EFFECTS OF TWO-PARTICLE–TWO-HOLE GROUND-STATE CORRELATIONS ON SPIN-DIPOLE TRANSITIONS IN ¹²N

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We have studied the effect of two-particle-two-hole ground-state correlations on the spin-dipole transitions in ¹²N. We found that the transition strengths in the energy range $E_x = 2-12$ MeV are quenched 25% by the ground state correlations. The tensor correlation is important for 1⁻ and 2⁻ states, while the spin-orbit force has an appreciable effect on 1⁻ states only.

The (p, n) reaction at intermediate energy is an extremely useful probe for the study of $\sigma \cdot \tau$ correlations in nuclei [1]. The cross section at zero degrees has been studied in detail and the Gamow-Teller (GT) strength for l = 0 transition has been extracted systematically in many nuclei throughout the periodic table [1,2]. Moreover, the neutron spectra at larger angles ($\theta = 5-15^{\circ}$) show strong transitions characterized by l = 1 angular distributions with a large width of around 10 MeV for nuclei $A \ge 40$ [1]. These broad resonances are interpreted by an envelope of collective states with spin parities 2^{-} , 1^{-} and 0^{-} excited through the transition operators [3],

$$T_{\lambda\mu} = \sum_{i} r_{i} \left[Y_{i=1}(\hat{r}_{i}) \times \boldsymbol{\sigma}_{i} \right]_{\lambda\mu} \boldsymbol{\tau}_{-1}, \quad \lambda = 2^{-}, 1^{-}, 0^{-}.$$
(1)

We refer to them as spin-dipole transition operators. The particle-hole matrix elements for these operators are given by

$$\begin{split} &\langle \left(j_{h}^{-1}j_{p}\right)\lambda \|r'[Y_{l}\times\sigma]_{\lambda}\|0\rangle \\ &= \left[(2j_{h}+1)/4\pi\right]^{1/2}\langle j_{h}\frac{1}{2}\lambda\,0|j_{p}\frac{1}{2}\rangle\langle l_{p}|r'|l_{h}\rangle \\ &\times 1, \quad \lambda = 0^{-}, \\ &\times \left\{-\left\{(2\lambda+1)/[\lambda(\lambda+1)]\right\}^{1/2}\right\} \\ &\times \left\{(-)^{l_{p}+1/2-j_{p}}\left(j_{p}+\frac{1}{2}\right) \\ &-(-)^{l_{h}+1/2-j_{h}}\left(j_{h}+\frac{1}{2}\right)\right\}, \quad \lambda = 1^{-}, \end{split}$$

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$$\times [\lambda]^{1/2} \Big\{ 1 + (-)^{l_{p} + 1/2 - j_{p}} \Big(j_{p} + \frac{1}{2} \Big) \\ + (-)^{l_{h} + 1/2 - j_{h}} \Big(j_{h} + \frac{1}{2} \Big) \Big\}, \quad \lambda = 2^{-}.$$
(2)

It has not been possible to resolve the broad peaks at angles ($\theta = 5-15^{\circ}$) in heavy nuclei into their components and confirm each transition strength partly due to the poor energy resolution of neutron spectra. Recently, in the light nucleus ¹²N, the spin-dipole states with $\lambda^{\pi} = 1^{-}$, 2^{-} have been observed separately in the energy region E_x = 2-12 MeV [4]. From the experimental neutron spectrum for the ¹²C(p, n)¹²N reaction at $E_p = 160$ MeV and $\theta = 8^{\circ}$, the extracted cross section obtained by adding the total yield (corrected for cosmic ray background) is given by

$$\sum_{E_x} \frac{d\sigma}{d\Omega} (E_x = 2 - 12 \text{ MeV})_{exp} = (12.0 \pm 1.8) \text{mb/sr},$$
(3)

while a shell-model calculation [4] gives the summed cross section,

$$\sum_{E_x} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} (E_x = 2 - 12 \text{ MeV})_{\mathrm{theory}} = 20 \text{ mb/sr.} \qquad (4)$$

The model prediction in ref. [4] took into account one-particle-one-hole (1p-1h) states built on the lowest six odd-parity states of the A = 11 system and the excitations from the $0s_{1/2}$ -orbit were not allowed.

