

Gamow-Teller and $M1$ strength in the $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction at 135 MeV

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(Received 28 May 1987)

The $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction was studied at 135 MeV by the time-of-flight technique. The overall energy resolution obtained was about 270 keV. The forward-angle spectra are dominated by the excitation of 1^+ states with characteristic $\Delta L = 0$ angular distributions peaked at 0° . The 1^+ spectrum is described well by distorted-wave Born approximation calculations with full S - D shell-model wave functions and a normalization factor of 0.60. The strengths of the 1^+ excitations are interpreted as being equivalent to Gamow-Teller strengths excited in beta decay. The (p,n) cross sections are converted to reduced transition probabilities, $B(\text{GT})$, by means of a "universal" relationship obtained from other nuclei where analog beta decay measurements are available. The general distribution and total strength so obtained are represented well by full S - D shell-model calculations with Gamow-Teller matrix elements adjusted to reproduce the strengths of Gamow-Teller beta decays of S - D shell nuclei. The 1^+ (p,n) cross sections are interpreted also in terms of magnetic-dipole ($M1$) strength and compared with inelastic-electron scattering measurements of such strength in ^{32}S . The strength observed in individual states is seen to vary considerably between the two reactions; these differences are ascribed to orbital-current contributions in the electron-scattering reaction not present in the (p,n) reaction. The (p,n) measurements extend to higher excitation energies than the electron-scattering measurements.

I. INTRODUCTION

The quenching of Gamow-Teller (GT) strength in nuclei is a topic of high current interest. This quenching has been observed both directly in beta decay and also via various nuclear reactions, especially the (p,n) reaction at medium energies. One of the first indications of GT quenching was seen in the renormalization of the GT matrix elements required to reproduce the magnitudes of observed beta decay strengths in the S - D shell. Using full S - D shell-model wave functions, Brown *et al.*¹ found that GT matrix elements are quenched systematically relative to the values required for consistency with the decay of the free neutron. In a recent analysis,² which includes all of the S - D shell β -decay data, the experimental GT decay strength was found to be 60% of the calculated value. Beta decays usually have small Q values so that only the lowest few levels of the residual nuclei are sampled. The strong spin-flip nature of GT excitations is expected to push much of the GT strength to excitation energies characteristic of the spin-orbit splitting, namely 7–10 MeV; as a consequence, beta-decay studies sample only a fraction of the total strength. Thus, one possible explanation for the

"quenching" of GT strength observed in the beta decays of the S - D shell is that more of the GT strength is pushed to higher excitation energies, where it is inaccessible in beta decay, than the shell-model calculations predict.

Insofar as the (p,n) reaction proceeds predominantly via one-step processes, transitions to 1^+ states from even-even nuclei must proceed via the isovector spin-transfer term of the nucleon-nucleon effective interaction. At low momentum transfer, the strengths of these transitions are similar to those of GT beta decays. The usefulness of (p,n) studies of GT strength lies in the fact that the (p,n) reaction is not limited by Q -value restrictions and can excite the entire profile of GT strength. Thus the (p,n) reaction provides a more complete sampling of the distribution of GT strength. The distribution of strength will be directly related to the configuration mixing in the residual nucleus, so that this mixing must be addressed in the theoretical calculations. In earlier work, Anderson *et al.*³ and Madey *et al.*⁴ presented detailed comparisons of the experimental 1^+ , GT spectra in the $^{18}\text{O}(p,n)^{18}\text{F}$ and $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reactions at 135 MeV with distorted-wave-impulse-approximation (DWIA) calculations based on the S - D

shell-model wave functions of Wildenthal *et al.*⁵ In both of these studies, generally good agreement between the experimental results and the theoretical predictions was observed with an overall normalization factor of about 0.60 required, similar to that required for the beta decay comparisons in the *S-D* shell. The general distribution of GT strength was in good agreement with the shell-model predictions.

We present here an analysis of the $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction similar to the earlier analyses of the (p,n) reaction on ^{18}O and ^{26}Mg . In the simple shell model, the ^{32}S nucleus has orbitals filled up through the $2s_{1/2}$ orbital so that, similar to the earlier studies, positive-parity excitations should be dominated by the *S-D* shell degrees of freedom. Both ^{18}O and ^{26}Mg are $T=1$ nuclei, so that the (p,n) reaction (with $\Delta T=1$) can excite $T=0, 1$, and 2 strengths in the residual nucleus. The overall agreement of the theoretical predictions with the experimental results was good in both cases; however, it is difficult to make unambiguous isospin assignments for the high-lying excitations where the different isospin components are expected to overlap. Because ^{32}S is a self-conjugate nucleus, the (p,n) reaction will excite only $T=1$ strength in the residual nucleus, ^{32}Cl . Thus, the study of GT strength in the $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction nicely extends the earlier studies on ^{18}O and ^{26}Mg ; it provides not only another test case in the *S-D* shell, but also a somewhat simpler spectrum for theoretical analysis.

The GT strength distribution in ^{32}Cl was studied recently by Bjornstad *et al.*⁶ via the beta decay of ^{32}Ar . By comparison with an earlier version⁷ of the *S-D* shell wave functions, they concluded that they observed $49\pm 5\%$ of the expected GT strength within the kinematic window available. Although this particular beta decay has a relatively large kinematic window, it still cuts off at about 8.75 MeV of excitation in ^{32}Cl . It will be important to compare the (p,n) results with these measurements as well as to the (p,n) measurements on the other *S-D* shell nuclei.

A subject closely related to the excitation or GT strength is the excitation of $\Delta T=1$ magnetic dipole (*M1*) strength. The latter has been studied extensively via backward-angle inelastic electron scattering and has been observed also to be quenched relative to simple expectations; for example, Burt *et al.*⁸ show that the total *M1* strength observed from inelastic electron scattering on the four self-conjugate *S-D* shell nuclei ^{20}Ne , ^{24}Mg , ^{28}Si , and ^{32}S is between 57 and 76% of the Kurath energy-weighted sum rule.⁹ Note that it is important to keep in mind that GT and *M1* excitations are inherently different, although related. A true GT transition, such as GT beta decay, involves only spin-transfer strength, whereas inelastic electron scattering can involve orbital-current contributions also. Insofar as the (p,n) reaction is dominated by spin-transfer strength, it is more analogous to beta decay than to inelastic electron scattering. In an earlier study,¹⁰ it was shown for various *S-D* shell nuclei there is not a simple one-to-one relationship between 0° (p,n) cross sections and $B(M1)$ values obtained from electron scattering. In fact, the comparison of 0° (p,n) cross sections with $B(M1)$ values can be used to

identify experimentally the presence of significant orbital contributions. The earlier work showed that shell-model predictions for a number of relatively strong *M1* transitions in the *S-D* shell were in reasonable agreement with the observations. Thus, it is important for the present work to compare the 0° (p,n) measurements with the $B(M1)$ values obtained by Burt *et al.*⁸ from electron scattering on ^{32}S . In particular, they note that the total $B(M1)$ strength observed in ^{32}S is a significantly smaller fraction of the expected strength than for the other self-conjugate nuclei in the *S-D* shell. The (p,n) measurements can provide a related determination of this strength.

