$E2$ Effective Charges of $g_{9/2}$ Nucleons Derived from Quadrupole Moments of High-Spin Isomers in ^{88,90,91}Zr and ^{90,92,94}Mo

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Static quadrupole moments (Q) of 8^+ isomers in ^{88,90}Zr and the $\frac{21}{2}^+$ isomer in ^{91}Zr, as well as Q ratios of 8^+ isomers in ^{90,92,94}Mo, have been measured with the time-differential perturbed-angular-distribution technique. The isomers were excited by pulsed heavy-ion beams and recoil implanted into single crystals of Zr in which their quadrupole precession was observed. Our results, the first such data for high-spin isomers in the $g_{9/2}$ shell, yield large $E2$ effective charges for the $g_{9/2}$ proton and the $g_{9/2}$ neutron. The systematics of these charges are comprehensively explained by particle-vibration coupling and the blocking effect.

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The effective charges e_{eff} of valence nucleons, which describe their polarizing effect on the nuclear core, play a key role in bridging the gap of our understanding between the simple shell model and the collective model of nuclei. The e_{eff} for the neutron is of interest because of its role in inducing core deformation[1] and the isospin dependence of e_{eff} is of interest because of its connection to the isospin structure of core vibrations.[2,3] The most reliable data on e_{eff} are obtained from measurements of static quadrupole moments (Q) of high-spin isomers in nuclei near closed shells, since they have relatively pure shell-model character. While values of e_{eff} may be estimated from transition $B(E2)$ values, such data are less uniquely defined than those obtained directly from static Q moments, since $B(E2)$'s connect two states, both of which have to be well characterized. In the $g_{9/2}$ shell, e_{eff} values have been previously derived from isolated $B(E2)$ data.[4,5] However, a definitive study of effective charges based on Q moments of a sizable family of high-spin isomers in this region has been lacking. In this Letter we present the first comprehensive study of e_{eff} in the $g_{9/2}$ shell, by measurements of Q moments of several high-spin isomers in Zr and Mo isotopes and extensive shell-model analyses of these results. Our work provides an important data base on the $E2$ electromagnetic structure of $g_{9/2}$ nuclei that complements the data from electron scattering which is now being widely used for deriving transition densities.[6]

The experimental work reported here comprises (1) model-independent determinations of the static Q moments of 8^+ isomers in ^{88}Zr and ^{90}Zr and the $\frac{21}{2}^+$ isomer in ^{91}Zr, and (2) ratios of Q moments of 8^+ isomers in ^{90,92,94}Mo. These states, involving the stretched-spin configurations $[\pi,\nu(g_{9/2})^{\pm n}]8^+$ or $[\pi(g_{9/2})^2\otimes(\nu d_{5/2})]\frac{21}{2}^+$, provide an excellent laboratory for studying effective charges. They are amenable to modern techniques for isomeric Q-moment measurements and are theoretically simple and well characterized. These isomers form a well-balanced ensemble with which e_{eff} for the proton and the neutron can be individually measured. Since the particles appear in the same orbital [for example, ^{88}Zr, $(\nu g_{9/2})^{-2}$ and ^{90}Zr, $(\pi g_{9/2})^2$], their relative e_{eff} values can be directly compared without uncertainties in the orbit dependence of e_{eff}. Further, the three-particle state and the Mo isomers afford critical tests of the influence of their different structures on e_{eff}. By use of shell-model calculations with current descriptions of these states, values of e_{eff} consistent with our measured Q's [as well as known $B(E2)$'s] were derived. These values, especially that for the neutron, are about the largest effective charges observed for nuclei near closed shells. We show that the particle-vibration coupling model explains not only these large e_{eff}, but predicts significant reductions in the e_{eff} when part of the vibrational excitations are blocked, an effect which is also clearly observed in our results.

