ISOSPIN-FORBIDDEN PROTON AND NEUTRON EMISSION IN 1s–0d SHELL NUCLEI

W E ORMAND and B A BROWN

National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, MI 48824, USA

Received 12 February 1986

Spectroscopic amplitudes for the decay of $T = 3/2$ states in $A = 4n + 1$ nuclei ($21 \leqslant A \leqslant 37$) by proton and neutron emission to the $T = 0$ ground state of $A = 4n$ daughter nuclei are calculated and compared with experiment The calculations were performed in the framework of the shell model using the full or nearly full 1s–0d configuration space and an isospin-nonconserving interaction which reproduces experimental isotopic mass shifts Contributions due to mixing with individual parent and daughter states are analyzed The question of periodicity in A for these amplitudes is addressed

One of the clearest indications of isospin mixing in light nuclei is the decay of $T= 3/2$ states in $A = 4n+1$ nuclei, by proton or neutron emission, to $T=0$ states in $A = 4n$ nuclei [1–6] Previous studies [1–6] concluded that the spectroscopic amplitudes $\Theta^{\mathrm{p(n)}}_{\mathrm{INC}}$ for these isospin-nonconserving (INC) transitions (1) systematically increase with mass A, (2) have an oscillation with period $\Delta A = 8$ superimposed on the proton amplitudes, (3) are generally greater for neutron emission, (4) generally do not increase as a function of excitation energy Perhaps the most striking of these features is the oscillation observed in $\Theta^{\mathrm{p}}_{\mathrm{INC}}$ One cannot explain this periodicity in terms of simple schematic models [6], and thus it has been speculated [5] that this phenomenon is due to isospin mixing mediated by Δ-isobars In this letter, we present for the first time results of detailed shell-model calculations of $\Theta^{\mathrm{p(n)}}_{\mathrm{INC}}$ in the region $21 \leqslant A \leqslant 37$, where the full or nearly full 1s–0d (sd) major-shell configuration space is used In our model, all the above features as well as the absolute magnitude of $\Theta^{\mathrm{p(n)}}_{\mathrm{INC}}$ can be qualitatively understood However, our results indicate that the variations from state to state are random or statistical in nature, rather than periodic

In this work, we take into account INC effects due to admixtures of essentially all sd-shell $T= 1/2$ states with the $T= 3/2$ parent, and $T= 1$ and 2 states with the $T=0$ daughter Contributions due to the giant isovector monopole have been evaluated previously [7] using an harmonic-oscillator basis, and were found to be “small but not negligible” [8] compared to experiment We note, however, that the harmonic-oscillator basis tends to predict more isospin mixing in the ground state than does a more realistic Hartree–Fock calculation [8] In this work, we concentrate on the contributions due to mixing within the sd shell, and assume that the isovector monopole contribution is negligible A typical level diagram which illustrates the relative location of the states of interest in both the parent and daughter nuclei is shown in fig 1

At present there has been only a very limited amoun of theoretical analysis dealing with the contribution due to mixing within a major-shell configuration [6,9, 10] The most detailed of these is that of Arima and Yoshida [9] for $A = 13$ Their approach was to take into account the entire 0p shell and to determine the INC mixing via perturbation theory Their results generally agree with experiment, and were found to depend on the location of the, then unknown, third $J^{\pi} = 3/2^{-}$, $T= 1/2$ state The approach of McDonald and Adelberger [6] and Auerbach and Lev [10] was to assume that the source of isospin impurity was due to mixing with the anti-analog configuration in the parent nucleus Within the framework of this simple model, they were able to reproduce the qualitative behavior of the data, but were unable to explain the observed oscillation

0370-2693/86/$ 03 50 © Elsevier Science Publishers B V
(North-Holland Physics Publishing Division)