II. EXPERIMENTAL PROCEDURE

The experiment was performed at the Indiana University Cyclotron Facility with the beam-slinger system. The basic experimental arrangement and data-reduction procedures were similar to those described previously.^{11,12}

Neutron kinetic energies were measured by the time-of-flight (TOF) technique. A beam of 135 MeV protons was obtained from the cyclotron in narrow beam bursts with a duration of typically 350 ps. Neutrons were detected in three detector stations at 0° , 24° , and 45° with respect to the undeflected proton beam. The flight paths were 125.2, 133.6, and 80.9 m, respectively. For the purposes of this work, only measurements from the 0° and 24° stations were considered. The neutron detectors were rectangular bars of fast plastic scintillator 10.2 cm thick. Three separate detectors each 1.02 m long by 0.51 m high were combined for a total frontal area of 1.55 m² in the 0° station. Two detectors were used in the 24° station, one was 1.02 m long by 1.02 m high and the other was 1.02 m long by 0.51 m high, for a combined frontal area of 1.55 m². Each neutron detector had tapered Plexiglas light pipes attached on the two ends and coupled to 12.7 cm diam phototubes. Timing signals were derived from each end and combined in a mean-timer circuit¹³ to provide the timing signal from each detector. Overall time resolutions of about 800 ps were obtained, including contributions from the beam burst width (~ 350 ps) and energy spread (~ 400 ps), energy loss in the target (~ 450 ps), neutron transit times in the detectors (~ 530 ps), and the intrinsic time dispersions of each detector (~ 300 ps). (Note that these contributions are not all Gaussian and do not combine simply in quadrature. See Ref. 14.) This overall time resolution provided an energy resolution of about 270 keV. The large-volume neutron detectors were described in more detail previously.¹⁴

The ^{32}S target was a 43.8 mg/cm² self-supporting foil of Li_2S , which was made by pressing Li_2S powder in a specially prepared steel die.^{8,15} The $^{32}\text{S}(p,n)$ spectra were obtained by subtraction of TOF spectra obtained with a 40.8 mg/cm² Li-foil target. The TOF spectra from the LiS and Li targets are shown in Fig. 1. The Li subtraction was normalized to the strong $^7\text{Li}(p,n)^7\text{Be(g.s.} + 0.43 \text{ MeV)}$ transition observed in the spectra. The $^7\text{Li}(p,n) Q$ value is -1.64 MeV compared to -13.98 MeV for the

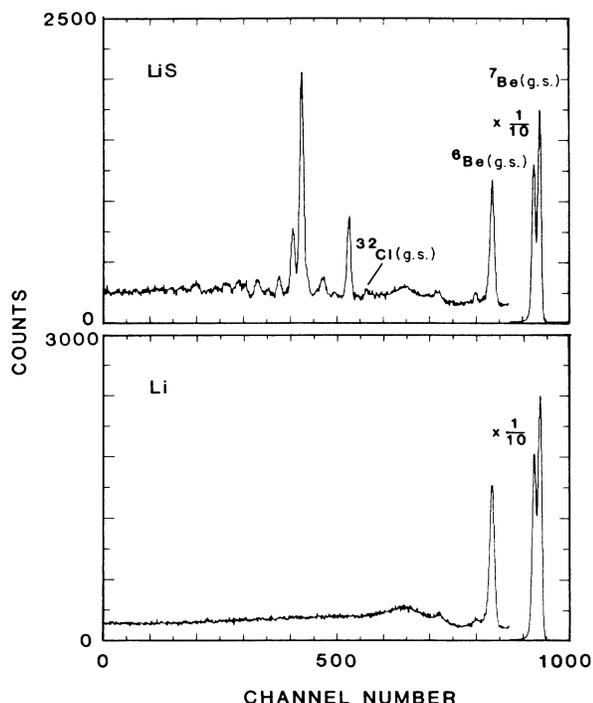


FIG. 1. Neutron time-of-flight (TOF) spectra from the (p,n) reaction on LiS and Li targets. The Li(p,n) TOF spectrum was subtracted from the LiS(p,n) spectrum to obtain the $^{32}\text{S}(p,n)$ spectrum shown in Fig. 2.

$^{32}\text{S}(p,n)$ reaction, so that the part of the subtracted Li(p,n) spectrum which overlapped the $^{32}\text{S}(p,n)$ spectra corresponded to relatively high excitation energy in ^7Be , where the spectrum is observed to be relatively smooth.

III. DATA REDUCTION

During the experimental run, neutron TOF and detector pulse-height spectra were recorded simultaneously at various pulse-height thresholds between 30 and 70 MeV equivalent-electron energy (MeVee). For a final analysis, a threshold of 55 MeVee was chosen as the best compromise between higher thresholds which reduce overlap of slow neutrons from the previous beam burst and lower thresholds which provide increased counting statistics. We obtained excitation-energy spectra from the measured TOF spectra using the known flight path and a calibration of the time-to-amplitude converter. The known Q value of the $^7\text{Li}(p,n)^7\text{Be}(g.s.)$ reaction served as a calibration point for determining absolute neutron energies. The excitation-energy spectrum at 0° is shown in Fig. 2.

Yields for transitions in the $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction were obtained by peak fitting of the Li-subtracted TOF spectra. Cross sections were obtained by combining the yields with the measured geometrical parameters, the beam integration, and the measured target thickness. The neutron detector efficiencies were obtained from a Monte Carlo computer code¹⁶ which has been tested extensively at these energies.^{17,18} The overall absolute

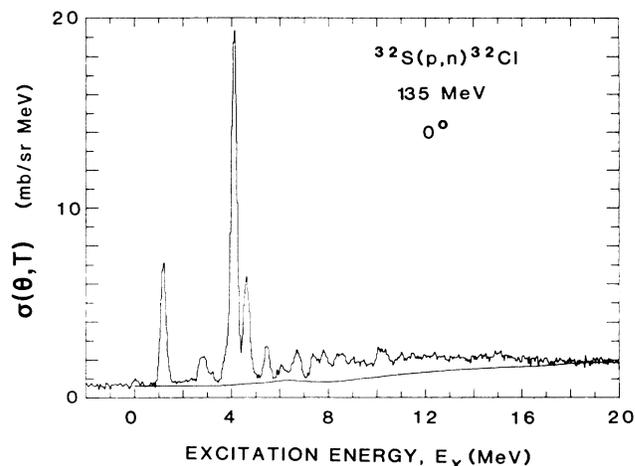


FIG. 2. The experimental $^{32}\text{S}(p,n)^{32}\text{Cl}$ excitation-energy spectrum at 135 MeV and 0° . The solid line represents the sum of the cosmic-ray and overlap neutron backgrounds plus a calculated quasifree scattering background (see text).

cross sections so obtained were checked by remeasuring the known $^{12}\text{C}(p,n)^{12}\text{N}(g.s.)$ reaction.¹⁷ The experimental procedure and data reduction is similar to that described in more detail in Refs. 10 and 11. The uncertainty in the overall scale factor is dominated by the uncertainty in the detector efficiencies and is estimated to be $\pm 12\%$.