The only technique presently available for the measurement of Q moments of isomeric states is that of time-differential perturbed angular distributions of the deexcitation γ rays of the isomer,[7] whereby the spin precession of the isomer in the electric field gradient (EFG) of a crystal lattice can be directly observed. Q-

moment measurements of high-spin isomers, unlike those of g factors, have faced two major challenges. First, while the coupling frequency $\omega_0 \propto e^2qQ/4I(2I-1)\hbar$ (I is the level spin, eq is the EFG) is measurable in a fairly straightforward way, the extraction of Q from this product requires a suitable host lattice with a calibrated EFG. The use of heavy-ion reactions elegantly solves this problem since they provide on the one hand, large excitation cross sections for high-spin isomers and on the other, large enough recoil momenta to the excited nucleus to be propelled out of fairly thick targets and recoil implanted into an independent and selected host lattice in which the EFG calibration is available. Secondly, since the observed quadrupole precession period $T_{\omega_0} = 2\pi/\omega_0$ varies approximately as I^2, high spins result in $T_{\omega_0} >> \tau$ the isomeric lifetime, and also reduce the modulation contrast in the observable time window, which makes it difficult to measure coupling frequencies in a reliable manner. However, as demonstrated first in our laboratory,[8] the quadrapole precession in a single crystal with its symmetry axis (c axis) set in the plane containing the beam and the γ-ray detectors and at 45° to the beam axis regains sufficient modulation contrast in the precession spectra even in the short time window available. The present work exploits both of these advances.

The isomers were excited by 48-MeV $^{12,13}C$ or 56-MeV ^{16}O pulsed beams from the Rutgers-Bell tandem accelerator incident on enriched Se targets. The targets were evaporated onto a Zr single crystal, which was mounted with its c axis oriented at 45° to the beam axis coplanar with the beam and detectors. The delayed γ rays were detected at 90° or 0° to the beam axis, and their intensities $I(t)$ recorded as a function of time t after the beam burst. These time spectra were used to construct the ratio $R(t) = \{I(0,t) - I(90,t)\}/\{I(0,t) + I(90,t)\}$. The modulation spectrum $R(t)$ contains all the information on the quadrupole precession and is described by a superposition of oscillatory terms of the type $\{c_n \cos(n\omega_0 t)\}$, where the values of n determining the frequency components and the weighting factors c_n are given theoretically. They depend on the spin I, the symmetry of the EFG, the geometry of the c axis relative to the beam axis and the detectors, and the γ-ray angular distribution properties. The theoretical function can be calculated by use of the general formulas of Alder[9] and fitted to the observed $R(t)$ spectra as shown for typical cases in Fig. 1. This yields the basic frequency ω_0 from which the coupling constants e^2qQ are obtained.

In the case of the Mo isomers,[10] the e^2qQ data yield the ratios of the Q moments only, since the EFG at Mo in Zr metal is not known. The Q moments of these isomers are given in Table I relative to the theoretical value (see below) for the ^{92}Mo isomer.