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Table 1

Sum rule values $B(l=1, \lambda^{\pi})$ for the spin-dipole transitions in ¹²N. The values in row 1 are obtained by the 1p-1h excitations from $0p_{3/2}$ and $0s_{1/2}$ -states assuming a $0p_{3/2}$ -closed shell. The shell-model calculation of ref. [4] in row 2 took into account only 1p-1h excitations from the lowest six odd-parity states of the A=11 system, while the present calculation in row [3] includes all configurations from 0p- and 0s-orbits. For details, see the text.

| | λ ^π | | | |
|----------------------------------------------|----------------|-------|-------|--|
| | 0- | 1- | 2- | |
| 1p-1h | 5.56 | 14.11 | 14.97 | |
| ref. [4] | 3.79 | 10.76 | 16.03 | |
| present | 4.56 | 12.61 | 17.48 | |
| ref. [4] ($E_x \leq 12 \text{ MeV}$) | 3.34 | 9.70 | 15.00 | |
| present ($E_x \leq 12 \text{ MeV}$) | 2.69 | 8.60 | 14.67 | |
| ref. [4] ($E_{\star} \leq 18 \text{ MeV}$) | 3.79 | 10.76 | 16.03 | |
| present ($E_x \leq 18 \text{ MeV}$) | 3.63 | 9.97 | 15.80 | |

We show in table 1 the results of a full 1 $\hbar\omega$ 1p-1h configuration space calculation based on the Cohen-Kurath wave function for the A = 11system in comparison with the result of ref. [4]. A Yukawa-type potential with central, spin-orbit and tensor components is used for the present calculation;

$$V(r) = V_{c}(r) + V_{LS}(r) + V_{T}(r),$$

$$V_{c}(r) = V_{c} \left(a_{c}^{11} P^{11} + a_{c}^{31} P^{31} + a_{c}^{13} P^{13} + a_{c}^{33} P^{33} \right)$$

$$\times (\mu_{c}/r) \exp(-r/\mu_{c}),$$

$$V_{LS}(r) = V_{LS} \left(a_{LS}^{13} P^{13} + a_{LS}^{33} P^{33} \right) L \cdot S(\mu_{LS}/r)$$

$$\times \exp(-r/\mu_{LS}),$$

$$V_{T}(r) = V_{T} \left(a_{T}^{13} P^{13} + a_{T}^{33} P^{33} \right) S_{12}(\mu_{T}/r)$$

$$\times \exp(-r/\mu_{T}),$$
(5)

where $P^{2T+1,2S+1}$ is the projection operator of the (T, S) channel and S_{12} is the tensor operator. We



Fig. 1. Diagrammatical representation of the 2p-2h ground-state correlation.

adopted the parameter set of Millener and Kurath [5] which is listed in table 2. The summed cross section $d\sigma/d\Omega$ ($E_x = 2-12$ MeV) for the present calculation is 19 mb/sr compared to 20 mb/sr obtained in the restricted space calculation [4]. Both calculations show about 10% of the total transition strengths in the energy region $E_x = 12-18$ MeV. However, the full space calculation shows 15% of the total strength above $E_x = 18$ MeV, while there is no transition strength in this region in the calculation of ref. [4].

We study in this letter the effect of 2p-2hground-state correlations on the spin-dipole transitions in ¹²N using perturbation theory. The 2p-2hcorrelation is claimed as an important effect for the quenching of magnetic dipole and Gamow –Teller transition strengths in recent microscopic calculations [6,7]. The ground-state correlation is shown diagrammatically in fig. 1. The first-order perturbation theory gives the following wave function;

$$\begin{split} \tilde{0} \rangle &= |0\rangle + \sum_{\substack{\text{ph}, p'\mathbf{h}'\\J}} \frac{\langle (\mathbf{ph}^{-1})J, (\mathbf{p'}\mathbf{h'}^{-1})J; 0^+ | V | 0^+ \rangle}{E_0 - E_J(\mathbf{ph}, \mathbf{p'h'})} \\ &\times |(\mathbf{ph}^{-1})J, (\mathbf{p'h'}^{-1})J; 0^+ \rangle. \end{split}$$
(6)

Using this perturbed wave function (6), we obtain the modified transition strength,

$$\langle (\mathrm{ph}^{-1})\lambda \| T_{\lambda} \| \tilde{0} \rangle = \langle (\mathrm{ph}^{-1})\lambda \| T_{\lambda} \| 0 \rangle (1-\alpha), \quad (7)$$

| | a ¹¹ | a ³¹ | a ¹³ | a ³³ | <i>b</i> /μ | V(MeV) | |
|------------|-----------------|-----------------|-----------------|-----------------|-------------|---------|--|
| central | - 0.714 | 0.6 | 1 | -0.286 | 1.18 | - 44.8 | |
| tensor | 0 | 0 | 1 | -0.38 | 1.18 | - 16.25 | |
| spin-orbit | 0 | 0 | 1 | 3.5 | 2.36 | - 26 | |