The spectra were fitted with an improved version of the peak-fitting code of Bevington.¹⁹ For each angle, the spectrum was fitted in two regions: (1) the low-lying region from 0 to 6 MeV of excitation (in ^{32}Cl) and (2) the high-lying region from 6 to 12 MeV of excitation. As seen in Fig. 2, the peaks in the low-lying region are separated relatively well and have a small, nearly flat background. Because the widths of the peaks in this region are dominated by the instrumental resolution, they are observed to be equal; consequently, the peaks in this region were fitted by simple Gaussian peaks all constrained to have the same width, on top of a smooth background. The fits were good (as judged by small reduced chi-squared values) and appear to be unambiguous.

The peaks in the high-lying region (≥ 6 MeV) are seen to be broader and overlapping and to be on top of a rising background. It is possible to describe this background as a combination of neutron yield from quasifree scattering (QSF) [i.e., $^{32}\text{S}(p,pn)$] and from excitation of the $\Delta L = 1$ giant resonance in the (p,n) reaction. This procedure is similar to that in the earlier analyses of GT strength in the continuum above the GT giant resonance in the $^{48}\text{Ca}(p,n)$ reaction at this energy.¹² In $^{32}\text{S}(p,n)$ the QFS contribution to this background was estimated by performing a plane-wave-impulse-approximation (PWIA) calculation which includes a partial accounting for Pauli blocking.¹² The results of this calculation are shown in Fig. 2, normalized to the continuum near $E_x = 22$ MeV, which is near the peak in the calculated QFS spectrum.

The remaining "background" strength (above the QFS calculation) is observed to be peaked away from 0° , as

seen from the comparison of the 6° spectrum with the 0° spectrum shown in Fig. 3. Gamow-Teller strength has $\Delta L=0$ and is known to be peaked at 0° in the (p,n) reaction at this energy.¹² The entire spectrum above 6 MeV is seen to increase at the wider angle. The strength above the QFS calculation can be described by a slightly skewed Gaussian and the angular distribution for this strength is consistent with that expected for $\Delta L=1$ transitions. We note that this result is different than that observed in the case of $^{48}\text{Ca}(p,n)$, where the continuum immediately above the giant GT resonance is actually peaked at 0° . One expects, on general grounds, that there may be some GT (1^+) strength in the continuum in the $^{32}\text{S}(p,n)$ reaction also. As the first step in determining GT strength in ^{32}Cl , we consider only the peaks seen in the (p,n) spectrum. Because the peaks in the 6–12 MeV region are broader than the low-lying peaks, they were fitted with Gaussians whose widths were allowed to vary. The location of the peaks in energy were chosen to provide consistency from angle to angle. From these fits, angular distributions were extracted. These angular distributions were then analyzed to extract $\Delta L=0$ strength at 0° . Only a few peaks in this region were observed to have obvious $\Delta L=0$ (i.e., peaked at 0°) angular distributions. Several other transitions were seen to be complexes apparently involving some $\Delta L=0$ strength.

The extracted $\Delta L=0$ cross sections at 0° , associated with peaks in the entire 0–12 MeV range of excitation energy, are listed in Table I. The uncertainties presented for each peak are relative uncertainties only. For transitions up to 6 MeV, the uncertainties are taken from the error matrix of the fitting code. Above 6 MeV, because of the additional uncertainty in the fitting procedure, an estimated uncertainty of $\pm 30\%$ is indicated for each transition. For a few of the smaller peaks, the transitions appear to be mixed complexes of $\Delta L=0$ and 1 transitions; in such cases we extracted the $\Delta L=0$ contribution by fitting the angular distribution with “standard” $\Delta L=0$ and 1 shapes. The total amount of GT

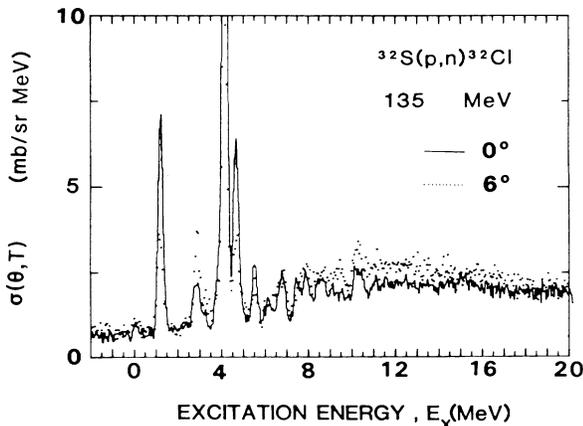


FIG. 3. Comparison of the experimental excitation-energy spectra from the $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction at 0° (histogram) and 6° (data points). The vertical scale is increased from Fig. 2.

TABLE I. Experimental $^{32}\text{S}(p,n)^{32}\text{Cl}(1^+)$ cross sections at 135 MeV.

E_x (MeV)	$\sigma_{\text{GT}}(0.2^\circ)$ (mb/sr)	$\sigma_{\text{GT}}(q=0)$ (mb/sr)	$B_{\text{pn}}(\text{GT})$
0.00	0.07 (0.04 ^a)	0.09	0.014
1.15	1.64 (0.02)	2.18	0.344
2.79	0.34 (0.01)	0.45	0.071
3.73	0.12 (0.03)	0.16	0.025
4.06	4.78 (0.03)	6.36	1.005
4.58	1.47 (0.02)	1.96	0.310
5.41	0.41 (0.01)	0.55	0.087
6.06	0.16 (0.01)	0.21	0.033
6.29	0.10 (0.03)	0.13	0.021
6.65	0.32 (0.11)	0.43	0.068
7.36	0.28 (0.09)	0.37	0.058
7.72	0.26 (0.09)	0.35	0.055
8.30	0.14 (0.04)	0.19	0.030
8.60	0.15 (0.05)	0.20	0.032
10.00	0.15 (0.05)	0.20	0.032
10.30	0.13 (0.04)	0.17	0.027
$\Sigma = 10.52 (0.20)$ (1.58) ^b		14.00	2.212

^aIndividual uncertainties are relative only (see text).