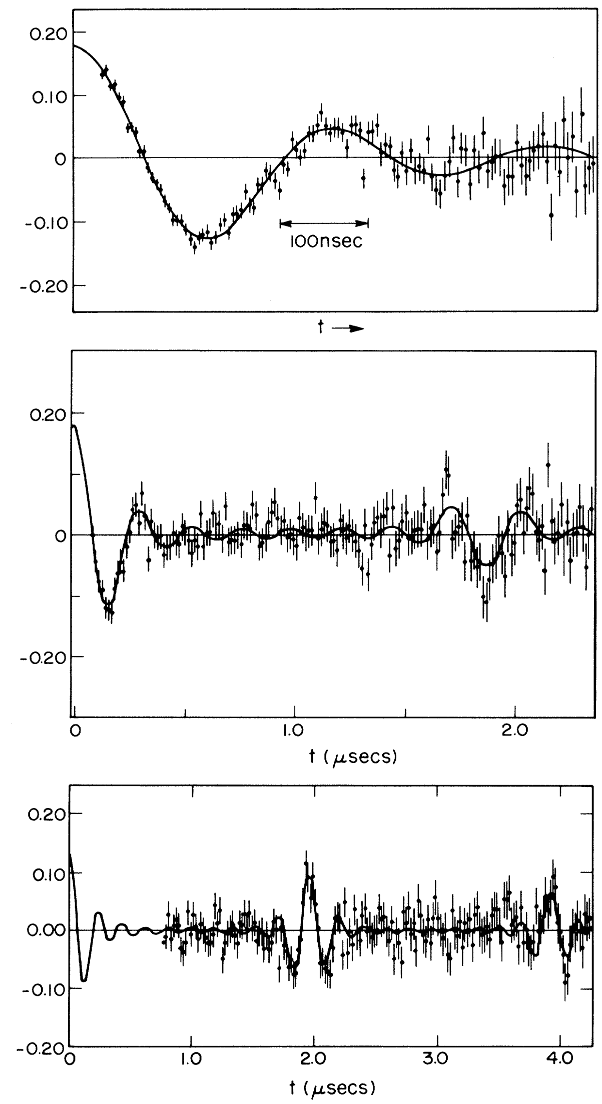

FIG. 1. Quadrupole precession patterns for 8^+ isomers in ^{90}Zr (top) and in ^{88}Zr (middle), and for the $\frac{21}{2}^+$ isomer in ^{91}Zr (bottom). The host material was a single crystal of Zr metal set in the c-45° geometry.

With the known Q moment of the ground state of ^{91}Zr [11] and recent NMR data on e^2qQ for this isotope[12] in Zr metal, the EFG for Zr in Zr is calibrated[13] and thus we obtain model-independent values of Q for the Zr isomers given in Table I.

Shell-model calculations were carried out with use of the model-space assumptions and interactions of Gloeckner and co-workers.[14–16] The proton particles were restricted to the $1p_{1/2}$ and $0g_{9/2}$ orbits. The two neutron holes for the $N=48$ nuclei were also restricted to these orbits while the neutron particles in nuclei with $N=51, 52$ were confined to the $2s_{1/2}$, $1d_{5/2}$, and $0g_{7/2}$ orbits. The wave functions and one-body transi-

TABLE I. Comparison of experimental and theoretical $E2$ moments.

Nuc.	J_i	J_f	$B(E2)^{1/2}$ or Q				Ref.
			Theory[a]			Expt	
			A (fm^2)	B (fm^2)	Moment (e fm^2)	(e fm^2)	
^{90}Zr	8	6	4.07[b]	0	8.1	7.8(2)	d
^{90}Zr	8		−25.2[b]	0	−50.4	|51(3)|	e
^{88}Zr	8(1) 8(2)		4.2[c] −22.6[c]	24.0 −11.4	51.6 −65.7	|51(3)|	e
^{86}Sr	8	6	0[c]	4.07	7.3	7.9(3)	i
^{91}Zr	21/2	17/2	3.85[b]	2.28	11.8	11.6(3)	g
^{91}Zr	21/2		−25.2[b]	−14.8	−77.0	|86(5)|	e
^{90}Mo	8(2) 8(1)		15.0[c] −17.0[c]	22.9 −12.0	51.5 −41.6	|58(3)|	f
^{92}Mo	8	6	3.49[b]	0	5.6	5.69(11)	d
^{92}Mo	8		−21.6[b]	0	−34.6	|34|	e
^{94}Mo	8		−19.2[b]	−15.7	−49.6	|47(1)|	f
^{91}Zr	5/2		−2.1[b]	−14.7	−21.0	−20.6(10)	h

[a] $B(E2)^{1/2}$ or $Q = Ae_{\text{eff}}(p) + Be_{\text{eff}}(n)$ with $e_{\text{eff}}(p) = 2.0e$ and $e_{\text{eff}}(n) = 1.8e$ for the high-spin states in the Zr and Sr isotopes and $e_{\text{eff}}(p) = 1.6e$ and $e_{\text{eff}}(n) = 1.2e$ for the Mo high-spin states and the ground state of ^{91}Zr.

[b] Based on "seniority" p-p interaction of Ref. 14, and "$N=51$ free-fit" p-n interaction of Ref. 15.