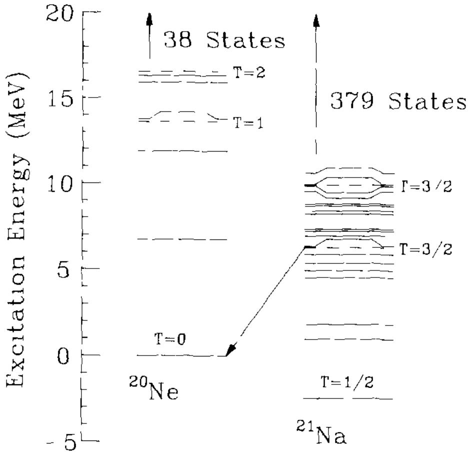

Fig 1 Level scheme of the relevant states in the decay of $J^\pi = 5/2^+$, $T = 3/2$ state in ^{21}Na via isospin-forbidden proton emission The isospin of the $T = 1/2$ parent and $T = 0$ daughter states are represented by dashed lines and are labeled in the figure The relative energies of the two ground states and the $T = 3/2$ state are taken from experiment, while theoretical energies were used for the remaining states

Our calculations start with the states $\Psi_0(k)$ which have definite isospin, and are proton–neutron shell-model wave functions which consist of the many-body Slater determinants obtained within a spherical single-particle basis The model space consisted of $A - 16$ nucleons outside an inert ^{16}O core restricted to the $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$ orbits[‡1] The isoscalar hamiltonian was taken to be the mass-dependent sd-shell hamiltonian of Wildenthal [11]

Since the purpose of this investigation is to determine the influence of contributions due to mixing with individual parent and daughter states, a perturbation expansion for $\Theta^{p(n)}_{INC}$ was used In this formalism, $\Theta^{p(n)}_{INC}$ is the sum of terms whose contribution is given by the product of the isospin-mixing amplitude and allowed spectroscopic amplitude In the present work, essentially all sd-shell states in both the parent and daughter nuclei were included in the expansion In first-order perturbation theory, the isospin-mixing amplitude between the states $\Psi_0(i)$ and $\Psi_0(k)$ is given by the matrix element $V_{ik} = \langle \Psi_0(i) | V_{INC} | \Psi_0(k) \rangle$ divided by the separation energy $\Delta E = E(i) - E(k)$, where $E(i) = E_0(i) + V_{ii}$ The isospin-nonconserving potential V_{INC} was determined empirically by requiring that it reproduce experimental b- and c-coefficients of the isobaric mass multiplet equation (IMME, Mass $= a + bT_Z + cT_Z^2$) for sd-shell nuclei[‡2] It was found that in order to reproduce experimental c-coefficients, a nuclear charge-dependent (isotensor) interaction was necessary Details of the IMME calculation and the resulting V_{INC} potential are given in ref [12]

Shown in table 1 are the individual contributions to Θ^{p}_{INC} for the decay of the lowest $T = 3/2$ state due to (1) the lowest $T = 1$ and 2 daughter states, (2) the lowest $T = 1/2$ parent state, and (3) those $T = 1/2$ states which are within ±500 keV of the $T = 3/2$ state (except for $A = 37$, where the contribution due to the two closest $T = 1/2$ states is given) The isospin-mixing amplitudes were evaluated using the experimental excitation energies for the $T = 3/2$ states in the energy denominator ΔE The states shown in the table are the principal contributors to the total value, and the remaining contributions are typically less than unity and tend to cancel

As can be seen from table 1, those $T = 1/2$ parent states which lie within ±500 keV of the $T = 3/2$ state contribute significantly to the total Unfortunately the excitation energies and allowed spectroscopic amplitudes for these states are uncertain both experimentally and theoretically At present, these quantities have been determined experimentally in only one case [4] In addition, theoretical estimates of excitation energies are generally uncertain by several hundred keV To account for these uncertainties in the properties of the $T = 1/2$ states, the $T = 1/2$ excitation-energy spectrum was shifted relative to its original position by ±500 keV in small steps We take as our "best" estimate of $\Theta^{p(n)}_{INC}$, the average of the values obtained at each step The upper and lower limits on the range of $\Theta^{p(n)}_{INC}$ were determined by evaluating both the upper and lower rms deviations away from this "best" value

