_{1/2}0s_{1/2}$ nucleons and leave a spin-0 irregular-parity α particle, but in the six-nucleon system the required core excitation is Pauli blocked. Thus $1\text{-}\hbar\omega$ excitations of the core, although possible in both ${}^6\text{Li}$ and the α particle, play no role in enabling the decay.

Performing the Moshinsky transformation and the appropriate $LS - jj$ recoupling, we find the following wave functions:

$$\begin{aligned} |\alpha d\rangle &= A |0p_{3/2}0d_{3/2}\rangle + B |0p_{1/2}1s_{1/2}\rangle \\ &\quad - \sqrt{1/3}\delta \sin\theta |(0p_{3/2})^2\rangle + \sqrt{2/3}\delta \sin\theta |(0p_{1/2})^2\rangle, \\ |{}^6\text{Li}\rangle &= a |(0p_{3/2})^2\rangle + b |(0p_{1/2})^2\rangle + c |0p_{3/2}0d_{3/2}\rangle \\ &\quad + d |0p_{1/2}1s_{1/2}\rangle, \end{aligned}$$

where

$$A = -\sqrt{2} 0.236\gamma \sin\theta - \sqrt{2} 0.408\alpha \cos\theta + \sqrt{2} 0.527\beta \sin\theta ,$$

$$B = -\sqrt{2} 0.527\gamma \sin\theta + \sqrt{2} 0.456\alpha \cos\theta + \sqrt{2} 0.118\beta \sin\theta .$$

The reduced amplitude for the parity-forbidden decay is the overlap of these two wave functions:

$$X_{\alpha d} = +Ac + Bd - \sqrt{1/3}\delta a \sin\theta + \sqrt{2/3}\delta b \sin\theta .$$

The minor-component amplitudes c and d can be evaluated by perturbation theory:

$$-\hbar\omega c = a \langle 0p_{3/2}0d_{3/2} | V_{PV} | (0p_{3/2})^2 \rangle$$

$$+ b \langle 0p_{3/2}0d_{3/2} | V_{PV} | (0p_{1/2})^2 \rangle ,$$

$$-\hbar\omega d = a \langle 0p_{1/2}1s_{1/2} | V_{PV} | (0p_{3/2})^2 \rangle$$

$$+ b \langle 0p_{1/2}1s_{1/2} | V_{PV} | (0p_{1/2})^2 \rangle .$$

$$\hbar\omega\delta \sin\theta = A\sqrt{2/3} \langle (p_{1/2})^2 | V_{PV} | p_{3/2}d_{3/2} \rangle + B\sqrt{2/3} \langle (p_{1/2})^2 | V_{PV} | p_{1/2}1s_{1/2} \rangle$$

$$- A\sqrt{1/3} \langle (p_{3/2})^2 | V_{PV} | p_{3/2}d_{3/2} \rangle - B\sqrt{1/3} \langle (p_{3/2})^2 | V_{PV} | p_{1/2}1s_{1/2} \rangle .$$

Here no core contributions are to be included.

The matrix elements have been evaluated in $j-j$ coupling because a set of codes had previously been written in this scheme.⁵ One- and two-body contributions to the matrix elements have been calculated for $\hbar\omega = 14$ MeV, with the inclusion of short-range correlations in the form of a damping factor¹¹

$$1 - (1 - 0.68r^2)\exp(-1.1r^2)$$

in the radial matrix elements. With $f_\pi = 12$, the results are

$$10^8 c = 10.09a - 3.93b ,$$

$$10^8 d = -0.79a - 7.06b ,$$

$$10^8 \delta = (4.81A - 1.36B) \csc\theta .$$

Typical values for the coefficients are

$$a = 0.958, \quad b = 0.283, \quad \alpha = 0.964, \quad \beta = 0.0, \quad \gamma = 0.265 ,$$

the ${}^6\text{Li}$ parameters being the result of a calculation by Vergados,¹² and the deuteron parameters being chosen to give a 7% D -state probability. In order to determine the mixing angle θ , we performed a distorted-wave calculation for the relative motion of α -d clusters having the appropriate energy (2.08 MeV in the center of mass). The McIntyre-Haerberli potential,¹³ which reproduces the α -d phase shifts accurately, was used. The overlap between the continuum wave function thus generated and the harmonic oscillator wave functions suggested that (for $\hbar\omega = 14$ MeV) $\theta \approx 215^\circ$. For the parameters chosen,

$$c = 7.2 \times 10^{-9} f_\pi ,$$

$$d = -2.3 \times 10^{-9} f_\pi ,$$

$$\delta = -4.3 \times 10^{-9} f_\pi ,$$

and

$$X_{\alpha d} = 3.72 \times 10^{-9} f_\pi .$$

Despite the unorthodox method of calculation, the value for δ is in reasonable agreement with 3P_1 amplitudes calculated by others [interaction (12a) of Lassey and McKellar¹⁴ corresponds roughly to the DDH "best value" and yields $|\delta| = 4.1 \times 10^{-8}$].