^bNet uncertainty includes scale uncertainty (see text).

strength obtained from such mixed transitions is small, less than 5% of the total, and is not sensitive to the “standard” shapes assumed. The net relative uncertainty in the sum is seen to be only about 2% of the total. It is so small because 75% of the total sum is in the three largest peaks and these are fit very well; only 15% of the total strength is above 6 MeV. To the relative uncertainty we must add a scale uncertainty of $\pm 12\%$, arising from uncertainties in the target thickness, beam integration, and detector efficiencies (see above). Also if we allow as much as a 33% error in the total strength above 6 MeV because of the choice of background in this region, we have a net uncertainty in the total 1^+ “peak” strength of about $\pm 15\%$.

IV. THEORETICAL ANALYSIS AND INTERPRETATION

We will consider the theoretical analysis of the (p,n) measurements in three separate ways. For the reaction considered here, no reliable analog beta decay is available for normalizing 0° (p,n) cross sections in units of $B(\text{GT})$ (such as was possible for the earlier studies^{3,4} on ^{18}O and ^{26}Mg). There does exist²⁰ an experimental $\log ft$ value for the beta decay of the ground state of ^{32}Cl back to the ground state of ^{32}S ; however, because this transition is highly hindered, it is not reliable for this purpose. (For example, the $\log ft$ value²⁰ of the analog β^- decay of ^{32}P to the ground state of ^{32}S differs significantly.) Also, since there is no transition to the isobaric analog state (IAS) of the target ground state, one cannot compare the strength observed to 1^+ GT states with the Fermi strength assumed to be concentrated in the IAS tran-

sition, similar to the analysis performed for the $^{48}\text{Ca}(p,n)^{48}\text{Sc}$ reaction.¹² [This latter analysis is based on the determination by Taddeucci *et al.*²¹ of the ratio of isovector spin-flip to non-spin-flip strengths in the nucleon-nucleon central interaction, $(t_{\sigma\tau}/t_{\tau})$.]

We will begin by simply comparing the experimental results with DWIA calculations using full S - D shell structure wave functions. In the second analysis, we will obtain a “universal” relationship between 0° (p,n) cross sections and $B(\text{GT})$ values from several other reactions where reliable analog beta decays exist. With this relationship we can convert the (p,n) cross sections directly to $B(\text{GT})$ values and compare with values expected from shell-model calculations. Finally, in the third analysis we will compare the 0° (p,n) cross sections with $B(M1)$ values obtained from inelastic electron scattering. As discussed in the Introduction, such comparisons may be especially useful to identify orbital-current contributions in the $M1$ transitions.

A. Comparison with DWIA calculations

As discussed above, we begin by considering only GT strength in the observed peaks; we will return later to discuss possible contributions from the background and continuum. The 1^+ experimental cross sections at 0° (from Table I) are shown in the top half of Fig. 4. The bottom half of Fig. 4 shows the result of DWIA calculations, with full S - D shell wave functions,⁵ for 1^+ excitations in the $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction. The DWIA calculations were performed with the computer code DW81,²² the nucleon-nucleon effective interaction at 140 MeV of Franey and Love,²³ and the global optical-model parameters of Schwandt *et al.*²⁴ Harmonic-oscillator wave functions were assumed for the single-particle orbitals with an oscillator parameter $b = 1.88$ fm, consistent with

that required to reproduce the rms charge radius of ^{32}S .²⁵ The general agreement between the predictions and the experimental spectrum is seen to be good. The locations of the three strongest excitations are described well and the gradual falloff of strength between 5 and 10 MeV is indicated. Although the predictions have the relative strengths of the two strongest excitations (near $E_x = 4$ MeV) reversed from that observed, this is not so significant because the mixing between two states less than 1 MeV apart is sensitive to small details of the structure wave functions; however, one might hope that this defect will be removed in future generations of S - D shell wave functions. The general agreement between the theoretical calculations and the experimental results shown in Fig. 4 would be observed also if we compared the measurements with the earlier S - D shell-model calculations for GT strength in ^{32}S due to Muller *et al.*²⁶ These calculations were based on the S - D matrix elements of Wildenthal and Chung⁷ which do not reproduce the total set of S - D experimental parameters as well as the later “universal” S - D matrix elements of Wildenthal⁵ used in the calculations presented here.

The experimental angular distributions of the three largest excitations are shown in Fig. 5 compared with DWIA calculations for these transitions. All three are peaked at 0° and have shapes which are described well by DWIA calculations for transitions to 1^+ states in ^{32}Cl . The individual normalization factors vary and are sensitive to the details of the configuration mixing in the shell-model wave functions. The strength summed over all final states is much less dependent on the details of the final-state wave functions. From Table I, the total 1^+ strength observed in this experiment is 10.5 ± 1.6 mb/sr. The full S - D shell model calculations predict 30 1^+ states between 0 and 10.2 MeV with a total strength

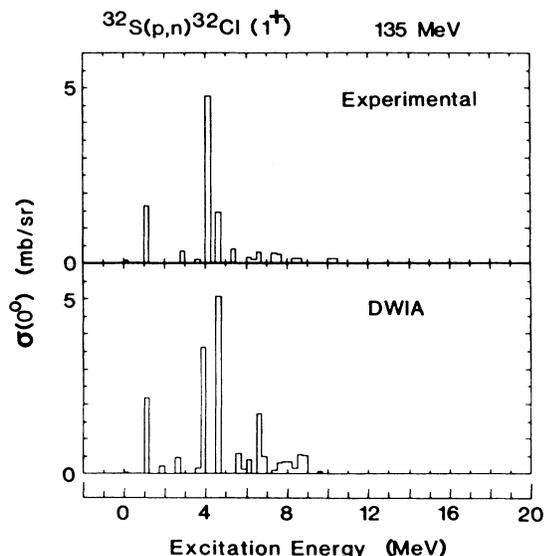


FIG. 4. Comparison of the experimental and DWIA predicted 1^+ distribution functions at 0° . Both spectra are binned in 0.25 MeV steps, which is about equal to the experimental resolution (0.27 MeV).