Table 2

Interaction parameters.

where

$$\alpha = \sum_{\mathbf{p}'\mathbf{h}'} \frac{\langle (\mathbf{p}\mathbf{h}^{-1})\lambda | V | (\mathbf{p}'\mathbf{h}'^{-1})\lambda \rangle}{E_{\lambda}(\mathbf{p}\mathbf{h}, \mathbf{p}'\mathbf{h}') - E_{0}} \langle (\mathbf{p}'\mathbf{h}'^{-1})\lambda \| T_{\lambda} \| 0 \rangle.$$
(8)

We calculate the particle-hole (p-h) matrix elements with the Yukawa-type potential with central, spin-orbit and tensor components as given in eq. (5). The p-h energy difference is taken to be 15.4 MeV. The oscillator length is determined as b = 1.64 fm in order to reproduce the mean charge radius of ¹²C.

The calculated values of α are given in table 3. The central interaction gives $\alpha = 0.13$ which decreases the spin-dipole transition strength by about 25%. This positive value of α is due to the fact that the spin-isospin p-h interaction is repulsive. We also calculated α by using the parameter set for the central interaction given by Ferrell and Visscher [8]. This gave a slightly larger value of $\alpha = 0.15$.

The tensor correlation increases the values of α for the state with $\lambda^{\pi} = 1^-$ by 10–20%, while decreasing the value of α for $\lambda^{\pi} = 0^-$ by about the same amount. The transition strength of the 2⁻ state is not changed much by the tensor correlation. This result can be understood intuitively by the following argument given by Mottelson [9].

The tensor interaction can be given in the form

$$V_{\rm T}(r) = F(r) \left\{ \left[\sigma_1 \times \sigma_2 \right]^{(2)} \times \left[r^2 Y_2(\hat{r}) \right]^{(2)} \right\}^{(0)}.$$
 (9)

Table 3

Normalization factors α for spin dipole transitions.

Since the solid spherical harmonics $r^2 Y_{2m}(\hat{r})$ can be expanded by the formula

$$r^{2}Y_{2m}(\hat{r}) = \sum_{\lambda'\mu'} \sqrt{4\pi} (-)^{\lambda'} \\ \times \left[5!/(2\lambda'+1)!(5-2\lambda')! \right]^{1/2} \\ \times \langle 2-\lambda'\lambda'm-\mu'\mu'|2m\rangle r_{1}^{2-\lambda'}r_{2}^{\lambda'} \\ \times Y_{2-\lambda',m-\mu'}(\hat{r}_{1})Y_{\lambda'\mu'}(\hat{r}_{2}),$$
(10)

the tensor interaction (9) can be expressed in terms of the spin-dipole operators,

$$V_{\mathrm{T}}(r) = F(r) \sum_{\lambda} (-)^{\frac{1}{6}} \sqrt{4\pi} \left(\frac{10}{3}\right)^{1/2} \begin{pmatrix} 2\sqrt{5} \\ -\sqrt{15} \\ 1 \end{pmatrix}$$
$$\times \left\{ r_{1} \left[\sigma_{1} \times Y_{1}(\hat{r}_{1}) \right]^{(\lambda)} \\\times r_{2} \left[\sigma_{2} \times Y_{1}(\hat{r}_{2}) \right]^{(\lambda)} \right\}^{(0)},$$
$$\lambda^{\pi} = \begin{pmatrix} 0^{-} \\ 1^{-} \\ 2^{-} \end{pmatrix}, \qquad (11)$$

where we choose $\lambda' = 1$ only in the expansion (10). Since the coupling strength F(r) is repulsive for the tensor interaction, the tensor correlation for the 1⁻ state is additive to that of the central interaction. On the other hand, the two contributions tend to cancel each other in the case of the 0⁻ state. A smaller tensor correlation for the 2⁻