---

[‡1] For $A = 25$ and 29 a model-space truncation based on the monopole energy of the diagonal matrix element was used With this truncation, the number of $5/2^+$ states in $A = 29$ was reduced from the full space value of 12878 to 2236 In addition, excitation energies obtained with these truncations were found to deviate systematically from the full space values, and, therefore, were shifted relative to the ground state by the average deviation from the full space values

[‡2] See ref [12] The parameters used in this work are given in the first and seventh rows of tables 2 and 3, respectively

Table 1
Contribution to Θ^{p}_{INC} (× 1000) for the lowest J^π $T = 3/2$ state due to mixing with states in the parent ($T = 1/2$) and daughter ($T = 1$ and 2) For the $T = 3/2$ state the experimental excitation energy E_x (MeV) is given, while theoretical excitation energies (in MeV) for the remaining states are given The sum of contributions due to states listed is given, as well as the total which includes essentially all states in the parent and daughter nuclei

A	J^π	E_x (MeV)	T	E_x (MeV)	Θ_{INC} (× 1000)	A	J^π	E_x (MeV)	T	E_x (MeV)	Θ_{INC} (× 1000)
21	$5/2^+$	8 970	1	13 61	0 0	33	$1/2^+$	5 548	1	7 32	2 3
			2	16 57	−2 3				2	11 99	3 5
			1/2	0 24	1 6				1/2	0 82	−4 4
			1/2	8 58	1 8				1/2	5 60	8 4
			1/2	9 06	4 2				1/2	6 02	2 7
			1/2	–	–				1/2	–	–
			1/2	–	–				1/2	–	–
			1/2	–	–				1/2	–	–
			sum		5 3				sum		12 5
			total		5 3				total		12 1
25	$5/2^+$	7 898	1	12 84	−0 1	37	$3/2^+$	5 052	1	10 87	3 6
			2	15 28	−2 4				2	15 50	0 2
			1/2	0 00	1 1				1/2	0 00	−1 7
			1/2	7 61	−0 1				1/2	5 06	−15 3
			1/2	7 81	2 4				1/2	6 83	0 0
			1/2	7 87	1 1				1/2	–	–
			1/2	8 30	2 4				1/2	–	–
			1/2	8 37	0 1				1/2	–	–
			sum		4 5				sum		−13 2
			total		5 8				total		−13 7
29	$5/2^+$	8 384	1	10 71	−0 7						
			2	15 23	−1 6						
			1/2	1 96	1 2						
			1/2	7 71	0 0						
			1/2	8 31	0 2						
			1/2	8 54	7 4						
			1/2	–	–						
			1/2	–	–						
			sum		6 5						
			total		7 0						

Sufficient accuracy was obtained by taking steps of 10 keV

A comparison between experiment and our calculations is shown in fig 2 The experimental results were determined from (n,n) [1] and (p,p) [2,5] resonance data The spectroscopic amplitudes are given by $\Theta^{p(n)}_{INC} = [\gamma^2_{p(n)}/\gamma^2_{sp}]^{1/2}$, where $\gamma^2_{p(n)}$ is the reduced width for the observed resonance, and γ^2_{sp} is the single-particle reduced width[‡3] The experimental errors are typically less than 10% and have been suppressed from the

---

[‡3] In this work, as in ref [5], we have used $\gamma^2_{sp} = \hbar/MR^2$ with $M = A/(A + 1)$, and $R = 1\ 4A^{1/3}$ fm There is some uncertainty in this method, sometimes 1 5 times this value is used [1] This uncertainty, however, should be systematic and would tend to shift all the experimental values by the same amount