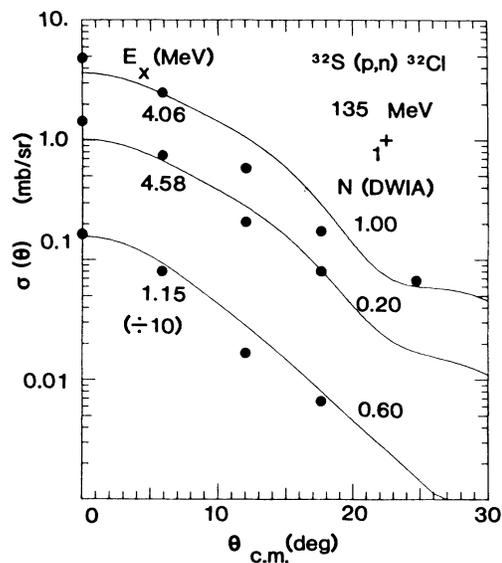


FIG. 5. Angular distributions for the three largest 1^+ transitions in the $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction is 134 MeV. The solid lines represent DWIA calculations for each transition, with the normalizations indicated (see text).

calculated in the DWIA to be 17.6 mb/sr at 0° . Thus the observed ratio of experimental to theoretically predicted 1^+ , GT strength in this reaction is 0.60 ± 0.09 . This ratio is consistent with that observed for the quenching of beta-decay strengths in the S - D shell² and also in the (p,n) reactions on the S - D shell nuclei ^{18}O and ^{26}Mg reported earlier.^{3,4}

The total observed strength might be increased slightly by consideration of possible GT strength in the background and continuum. In an analysis of the $^{48}\text{Ca}(p,n)^{48}\text{Sc}$ reaction at this same energy,¹² it was found that the background under the Gamow-Teller giant resonance (GTGR) and the continuum just above the GTGR displayed angular distributions consistent with there being a significant amount of GT (i.e., $\Delta L=0$) strength in these regions. Such contributions appear to be much weaker in the $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction. First of all, the GT strength in this reaction is concentrated primarily in three discrete transitions seen between 1 and 6 MeV of excitation. The background in this region is small and clearly dominated by the wrap-around neutron and cosmic-ray backgrounds; the level of this background can be seen below the ground state in Fig. 2. No significant amount of GT strength can be ascribed to this background.

As seen in Fig. 3, the continuum region above 6 MeV is peaked away from 0° , consistent with strength dominated by $\Delta L=1$ transitions. This is the region where we expect the $\Delta L=1$ giant resonance to appear, and indeed, this region can be fit by a broad (skewed) Gaussian with an angular distribution consistent with that expected for $\Delta L=1$ transitions. As discussed above, this reaction appears to be different than the $^{48}\text{Ca}(p,n)$ reaction, where the continuum just above the giant GT resonance is actually peaked at 0° , indicating the presence of a significant amount of GT (1^+) strength.¹² For $^{48}\text{Ca}(p,n)$ it was found that about 70% of the continuum, above the QFS background could be considered to be GT strength. Note that the QFS calculation is constrained to fit the continuum in the region of the peak observed in the calculations, viz., about 20 MeV above the giant GT resonance; therefore the GT strength extracted for the continuum this way is confined to this region. For the $^{32}\text{S}(p,n)$ reaction, where there appears to be less evidence for such strength in the continuum, we estimate that no more than one-half of the background above the QFS contribution could be considered to be GT strength, analyzed in a manner similar to that for the $^{48}\text{Ca}(p,n)$ reaction. If we use one-quarter of this “residual” background as our rough estimate, we obtain the result that the GT strength in the continuum, up to about 20 MeV of excitation, is 1 ± 2 mb/sr. (The uncertainty reflects both the uncertainty associated with the assumed fraction for the “residual” continuum and the uncertainty associated with the QFS subtraction.) Note that this contribution from the continuum is a small fraction of the total observed in discrete peaks. The results observed in the $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction seem to be more like those observed in the (p,n) reaction on the S - D shell nuclei ^{18}O and ^{26}Mg .^{3,4} Typically, the GTGR is observed to be primarily in discrete peaks in light nuclei;

and in a broad, coherent resonance in heavy nuclei. The GTGR in the $^{48}\text{Ca}(p,n)^{48}\text{Sc}$ reaction appears to be an intermediate case where the discrete peaks are moving together and overlap considerably. The GT strength distribution observed in the $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction appears more like that observed in light nuclei.

B. Gamow-Teller strength

Because many reservations exist about a DWIA analysis, we wish now to present an alternative analysis which converts the 0° (p,n) cross sections more directly to $B(\text{GT})$ values. The basic idea is that we can use the cases where reliable analog beta decays exist to establish a “universal” relationship between a 0° (p,n) cross section and a $B(\text{GT})$ value. This method is similar to the earlier analysis performed by Goodman *et al.*²⁷ To review the argument, recall that in the factorized DWIA, the (p,n) cross section can be written as

$$\sigma_{\text{GT}}(0^\circ) = 8\pi N_D \left[\frac{\mu}{2\hbar^2} \right]^2 \left[\frac{k_f}{k_i} \right] B(\text{GT}) |V_{\sigma\tau}(q)|^2, \quad (1)$$

where N_D is a distortion factor, k_f and k_i are the final-state and initial-state wave numbers, and $V_{\sigma\tau}$ is the strength of the spin-transfer, isospin-transfer term in the nucleon-nucleon effective interaction. The GT matrix element, $B(\text{GT})$, contains the nuclear-structure overlap integral and is the same as the matrix element sampled in beta decay. The beta decay matrix element can be obtained from the experimental ft value using the relationship for a pure GT transition⁴

$$B_\beta(\text{GT}) = \frac{3874}{ft}. \quad (2)$$

In order to obtain a universal relationship, we extrapolate the (p,n) cross sections to zero momentum transfer because then we always involve the same value of $V_{\sigma\tau}(q=0)$, and $k_f/k_i=1$. Then, from Eq. (1), we obtain

$$B_{\text{pn}}(\text{GT}) = \frac{\sigma_{\text{pn}}(q=0)}{N_D} C_{\text{GT}}, \quad (3)$$

where C_{GT} is a “universal” constant to be obtained from comparison with the analog beta decay results. This “constant,” of course, depends on energy; however, the present analysis will be only for 135 MeV. The distortion factor N_D can be estimated in the usual way from

$$N_D = \frac{\sigma_{\text{DW}}(0^\circ)}{\sigma_{\text{PW}}(0^\circ)}, \quad (4)$$

where the distorted-wave (DW) and plane-wave (PW) cross sections are calculated with a standard DWIA code. If we take the comparison of the $^{26}\text{Mg}(p,n)^{26}\text{Al}(1.06 \text{ MeV}, 1^+)$ reaction with the analog beta decay of $^{26}\text{Si}(\beta^+)^{26}\text{Al}(1.06 \text{ MeV}, 1^+)$ to determine the value of the constant in Eq. (3), we obtain $C_{\text{GT}}=0.0640$, as indicated in Table II. Using this value, we then obtain the other three results presented in Table II, which involve relatively strong GT transitions and for which analog beta-decay measurements are available.

TABLE II. Comparison of 0^+ (p,n) cross sections with analog beta decays.