| λ ^π | (p-h) | $\langle (\mathbf{h}^{-1}\mathbf{p})\lambda \ T \ 0 \rangle$ | $\alpha(V_{\rm C})$ | $\alpha(V_{\rm C}+V_{\rm T})$ | $\alpha(V_{\rm C} + V_{\rm T} + V_{\rm LS})$ |
|----------------|-----------------------|----------------------------------------------------------------|---------------------|-------------------------------|----------------------------------------------|
| 0- | $(2s1/2, 1p1/2^{-1})$ | - 0.654 | 0.093 | 0.077 | 0.077 |
| | $(1d3/2, 1p3/2^{-1})$ | 1.463 | 0.135 | 0.095 | 0.095 |
| 1- | $(1d3/2, 1p1/2^{-1})$ | 1.035 | 0.135 | 0.158 | 0.133 |
| | $(2s1/2, 1p1/2^{-1})$ | 0.925 | 0.093 | 0.101 | 0.116 |
| | $(1d3/2, 1p3/2^{-1})$ | 1.851 | 0.135 | 0.154 | 0.147 |
| | $(1d5/2, 1p3/2^{-1})$ | -1.388 | 0.135 | 0.156 | 0.180 |
| | $(2s1/2, 1p3/2^{-1})$ | 0.654 | 0.093 | 0.101 | 0.072 |
| 2- | $(1d3/2, 1p1/2^{-1})$ | -0.463 | 0.135 | 0.085 | 0.089 |
| | $(1d5/2, 1p1/2^{-1})$ | 2.266 | 0.135 | 0.133 | 0.133 |
| | $(1d3/2, 1p3/2^{-1})$ | -0.925 | 0.135 | 0.145 | 0.147 |
| | $(1d5/2, 1p3/2^{-1})$ | 2.120 | 0.135 | 0.129 | 0.127 |
| | $(2s1/2, 1p3/2^{-1})$ | 1.463 | 0.093 | 0.091 | 0.091 |

state is also quite reasonable since the coefficient for 2^- in eq. (11) is several times smaller than those for 0^- and 1^- . The results given in table 3 are slightly different from that expected from eq. (11) because of the exchange term contribution.

As can be seen in the last column of table 3, the spin-orbit two-body force has an appreciable effect only on 1^- states. The spin-orbit two-body operator is given by

$$\boldsymbol{L} \cdot \boldsymbol{S} = \frac{1}{2} (\boldsymbol{r}_1 - \boldsymbol{r}_2) \times (\boldsymbol{p}_1 - \boldsymbol{p}_2) \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2).$$
(12)

When eq. (12) is expanded in terms of the spin-dipole operators, the important terms for the direct two-body matrix elements in eq. (8) are

$$\boldsymbol{L} \cdot \boldsymbol{S} \propto \boldsymbol{r}_1 \left\{ \left[\boldsymbol{\sigma}_1 \times \boldsymbol{Y}_1(\hat{\boldsymbol{r}}_1) \right]^{(1)} \times \boldsymbol{p}_2 \right\}^{(0)} \\ + \boldsymbol{r}_2 \left\{ \left[\boldsymbol{\sigma}_2 \times \boldsymbol{Y}_1(\hat{\boldsymbol{r}}_2) \right] \times \boldsymbol{p}_1 \right\}^{(0)},$$
(13)

where the spin-dipole operator can couple to $\lambda^{\pi} = 1^{-}$ only. This is the reason why the spin-orbit force contributes appreciably to the value of α for the states with $\lambda^{\pi} = 1^{-}$, but not for the states with $\lambda^{\pi} = 0^{-}$ and 2^{-} .

In summary, we have studied the effect of the 2p-2h ground state correlation on the spin-dipole transition in ¹²N. We found that the net effect of the central, tensor and spin-orbit forces, averaging over the transition matrix elements, gives the values $\alpha = 0.089$, 0.139 and 0.122 for $\lambda^{\pi} = 0^{-}$, 1⁻ and 2⁻, respectively. Because the experimental data in the region $E_x = 2-12$ MeV is dominated by 1⁻ and 2⁻ resonances, the value α substantially decreases the spin-dipole transition strength by about 25%. The tensor correlation contributes 10-20% to the renormalization factors α for 0⁻ and 1⁻ states, while the 2⁻ state is insensitive to the tensor force,

the effect of the two-body spin-orbit correlation is appreciable only in the transition strength of the 1^- state. The experimental cross section in the energy region $E_x = 2-12$ MeV is 40% less than the 1p-1h shell-model prediction in the full configuration space and the present study suggests that the 2p-2h ground-state correlations explain the major part (60%) of this missing strength. The transition strengths might be decreased further by meson-exchange currents [7] and Δ -hole couplings [10] together with higher 2p-2h excitations above $1\hbar\omega$ configuration space [5].

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