[c] Based on "seniority fit–total energy" interaction of Ref. 16 plus a n-n interaction based on the ^{86}Sr energy levels.

[d] See Table IV of Ref. 4.

[e] Present work.

[f] Present work relative to (theoretical) $Q(^{92}Mo)$ (line 11).

[g] Based on $T_{1/2} = 3.6(1)$ μsec (present work).

[h] Reference 11.

[i] Reference 5.

tion densities were calculated with the shell-model code OXBASH.[17] Harmonic-oscillator radial wave functions with $\hbar\omega = 8.78$ MeV were used. The results for transition and static Q moments are given in Table I.

From a comparison of theory with the results of the present work (columns 6 and 7), the optimum set of effective charges for the $g_{9/2}$ nucleons in the high-spin states of Sr and Zr isotopes (top half of Table I) is $e_{\text{eff}}(p) = (1+\delta_p)e = 2.0e$ and $e_{\text{eff}}(n) = \delta_n e = 1.8e$. However, this set is not consistent with the results for the Mo high-spin states and especially the ground state of ^{91}Zr. They yield (lower half of Table I) the smaller values $e_{\text{eff}}(p) = 1.6e$ and $e_{\text{eff}}(n) = 1.2e$. Overall, our results show that the polarization charge δ_n for the $g_{9/2}$ neutron is almost twice that for the proton.

The need for effective charges is a result of truncation of the proton model space to the $(1p_{1/2}, 0g_{9/2})$ orbits. A semiquantitative understanding of the present results can be obtained in a much simpler way than expanding the model space, by considering the coupling of the valence particles to core excitations via first-order perturbation theory. Then, contributions of the type $\Delta N = 2$, where $N = (2n + l + \frac{3}{2})$, arise from the coupling to the isoscalar (IS) and isovector (IV) giant resonances. In addition, we need to include the proton excitations, with $\Delta N = 0$, of the type (a) $1p_{3/2}, 0f_{5/2} \to 1p_{1/2}$ and (b) $0g_{9/2} \to 1d_{5/2}, 0g_{7/2}$. We estimate that those of type (b) are relatively unimportant. Since the $p_{1/2}$ orbit is empty for the high-spin states of Sr and Zr isotopes in our model space, the $\Delta N = 0$ excitations (a) can be associated with the low-lying particle-hole state in ^{88}Sr which is the nearest closed-shell nucleus with an empty $1p_{1/2}$ orbit. In the case of the Mo isotopes and the low-spin states of Zr isotopes, the $1p_{1/2}$ orbit is partially filled and excitations of type (a) will be partially blocked. We will consider the blocking effect separately.

In the framework of the particle-vibration coupling model,[2] and with use of a schematic quadrupole-quadrupole interaction,[18] the contribution to the effective charge for a given transition to a 2^+ state at excitation energy E can be estimated from the expression[19]

$$\delta_{p/n} = \frac{\frac{1}{2}[1 + R \pm (R-1)C_1]C_0 B(E2, 0 \to 2)}{Ze^2 A^{2/3} \langle r^2 \rangle_{vp/n} E}, \quad (1)$$

where $\langle r^2 \rangle_{vp/n}$ is the mean square radius of the valence $g_{9/2}$ proton/neutron orbit. In our model, where $\hbar\omega = 8.78$ MeV has been adjusted to reproduce the rms charge radius of ^{90}Zr, we use $\langle r^2 \rangle_{vp} = \langle r^2 \rangle_{vn} = 26.0$ fm^2. $R = (M_n/M_p)(N/Z)$; $M_{n/p}$ are the $E2$ matrix elements[20] for the neutrons/protons in the $0 \to 2$ transition $[B(E2, 0 \to 2) = M_p^2]$.