The primes indicate that core contributions are to be included. It may appear that the results might be excessively sensitive to the choice of the oscillator energy $\hbar\omega$, but, in fact, the matrix elements themselves are approximately proportional to $\hbar\omega$, as was pointed out by Michel¹⁰ in 1964 for $\vec{\sigma} \cdot \vec{p}$ forces. One is thus relatively insensitive to the details of where the $1-\hbar\omega$ strength is located in ${}^6\text{Li}$, because the parity impurity induced in the $0^+, T=1$ state is largely independent of the single-particle energies. There may be, of course, some sensitivity to the residual interaction (which, apart from using configuration-mixed wave functions, we neglect), but even this is attenuated by the preservation of the center of gravity of the independent-particle states.

Similarly, the parity-irregular amplitude δ in the deuteron can be evaluated:

To relate the reduced width to the actual width $\Gamma_{\alpha d}$ we make a comparison with the known widths of the 3^+ and 2^+ states of ${}^6\text{Li}$. The reduced widths of these states in the model we use here are both given by

$$X_2 = (\alpha - \gamma) 0.707 .$$

Taking $L=2$ penetrabilities from Sharp, Gove, and Paul,¹⁵ we find the ratio of penetrabilities for the 2^+ , 4.31-MeV state [$(\Gamma = 1700 \pm 200$ keV (Ref. 16)] to the 3^+ , 2.185-MeV state [$\Gamma = 24$ keV (Ref. 16)] is 31, while the actual width ratio is 70 ± 9 . The difference is perhaps indicative of the uncertainties involved in this approach. However, there is good evidence¹⁷ that the 3^+ state has a large parentage in $\alpha + d$, about 85%, whereas the 2^+ state is wide and poorly characterized. In light of the 3^+ data, we write

$$\Gamma_{\alpha d} = \Gamma_3 + \left(\frac{X_{\alpha d}}{X_2} \right)^2 \frac{P_1}{P_2} ,$$

where P_L is the L -wave penetrability. The ratio P_1/P_2 is found to be 68. Then

$$\Gamma_{\alpha d} = 9.26 \times 10^{-11} f_\pi^2 \text{ eV} .$$

With $f_\pi = 12$, as suggested by DDH,

$$\Gamma_{\alpha d} = 1.3 \times 10^{-8} \text{ eV} .$$

We have investigated the sensitivity of this result to various reasonable changes in input parameters. Reducing $\hbar\omega$ from 14 to 11 MeV does modify the results through the mixing angle θ (which is reduced to 197°), and leads to an increase in $\Gamma_{\alpha d}$ of about 33%. Adding a 20% amplitude of $|10\rangle$ state in the deuteron (at the expense of the $|00\rangle$ component) decreases $\Gamma_{\alpha d}$ by 21%. Reducing the D -state amplitude to zero has a substantial effect, decreasing $\Gamma_{\alpha d}$ by 47%. Reducing the $(0p_{1/2})^2$ amplitude in ${}^6\text{Li}$ to zero decreases $\Gamma_{\alpha d}$ 31%.

It may be useful to summarize the salient features of this calculation. Only the weak pion-exchange force has been considered, but similar techniques could be used in a treatment including vector mesons. While the deuteron and α -particle parity impurities are found not to contribute greatly to the width, there are no significant cancellations in the ${}^6\text{Li}$ impurity. We therefore expect that vector meson contributions will be minor provided f_π is not too small. A shell-

model calculation has been used to estimate the parity admixture in the 0^+ , $T=1$ state, and there are the usual approximations inherent in the method—harmonic oscillator wave functions, single-particle energies, and truncation of the basis. We have used an empirical technique to normalize the reduced width, but we are encouraged by the success of α -d cluster models in other applications to ${}^6\text{Li}$.

Measurement of such a small width is a formidable experimental challenge, but a width two orders of magnitude smaller has successfully been measured¹⁸ in ${}^{16}\text{O}$. Consider-

able progress has also been made in ${}^6\text{Li}$ since Wilkinson's 1958 experiments,¹⁹ in which he found $\Gamma_{ad} \leq 0.2$ eV. In 1975, Bellotti *et al.*²⁰ reduced the limit to 8×10^{-4} eV, and more recently²¹ the limit has been pushed below 10^{-5} eV.

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