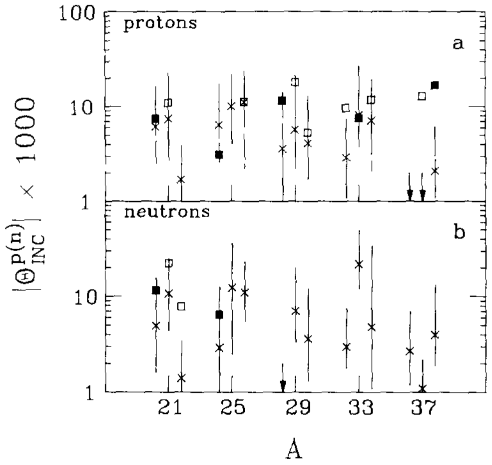

Fig. 2. Comparison between experimental and theoretical spectroscopic amplitudes for the isospin-forbidden decay of $T = 3/2$ states by (a) proton and (b) neutron emission. For each A, one value of each spin is given and is plotted in the order $2J^{\pi} = 5^{+}, 1^{+}, 3^{+}$. Experimental data are represented by square (filled in for the lowest $T = 3/2$ state for each A, and open for the remaining states). Crosses with error bars denote our "best" estimate and its upper and lower limits as described in the text.

figure. Experimental excitation energies for the $T = 3/2$ states were used to evaluate the isospin-mixing amplitudes. In the absence of experimental values, excitation energies of the analog states in the neighboring $T_Z = -3/2$ (neutron rich) nuclei were used.

We find generally good agreement between experiment and our calculations, with the noted exception of $A = 37$. However, in this region, the level density of sd-shell configurations is low compared to experiment, indicating that fp-shell configurations are important. Our results show that Θ^{p}_{INC} is roughly constant as a function of A, with fluctuations about this constant value. This is consistent with the known experimental data. The data for the lowest $T = 3/2$ states in each nucleus show an oscillation with period $\Delta A = 8$. However, the data for the remaining $T = 3/2$ states fail to show this feature. Our results indicate that the observed variations in Θ^{p}_{INC} are due to random or statistical properties of the locations, the INC matrix elements, and the allowed spectroscopic factors of the nearby $T = 1/2$ states.

Any difference between Θ^{p}_{INC} and Θ^{n}_{INC} which is not due to the different energy denominators used to evaluate the isospin-mixing amplitudes is attributable to the isotensor component of the INC interaction. This is because the isovector interaction can cause only $\Delta T = 1$ mixing, and, therefore, would give equal isospin-mixing matrix elements in both the proton- and neutron-rich systems. The sd-shell experimental neutron amplitudes are 2 to 3 times greater than their proton counterparts. Our calculations do not show a systematic asymmetry between the proton and neutron amplitudes, although, individual cases may differ significantly.

The forbidden amplitudes for higher excited states have been evaluated, and were found to decrease slightly with excitation energy and to fluctuate in the same manner as the lower states. This is in agreement with experiment [5]. Although the density of background states is higher, and the isospin-mixing matrix elements are of the same order of magnitude as for lower states, we find that the decrease in $\Theta^{p(n)}_{INC}$ is due to a reduction in the magnitude of the allowed spectroscopic amplitudes.

The exact location and allowed spectroscopic amplitudes for nearby $T = 1/2$ states have been measured [4] for only one case under investigation in this work, the lowest $J^{\pi} = 1/2^{+}$, $T = 3/2$ state in ^{33}Cl. From table 1, it is seen that only three $T = 1/2$ parent states contribute significantly to the total. The experimental excitation energies for these states are 0.8107 [13], 5.451 [4], and 5.740 [4] MeV, respectively. The experimental values for the allowed spectroscopic amplitudes for these states are $-0.7(2)$ [13], 0.135(9)[‡4] and 0.143(8)[‡4] (the signs are taken from the theoretical values), while the theoretical values for these quantities are -0.480, 0.190, and 0.092. In addition, the theoretical isospin-mixing matrix elements for these states are 43.7, -2.2, and -13.7 keV, respectively.