β decay	$\log ft$	$B_\beta(\text{GT})^a$	(p,n)	σ (0^+) (mb/sr)	σ ($q=0$) (mb/sr)	N_D^b	$B_{\text{pn}}(\text{GT})^c$
$^{26}\text{Si}(\beta^+)^{26}\text{Al}$	3.550	1.09	$^{26}\text{Mg} \rightarrow ^{26}\text{Al}$	7.43	7.80	0.458	1.09
$^{12}\text{N}(\beta^+)^{12}\text{C}$	4.119	0.885 ^d	$^{12}\text{C} \rightarrow ^{12}\text{N}$	5.90	7.54	0.527	0.916
$^{14}\text{O}(\beta^+)^{14}\text{N}$	3.15	2.74	$^{14}\text{C} \rightarrow ^{14}\text{N}$	19.96	20.95	0.518	2.59
$^{18}\text{Ne}(\beta^+)^{18}\text{F}$	3.088	3.16	$^{18}\text{O} \rightarrow ^{18}\text{F}$	23.92	25.11	0.495	3.25

^a $B_\beta(\text{GT})=3874/ft$ (see Ref. 4).

^b $N_D=\sigma_{\text{DW}}/\sigma_{\text{PW}}$.

^c $B_{\text{pn}}(\text{GT})=[\sigma_{\text{pn}}(q=0)/N_D]0.0640$.

^d0.885=0.295 \times 3 for detailed balance for inverse reaction.

The comparisons are seen to be excellent; all four agree to better than 7%. In addition, we note that if the relationship of Eq. (3) is used to convert the $^{48}\text{Ca}(\text{p},\text{n})^{48}\text{Sc}(1^+)$ peak cross sections reported earlier,¹² one obtains $B_{\text{pn}}(\text{GT})=10.7$, which is 44% of the simple $3(N-Z)$ sum rule for ^{48}Ca . This result agrees with that obtained for this nucleus in the analysis performed in Ref. 12, where the GT strength is considered relative to the observed Fermi strength assumed to be concentrated in the 0^+ isobaric analog state. Similarly, results obtained using the universal conversion of Eq. (3) for several heavy nuclei, including ^{208}Pb , are in good agreement with the analyses performed by Madey *et al.*,²⁸ which also uses the comparison with the observed Fermi strength [viz., about 50 to 60 % of the $3(N-Z)$ sum rule is observed in these nuclei]. Thus, Eq. (3) does appear to be generally valid.

The above comparisons are all for relatively strong transitions from even-even target nuclei. Certain other cases may not be described well by Eq. (3). Certainly the (p,n) reaction, mediated by the strong nuclear force, is not identical to β decay. Weak transitions are naturally suspect. Two-step processes may be significant in such cases and these processes may be quite different in the two different reactions. The case of the ground-state transition in the $^{32}\text{S}(\text{p},\text{n})^{32}\text{Cl}$ reaction discussed above is a good example of such difficulties in a weak transition. Additionally, there may be certain singular transitions which deviate significantly from Eq. (3), even for relative strong transitions. Watson *et al.*²⁹ showed that such deviations may be observed in the strong GT transitions in the $A=15$ and 39 systems. Thus, one might use Eq. (3) only as a starting point, keeping in mind that it may not be a truly "universal" relationship.

Accepting the caveats of the above paragraph, we apply Eq. (3) to the $^{32}\text{S}(\text{p},\text{n})^{32}\text{Cl}$ cross sections. The results are presented in Table I. The total $B_{\text{pn}}(\text{GT})$ is seen to be 2.21. If one assumes the simple shell model for ^{32}S , the standard GT matrix elements⁷ predict $B(\text{GT})=9.60$. If one uses a shell-model calculation involving the full S - D model space with the matrix elements of Wildenthal⁵ and a GT transition operator consistent with the beta decay of the free neutron, one obtains a prediction of $B(\text{GT})=4.00$. If one uses the full S - D model space with the matrix elements of Wildenthal and the renormalized

beta decay operator of Brown and Wildenthal,¹ which was adjusted to reproduce the magnitudes of beta decay $B(\text{GT})$ values in the S - D shell, then one obtains a prediction of $B(\text{GT})=2.11$, which agrees with the experimental $B_{\text{pn}}(\text{GT})$ in Table I.

Note that the total GT strength observed is only a small amount of that expected in the simple shell model. Using a full S - D model space calculation reduces the amount predicted by more than a factor of two. Finally, using the renormalized GT operators empirically determined from beta decays in the S - D shell reduces the predicted amount by almost another factor of two. The final prediction is in good agreement with the amount observed, both in terms of total strength and the distribution of this strength in excitation energy. This agreement is illustrated in Fig. 6 where the differential and integral distributions of experimental strength are compared with theory.

The GT strength function in ^{32}Cl was studied also via the beta decay of ^{32}Ar .⁶ It is worthwhile to compare the (p,n) results with the beta decay results; however, it is necessary to realize that this beta decay is not the simple analog of the $^{32}\text{S}(\text{p},\text{n})^{32}\text{Cl}$ reaction. The ^{32}Ar and ^{32}S ground states have completely different, independent, wave functions, with $T=2$ and 0, respectively. The β^+ decay of ^{32}Ar to ^{32}Cl is a $\Delta T=1$ transition from a $T_z=2$ nucleus to a $T_z=1$ nucleus which can populate $T=1, 2,$ and 3 states. The $^{32}\text{S}(\text{p},\text{n})^{32}\text{Cl}$ reaction is a $\Delta T=1$ transition from a $T=0$ nucleus and populates only the $T=1$ states in ^{32}Cl . Thus, the (p,n) reaction is expected to populate only a fraction of the states observed in the beta decay. The β^+ decay of ^{32}Ar to ^{32}Cl is somewhat unusual in that the kinematic window allows the decay to excite a large fraction of the expected distribution of GT strength in the residual nucleus, viz., up to 8.75 MeV of excitation in ^{32}Cl . By comparison with an earlier version⁷ of the S - D shell-model calculation, Bjornstad *et al.*⁶ reported that they observed $49\pm 5\%$ of the amount of GT strength expected up to this excitation energy. Using the later version of the S - D wave functions,⁵ we find that the beta decay measurements see approximately $69\pm 10\%$ of the expected strength; furthermore, the shape of the calculated strength distribution is in qualitative agreement with the experimental distribution. The new S - D wave functions