The constants C_0 and C_1 are related to the effective interaction between the valence nucleon and the vibration. The form of Eq. (1) is derived from Eq. 6-218 of Ref. 2 together with the assumption[18] that the valence neutrons couple to the vibration with a strength $\propto V_{np}(M_p/Z) + V_{nn}(M_n/N)$ and the valence protons couple to the vibration with a strength $\propto V_{pp}(M_p/Z) + V_{pn}(M_n/N)$, where the V's are average interaction strengths between the valence particle and the particle excitation in the vibration. We assume that $V_{pn} = V_{np}$ and $V_{pp} = V_{nn}$. In Eq. (1), the constant $C_0 \propto V_{pn} + V_{pp}$ and the constant C_1 is given by $(V_{pn} - V_{pp})/(V_{pn} + V_{pp})$. Realistic values for C_0 and C_1 may be obtained by matching our Eq. (1) above to the results for $B(E2)$, E, and δ obtained from microscopic calculations for the IS and IV $E2$ giant resonances. Thus, we adopt the values $C_0 = 56$ MeV fm^{-2} and $C_1 = 1.0$, based on random-phase-approximation–Hartree-Fock calculations with the SG-II

Skyrme-type interaction of Sagawa and van Giai for the coupling of valence particles in the *sd* shell to the IS and IV resonances of ^{40}Ca.[21] We expect to obtain essentially the same results for C_0 and C_1 if similar random-phase-approximation–Hartree-Fock calculations were carried out for ^{88}Sr.

Considering first the high-spin states of Zr and Sr (8^+ and $\frac{21}{2}^+$), the case with an empty $p_{1/2}$ orbit, we calculate contributions from three states in ^{88}Sr: The IS and IV giant resonances and the low-lying 2^+ state at 1.836 MeV. Using measured values for the $[B(E2), 0 \rightarrow 2]$[22] and $R \approx 0.68$[20] for this 2^+ state, we obtain $\delta_p(1.836) = 0.92$ and $\delta_n(1.836) = 1.36$. The δ's due to the coupling to the giant resonances are relatively insensitive to shell effects; we assume that $\delta(\mathrm{IS})$ varies smoothly as Z/A and $\delta(\mathrm{IV})$ is a constant.[2] The well-established results for δ for the s-d orbits near ^{40}Ca[21] can thus be scaled to yield $\delta_p(\mathrm{IS}) = \delta_n(\mathrm{IS}) = 0.29$ and $\delta_n(\mathrm{IV}) = -\delta_p(\mathrm{IV}) = 0.09$. Thus the totals are $\delta_p = 1.12$ and $\delta_n = 1.74$, in excellent agreement with our experimental results.

In the case of the Mo isotopes and the ground states of ^{90}Zr and ^{91}Zr, the $p_{1/2}$ orbit is (60–70)% full in our model space. The value of $R < 1$ for the 2^+ state of ^{88}Sr (see above) shows that proton excitations dominate, and as mentioned earlier, part of the $\Delta N = 0$ proton excitations will be blocked by the partially full $p_{1/2}$ orbit. We obtain a qualitative estimate of the blocking effect, by considering the sum of the factors $\{B(E2)/E\}$ for the $0 \rightarrow 2$ transitions in ^{90}Zr[23] in excess of that calculated within the model space. This estimate indicates a reduction of about 0.4 for δ_p and 0.5 for δ_n, in good agreement with the difference between our two sets of e_{eff}.

Our work demonstrates that $E2$ effective charges in the $g_{9/2}$ shell can be satisfactorily understood in a semiquantitative manner. The most important cause of the large e_{eff} that we observe is the excitation to the low-lying 2^+ state in ^{88}Sr. When essentially all the low-lying states are contained explicitly in the model space, as in the full *sd* shell-model calculations,[24] the empirical e_{eff} are much smaller and are consistent with just the giant resonance contributions. Our relatively large neutron effective charge stems from the fact that the ^{88}Sr low-lying state is predominantly a proton excitation ($R < 1$). In contrast, for ^{48}Ca,[20] where $R \approx 3$ for the lowest 2^+ state, one expects a larger contribution to protons than to neutrons. The effective charges needed for ^{46}Ca and ^{50}Ti[25] are consistent with this expectation.

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