The experimental spectroscopic amplitudes for the neighboring $T = 1/2$ states differ somewhat from the theoretical values. However, the sum of the squares of the experimental values (0.039) does not differ much from the sum of the squares of the theoretical values (0.045), indicating that perhaps these states should be mixed. Therefore, we construct the states $\tilde{\Psi}(4) = \alpha\Psi(4) + \beta\Psi(5)$ and $\tilde{\Psi}(5) = -\beta\Psi(4) + \alpha\Psi(5)$, by re-

---

[‡4] The experimental [4] decay widths for the fourth and fifth $T = 1/2$ states are 30 ± 4 and 40 ± 5 keV, respectively. The Coulomb penetrabilities were taken from ref. [14] and $\gamma^{2}_{sp} = 2.16$ MeV.

quiring that the ratio of the square of the spectroscopic amplitudes for these states be 0 891, giving two solutions (I) $\alpha = 0.9339$ and $\beta = -0\ 3575$, and (II) $\alpha = -0\ 3023$ and $\beta = -0\ 9532$ We find $|\Theta^{p}_{INC}| \times 1000$ to be 12 9 ± 1 8 and 21 0 ± 1 8 for solutions I and II, respectively, while the experimental value is 7 5 ± 0 3 Since solution II gives a value which is three times the experimental value, and represents a crossing between the fourth and fifth levels, we can reject this solution Since the 5 451 MeV state is only 97 keV away from the $T = 3/2$ state, Θ^{p}_{INC} is very sensitive to the small isospin-mixing matrix element with this state Unfortunately, the INC interaction is not known well enough to accurately evaluate small isospin-mixing matrix elements In fact, we estimate an uncertainty of about 5 keV in the off-diagonal INC matrix elements (this is based on the fact that a theoretical error of about 5 keV must be added in the fit to the *c*-coefficients of the IMME in order to obtain a reduced chi-squared deviation between experiment and theory of unity [12])

From the above analysis, it is apparent that rather large variations in $\Theta^{p(n)}_{INC}$ are associated with the properties of the nearby $T = 1/2$ states We propose that the oscillation observed in Θ^{p}_{INC} is in fact part to these random variations, rather than the presence of other more exotic processes such as Δ-isobar configurations

Finally, we conclude with some remarks regarding possible future work One possibility is the decay of $T = 5/2$ states in $A = 4n + 1$ nuclei The level density of background $T = 1/2$ states, however, is rather high, indicating that these decays again may be difficult to calculate accurately An enterprise which is perhaps more fruitful than the decay of $T = 3/2$ states from the theoretical standpoint, is the decay $T = 2$ states in $T_Z = 0$ and -1 nuclei Here the density of background sd-shell states is considerably lower With this in mind, we plan to make a similar theoretical investigation of the INC proton, neutron, and alpha decays of $T = 2$ states

We wish to acknowledge several useful discussions with T B Clegg, E J Ludwig, and J F Wilkerson This research was supported in part by the National Science Foundation Grant No PHY-83-12245

References

[1] R Weigman, R L Macklin and J A Harvey, Phys Rev C 14 (1976) 1328,
F Hinterberger et al , Nucl Phys A352 (1981) 93, A424 (1984) 200
[2] A B McDonald, J R Patterson and H Winkler, Nucl Phys A137 (1969) 545
[3] J C Adloff et al , Phys Rev C 5 (1972) 664
[4] P G Ikossi et al , Nucl Phys A274 (1976) 1
[5] J F Wilkerson et al , Phys Rev Lett 51 (1983) 2269
[6] A B McDonald and E G Adelberger, Phys Rev Lett 40 (1978) 1692
[7] A Lev and N Auerbach, Nucl Phys A206 (1973) 563
[8] N Auerbach, Phys Rep 98 (1983) 273
[9] A Arima and S Yoshida, Nucl Phys A161 (1971) 492
[10] N Auerbach and A Lev, Phys Lett B 34 (1971) 13
[11] B H Wildenthal, Prog Part Nucl Phys , Vol 11, ed D H Wilkinson (Pergamon, Oxford, 1984) p 5
[12] W E Ormand and B A Brown, Nucl Phys A440 (1985) 274
[13] P M Endt and C Van Der Leun, Nucl Phys A310 (1978) 1
[14] J B Marion and F C Young, Nuclear reaction analysis (Wiley, New York, 1968)