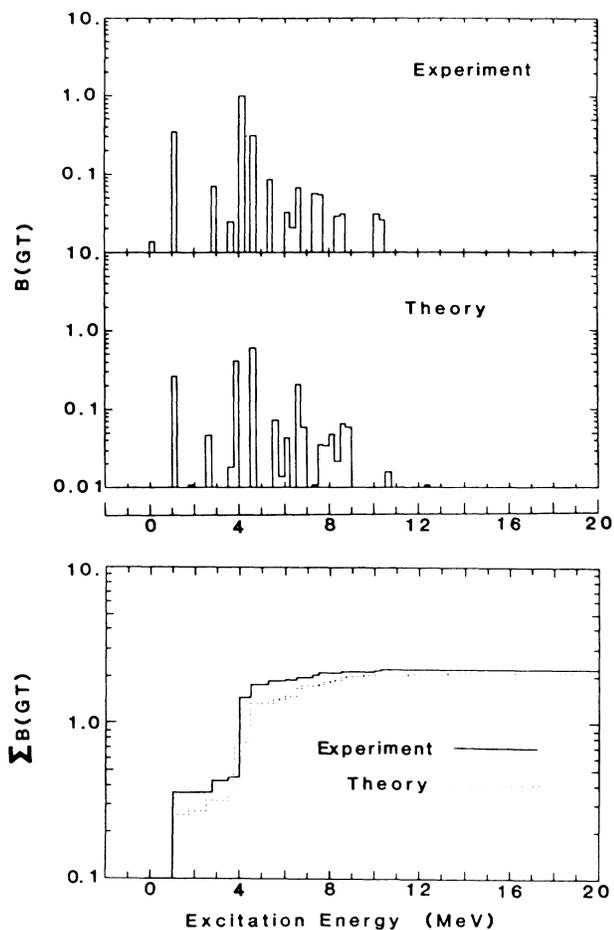


FIG. 6. (a) Comparison of the experimental $B_{pn}(GT)$ distribution function with the theoretical $B(GT)$ distribution function obtained with full S - D shell-model wave functions and an empirical GT operator adjusted to fit β -decay strengths. (b) Comparison of the integral experimental and theoretical $B(GT)$ distribution functions.

appear to describe the general distribution of strength observed in both beta decay and the (p,n) reaction reasonably well.

C. $M1$ strength

It is worthwhile also to consider the relationship of the forward-angle (p,n) cross sections to $M1$ strength, which can be measured with electromagnetic probes; however, the relationship of the forward-angle (p,n) cross sections to $M1$ strength is more complicated than the relationship to GT strength. Electromagnetic excitation of $M1$ strength can involve orbital- and exchange-current contributions not expected in the (p,n) excitations. These additional contributions can interfere either constructively or destructively with the spin component and with each other; the result is that there is not a simple one-to-one relationship, even to first order, between forward-angle (p,n) cross sections and $B(M1)$ values obtained from electromagnetic excitations. These differences were seen clearly in the comparisons of

forward-angle (p,n) cross sections with $B(M1)$ values for several transitions in various S - D shell nuclei reported earlier.¹⁰ Ratio differences of up to an order of magnitude were observed and were in general agreement with S - D shell-model predictions which considered explicitly the effect of orbital-current contributions; however, even though one does not expect a one-to-one relationship, it is still useful to make the comparisons between the (p,n) cross sections and $B(M1)$ values obtained from electromagnetic excitations. The differences indicate directly the transitions with significant orbital- or exchange-current contributions. If one hopes to understand these contributions quantitatively, such comparisons are important.

Perhaps the most direct way to convert the forward-angle (p,n) measurements to $B(M1)$ values is to use the simple relationship between the $M1$ and GT operators obtained by assuming pure spin-transfer transitions as expected for the (p,n) reaction. In terms of spherical tensors, these operators have the forms³⁰

$$\Theta_{M1} = \frac{(\mu_p - \mu_n)}{2} \sqrt{3/4\pi} \sigma^m \tau^0, \quad (5)$$

and

$$\Theta_{GT} = \frac{\sigma^m \tau^{-1}}{\sqrt{2}}. \quad (6)$$

The ratio of the corresponding reduced transition probabilities for isovector ($\Delta T = 1$) transitions is therefore

$$\frac{B_{pn}(M1)(J_f T_f \leftarrow J_i T_i)}{B_{pn}(GT)(J_f T_f \leftarrow J_i T_i)} = \frac{8\pi}{3(\mu_p - \mu_n)^2} \frac{\langle T_f T_f^z | 1, -1; T_i T_i^z \rangle^2}{\langle T_f T_f^z | 1, 0; T_i T_i^z \rangle^2}. \quad (7)$$

For $T_i = 0$, the ratio of the isospin Clebsch-Gordan coefficients is unity; thus for the case considered here, viz., ^{32}S , we have

$$B_{pn}(M1) = \frac{8\pi}{3(\mu_p - \mu_n)^2} B_{pn}(GT) = 2.643 B_{pn}(GT). \quad (8)$$

Hence this rescaling of the $B(GT)$ values will give the corresponding contribution to $B(M1)$ strength. Values of $B(M1)$ from electromagnetic excitations that differ from those in Eq. (8) signal the importance of orbital or exchange contributions.

In Table III we compare the $B_{pn}(M1)$ values obtained by using Eq. (8) to rescale the $B_{pn}(GT)$ values of Table I with $B_{ec}(M1)$ values obtained from inelastic-electron scattering by Burt *et al.*⁸ The (p,n) data extend to higher energies than the (e,e') data because the (p,n) measurements have smaller backgrounds and a relatively clear experimental signature for spin-transfer strength, viz., the peaking at 0° of the $\Delta L = 0$ angular distributions. Note that the three strongest excitations observed in the electron-scattering experimental (at $E_x = 8.11$, 11.12, and 11.63 MeV) are the analog states of the three strongest excitations seen in the (p,n) experiment (at $E_x = 1.15$, 4.06, and 4.58 MeV). The $B_{ec}(M1)$ and $B_{pn}(M1)$ values are comparable for these three transi-

TABLE III. Comparison of (p,n) and (e,e') $B(M1)$ values.

(e,e')		(p,n)	
E_x (MeV)	$B_{ee}(M1)$ (μ_0^2)	E_x (MeV)	$B_{pn}(M1)$ (μ_0^2)
7.00 (0.00)	(< 0.09)	0.00	0.037
8.11 (1.11)	1.14	1.15	0.909
9.68 (2.68)	0.69	2.79	0.188
10.05 (3.05)	(0.57)	3.73	0.066
11.12 (4.12)	2.40	4.06	2.656
11.63 (4.63)	1.26	4.58	0.819
	$\Sigma = 5.49$		$\Sigma = 6.75$
		5.41	0.230
		6.06	0.087
		6.29	0.056
		6.65	0.180
		7.36	0.153
		7.72	0.145
		8.30	0.079
		8.60	0.085
		10.00	0.085
		10.30	0.071
		Total $\Sigma = 5.846$	

tions: both experiments see the state at 11.12/4.06 MeV as the strongest excitation and the $B(M1)$ values agree to within 10%. However, note that the 9.68/2.68 MeV transition has a $B_{pn}(M1)$ value only about $\frac{1}{4}$ of the $B_{ee}(M1)$ value, suggesting significant orbital- or additional exchange-current contributions in the electromagnetic excitation of this state.

It is significant that the (p,n) measurements can extend the spin-transfer measurements to higher excitation energies. Burt *et al.*⁸ note that the total $B_{ee}(M1)$ strength they observe in ^{32}S is significantly lower than that observed for other S - D shell nuclei. The additional strength observed in the (p,n) measurements above the highest state reported in the (e,e') measurements is about 25%. If approximately this amount is also present (but unable to be extracted) in the electron-scattering experiment, the total strength would be in good agreement with the results for the other S - D shell nuclei.

We find that the total $B_{pn}(M1)$ strength listed in Table III is 58% of that obtained from a full S - D shell-model calculation of $M1$ strength with orbital contributions removed. This, of course, is just the same result as for the comparison of experimental and theoretical GT strength since both the experiment and theory are simply rescaled by the factor 2.643 of Eq. (8). We note that the S - D shell-model calculations predict a total $B_{ee}(M1)$ for the first six states of $6.31 \mu_0^2$ and a total $B_{pn}(M1)$ of $6.51 \mu_0^2$; the fact that the calculations yield a slightly larger result for the B_{pn} sum than for the B_{ee} sum is in contrast to the experimental results which show that the B_{pn} sum is somewhat smaller than the B_{ee} sum. Since the calculations consider orbital contributions, the difference is probably due to the neglect of exchange terms, which can be different for the two reactions.

V. CONCLUSIONS

We measured the strength and distribution of 1^+ excitations in the $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction at 135 MeV. This strength dominates the forward-angle spectra and is clearly identifiable from the characteristic $\Delta L = 0$ angular distributions which are peaked at 0° . The 1^+ strength is seen to be fragmented into at least 16 states from the ground state up to 10.3 MeV of excitation in ^{32}Cl ; however, 75% of the 1^+ strength (i.e., cross section at 0°) is concentrated into three states at $E_x = 1.15, 4.06,$ and 4.58 MeV. The strength above $E_x = 6$ MeV is relatively weak and is in the presence of increasing strength from quasifree scattering (neutron knockout) and the broad $\Delta L = 1$ giant resonance, which is seen to increase at wider angles. The observed distribution of 1^+ strength is described well by DWIA calculations with full S - D shell-model wave functions. The total 1^+ strength is $60 \pm 9\%$ of that predicted by these DWIA calculations.

The 1^+ strength observed in the $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction is interpreted as being (essentially) equivalent to GT strength excited in beta decay. The measured 0° (p,n) cross sections were converted into units of $B_{pn}(\text{GT})$ by consideration of other (p,n) reactions for which analog beta decays exist. Using the universal relationship [Eq. (3)] so obtained, the total GT strength observed in the peaks of the $^{32}\text{S}(p,n)^{32}\text{Cl}$ reaction is found to be 56% of that predicted using the same full S - D shell-model wave functions considered for the DWIA comparison and GT matrix elements consistent with the beta decay of the free neutron. The normalized GT matrix elements of Brown and Wildenthal,² which were adjusted to describe GT beta decays in the S - D shell nuclei, yield theoretical predictions of GT strength in good agreement with the experimental results. If we consider possible GT strength in the background and continuum just above the strong 1^+ excitations, we estimate that there is, up to 22 MeV, less than 20% of the GT strength observed in the peaks. This is in contrast to the case for the $^{48}\text{Ca}(p,n)^{48}\text{Sc}$ reaction where significant continuum strength can be seen, but more similar to light nuclei where the GT strength appears primarily in the discrete peaks.

We considered also the interpretation of the 0° (p,n) cross sections in terms of magnetic dipole ($M1$) strength. Similar to an earlier analysis for other targets in the S - D shell, it is found that there is not a simple one-to-one correspondence between 0° (p,n) cross sections and $B(M1)$ values obtained from inelastic electron scattering. The electron-scattering process can involve orbital- and exchange-current contributions which can be significant in some transitions. The comparison of the $B_{pn}(M1)$ values with $B_{ee}(M1)$ values from inelastic electron scattering on ^{32}S is seen to be good for the strongest excitations; however, one sees significant differences for some of the weaker excitations. Clearly, orbital contributions, which can interfere either constructively or destructively, are significant.

The (p,n) experiment is able to probe higher excitation energies than electron scattering because the (p,n) exper-

iment has smaller backgrounds and a distinct signature for 1^+ ($M1$) excitations, viz., strong peaking at 0° . Although each of the 1^+ transitions above about 5 MeV (in ^{32}Cl) are weak, in total they add 25% to the amount of strength observed. This additional strength observed in the (p,n) reaction suggests that a more sensitive electron-scattering experiment might find more $M1$ strength at higher excitation energies.

It is significant that the normalization factor required to make the DWIA calculations fit the experimental cross sections, viz., 0.60, is essentially the same normalization factor required to make the shell-model predictions for the total $B(\text{GT})$ agree in magnitude with the experimental results if the operators assumed are the so called free-nucleon operators. This consistency between normalization factors required for DWIA calculations and for other methods of measuring the amount of GT strength observed is found also in several other nuclei. For ^{18}O and ^{26}Mg , where analog beta decays are available to normalize the 0° cross sections in units of $B(\text{GT})$, the same normalization factor (to within 5%) for the DWIA is required as for the predicted $B(\text{GT})$, if the same wave functions are used. For ^{48}Ca , where the observed GT strength was compared to the observed Fermi strength, assumed to be concentrated in the IAS transition, again the same normalization factor is required for the DWIA as required to explain the observed ratio of GT to Fermi strength. The general result seems to be

that the normalization factor required to describe GT strength in these various nuclei is about the same whether one uses the method of normalizing to analog beta decay strengths, compares with the observed Fermi strength, or simply uses the DWIA normalization. In the present case of ^{32}S , we see that the normalization required for DWIA calculations, for the observed total GT strength normalized by means of a "universal" conversion factor, or for the observed $M1$ strength, are all about the same, viz., about 0.55–0.60 of predictions based on full S - D shell wave functions. This result is consistent also with that observed for GT or $M1$ strengths measured in other S - D shell nuclei, and even in heavier nuclei. It would appear that this normalization factor for strength observed in peaks is, to first order, universal.

In conclusion, the (p,n) reaction on ^{32}S is seen to provide a good measure of the GT and $M1$ strengths in the $A = 32$ system. Although neither of these strengths obtained from the (p,n) measurements are identical to the GT or $M1$ strengths measured with the other probes (i.e., beta decay and electron scattering), the (p,n) results are clearly related and can provide important complementary information. The (p,n) results are seen to be in good agreement with the total strength predicted using full S - D shell model wave functions with GT matrix elements adjusted to reproduce beta-decay strengths for S - D shell nuclei.

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